Lattice simulation of ultracold atomic Bose-Fermi mixtures

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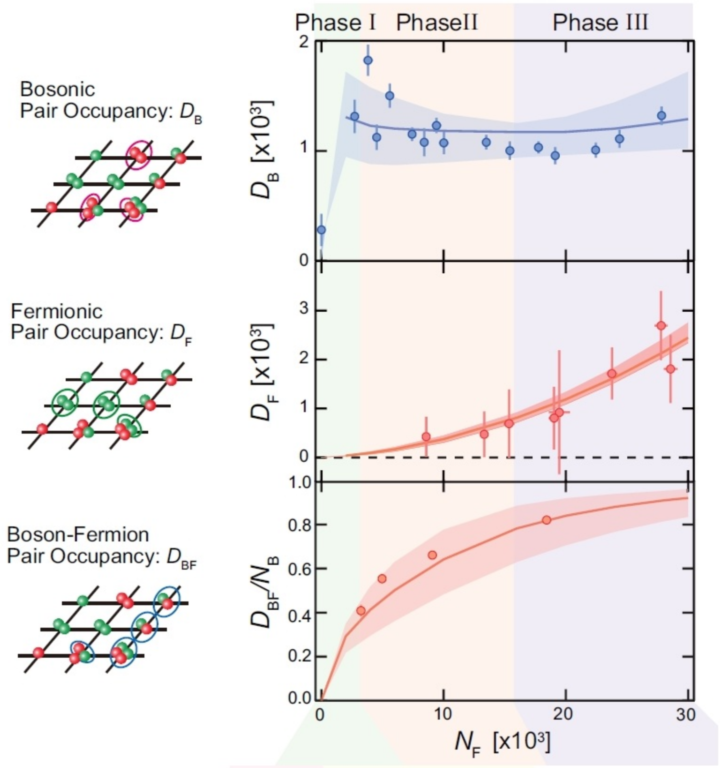
Bose-Fermi Mixtures

^4He-^3He mixture

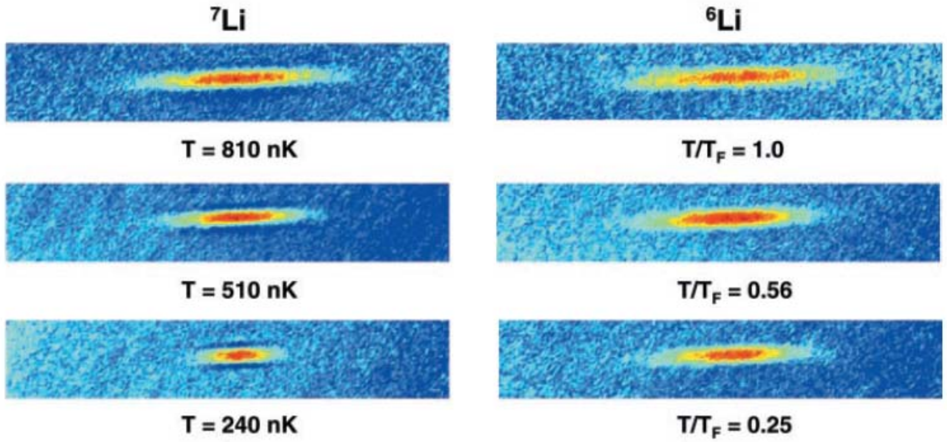
^{170}Yb-^{173}Yb in 3D optical lattice

[Sugawa et al. (2011)]



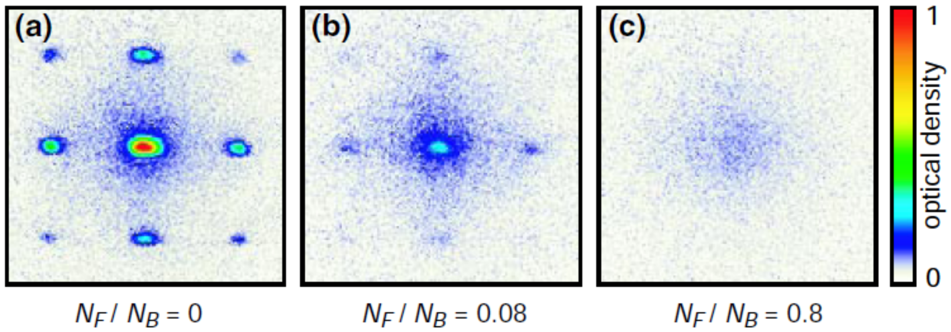
^7Li-^6Li in trapping potential

[Truscott et al. (2001)]



