The QCD strong coupling from the lattice three gluon vertex using 2+1 flavour domain wall fermions

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Recent lattice calculations

- **Wilson loop**
  HPQCD : Davies et al. (2008) (Nf=2+1) .
- **Heavy current correlator**
  HPQCD : Allison et al. (2008), McNeile et al. (2010) (Nf=2+1).
- **Light current correlator**
  JLQCD : Shintani et al. (2009) (Nf=2+1).
- **Schrödinger functional**
  ALPHA : DellaMorte et al. (2005) (Nf=2).
- **Vertex functions**
  Ghost-Gluon ETMC : Blossier et al. (2010) (Nf=1+1). (Nf=2+1+1) See K. Petrov's talk.
  Triple-gluon RBC/UKQCD : R.J.H , Boyle & Del Debbio (Nf=2+1).

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$p_1^2 = p_2^2 = \mu^2, p_3^2 = 0.$

Non-exceptional kinematics :
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- Amputate the three point function's legs & perform gluon field renormalisation.
- Match to $\overline{\mathrm{MS}}$ at given scale μ. Run to the Z mass.

Ensembles

- RBC/UKQCD ensembles, Nf=2+1 DWF.
- Iwasaki gauge action.

"Fine"

1. $32^3 \times 64 \times 16$ $(2.75\mathrm{fm}^3 \times 5.5\mathrm{fm})$.
2. $a^{-1} = 2.282(28)$ GeV.
3. $am_s = 0.03$.
   - $am_u = 0.004$ (519),
   - $am_u = 0.006$ (745),
   - $am_u = 0.008$ (308).

"Coarse"

1. $24^3 \times 64 \times 16$. $(2.74\mathrm{fm}^3 \times 7.3\mathrm{fm})$
2. $a^{-1} = 1.730(25)$ GeV.
3. $am_s = 0.04$
   - $am_u = 0.005$ (1555),
   - $am_u = 0.01$ (803),
   - $am_u = 0.02$ (573).

Fixing to Landau gauge

- Fix to Landau gauge using a Fourier-accelerated algorithm.
- Gluon fields defined as

$$A_\mu(x + a\hat{\mu}/2) = \frac{1}{2i}\left(U_\mu(x + a\hat{\mu}/2) - U_\mu(x + a\hat{\mu}/2)^\dagger\right)_{\mathrm{Trf}},$$

$$(X)_{\mathrm{Trf}} = X - \frac{1}{Nc}\mathrm{Tr}\left[X\right] I_{Nc\times Nc}.$$

- We fix to an accuracy of

$$\frac{1}{V}\sum_x Tr\left[\left|\partial_\mu A_\mu(x + a\hat{\mu}/2)\right|^2\right] < 10^{-20}.$$

- And Fourier transform our fields to momentum space.

We use the following momentum definitions,

$$ap_\mu = \frac{2\pi n_\mu}{L_\mu}, \; \tilde{p}_\mu = 2\sin\left(\frac{ap_\mu}{2}\right).$$

The Landau condition $\tilde{p}_\mu A_\mu(p) = 0$ is not a constraint for p=0!

Gluon propagator

We amputate the external legs of our three point function using the gluon propagator.

- Two point gluonic correlation function, has tensor structure in Landau gauge

$$G^{ab}_{\mu\nu}(p^2) \;=\; \left(\delta_{\mu\nu} \;-\; \frac{p_\mu p_\nu}{p^2}\right) G^{ab}(p^2).$$

- Measured on the lattice as

$$G^{(2)}(p^2) = \frac{1}{V}\frac{2}{(Nd-1)(Nc^2-1)}G_{\mu\mu}(p^2),$$

$$G_{\mu\nu}(p^2) \;=\; \langle \mathrm{Tr}\left[A_\mu(p)A_\nu(-p)\right]\rangle.$$

The $(Nd-1)$ factor is valid only for $p \neq 0$, conventionally use

$$G^{(2)}(0) \;=\; \frac{1}{V}\frac{2}{Nd(Nc^2-1)}G_{\mu\mu}(0).$$

- Ambiguity in the gluon normalisation at zero momentum?

Testing the anisotropic gluon propagator

Are all polarisations equivalent for the zero momentum gluon propagator on an asymmetric $(L_t > L_{x,y,z})$ lattice?

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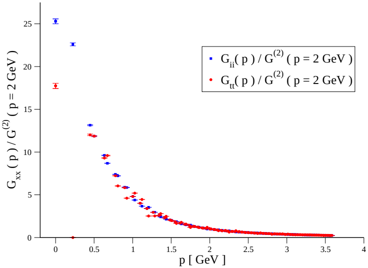
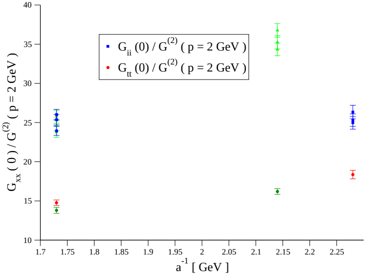
Are all polarisations equivalent for the zero momentum gluon propagator on an asymmetric ($L_t > L_{x,y,z}$) lattice?

