Gradient flow and scale setting for twisted mass fermions

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The gradient flow method

Workflow

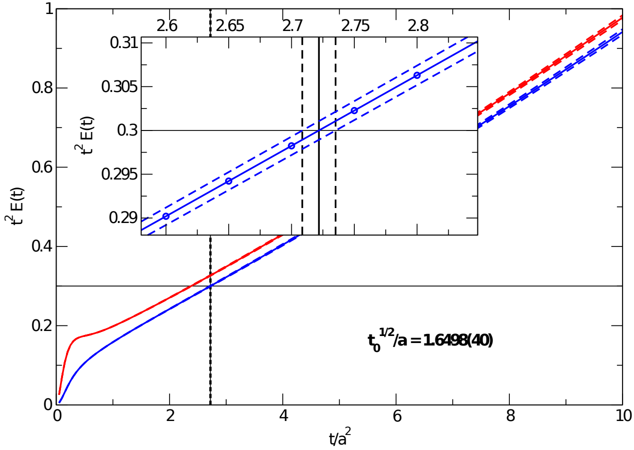
- Define
$$\frac{\partial}{\partial t} V_t\left(x,\mu\right) = -g^2\left[\frac{\partial}{\partial_{x,\mu}} S(V_t)\right] V_t(x,\mu),$$
with the boundary condition $V_0 = U\left(x,\mu\right)$.
- Intregrate this differential equation numerically through
$$V_{t+\epsilon}\left(x,\mu\right) = \exp\left[\epsilon T^a \partial^a_{x,\mu} P\left(V_t(x,\mu)\right)\right] V_t(x,\mu).$$
- Calculate observables $\langle\mathcal{O}\rangle_t \equiv \langle\mathcal{O}\left(V_t\left(x\mu\right)\right)\rangle$.

See also talks by Stefan Schaefer, Thorsten Kurth and Daniel Nogradi.

Gradient flow properties

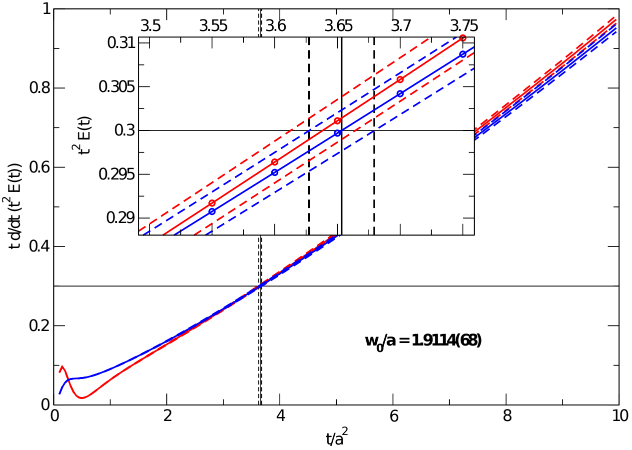
- Has a well-defined perturbative expansion.
- Observables are renormalized along the flow [Lüscher, JHEP 08 (2010) 071].
- Moves towards a (local) minimum of the gauge action.
- Equivalent to repeated infinitesimal (reversible) smearing steps. $\rightarrow$ Useful in defining smeared operators.
- Fictitious time t defines a renormalization scale $\Lambda \propto \sqrt{t}$.
- Useful for scale setting purposes.

Scale from energy density



$$E(t) = 2 \sum_{p \in P_x} \mathrm{Re}\,\mathrm{tr}\,[1 - V_t(p)] \qquad t^2 \langle E(t) \rangle|_{t_0} = 0.3$$

BMW's proposal: w_0



$$W(t) = t\frac{\mathrm{d}}{\mathrm{d}t}\left[t^2\langle E(t)\rangle\right] \qquad W(t = w_0^2) = 0.3$$

Properties of w_0

The quantity w_0...

- ...is cheap to calculate.
- ...is straightforward to determine precisely.
- ...introduces negligible systematic error from fitting.

BMW's results [BMW, arXiv:1203.4469v1]

Value in the continuum, at the physical quark mass (from m_Ω)

$$w_0 = 0.1755(18)(04)\,\mathrm{fm}$$

Measured with $N_f = 2 + 1$, consistent under change of fermion action.

