The chirally rotated Schrödinger Functional

Perturbative analysis, work in progress!

Stefan Sint

Trinity College Dublin

Lattice 2012

Cairns, 27 June 2012

Contents

- Motivation for chirally rotated SF (χSF)
- SF boundary conditions and chiral rotations
- SF vs. χSF correlation functions
- χSF on the lattice & counterterms
- Tuning of m_0 and z_f at tree-level
- Sensitivity to z_f and $m_{\rm critical}$
- Determination of Z_A, O(a) vs. O(a^2) uncertainty
- Conclusions

Motivation for chirally rotated SF (χSF)

- In the continuum limit the χSF is equivalent to the SF with 2 (4,6,...) fermions

$\Rightarrow$ obtain 2 different lattice regularizations of the SF!

- Main advantage of χSF: compatibility with automatic O(a) improvement in the bulk after proper tuning of z_f (dimension 3 boundary counterterm)

$\Rightarrow$ bulk O(a) improvement of step-scaling functions without $c_{\rm sw}$ and operator improvement ($c_{\rm A}$, $c_{\rm V}$,...); interesting both for QCD and for technicolor models

- Universality relations offer new ways to determine finite renormalization constants otherwise fixed by Ward identities ($Z_{\rm A}$, $Z_{\rm P}/Z_{\rm S}$,...) & bulk improvement coefficients ($c_{\rm sw}$, $c_{\rm A}$,$c_{\rm V}$,...)
- Need perturbation theory for consistency checks & to develop non-perturbative strategies & to improve non-perturbative data

SF boundary conditions and chiral rotations

Consider isospin doublets ψ' and $\overline{\psi}'$ satisfying homogeneous SF boundary conditions ($P_\pm = \frac{1}{2}(1 \pm \gamma_0)$),

$$P_+\psi'(x)|_{x_0=0} = 0, \qquad P_-\psi'(x)|_{x_0=T} = 0,$$
$$\overline{\psi}'(x)P_-|_{x_0=0} = 0, \qquad \overline{\psi}'(x)P_+|_{x_0=T} = 0.$$

perform a chiral field rotation,

$$\psi' = \exp(i\alpha\gamma_5\tau^3/2)\psi, \qquad \overline{\psi}' = \overline{\psi}\exp(i\alpha\gamma_5\tau^3/2),$$

the rotated fields satisfy chirally rotated boundary conditions

$$P_+(\alpha)\psi(x)|_{x_0=0} = 0, \qquad P_-(\alpha)\psi(x)|_{x_0=T} = 0,$$
$$\overline{\psi}(x)\gamma_0 P_-(\alpha)|_{x_0=0} = 0, \qquad \overline{\psi}(x)\gamma_0 P_+(\alpha)|_{x_0=T} = 0,$$

with the projectors

$$P_\pm(\alpha) = \frac{1}{2}\left[1 \pm \gamma_0 \exp(i\alpha\gamma_5\tau^3)\right].$$

SF boundary conditions and chiral rotations

$$P_\pm(\alpha) = \frac{1}{2}\left[1 \pm \gamma_0 \exp(i\alpha\gamma_5\tau^3)\right].$$

Special case $\alpha = pi/2$:

$$P_\pm(\pi/2) \equiv \tilde{Q}_\pm = \frac{1}{2}(1 \pm i\gamma_0\gamma_5\tau^3) \equiv \mathrm{diag}(Q_\pm, Q_\mp),$$

The chiral rotation introduces a mapping between correlation functions:

$$\langle O[\psi,\bar{\psi}]\rangle_{(P_\pm)} = \langle \tilde{O}[\psi,\bar{\psi}]\rangle_{(P_\pm(\alpha))}$$

with $\quad \tilde{O}[\psi,\bar{\psi}] \;=\; O\left[\exp(i\alpha\gamma_5\tau^3/2)\psi, \bar{\psi}\exp(i\alpha\gamma_5\tau^3/2\right]$

where SF boundary quark and anti-quark fields are included by replacing

$$\bar{\zeta}(\mathbf{x}) \leftrightarrow \bar{\psi}(0,\mathbf{x})P_+ \qquad \zeta(\mathbf{x}) \leftrightarrow P_-\psi(0,\mathbf{x})$$

