Pseudoscalar mesons with 4-flavor HISQ action

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[1] speaker

Introduction

- Symanzik-tadpole improved gauge action
- Highly Improved Staggered Quark (HISQ) action
- Two level smear-unitarize-smear suppresses taste violation
- Fix tree-level charm quark dispersion relation ("Naik ϵ")
- 2+1+1 flavors of sea quarks (dynamical charm)
- Ensembles at physical light quark mass
- Volumes up to $(5.6\ \mathrm{fm})^3$
- 1000 lattices in completed ensembles

Ensembles used

a(fm)	$M_{\pi\,\text{(MeV)}}$	M_{RMS}	N_{lats}	L(fm)	comment
0.1549(5)	307	416	1020	2.48	
0.1525(3)	215	353	1000	3.66	
0.1496(2)	132	310	1000	4.79	physical quark mass
0.1243(3)	305	365	1040	2.98	
0.1219(3)	217	296	1000	3.90	
0.1197(2)	132	240	880	5.75	physical quark mass
0.1224(4)	216	295	1020	2.94	finite size check
0.1218(2)	215	294	1032	4.87	finite size check
0.0906(3)	305	326	1011	2.90	
0.0884(2)	218	247	900	4.24	
0.0863(2)	128	173	546	5.52	physical quark mass
0.0617(5)	312	316	1000	2.96	
0.0583(3)	229	234	609	3.73	

M_π is the Goldstone pion mass, M_{RMS} is the RMS pion mass.
Note scale setting uncertainty (later).

Divide and conquer

Stage 1: Fit correlators, find masses and amplitudes (Jongjeong Kim's talk)

Stage 2: Find quark masses and decay constants on each ensemble

Stage 3: Continuum limit and sea quark mass adjustments

- A self-contained analysis — lattice spacing set from same correlators, so no intermediate scales like r_1 etc.
- Physical quark mass ensembles allow simple chiral interpolations. But χPT will help understand quark mass dependence, and likely lead to better controlled fits. See Javad Komijani's talk for more info.

Pseudoscalar decay constants

- Partially quenched pseudoscalar correlators, including charm quarks
- Random wall and Coulomb wall sources
- $f_{pseudo} = (m_A + m_B)\sqrt{\frac{3VA_{pt-pt}}{2M^3_{pseudo}}}$
- Fit to common mass, random wall amplitude gives f_{pseudo}
- "1+0" state fits for light-light, "2+1" state fits for charm-light
- See Jongjeong Kim's talk for more information on mass and amplitude computations.

f_D, f_{D_s} etc. on each ensemble

- "Fpi tuning": Interpolate/extrapolate in m_l such that $\frac{M_\pi}{f_\pi}$ has its physical[2] value. This fixes am_l and a
- Find am_s where $2M_K^2 - M_\pi^2$ has its physical value. This fixes m_s.
- Find charm valence mass where M_{D_s} is correct. This fixes m_c.
- cf. "FP4S tuning": Use $\frac{M^2}{F^2}$ to fix a and m_{p4s} . . . , Take f_{p4s} from Asqtad program.
- cf. "FK tuning": Use $\frac{2M_K^2 - M_\pi^2}{f_K^2}$

[2] adjusted for E&M and finite size — see later

f_D, f_{D_s} etc. on each ensemble

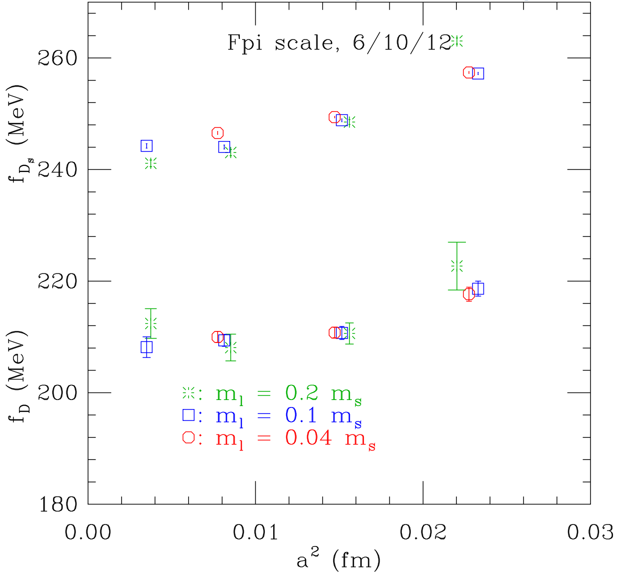
- Find $m_d - m_u$ from E&M adjusted $K^0 - K^+$ mass difference. See Ludmila Levkova's talk.
- Find (interp./extrap.) f_K at adjusted light quark mass.
- Find (interp./extrap.) f_D and M_D (a check) at adjusted light and charm masses.
- Find (interp./extrap.) f_{D_s} at adjusted strange and charm masses.
- Find (interp./extrap.) M_{η_c} (check) at adjusted charm mass.
- Do this whole procedure inside a jackknife resampling