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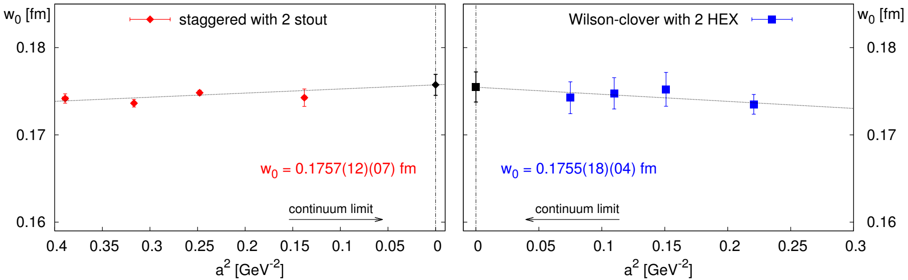
BMW: consistent for different actions



[BMW, arXiv:1203.4469v1]

Ensembles used

Re-analysis of published ensembles

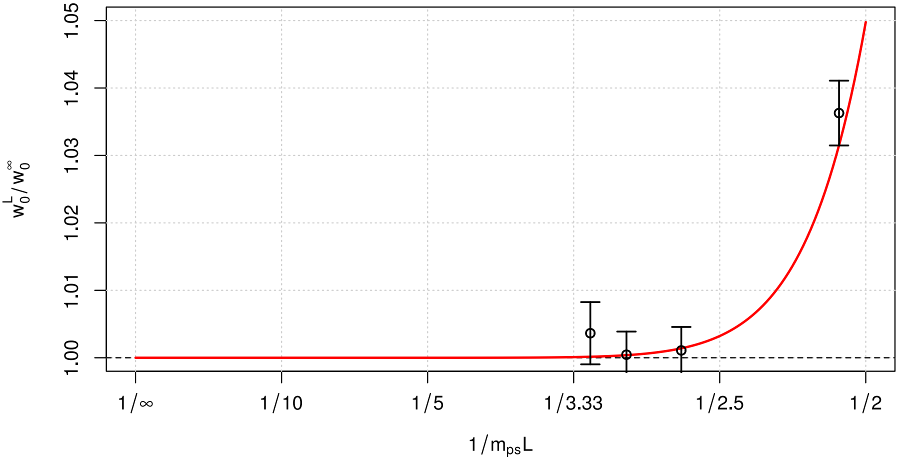
$N_f = 2$ [JHEP 08 (2010) 97]

tree-level Symanzik improved gauge action.
$\beta = 3.90$ (SB), 4.05 (SC)
$m_{\rm ps} = 291 – 646$ MeV (SB), $325 – 634$ MeV (SC)
$L = 1.90 – 2.52$ fm (SB), 2.02 fm (SC)
$m_{\rm ps}L = 3.7 – 6.2$ (SB), $3.3 – 6.5$ (SC),

$N_f = 2 + 1 + 1$ [PoS LAT2010 (2010) 123]

Iwasaki gauge action.
$\beta = 1.90$ (IA), 1.95 (IB), 2.10 (ID)
$m_{\rm ps} = 285 – 512$ MeV (IA), $269 – 489$ MeV (IB), $228 – 394$ MeV (IC)
$L = 2.06 – 2.75$ fm (IA), 2.50 fm (IB), $1.94 – 2.91$ fm (IC)
$m_{\rm ps}L = 4.0 – 4.8$ (IA), $3.4 – 5.8$ (IB), $3.4 – 4.7$ (IC)