SF vs. χSF correlation functions

Consider 2-point functions with quark bilinear operators $X^{f_1 f_2}(x)$ ($X = A_0, P, V_0, S$):

- Standard SF correlators:

$$f_{\rm X}(x_0) = -\tfrac{1}{2}\langle X^{f_1 f_2}(x)\mathcal{O}_5^{f_2 f_1}\rangle_{(P_\pm)}, \qquad \mathcal{O}_5^{f_1 f_2} = a^6 \sum_{\rm y,z} \bar\zeta_{f_1}(\mathbf{y})\gamma_5 P_- \zeta_{f_2}(\mathbf{z})$$

- χSF correlators, main difference: two-point functions now depend on the flavour indices

$$g_{\rm X}^{f_1 f_2}(x_0) = -\tfrac{1}{2}\langle X^{f_1 f_2}(x)\mathcal{Q}_{5,\pm}^{f_2 f_1}\rangle_{(\tilde{Q}_+)}$$

Boundary sources defined such that they rotate into the standard SF sources:

$$\mathcal{Q}_5^{uu'} \;=\; a^6 \sum_{\rm y,z} \bar\zeta_u(\mathbf{y})\gamma_0\gamma_5 Q_- \zeta_{u'}(\mathbf{z}), \quad \mathcal{Q}_5^{ud} = a^6 \sum_{\rm y,z} \bar\zeta_u(\mathbf{y})\gamma_5 Q_+ \zeta_d(\mathbf{z})$$

Dictionary: SF $\leftrightarrow$ χSF correlation functions

- Chiral rotation by $\alpha = \pi/2$; 4 possibilities: $f_1 f_2 = uu', dd', ud, du$; assume existence of a second doublet (u', d') $\Rightarrow$ no disconnected diagrams
- Non-vanishing correlations functions with pseudo-scalar source

$$f_{\mathrm{A}} = g_{\mathrm{A}}^{uu'} = -ig_{\mathrm{V}}^{ud}, \qquad f_{\mathrm{P}} = ig_{\mathrm{S}}^{uu'} = g_{\mathrm{P}}^{ud},$$

- All other correlation functions vanish by parity or flavour symmetries!

$$f_{\mathrm{S}} = ig_{\mathrm{P}}^{uu'} = g_{\mathrm{S}}^{ud} = \quad 0 \quad = f_{\mathrm{V}} = g_{\mathrm{V}}^{uu'} = -ig_{\mathrm{A}}^{ud},$$

- use standard SF as "physical basis" $\Rightarrow$ parity and flavour symmetries take a chirally rotated form in χSF basis, e.g. parity:

$$P_5 : \begin{cases} \psi(x) & \rightarrow i\gamma_0\gamma_5\tau^3\psi(x_0, -\mathbf{x}), \\ \overline{\psi}(x) & \rightarrow -\overline{\psi}(x_0, -\mathbf{x})i\gamma_0\gamma_5\tau^3 \end{cases}$$

χSF on the lattice & counterterms

- Replace $\mathcal{D}_W \to \mathcal{D}_W + \delta\mathcal{D}_W$,

$$\begin{aligned}\delta\mathcal{D}_W\psi(x) &= (\delta_{x_0,0} + \delta_{x_0,T})\left[(z_f - 1) + (d_s - 1)\, a\mathbf{D}_s\right]\psi(x). \\ \mathbf{D}_s &= \tfrac{1}{2}(\nabla_k + \nabla_k^*)\gamma_k.\end{aligned}$$

- Parameterize the basic fermionic 2-point functions, e.g.

$$\begin{aligned}[\psi(x)\overline{\zeta}(\mathbf{y})]_F &= S(x; a, \mathbf{y})U_0(0, \mathbf{y})^\dagger \left(1 - \bar{d}_s a\overleftarrow{\mathbf{D}}_s\right) \tilde{Q}_-\,, \\ [\zeta(\mathbf{x})\overline{\psi}(y)]_F &= \tilde{Q}_- \left(1 + \bar{d}_s a\mathbf{D}_s\right) U_0(0, \mathbf{x})S(a, \mathbf{x}; y)\,,\end{aligned}$$