- Only asymmetric aspect of measurement at zero momentum is the volume.
- A spatial-temporal difference **must** be a finite volume effect.
- Expect G_{ii} (spatial propagator) and G_{tt} (temporal propagator), to be equivalent at higher momenta.
- Remove field renormalisation by taking the ratio $\frac{G_{xx}(p_1)}{G^{(2)}(p_2)}$, a direct measure of anisotropy effects. Allowing comparisons of anisotropies at different lattice spacings.

Zero momentum gluon propagator breaks Euclidean symmetry



The *fine* lattice gluon anisotropy.



The ratio $G_{xx}(0)/G^{(2)}(p = 2\,\mathrm{GeV})$.

Does this compromise the measurement in the exceptional scheme?

There is no *obvious* way to control finite volume effects for the zero momentum gluon propagator. Large breaking of Euclidean geometry by our volume. $G^{(2)}(0)$ **is finite volume compromised**.

The exceptional scheme

The $\widetilde{MOM}_{gg}$ scheme

Used in Parinello et al. (1994,1996) [1], Boucaud et al. (1998)[2] & perturbative matching from Chetyrkin & Rétey (2000) [3].

- $p_1^2 = p_2^2 = \mu^2, p_3^2 = 0.$
- Project out the scalar amplitude,

$$G^{(3)}(p^2) = \frac{4}{(Nd-1)Nc(Nc^2-1)} \left(\delta_{\mu\nu} - \frac{\tilde{p}_\mu \tilde{p}_\nu}{\tilde{p}^2} \right) \frac{\tilde{p}_\rho}{\tilde{p}^2} G_{\mu\nu\rho}(p^2),$$

$$G_{\mu\nu\rho}(p^2) = \langle \mathrm{Tr}\left[A_\mu(p) A_\nu(-p) A_\rho(0) \right] \rangle.$$

- Renormalise the gluon fields using $Z_{A_\mu}(p^2) = \tilde{p}^2 G^{(2)}(p^2)$,

$$g_R^{\widetilde{MOM}_{gg}}(\mu^2) \; = \; Z_{A_\mu}(\mu^2)^{3/2} \frac{G^{(3)}(\mu^2)}{\left(G^{(2)}(\mu^2)\right)^2 G^{(2)}(0)}.$$

[1] arXiv:hep-lat/9405024v1 , arXiv:hep-lat/9605033v2
[2] arXiv:hep-ph/9810322v2
[3] arXiv:hep-ph/0007088v1

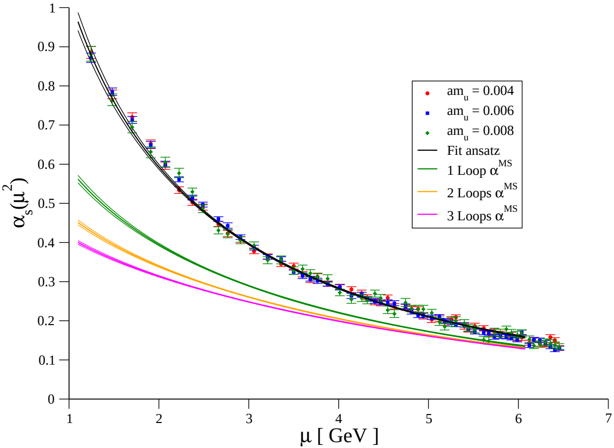
We use a fit ansatz.

- We assume that the coupling will have the following "perturbative" form

$$\alpha_s(p^2)_{\rm lattice} = a + b \log\left(\frac{p^2}{p_0^2}\right) + c \log\left(\frac{p^2}{p_0^2}\right)^2 .$$

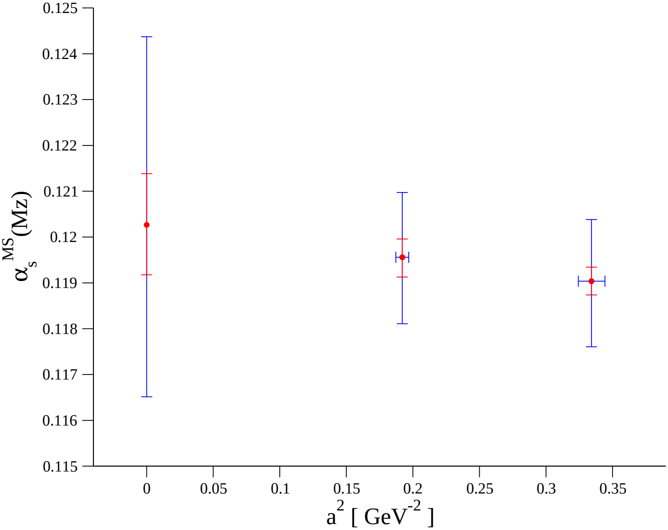
- Where the parameter "p_0" is a momentum point chosen to be in the middle of our fit range.
- "a" is the strong coupling from our fit at the point "p_0".
- We use a correlated fit.

The $\widetilde{MOM}_{gg}$ coupling from the *fine* lattice



No observed quark mass dependence.