The most important ensemble

- $a = 0.09$ fm physical quark mass ensemble, "Fpi scale"
- with finitesize, isospin and (some) E&M corrections.
- Statistical errors only!!!

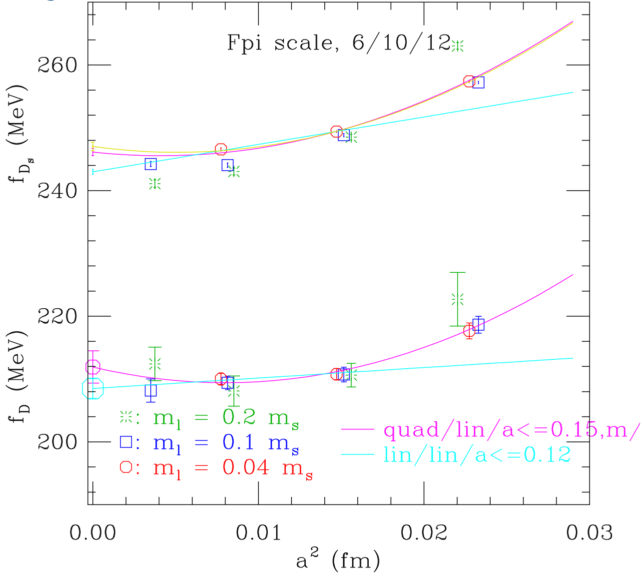
$a = 0.08794(11)$ fm
$am_l = 0.001336(6)$ $am_s = 0.03653(13)$ $am_c = 0.4325(8)$
$m_u/m_d = 0.487(7)^*$ $m_s/m_l = 27.34(5)$ $m_c/m_s = 11.841(21)$
$f_K = 155.00(21)$ MeV (cf 156.1)
$M_{D_0} = 1869.7(1.6)$ MeV (cf 1864.8)
$M_{D^+} = 1871.6(1.1)$ MeV (cf 1869.6)
$M_{\eta_c} = 2980.3(4)$ MeV (cf 2980.3(1.2))
$f_D = 210.00(98)$ MeV $f_{D_s} = 246.56(26)$ MeV $f_{D_s}/f_D = 1.174(5)$
$* \ \lambda = 1.65$,sensitive to E&M adjustments!!

Continuum extrap. and quark mass adj.



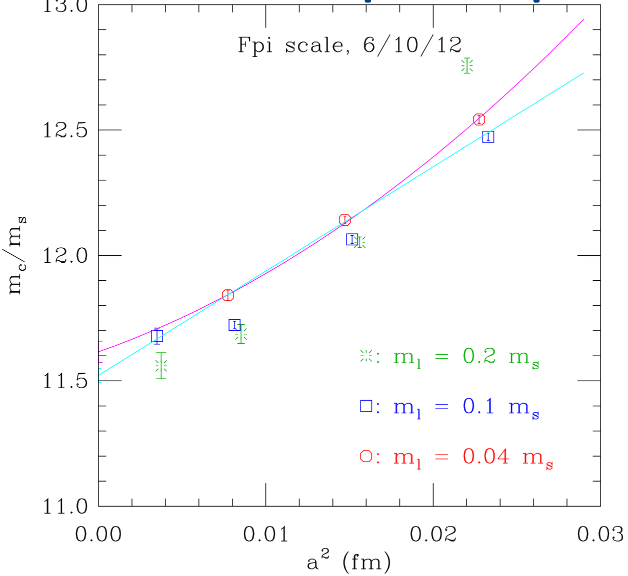
- Continuum extrapolation
- Mostly use physical sea quark mass runs, use $m_s/10$ and $m_s/5$ ensembles to correct for sea quark mass mistuning and to guide curvature as function of a^2.