- Renormalize the quark boundary fields:

$$\zeta_{\mathrm{R}} = Z_\zeta \zeta, \qquad \overline{\zeta}_{\mathrm{R}} = Z_\zeta \overline{\zeta}, \qquad \zeta'_{\mathrm{R}} = Z_\zeta \zeta', \qquad \overline{\zeta}'_{\mathrm{R}} = Z_\zeta \overline{\zeta}',$$

Some observations

- As usual one needs to tune the bare mass m_0 to $m_{\rm cr}$ by imposing the conservation of the axial current, e.g.

$$\tilde{\partial}_0 g_{\rm A}^{ud}(x_0)|_{x_0=T/2} = 0$$

- The correlation functions $g_{\rm X}^{f_1 f_2}$ are either even or odd under parity P_5
- P_5-odd correlation functions must vanish for parity to be restored

$\Rightarrow$ tuning condition for z_f, as the z_f-counterterm is P_5-odd:

$$\begin{aligned}\int \mathrm{d}^3\,\mathbf{x}\overline{\psi}(x)\psi(x) \quad &\rightarrow \quad -\int \mathrm{d}^3\mathbf{x}\,\overline{\psi}(x_0,-\mathbf{x})(i\gamma_0\gamma_5\tau^3)^2\psi(x_0,-\mathbf{x}) \\ &= -\int \mathrm{d}^3\mathbf{x}\,\overline{\psi}(x)\psi(x)\end{aligned}$$

- Tuning condition for z_f, example:

$$g_{\rm A}^{ud}(x_0)|_{x_0=T/2} = 0$$

- Proper choice of $m_0 = m_{\rm cr}$ and $z_f \Rightarrow$ parity and flavour symmetries are restored.

Some observations

- N.B. This counting of powers of a does not apply to boundary O(a) counterterms:
  - boundary O(a) counterterms $\propto c_{\rm t}$ and $\propto d_s$ are P_5-even
  - boundary O(a) counterterms $\propto \bar{d}_s$ is P_5-odd
- Universality relations:

$$Z_{\rm A} Z_\zeta^2 g_{\rm A}^{uu'}(x_0) = -i Z_{\rm V} Z_\zeta^2 g_{\rm V}^{ud}(x_0) \quad \Rightarrow \quad Z_{\rm A}/Z_{\rm V} = \frac{-i g_{\rm V}^{ud}(x_0)}{g_{\rm A}^{uu'}(x_0)}$$

- Question: How is $Z_{\rm A}/Z_{\rm V}$ affected by cutoff effects?
- P_5-odd correlators should have no O(a) with correct Symanzik improvement:

$$g_{\rm P}^{uu'}(x_0) = a(\bar{d}_s - \bar{d}_s{}^*) g_{\rm P}^{uu'}(x_0)_{\rm ins,b} + a(c_{\rm sw} - c_{\rm sw}^*) g_{\rm P}^{uu'}(x_0)_{\rm ins,v} + O(a^2)$$

$\Rightarrow$ improvement conditions to fix $c_{\rm sw}$, $\bar{d}_s$, $c_{\rm V}$, $c_{\rm A}$,....

Tuning of m_0 and z_f at tree-level (with P. Vilaseca)

- Expand in PT: $g_{\mathrm{X}} = g_{\mathrm{X}}^{(0)} + g_0^2 g_{\mathrm{X}}^{(1)} + \ldots$
- We set $T = L$ and $\theta = \pi/5$, $c_{\mathrm{sw}} = 0$, with the standard SU(3) abelian background field (fund. fermions) Impose simultaneously

$$\partial_0 g_{\mathrm{A}}^{ud}(x_0)^{(0)}|_{x_0=T/2} = 0, \qquad g_{\mathrm{A}}^{ud}(T/2)^{(0)} = 0$$

for given $L/a = 4, 5, 6 \ldots \Rightarrow m_{\mathrm{cr}}^{(0)}(L/a)$, $z_f^{(0)}(L/a)$

- Results:

L/a	$am_{\mathrm{cr}}^{(0)}(L/a)$	$z_f^{(0)}(L/a)$
6	-0.035243	1.07693
8	-0.020020	1.05344
12	-0.008969	1.03154

- N.B. No solution is found for $L/a = 4, 5$!