Continuum $\widetilde{MOM}_{gg}$ coupling.
(statistical)(pert+a^{-1} systematic)



The non-exceptional scheme

The MOM_{ggg} scheme

Project out the scalar part of the vertex, (J.A Gracey (2011)[4])

$$G_k^{(3)}(p^2) = \frac{4}{Nc(Nc^2-1)} \mathrm{Tr}\left[M_{k,l}\left(P_l^{\mu\nu\rho}(p_1,p_2) G^{(3)}_{\mu\nu\rho}(p_1,p_2)\right)\right],$$
$$G^{(3)}_{\mu\nu\rho}(p_1,p_2) = \langle A_\mu(p_1) A_\nu(p_2) A_\rho(-p_1-p_2)\rangle.$$

- $p_1^2 = p_2^2 = (-p_1-p_2)^2 = \mu^2$, requires triplets of external momenta. Which are only equal up to H(4) breaking effects.
- Evaluated using,

$$g_R^{MOM_{ggg}}(\mu^2) \;=\; Z_{A_\mu}^{3/2}(\mu^2) \frac{G_1^{(3)}(\mu^2)}{\left(G^{(2)}(\mu^2)\right)^3}.$$

- Matching to $\overline{\mathrm{MS}}$ known to two loops.

[4] arXiv:1108.4806v1 [hep-ph]

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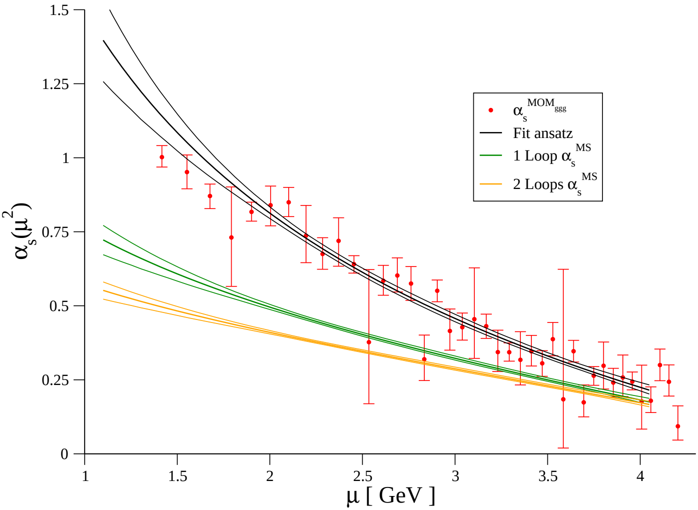
- Evaluated using,

$$g_R^{MOM_{ggg}}(\mu^2) \;=\; Z^{3/2}_{A_\mu}(\mu^2) \frac{G^{(3)}_1(\mu^2)}{\left(G^{(2)}(\mu^2)\right)^3}.$$

We are finally free of zero-momentum subtractions!

[4] arXiv:1108.4806v1 [hep-ph]

Fine lattice MOM_{ggg} coupling



Results

Continuum-extrapolated exceptional scheme yields the result, with an estimate of $\approx 5\%$ finite volume error,

$$\alpha_s^{\overline{\mathrm{MS}}}(Mz) = 0.1202(11)_{\mathrm{stat}}(2)_{\mathrm{a}}(39)_{\mathrm{pert}}(60??)_{\mathrm{finitevol}}.$$

We have no continuum limit for the non-exceptional case, this has to be included in the systematics,

$$\alpha_s^{\overline{\mathrm{MS}}}(Mz) = 0.1290(16)_{\mathrm{stat}}(60)_{\mathrm{pert}}(??)_{\mathrm{extrap}}.$$

Conclusions

- We have measured α_s in the exceptional scheme with good statistical accuracy using the triple gluon vertex.
- We have seen strong sensitivity to finite volume effects in the zero-momentum gluon propagator, making the applicability of the exceptional scheme **questionable**.
- The major contributions to the error come from finite volume effects and matching to perturbation theory.

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- The major contributions to the error come from finite volume effects and matching to perturbation theory.
- We have tried to address the finite volume effects by implementing the non-exceptional scheme, but we are limited by statistics.
- Controlling the perturbative errors could be performed by,
  1. More loops.
  2. Finer Lattices. MILC?

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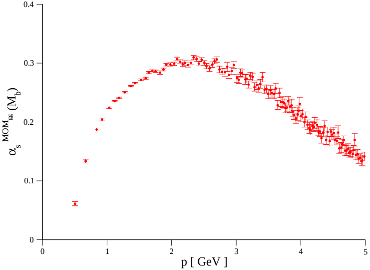
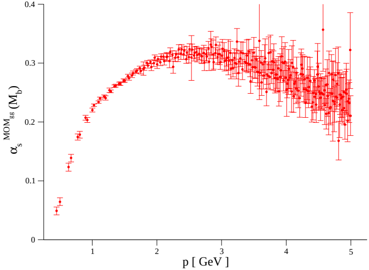
Back up slides

Locating the Rome-Southampton window



The *coarse* lattice coupling run to Mb



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