Continuum extrapolation and quark mass adjustment



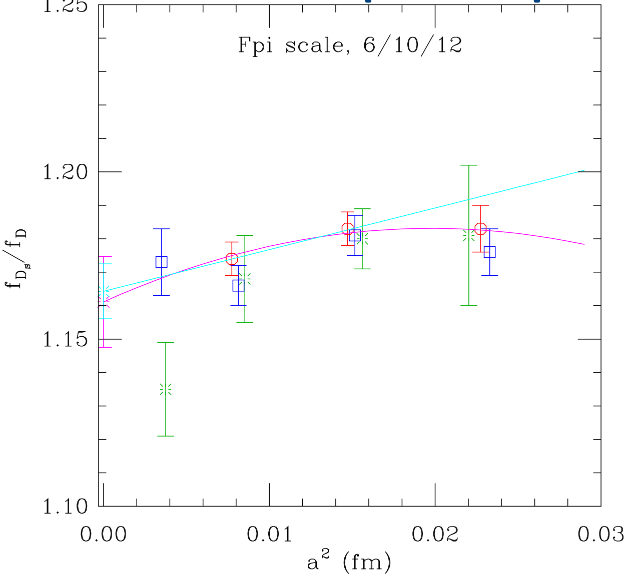
- Continuum extrapolation
- "Fpi" scale
- You can get different answers depending on your fit form
- Really need the $a = 0.06$ fm physical quark mass point

Continuum extrap. and quark mass adj.



- Continuum extrapolation
- "Fpi" scale
- m_c/m_s

Continuum extrap. and quark mass adj.



- "Fpi" scale
- Continuum extrapolation
- f_{D_s}/f_D

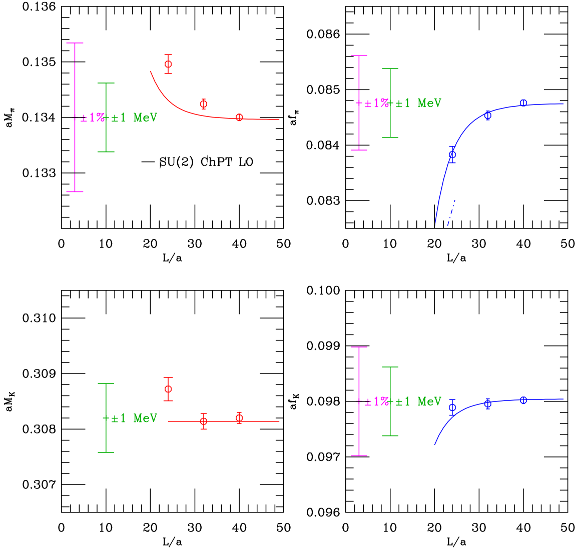
Systematic errors, short version

- Excited states: vary T_{min} and priors
- Finite volume: $1/2$ of change when adjustments turned off
- Isospin violation: $1/2$ of change from $m_u/m_d = 1$
- E&M: $1/2$ of change between $\lambda = 1$ and $\lambda = 2$. For m_c/m_s, change from adjusting average kaon mass.
- Scale setting: Try f_π, f_K, f_{p4s}. Use difference between f_π and f_K scales in error budget.
- a^2 fit form: Use average of quadratic, $a <= .15$ and linear, $a <= 0.12$ fits for central value. Use full difference between them in error budget.

