The $SU(3)$ β–function to twenty loops

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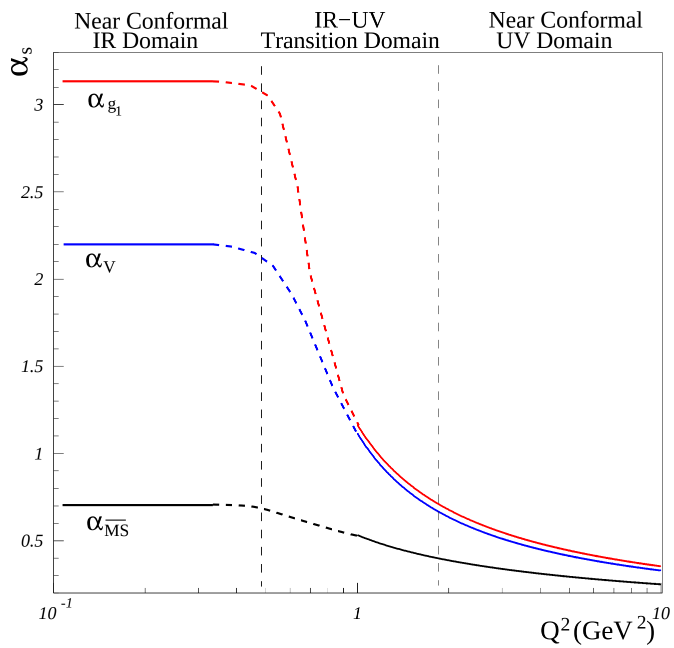
Motivation

Currently, the β–function is very much in the focus of our interest. At the extreme ends, two scenarios are possible:

$\mathbb{1}$. Infrared fixed point



Brodsky

Conformal window

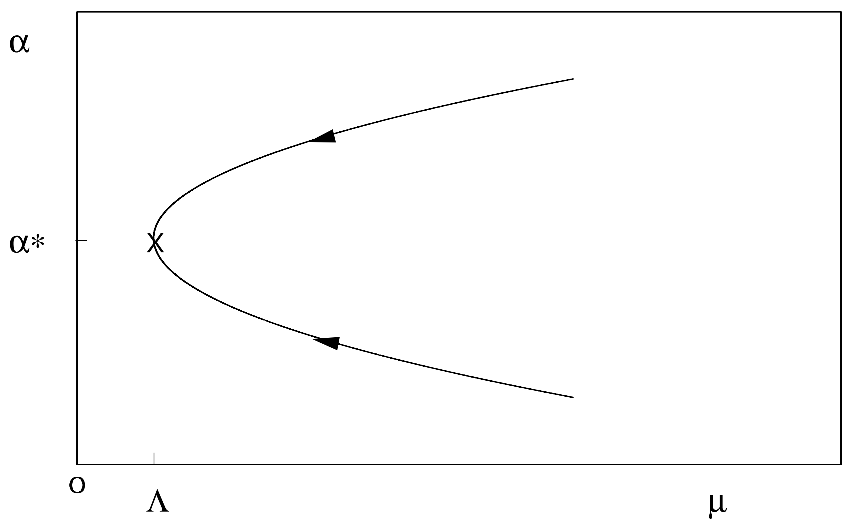
$\downarrow$

AdS/CFT correspondence

Holographic QCD

2. Infrared attractive point

Supersymmetric Yang-Mills



NSVZ

The theory has two phases, one which is asymptotically free and another which is strongly coupled in the infrared, with α^* being an infrared attractive point

The theory flows to α^* in the infrared, both from the small and large α domain

To find out what is happening in QCD, we need to map out the renormalization group flow of the running coupling constant over a wide range of scales from the ultraviolet to the infrared

The idea

We start from rectangular $L \times T$ Wilson loops $W(L,T)$ and the corresponding Creutz ratios $R(L,T)$

$$R(L,T) = \frac{W(L,T)\,W(L-1,T-1)}{W(L,T-1)\,W(L-1,T)} = 1 + \sum_{n=1}^{N} r_n(L,T)\,g^{2n}$$