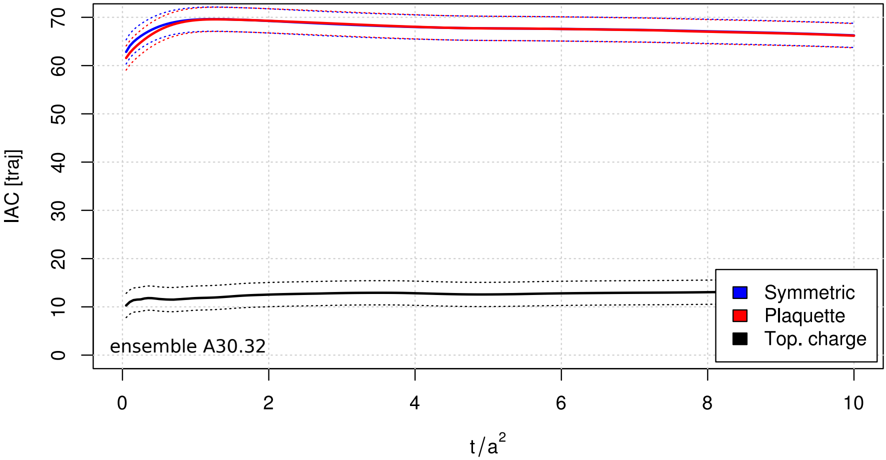
Finite volume effects on w_0



$L \approx 1.34 - 1.90$ fm $\qquad m_{\rm ps} \approx 300 - 330$ MeV

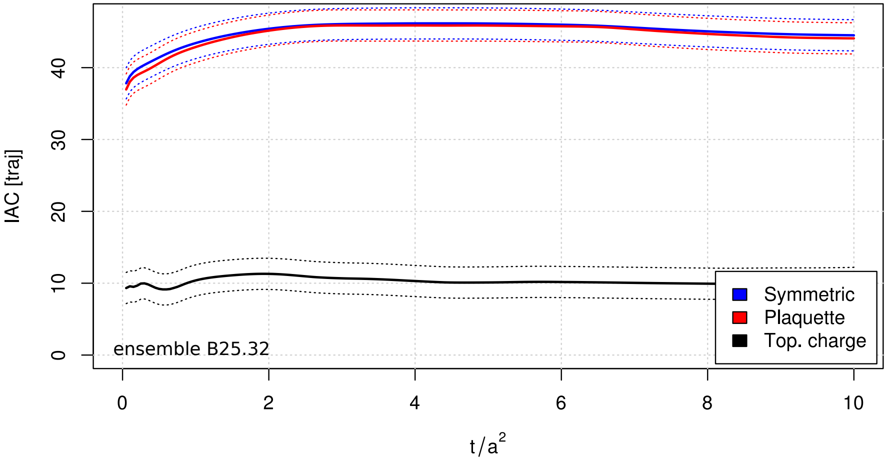
Finite volume effects negligible for all reasonable volumes...

Autocorrelation issues



The suppression of UV fluctuations systematically exposes large autocorrelations. Errors easily underestimated!

Autocorrelation issues



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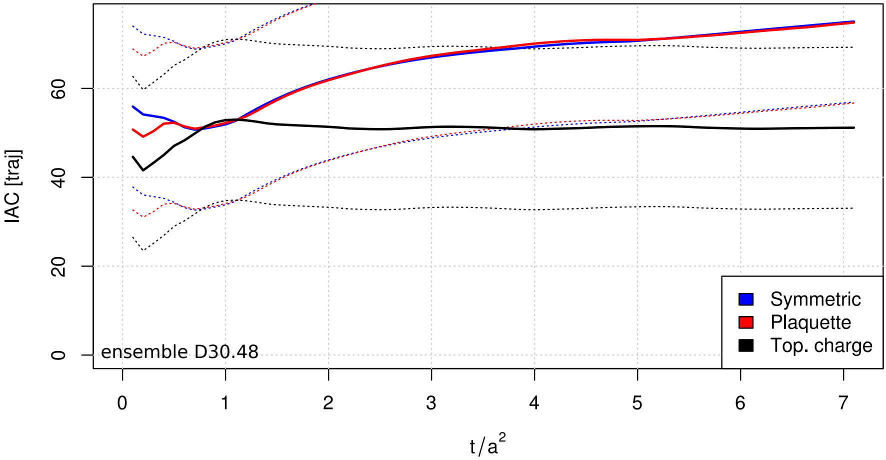