Finite size effects

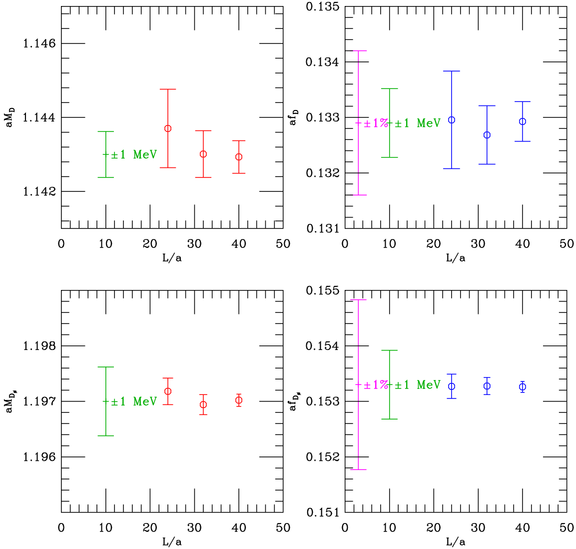
- Test for finite size effects
- $L = 24$, 32 and 40 at $a = 0.12$ fm, $m_l = m_s/10$, everything else the same
- Physical sizes of 2.88, 3.84 and 4.80 fm
- Effects are mostly in light pseudoscalars M_π, f_π and f_K
- But since we compute f_D/f_K etc., and use M_π in quark mass tuning, this matters for f_D etc. too.

Finite size effects



- Spatial size effects on M_π, M_K, f_π and f_K
- With χPT NLO forms
- Green error bar $= \pm 1$ MeV
- Magenta error bar $= \pm 1\%$

Finite size effects



- Spatial size effects on M_D, M_{D_s}, f_D and f_{D_s}
- With χPT NLO forms
- Green error bar $= \pm 1$ MeV
- Magenta error bar $= \pm 1\%$

Finite size effects

- Light pseudoscalar analysis - predict effect on M_π, f_π and f_K at physical quark mass, $L = 5.5$ fm. ("Boosted χPT", done by Matthew Lightman)
- Use finite size corrected values for M_π, f_π and f_K in stage II fitting. That is, we compute $\frac{f_D}{f_K(5.5\ \mathrm{f}m)}$.
- Then, use same factor to correct computed f_π or f_K to infinite volume.
- Including this changes f_D on "main" ensemble by 0.11 MeV, changes f_{D_s} by 0.09 MeV.
- Allow $1/2$ of this shift as residual systematic error

Isospin breaking

- Need an estimate of isospin breaking effects
- What would f_K be at $m_u = \hat{m}$?
- $f_K^{tune} = f_{K^+} + \frac{\partial f_K}{\partial m_l} (\hat{m} - m_u)$
- $f_K^{tune} \approx f_K + \frac{f_2 - f_1}{m_2 - m_1} (\hat{m} - m_u)$, where f_1 is f_K at lightest valence mass m_1, etc.
- Find $\hat{m}$ as above, then $m_d - m_u$ using E&M adjusted $K_0 - K^+$ mass difference
- Then evaluate f_D at m_d (instead of $\hat{m}$)
- Allow $1/2$ of shifts from including isospin splittings as residual systematic error

E&M

- Need an estimate of E&M effects
- Use π^0 mass. Alleged to be little affected by E&M.
- Adjusted K mass ("stretched" Dashen's theorem)
- $M^2_{K^+_{adj}} = M^2_{K^+} - \lambda \left(M^2_{\pi^+} - M^2_{\pi^0} \right)$
- $M_{\hat{K}} = \left(M_{K^+_{adj}} + M_{K^0} \right) /2$
- Used $\lambda = 1.65$ here. We will try changing λ (and $M_K(avg)$).
- See Ludmila Levkova talk this conference

Error budget

Source	f_D	f_{D_s}	f_{D_s}/f_D	m_c/m_s	m_u/m_d
Stat.	2.58 MeV	1.05 MeV	0.012	0.043	0.010
Excited	0.5 MeV	0.5 MeV	0.004	0.005	0.003
Volume	0.06 MeV	0.04 MeV	0.001	0.0015	0.0005
Isospin	1.1 MeV	0.7 MeV	0.003	0.021	na
E&M	0.3 MeV	0.16 MeV	0.001	0.022, 0.061	0.022
Scale set.	2.1 MeV	1.7 MeV	0.004	0.055	0.001
a^2 fit form	3.5 MeV	3.3 MeV	0.010	0.094	0.010

Note "Stat." includes valence quark mass tuning and continuum extrapolation with fixed fit form.

