Monte Carlo simulations of a supersymmetric matrix model of dynamical compactification in non perturbative string theory

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Outline

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1. Introduction
   - Motivation
   - The IIB Matrix Model

2. Monte Carlo Simulations
   - 6 Dimensional Model
   - Complex Action Problem
   - Results

IIB Matrix Model: Overview

- Non-perturbative definition of string theory
- A theory with only one scale, possibility to dynamically choose a unique vacuum
- Dynamical emergence of space–time and matter content
- Dynamical compactification of extra dimensions
- tackle cosmological questions, like expansion of $3+1$ dimensional space–time, resolution of cosmic singularity

The IKKT or IIB Matrix Model [Ishibashi,Kawai,Kitazawa,Tsuchiya ('96)]

Euclidean model:

$$Z = \int dA d\psi d\bar{\psi} e^{-S}$$

$$S = \underbrace{-\frac{N}{4g^2}\mathrm{tr}\left([A_\mu, A_\nu]^2\right)}_{=S_B} + \underbrace{\frac{N}{2g^2}\mathrm{tr}\left(\bar{\psi}_\alpha(\Gamma_\mu)_{\alpha\beta}[A_\mu, \psi_\beta]\right)}_{=S_F} .$$

$A_\mu (\mu = 1, \ldots, 10)$,
$\psi_\alpha (\alpha = 1, \ldots, 16)$ (10D Majorana-Weyl spinor),
$(A_\mu)_{ij}$, $(\psi_\alpha)_{ij}$, $i, j = 1, \ldots, N$ hermitian matrices.

- No space–time dependence
- SO(10) rotational invariance and SU(N) gauge invariance
- $\mathcal{N} = 2$ Supersymmetry

Relation to String Theory

- Matrix regularization of IIB string action in the large N limit:

$$S = \int d^2\sigma \sqrt{g} \left(\frac{1}{4} \{X_\mu(\sigma), X_\nu(\sigma)\}^2 - \frac{\imath}{2} \bar{\psi}(\sigma) \Gamma^\mu \{X_\mu(\sigma), \psi(\sigma)\} \right)$$

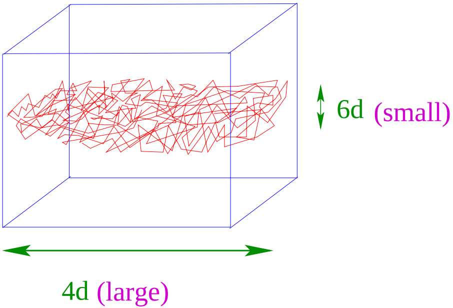
$$X_\mu(\sigma) \to (A_\mu)_{ij} \qquad \psi_\alpha(\sigma) \to (\psi_\alpha)_{ij}$$

$$\{\cdot,\cdot\} \to -\imath\,[\cdot,\cdot] \qquad \int d^2\sigma \sqrt{g} \to \mathrm{tr}$$

- non-commutative world sheet
- Block structure in matrices $\to$ second quantized string theory
- Reproduce interaction between D-branes
- Loop Equation for Wilson loops $\to$ Light cone IIB string field theory: $w(C) = \mathrm{tr} P \exp\left[\imath \int_C k^\mu A_\mu\right] \to \Psi\left[k(\cdot)\right]$

[Fukuma, Kawai, Kitazawa, Tsuchiya ('97)]

Space–Time Interpretation



- $\mathcal{N} = 2$ SUSY $\rightarrow A_\mu$ interpreted as space–time coordinates gives 10D $\mathcal{N} = 2$ spacetime SUSY

$$\left\{ \bar{\epsilon}_1 Q^{(i)}, \bar{\epsilon}_2 Q^{(j)} \right\} = -2\delta^{ij} \bar{\epsilon}_1 \Gamma_\mu \epsilon_2 \, p^\mu$$

