Strong Coupling Constant from First Principles

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$\Lambda_{\bar{MS}}$ and α_s from first principles

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Introduction

- number of parameters of QCD is small
- quark mass for every species
- and a scale Λ
- or, equivalently, strong coupling constant α_s
- once these are fixed, we can calculate everything else
- α_s is the last in the list of fundamental constants
- with biggest error of all

Introduction

- experimental determination error is dominated by the QCD error
- and it is important to improve on that
- as it matters for LHC crossection studies
- and exploring new physics
- it has to be done non-perturbatively
- with full control over systematic errors
- arXiv:1201.5770 [hep-ph] accepted to Phys.Rev.Lett.
- arXiv:1110.5829 [hep-lat] Phys.Rev. D85 (2012) 034503

Lattice

- calculation is done on ETMC ensembles
- with 2+1+1 dynamic quarks
- $\beta = 1.95$ simulated for $32^3 \times 64$
- $\beta = 2.10$ for a $48^3 \times 96$ lattice
- which have the same physical lattice volume
- $\beta = 1.95$ at $48^3 \times 96$ lattice
- to check finite-size effects
- of which none survives above $ap \approx 0.5$

Basics

- $\bar{MS}$ is not accessible on the lattice
- so we will perform calculation in so-called Taylor scheme
- and then convert to $\bar{MS}$

$$\alpha_T(\mu^2) \equiv \frac{g_T^2(\mu^2)}{4\pi} = \lim_{\Lambda\to\infty} \frac{g_0^2(\Lambda^2)}{4\pi} G(\mu^2, \Lambda^2) F^2(\mu^2, \Lambda^2) \ ,$$

where F and G stand for the ghost and gluon dressing functions

Propagators and Green Functions

$$A_\mu(x+\hat{\mu}/2) = \frac{U_\mu(x) - U_\mu^\dagger(x)}{2iag_0} - \frac{1}{3}\mathrm{Tr}\left(\frac{U_\mu(x) - U_\mu^\dagger(x)}{2iag_0}\right)$$

The 2-point Green functions is computed in momentum space by

$$\left(G^{(2)}\right)^{a_1 a_2}_{\mu_1 \mu_2}(p) = \langle A^{a_1}_{\mu_1}(p) A^{a_2}_{\mu_2}(-p)\rangle$$

$$\left(F^{(2)}\right)^{ab}(x-y) \;\equiv\; \langle \left(M^{-1}\right)^{ab}_{xy}\rangle \;,$$

as the inverse of the Faddeev-Popov operator, that is written as the lattice divergence,

$$M(U) = -\frac{1}{N}\nabla \cdot \widetilde{D}(U)$$

Diagonal Thinking

- Democracy is a popular choice (Leinweber'98)
- Pick momenta p such that
- $\frac{p^{[4]}}{(p^2)^2} < 0.3$ where
- $p^{[n]} = \sum_\mu p^n$, and $a^2 p^2 < 3$.
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- Voter turnout is small
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- We loose information from many momenta
- Also, democracy is mathematically impossible (Arrow'50)

H4

$$\alpha_T^{\mathrm{Latt}}\left(a^2p^2, a^2\frac{p^{[4]}}{p^2}, \ldots\right) = \widehat{\alpha}_T(a^2p^2) + \left.\frac{\partial \alpha_T^{\mathrm{Latt}}}{\partial\left(a^2\frac{p^{[4]}}{p^2}\right)}\right|_{a^2\frac{p^{[4]}}{p^2}=0} a^2\frac{p^{[4]}}{p^2} + \ldots$$