Results

These are preliminary

$f_D = 210.2(2.6)(4.2)$ MeV $\quad f_{D_s} = 244.5(0.6)(3.8)$ MeV

$f_{D_s}/f_D = 1.163(14)(12)$

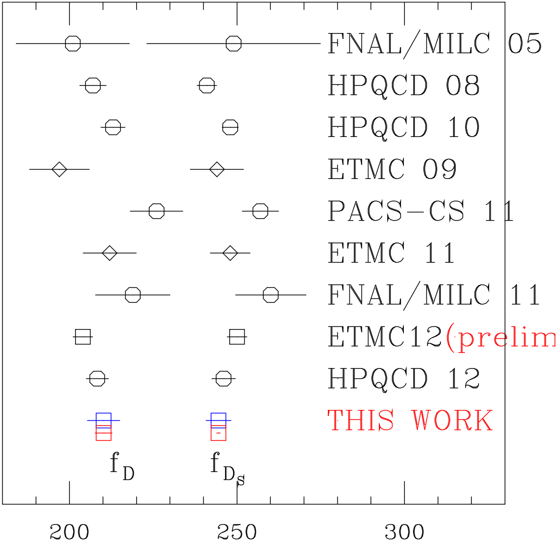
$m_c/m_s = 11.569(27)(129)$

$m_u/m_d = 0.508(10)(22)$ VERY preliminary

In progress:

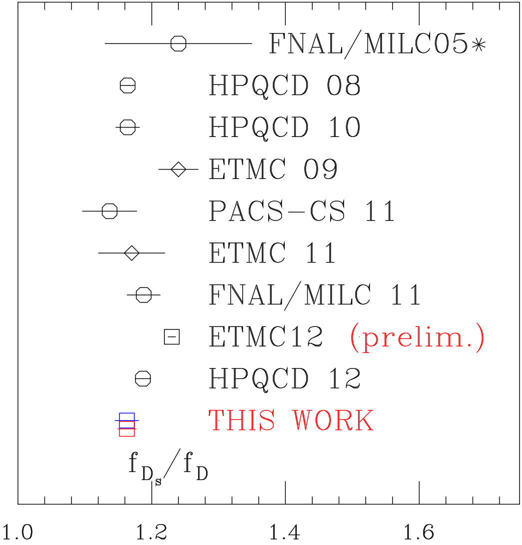
- Chiral PT to better control fit forms
- More study of continuum extrapolation
- $a = 0.06$ fm physical quark mass run

Other results



- Other values
- $\diamond$: $N_f = 2$
- $\circ$: $N_f = 2 + 1$
- $\square$: N_f = 2+1+1
- "ETMC 12" = BNL workshop, May 2012
- "HPQCD 12" = Heechang Na talk on Monday, arXiv 1206.4936

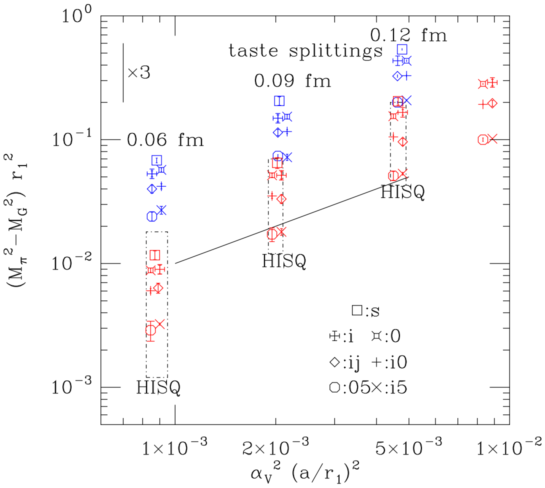
Other results



- Other values for f_{D_s}/f_D
- $\diamond$: $N_f = 2$
- $\circ$: $N_f = 2+1$
- $\square$: $N_f = 2+1+1$

EXTRA SLIDES

Taste symmetry



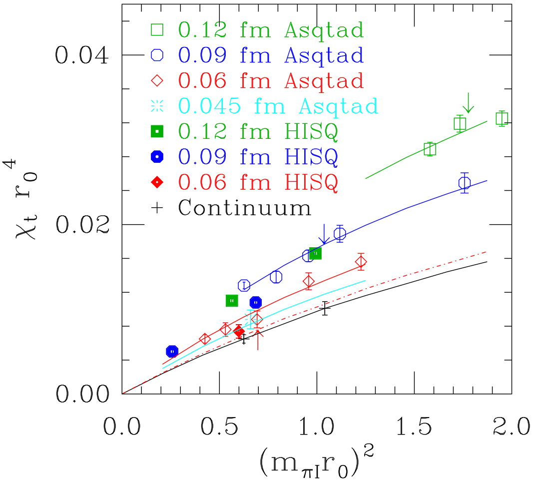
- At $m_l = 0.2 m_s$
- New results at 0.06 fm
- Note scaling looks a little better than expected