^{87}Rb-^{40}K in 3D optical lattice

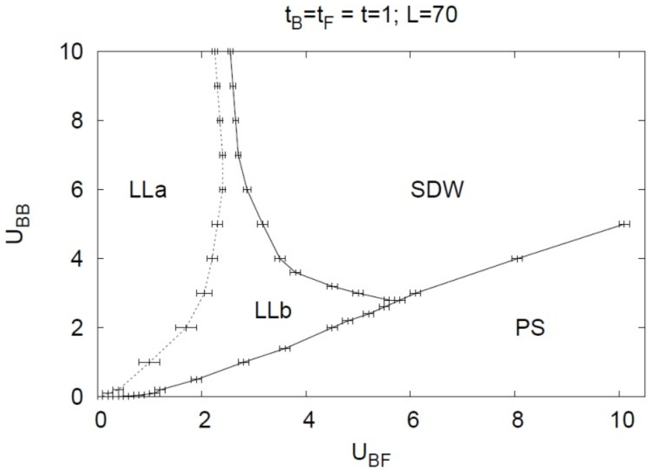
[Gunter et al. (2006)]



Quantum Monte Carlo simulations on (1+1)-dim. lattice

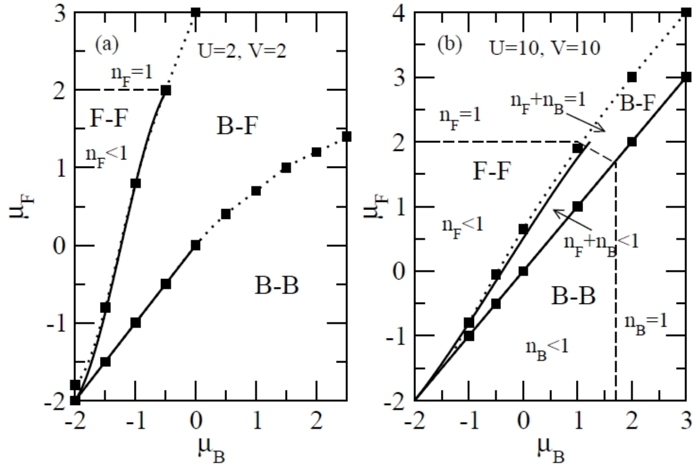
phase diagram

[Pollet, Troyer, Houcke, Rombouts (2006)]



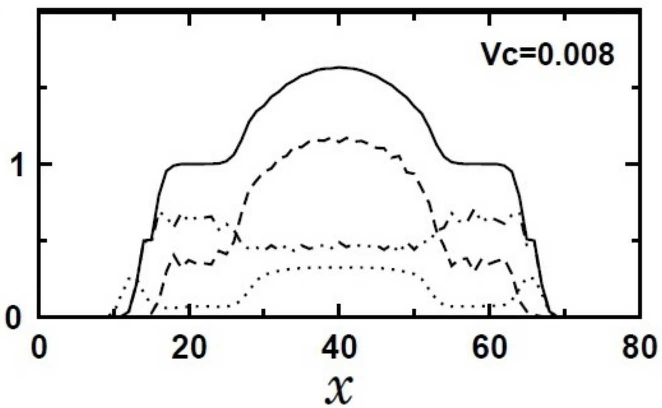
phase diagram

[Sengupta, Pryadko (2007)]



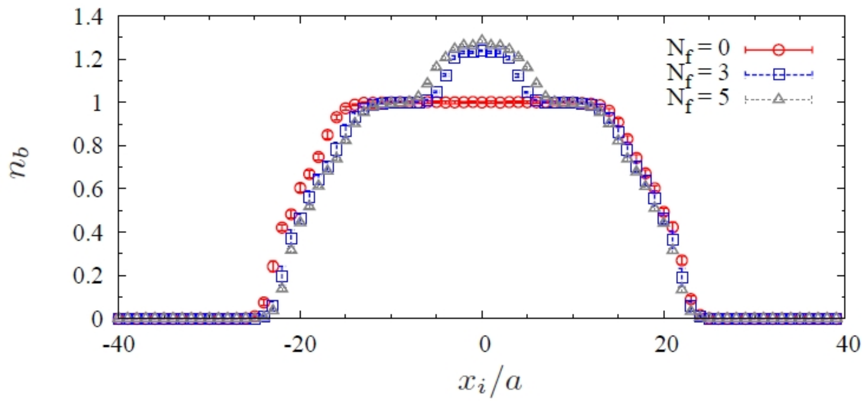
density profile

[Masaki, Tsukada, Mori (2008)]



density profile

[Varney, Rousseau, Scalettar (2008)]