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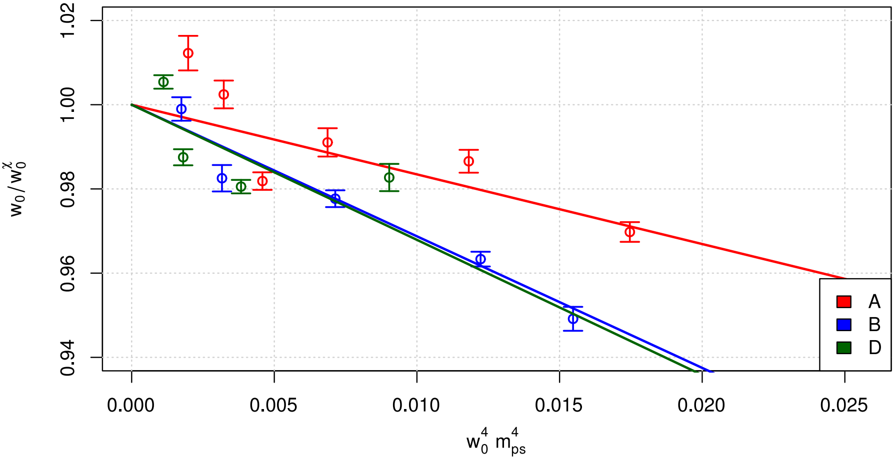
No magic here

First attempt – just plug in the value provided!

	a_{w_0}	a_{f_π}
$N_f = 2$	0.080(1) fm	0.064 fm
	0.104(3) fm	0.079 fm
	0.124(3) fm	0.086 fm
$N_f = 2+1+1$	0.068(1) fm	0.060 fm
	0.093(2) fm	0.078 fm
	0.107(1) fm	0.086 fm

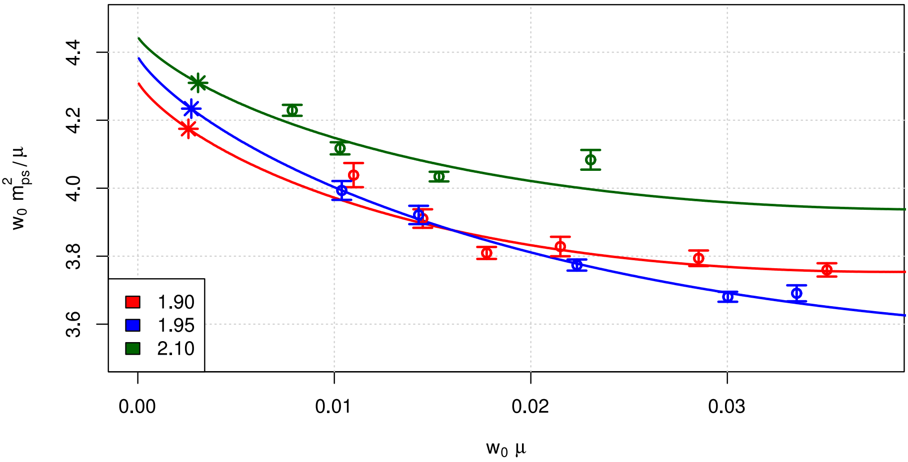
Disagreement between previous scale setting from pion decay constant.
Large chiral and/or lattice spacing artifacts?

Chiral extrapolation of w_0



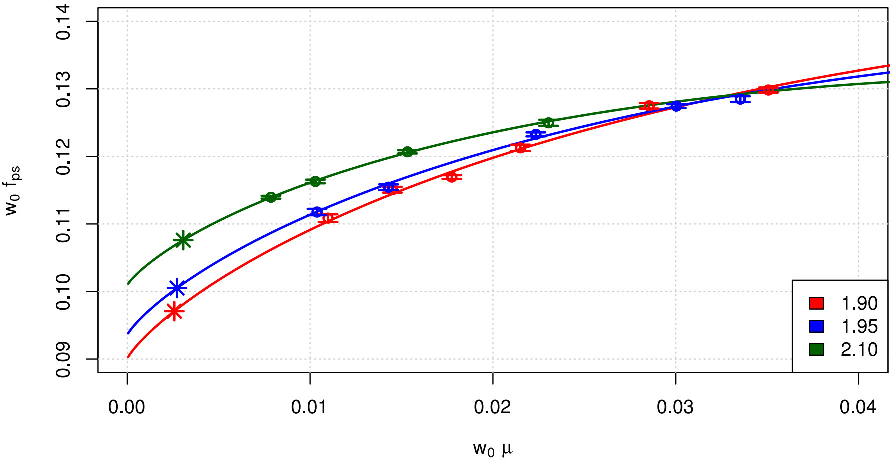
The chiral extrapolation is ambiguous.
Use non-extrapolated value instead.