Sensitivity to the definition of z_f and $m_0 = m_{\mathrm{cr}}$

- O(a) uncertainty in z_f and m_{cr} leads to $O(a^2)$ variation in P_5-even correlators:
- Change $z_f \to z_f + \Delta z_f$ and $m_0 \to m_0 + \Delta m_0$ and treat Δ's as O(a) insertions of P_5-odd operators

$\Rightarrow$ require at least double insertion to get P_5-even correlation function

$\Rightarrow$ O(a^2) effect in the correlators.

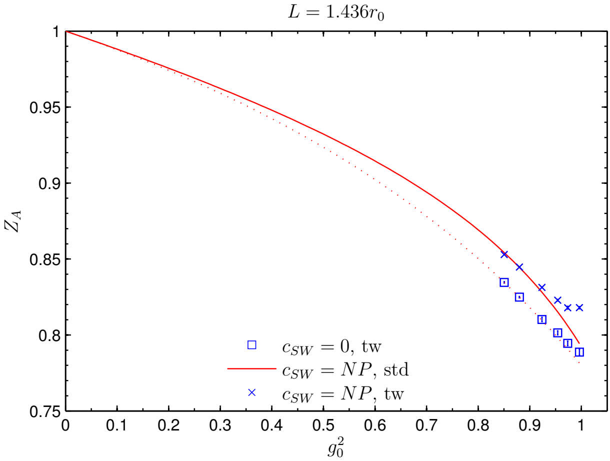
Cutoff effects in $Z_{\mathrm{A}}/Z_{\mathrm{V}}$

- $g_{\mathrm{A}}^{uu'}$ and g_{V}^{ud} are P_5-even correlators.
- Hence, no contribution from P_5-odd counterterms at $\mathrm{O}(a)$
- Only $\mathrm{O}(a)$ effects come from P_5-even $\mathrm{O}(a)$ boundary counterterms $\propto c_{\mathrm{t}}, d_s$
- universality $\Rightarrow$ counterterm insertions the same up to $\mathrm{O}(a^2)$

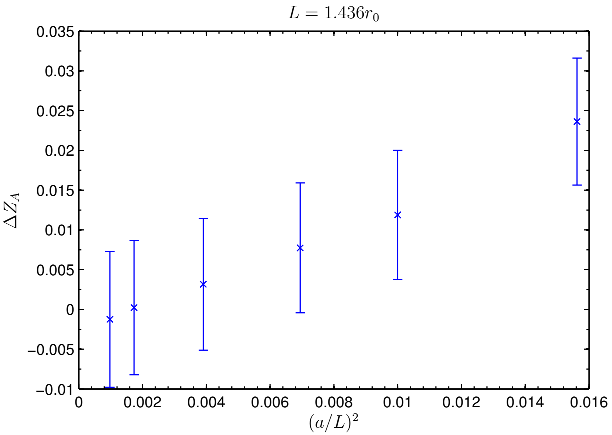
$\Rightarrow$ determination of $Z_{\mathrm{A}}/Z_{\mathrm{V}}$ up to $\mathrm{O}(a^2)$ uncertainty

- Consistent with non-perturbative data

Non-perturbative (quenched) determination of Z_{A}, (with B. Leder)



Difference to Z_{A} from Ward identity (ALPHA '96) vs. a^2



Conclusions

- χSF provides a useful alternative regularisation of the SF with Wilson quarks
- No bulk O(a) effects, reduced sensitivity to the definition of the chiral limit
- but requires tuning of z_f; necessitates $L/a > 5$!
- implementation for techicolor models almost finished (P. Vilaseca, S. S. A. Rago, A. Patella)
- Perturbation theory to one-loop (without background field) available soon; will provide some non-trivial checks of general expectations.