Two options

$$W(L,T) \underset{T \gg L}{=} C\,\mathrm{e}^{-V(L)\,T}$$

$$W(L,L) = C\,\mathrm{e}^{-V(L)\,L}$$

where g^2 is the bare lattice coupling. Using

$$V(L) = -\frac{4}{3}\frac{g_V^2(L)}{L}$$

and choosing the second option by writing $R(L) = R(L,L), r_n(L) = r_n(L,L)$, we then get the running coupling

$$g_V^2(L) = \frac{1}{r_1(L)} \ln R(L) = g^2 + \sum_{n=2}^{N} c_n(L)\,g^{2n}$$

From the expressions for the running coupling $g_V^2(L)$ we derive the β–function in the potential or V scheme

$$\beta(g_V(L{-}1)) \;=\; \frac{1}{2\,g_V(L{-}1)}\,\frac{g_V^2(L) - g_V^2(L{-}1)}{\ln L/(L{-}1)}$$

with

$$\beta(g_V) = \;-\frac{\beta_0}{16\pi^2}\,g_V^3 - \frac{\beta_1}{16\pi^2}\,g_V^5 \;-\;\cdots$$

$$\beta_0 = 11\,, \quad \beta_1 = 102$$

universal

In any other scheme S

$$\frac{\alpha_S}{\pi} = \frac{\alpha_V}{\pi} + P_2^S\left(\frac{\alpha_V}{\pi}\right)^2 + P_3^S\left(\frac{\alpha_V}{\pi}\right)^3 + \cdots$$

P_n's are known for several S

Computation of Wilson loops

Action

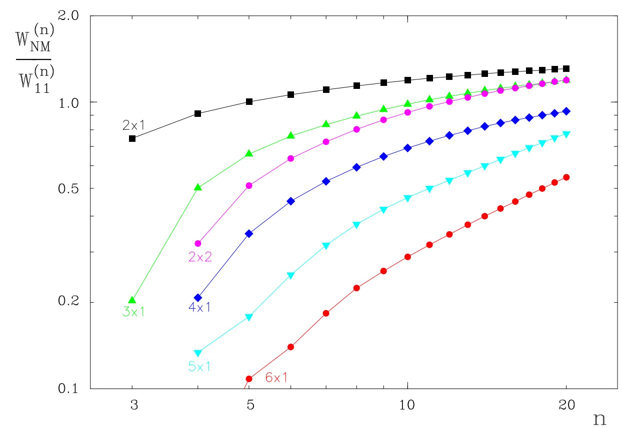
$$S = \beta \sum_P \left[1 - \frac{1}{6}\mathrm{Tr}\left(U_P + U_P^\dagger\right)\right]$$

We employ numerical stochastic perturbation theory (NSPT), in which the perturbed link variables $U_{x,\mu}$ are computed from a stochastic evolution in fictitious time τ driven by the Langevin equation

$$\frac{\partial}{\partial\tau} U_{x,\mu}(\tau;\eta) = \mathrm{i}\left[\nabla_{x,\mu}\, S(U) - \eta_{x,\mu}(\tau)\right] U_{x,\mu}(\tau;\eta)$$

$$V = 12^4\,, \qquad \boxed{N = 20}$$

arXiv:1205.1659 [hep-lat]



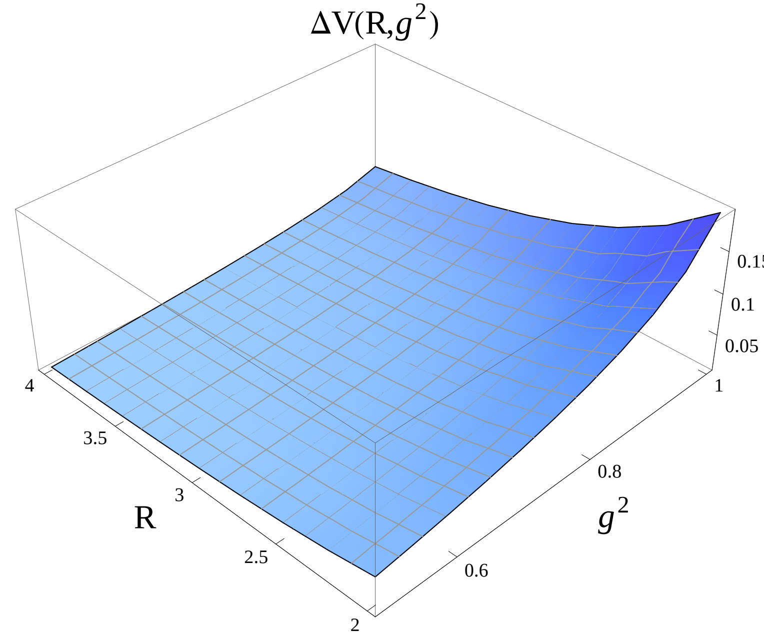
Valid definition of $g_V(L)$?