Quantum Monte Carlo Simulation

world-line formalism:

loop update

Fermion sign problem in higher dimension

Green-function Monte Carlo:

product of transfer matrices

Fermion sign problem

"Action" path integral Monte Carlo:

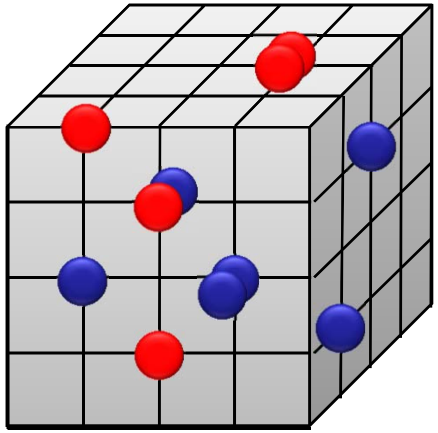
lattice QCD, lattice SUSY, etc.

Boson sign problem in non-relativity

one-component Boson + two-component Fermion on (3+1)-dim. lattice

$$Z = \int D\Psi_\uparrow^* D\Psi_\uparrow D\Psi_\downarrow^* D\Psi_\downarrow D\Phi^* D\Phi \; e^{-S}$$

$$= \int D\Phi^* D\Phi \; [\det K]^2 e^{-S_B}$$



No Fermion sign problem !

Hybrid Monte Carlo simulation, just like 2-flavor lattice QCD

$$\langle O \rangle = \frac{1}{Z} \int D\Phi^* D\Phi \; O \; [\det K]^2 e^{-S_B}$$

$$\simeq \frac{1}{N_{\rm conf}} \sum_{\rm conf} O$$

Bose-Fermi Hubbard Model

$$S$$

$$= \sum_x \Phi^*(x)\frac{\partial}{\partial\tau}\Phi(x) - \sum_{\langle xy\rangle} t_B[\Phi^*(x)\Phi(y) + \Phi^*(y)\Phi(x)] - \sum_x \mu_B n_B(x)$$

free Boson

$$+ \sum_x U_B n_B(x)[n_B(x) - 1]$$

Boson self-interaction (repulsive)

$$+ \sum_{x,\sigma} \Psi_\sigma^*(x)\frac{\partial}{\partial\tau}\Psi_\sigma(x) - \sum_{\langle xy\rangle,\sigma} t_F[\Psi_\sigma^*(x)\Psi_\sigma(y) + \Psi_\sigma^*(y)\Psi_\sigma(x)] - \sum_{x,\sigma} \mu_F n_{F\sigma}(x)$$

free Fermion

$$+ \sum_{x,\sigma} U_{BF} n_B(x) n_{F\sigma}(x)$$

Boson-Fermion interaction (repulsive or attractive)

Boson sign problem:

$$\sum_\tau \Phi^*(\tau)\frac{\partial}{\partial\tau}\Phi(\tau) \longrightarrow \sum_k i\omega_k \Phi^*(k)\Phi(k)$$

complex !