Taste symmetry, expected

Pion spectrum at $a \approx 0.06$ and 0.045 fm (MeV)
Splittings scaled from 0.06 fm, $m_l/m_s = 0.2$ masses

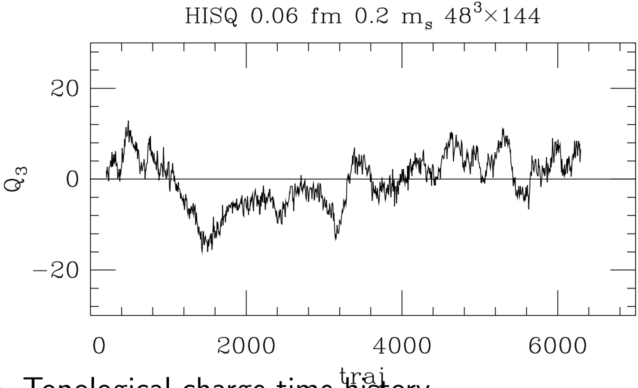
	0.06fm	0.06fm	0.045 fm
taste	$m_s/10$	$m_s/27$	$m_s/27$
π_5	220	135	135
π_{05}, π_{i5}	223	139	138
π_{ij}, π_{i0}	226	145	141
π_i, π_0	228	149	143
π_s	232	153	146
$\pi(\mathrm{RMS})$	226	145	141

Topology



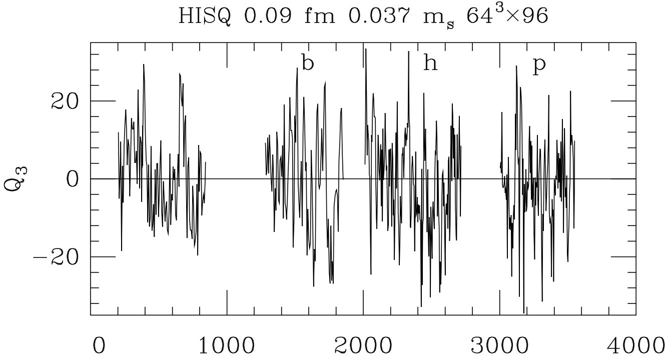
- arXiv:1003.5695, 1004.0342
- Really does improve the gauge configurations!!
- (Other tests involve valence quarks)

Topology



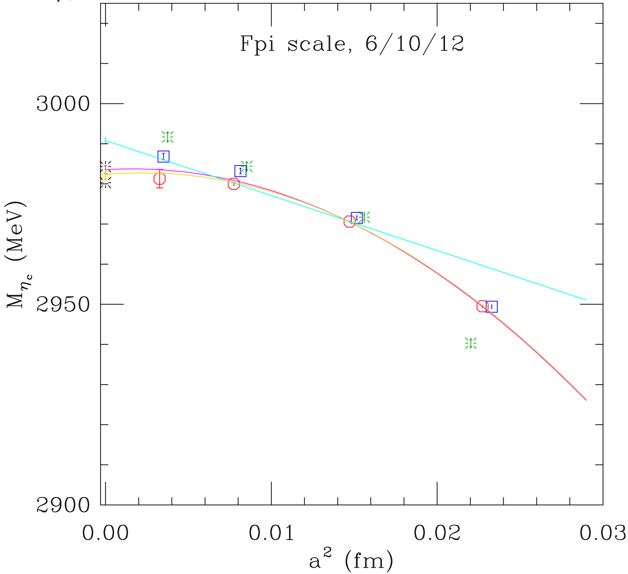
- Topological charge time history
- $a = 0.06$ fm, $m_l/m_s = 0.2$

Topology



- Topological charge time history
- $a = 0.09$ fm, physical quark mass

Continuum extrap. and quark mass adj., M_{η_c}



- Look at some things where we know the answer