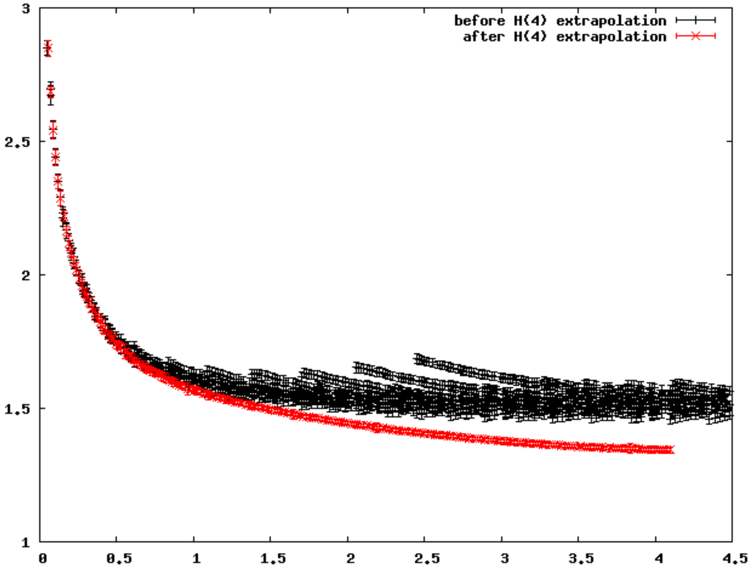
where $p^{[4]} = \sum_i p_i^4$ is the first $H(4)$-invariant (and the only one indeed relevant in our analysis).

H4

- average over any combination of momenta being invariant under $H(4)$ ($H(4)$ orbit)
- extrapolate then to the "continuum case"
- the effect of $a^2 p^{[4]}$ must vanish, by applying H4 for all the orbits sharing the same value of p^2
- with the only assumption that the slope depends smoothly on $a^2 p^2$
- $H(4)$-artefact-free lattice coupling, $\widehat{\alpha}_T(a^2p^2)$ might differ from the continuum coupling by some $O(4)$-invariants artefacts,

$$\widehat{\alpha}_T(a^2p^2) \; = \; \alpha_T(p^2) \; + \; c_{a2p2} \; a^2p^2 \; + \; \mathcal{O}(a^4) \; ,$$

Ghost Dressing Function



Lattice and Perturbation theory: an unlikely friendship

$$\alpha_T(\mu^2) = \alpha_T^{\rm pert}(\mu^2) \left(1 + \frac{9}{\mu^2}\, R\left(\alpha_T^{\rm pert}(\mu^2), \alpha_T^{\rm pert}(q_0^2)\right)\right.$$
$$\left. \times \left(\frac{\alpha_T^{\rm pert}(\mu^2)}{\alpha_T^{\rm pert}(q_0^2)}\right)^{1-\gamma_0^{A^2}/\beta_0} \frac{g_T^2(q_0^2)\langle A^2\rangle_{R,q_0^2}}{4(N_C^2-1)}\right), \qquad (1)$$

- derived from the OPE description of ghost and gluon dressing function
- $\gamma_0^{A^2}$ is calculated by perturbation theory (Gracey, Chetyrkin)
- $N_f = 4$, $1 - \gamma_0^{A^2}/\beta_0 = 27/100$,

Further Perturbations

$$\begin{aligned} R(\alpha, \alpha_0) &= \left(1 + 1.18692\alpha + 1.45026\alpha^2 + 2.44980\alpha^3\right) \\ &\times \left(1 - 0.54994\alpha_0 - 0.13349\alpha_0^2 - 0.10955\alpha_0^3\right) \end{aligned}$$

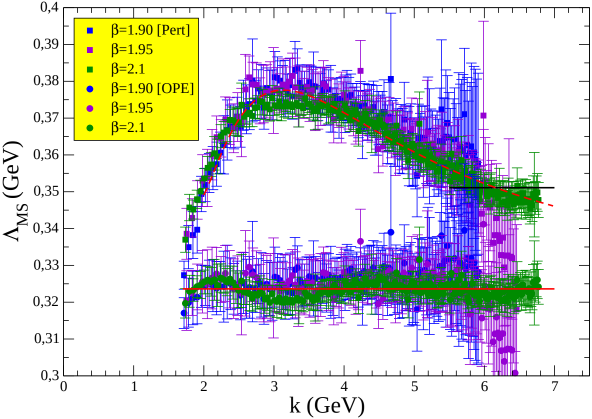
$$\begin{aligned} \alpha_T^{\mathrm{pert}}(\mu^2) &= \frac{4\pi}{\beta_0 t}\left(1 - \frac{\beta_1}{\beta_0^2}\frac{\log(t)}{t} + \frac{\beta_1^2}{\beta_0^4}\frac{1}{t^2}\left(\left(\log(t) - \frac{1}{2}\right)^2\right.\right. \\ &+ \left.\left.\frac{\beta_2\beta_0}{\beta_1^2} - \frac{5}{4}\right) + \frac{1}{(\beta_0 t)^3}\left(\frac{\beta_3}{2\beta_0} + \frac{1}{2}\left(\frac{\beta_1}{\beta_0}\right)^3\right.\right. \\ &\times \left(-2\log^3(t) + 5\log^2(t)\right. \\ &+ \left.\left.\left.\left(4 - 6\frac{\beta_2\beta_0}{\beta_1^2}\right)\log(t) - 1\right)\right)\right) \qquad (2) \end{aligned}$$