Chiral dynamics and w_0



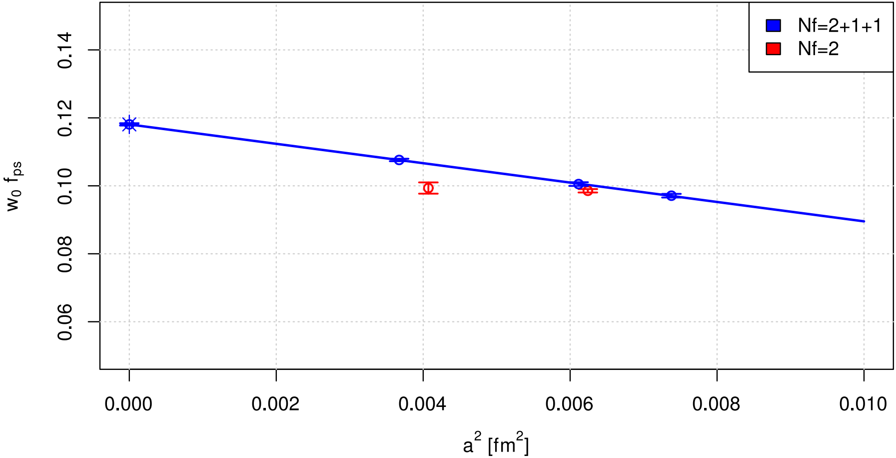
Chiral fits done separately – obtain $w_0 f_{\rm ps}$ at the physical point.

Chiral dynamics and w_0



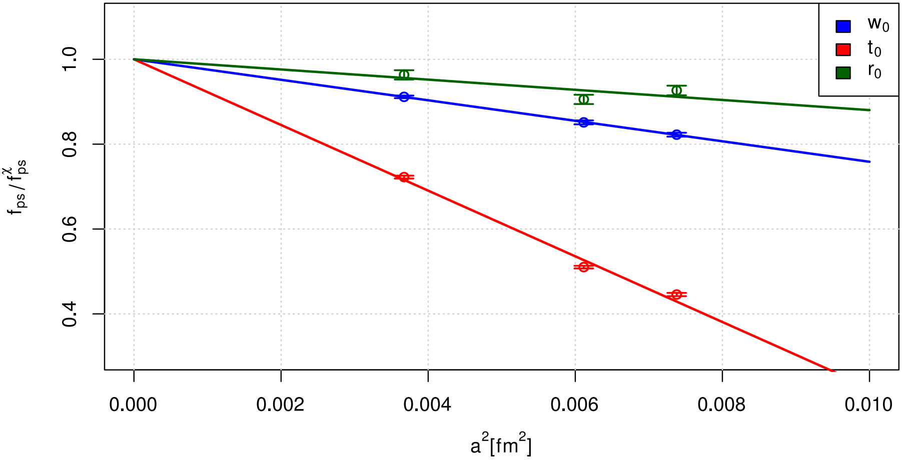
Chiral fits done separately – obtain $w_0 f_{\mathrm{ps}}$ at the physical point.

Lattice artifacts in w_0



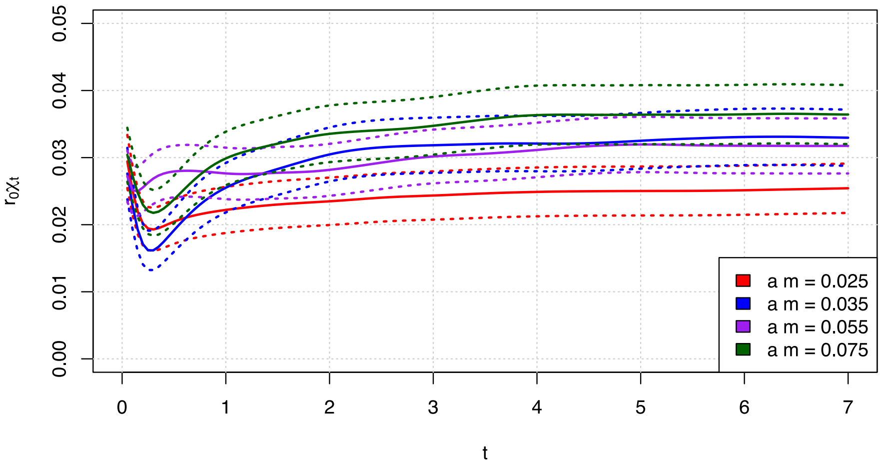
$$w_0 f_\pi = 0.1183(3) \quad \rightarrow \quad w_0 = 0.1782(7)\mathrm{fm} \quad (f_\pi^\pm)$$
$$= 0.179(7)\mathrm{fm} \quad (f_\pi^0)$$