- non-commutative space–time **[Iso,Kawai ('99)]**

Dynamical Compactification

Order Parameter

$$T_{\mu\nu} = \frac{1}{N}\mathrm{tr}\left(A_\mu A_\nu\right)$$

- Eigenvalues of $T_{\mu\nu}$: λ_n, $n = 1, \ldots, 10$

$$\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_{10}$$

- Extended d–dimensions if e.g. in the *large N limit*

$$\langle\lambda_1\rangle = \ldots = \langle\lambda_d\rangle \equiv R^2$$

Shrunk $(10 - d)$–dimensions if e.g.

$$\langle\lambda_{d+1} =\rangle \ldots = \langle\lambda_{10}\rangle \equiv r^2$$

- Breaking of SO(10) invariance

$$\mathrm{SO}(10) \to \mathrm{SO}(d)$$

Results using the Gaussian Expansion Method

[Aoyama, Nishimura, Okubo (arXiv:1007.0883)] [Nishimura, Okubo, Sugino (arXiv:1108.1293)]

- Compare free energy of ansatze for different d

$$d = 3$$

- The extent of the shrunken dimensions r is independent of d
- The extent of the large dimensions R depends on d so that the 10 dimensional volume is a finite constant and independent of d: $R^d r^{10-d} = l^{10}$

Space-time in the Euclidean model looks more like a “cigar” rather than an infinitely extended thin 3d space

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6 Dimensional Model

Study using GEM and Monte Carlo a simplified 6*D* SUSY matrix model

GEM results: **[Aoyama, Nishimura, Okubo ('10)]**

- Free energy minimizes for $d = 3$: SO(6)$\rightarrow$SO(3)
- Universal "compactification scale" $r^2 \approx 0.223$
  (in units of $g\sqrt{N}$, set to 1 in simulations)
- Constant volume property: $V = R^d \times r^{6-d} = l^6$
  For the phase quenched model $l^2 \approx 0.627$

Monte Carlo Simulation [KNA, Azuma, Nishimura (unpublished)]

$$Z = \int dA d\bar{\psi} d\psi \, \mathrm{e}^{-S_B - S_F} = \int dA \, \mathrm{e}^{-S_B} \, \det \mathcal{M} = \int dA \, \mathrm{e}^{-S_0} \, \mathrm{e}^{\imath \Gamma}$$

$$S_0 = S_b - \ln |Z_f| \; , \qquad Z_f = \int d\bar{\psi} d\psi \, \mathrm{e}^{-S_F} = \det \mathcal{M}$$

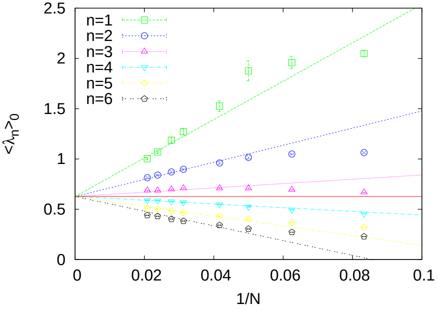
Phase Quenched Model

$$Z_0 = \int dA \, \mathrm{e}^{-S_0}$$

Monte Carlo simulations hard due to the strong complex action problem

$\det \mathcal{M}$ is Complex

Phase Quenched Model



- In the large–N limit $\langle\lambda_1\rangle_0 = \ldots = \langle\lambda_6\rangle_0$
- No SO(6) SSB: Phase fluctuations are important in inducing SSB as expected
- Supports constant volume property predicted by GEM: $l^2 \approx 0.627$

Factorization Method

Compute distribution functions of λ_n

[KNA, Nishimura ('01)], [Ambjørn, KNA, Nishimura, Verbaarschot ('02)], [KNA, Azuma, Nishimura (arXiv:1009.4504,1108.1534)]

$$\tilde{\lambda}_n = \frac{\lambda_n}{\langle \lambda_n \rangle_0}$$

- Deviation of $\langle \tilde{\lambda}_n \rangle$ from 1: effect of the phase
- Compute distribution functions