with $t = \ln \frac{\mu^2}{\Lambda_T^2}$

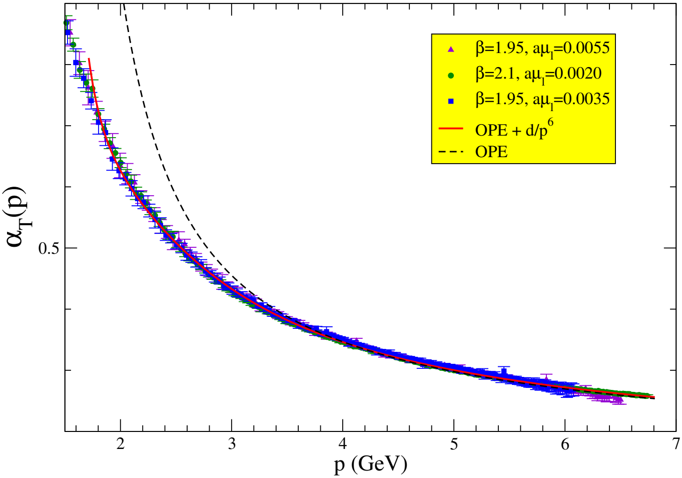
Fitting and Condensing

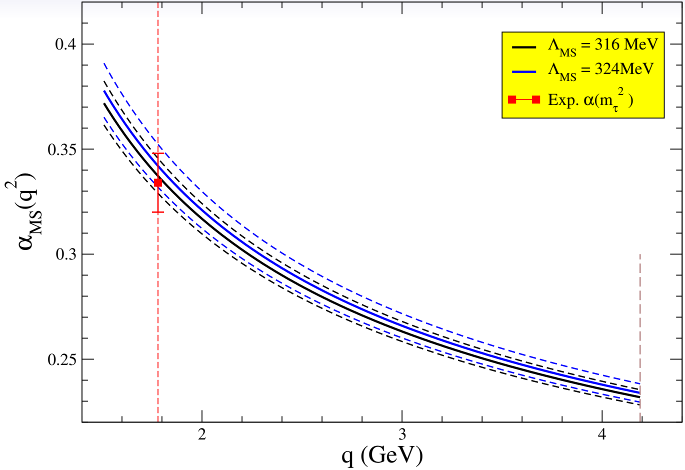
- So now we can compare prediction of OPE with lattice data
- and fit using two coefficients
- $g^2\langle A^2\rangle$, the Landau gluon condensate
- and Λ_T, the Λ_{QCD} parameter in Taylor scheme

Plots: Plateau of Saclay



Plots: Taylor scheme





	$\Lambda_{\overline{\rm MS}}^{N_f=4}$ (MeV)	$g^2\langle A^2\rangle$ (GeV^2)	$d^{1/6}$ (GeV)
OPE	316(13)	4.5(4)	
OPE$+1/p^6$	324(17)	3.8(1.0)	1.72(3)

Results

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- Systematic errors controlled at all steps
- Both on lattice and on perturbative side
- and the only thing you have to remember from this...

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- Systematic errors controlled at all steps
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- and the only thing you have to remember from this...

$$\alpha_{\bar{MS}}(m_\tau^2) \;=\; 0.339(13)$$

$$\alpha_{\bar{MS}}(m_Z^2) \;=\; 0.1200(14)$$