Comparison using other scales



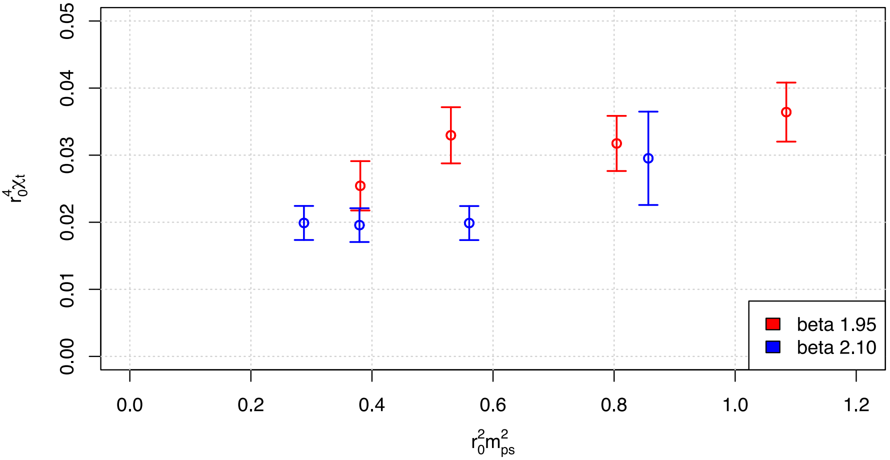
Mild dependence in a^2 at most for r_0, t_0 dominated by discretisation errors.

Chiral susceptibility from gradient flow



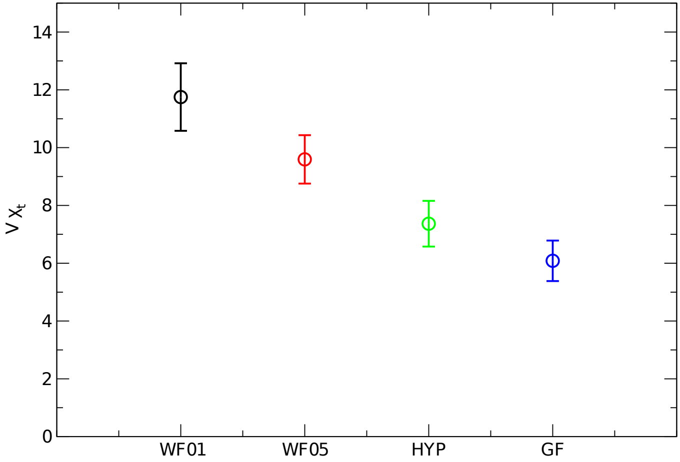
- Smooth configurations $\rightarrow F\tilde{F}$ charge definition.
- Renormalization properties imply $V\chi_t$ 'invariant' under gradient flow.

Chiral susceptibility suppressed?



Little indication of suppressed susceptibility in chiral limit. Little indication of topological slowing down, either!

Inconsistent results. . .



Different susceptibility definitions disagree – situation unclear.

Conclusions

w_0

- Large auto-correlations make measurements less precise than they may seem.
- Extrapolation to the chiral limit could introduce a large systematic error.
- Lattice artifacts are not negligible, complicating scale setting

Topological susceptibility

- Gradient flow definition of topological charge appears to satisfy renormalization conditions.
- Auto-correlation of topological charge is *not* the largest in the system.
- No topological slowing down seen in simulations.
- Different definitions remain in disagreement.