Boson Matsubara frequency: $\omega_k = 2\pi kT$

zero-frequency approximation : $k = 0$

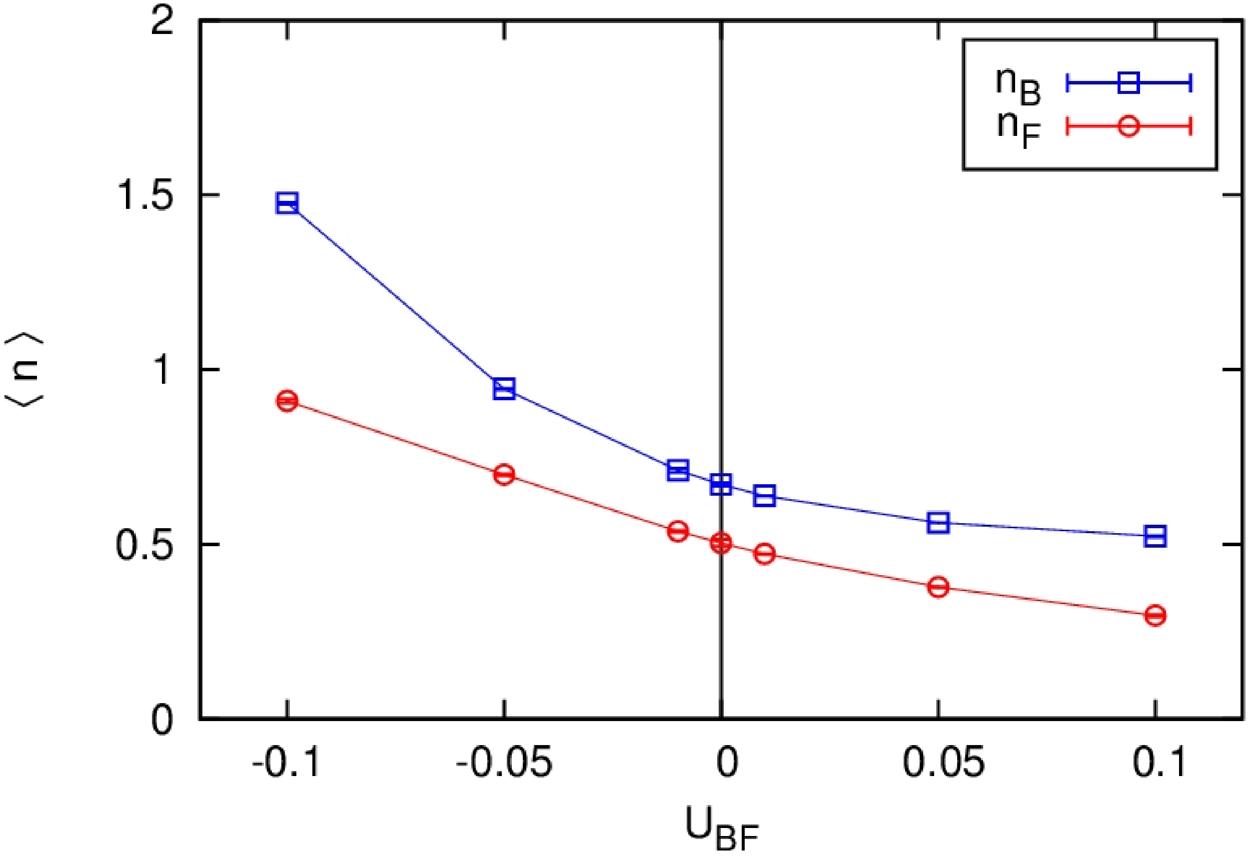
cf.) relativistic scalar theory

$$\sum_\tau (\partial_\tau \Phi(\tau))^* \partial_\tau \Phi(\tau) \longrightarrow \sum_k \omega_k^2 \Phi^*(k)\Phi(k)$$

real !

Number Density

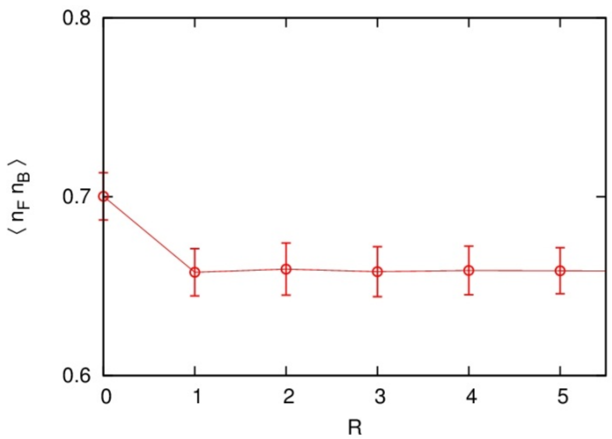
at $\mu_B = \mu_F = 0$ (fixed)