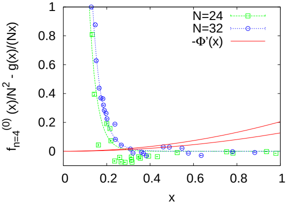
$$\rho_n(x) = \langle \delta(x - \tilde{\lambda}_n) \rangle \qquad \rho_n^{(0)}(x) = \langle \delta(x - \tilde{\lambda}_n) \rangle_0$$

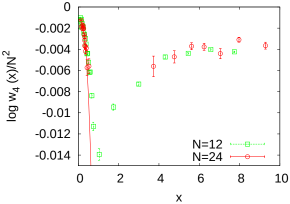
- The effect of the phase $w_n(x) = \langle e^{\imath\Gamma} \rangle_{n,x}$, $C = \langle e^{\imath\Gamma} \rangle_0$

$$\rho_n(x) = \frac{1}{C} \rho_n^{(0)}(x) w_n(x)$$

  shifts maximum of $\rho_n^{(0)}(x)$
- Estimate $\langle \tilde{\lambda}_n \rangle \equiv x_n \equiv \max \rho_n(x)$ (C not necessary in the computation)

Results





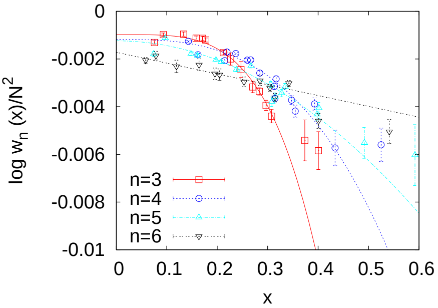
$$f_n^{(0)}(x) = -\frac{\partial}{\partial x} \ln \rho^{(0)}(x) \qquad \Phi(x) = \lim_{N\to\infty} \frac{1}{N^2} \ln w_n(x)$$

- Maximum of $\rho_n(x)$ computed from the solution of

$$\frac{1}{N^2} f_n^{(0)}(x) = -\frac{d}{dx}\Phi(x)$$

- The result for $n = 4$ is $x_n = 0.31(1)$ which can be compared to the GEM result $\langle\tilde{\lambda}_n\rangle = \frac{\langle\lambda_n\rangle}{\langle\lambda_n\rangle_0} = \frac{r^2}{l^2} \approx \frac{0.223}{0.627} = 0.355$

Results



In the large-N limit, free energies $\mathcal{F}_n(x_n) = \lim_{N\to\infty} \frac{1}{N^2} \ln \rho_n(x)$ differ by

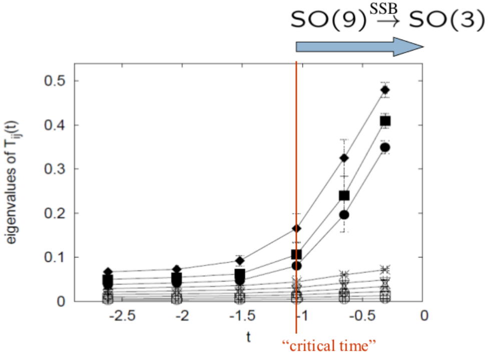
$$\Delta\mathcal{F} = \mathcal{F}_n(x_n) - \mathcal{F}_m(x_m) = -(\Phi_n(x_n) - \Phi_m(x_m))$$

Comparison for $n = 3, 4$, $N = 24$ is compatible with favoring 3d configurations at $x \approx 0.3$

Conclusions

- IKKT or IIB Matrix model can serve as a model for dynamical compactification of extra dimensions in string theory
- Recent GEM calculations in the Euclidean model show SSB of SO(10)→SO(3), finite volume for space–time. Similar result for the simplified 6D model.
- Monte Carlo simulations can play an important role in providing support for these important statements.
- Preliminary results from Monte Carlo simulations of the 6D model are consistent with GEM.
- The factorization method can be used to overcome the severe complex action problem of the model.

Conclusions



Simulations of Lorentzian model [Kim,Nishimura,Tsuchiya (arXiv:1108.1540)]

- dynamical time from A^0
- Expanding 3+1 universe after a critical time
- No sign problem