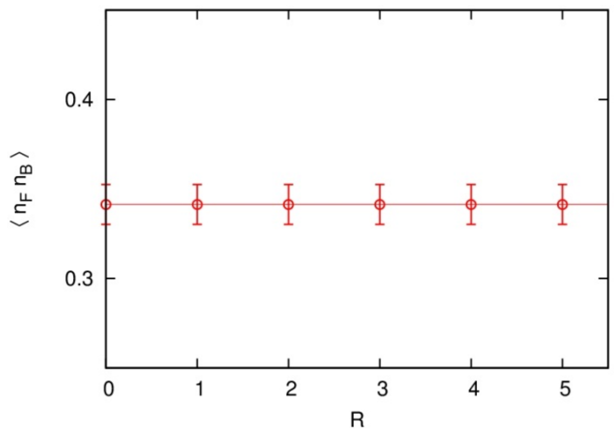
Pair Occupancy

$$\langle n_B(0) n_F(R) \rangle$$

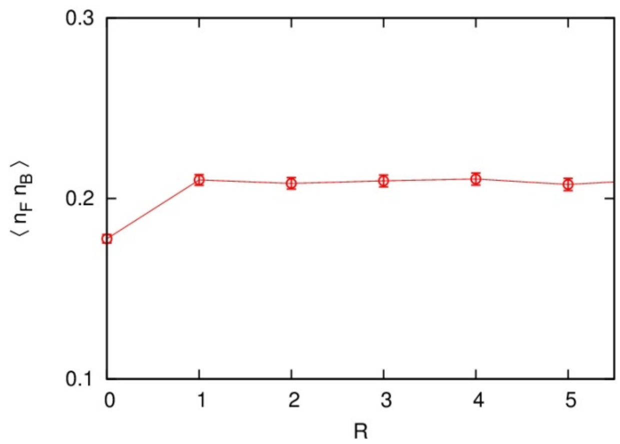
$U_{BF} < 0$

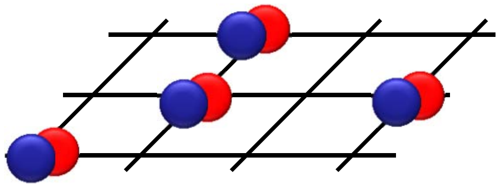


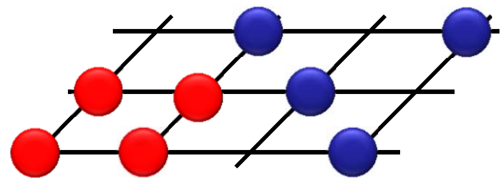
$U_{BF} = 0$



$U_{BF} > 0$

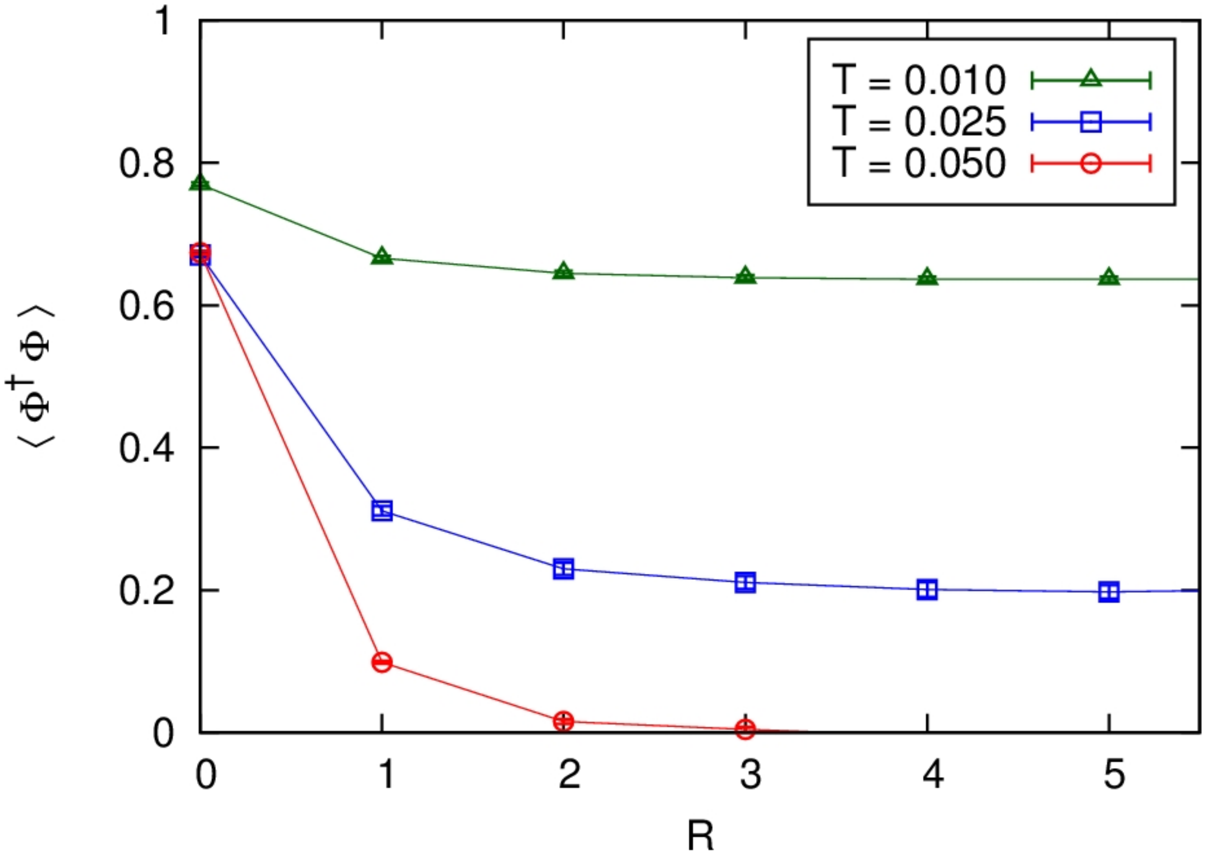






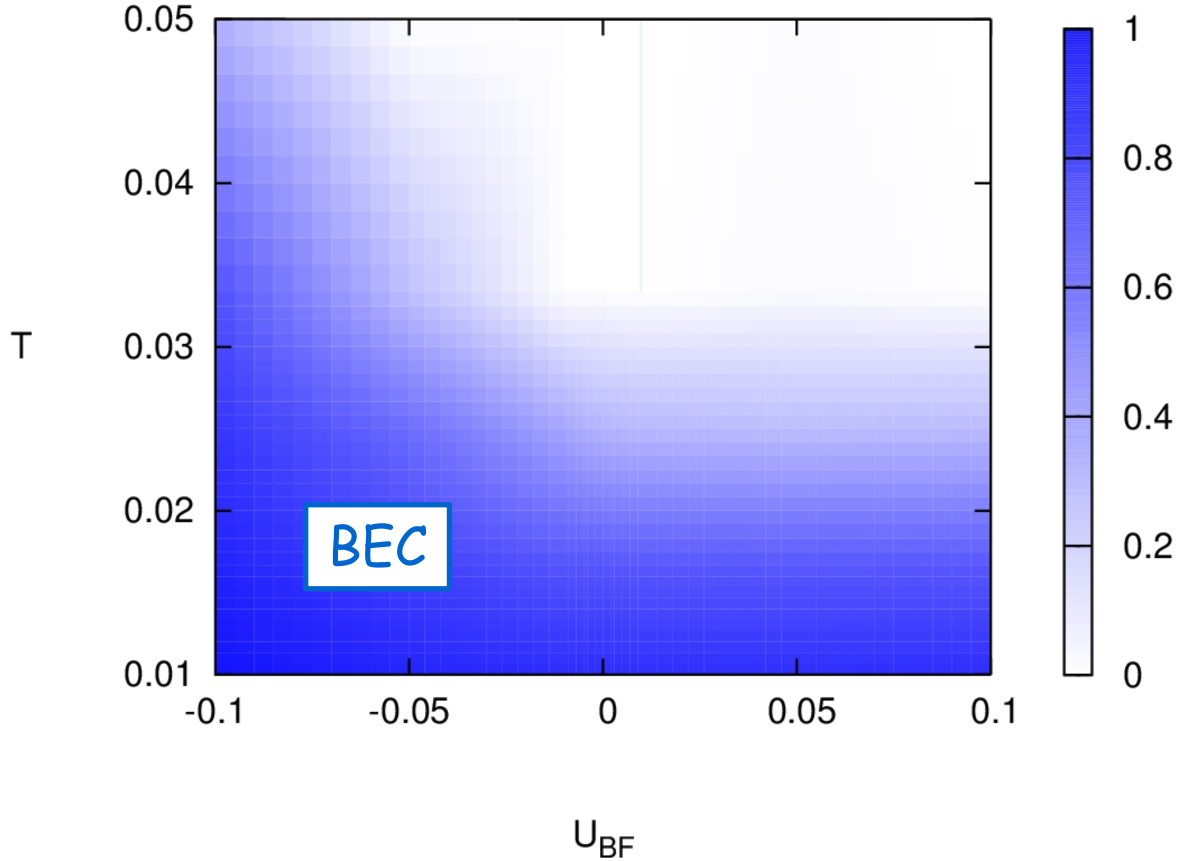
Boson Propagator

$$\langle \Phi^*(0)\Phi(R)\rangle = \begin{cases} n_B & (R=0) \\ n_{B0} & (R\to\infty) \end{cases}$$



Phase Diagram

condensate fraction: $\frac{n_{B0}}{n_B}$ at $U_B > 0$ (fixed)



Summary

- We have formulated the quantum Monte Carlo simulation of Bose-Fermi mixture on the (3+1)-dim. lattice.

- We have analyzed the Boson-Fermion density correlation and the BEC phase diagram.