$$\Delta V(L) = V(L) - V(L{-}1) = -\ln R(L)$$
$$\propto 1/(L{-}1/2)^2$$

$\underline{\Delta V(4)/\Delta V(3)}$

Expect 1.96

Obtain 1.93



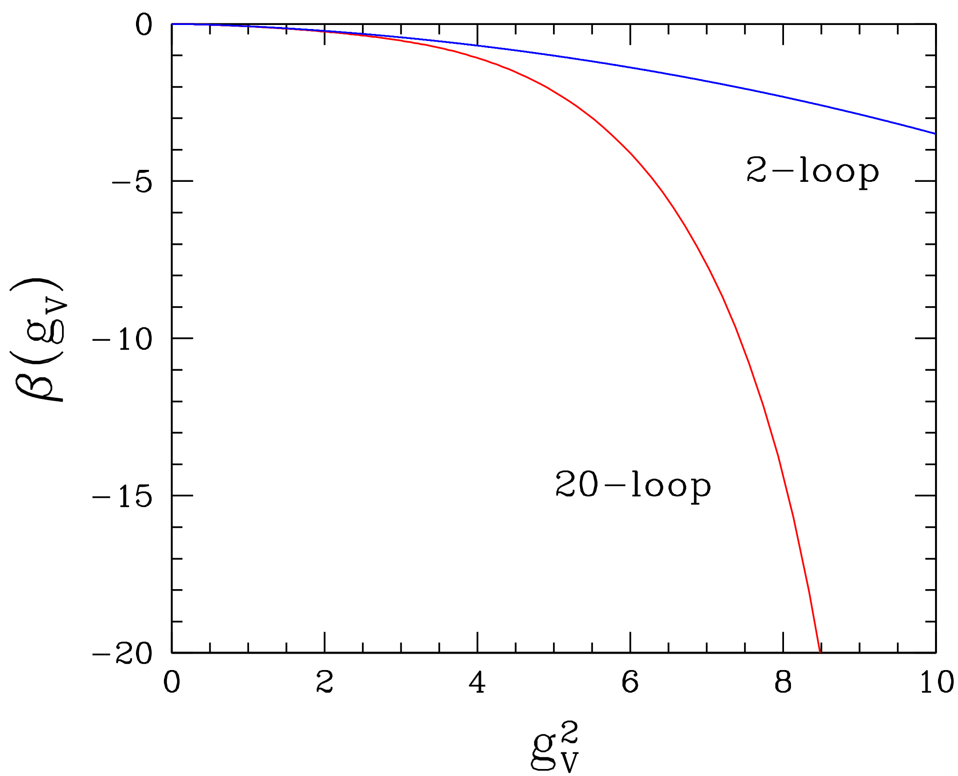
β–function

Test

First two coefficients

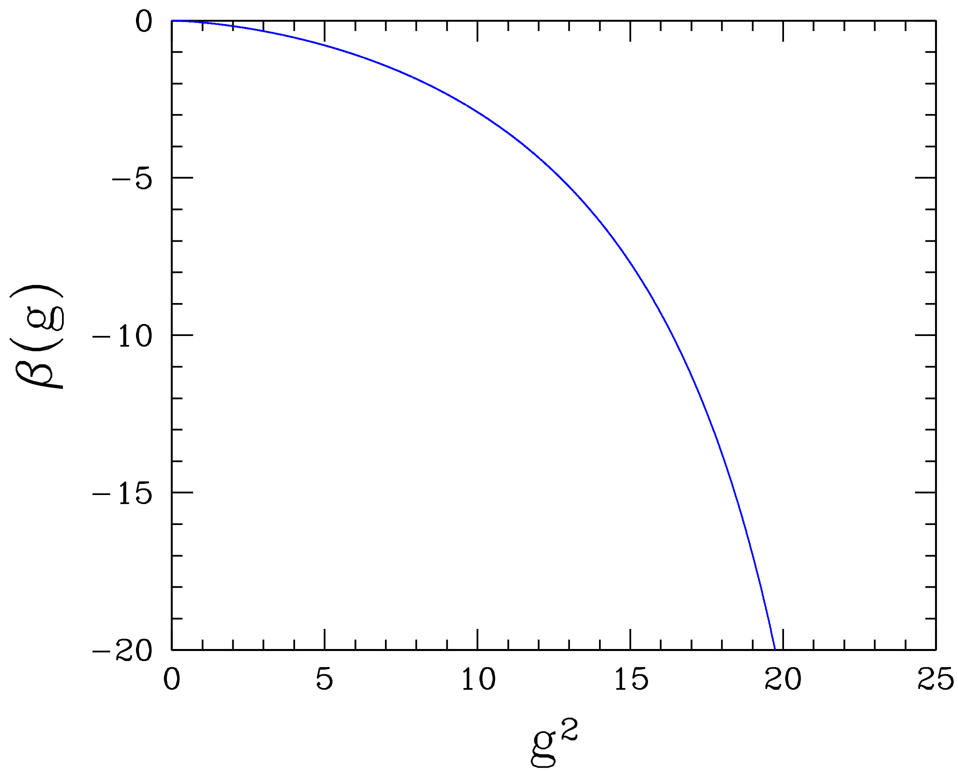
$L\|T$	β_0	β_1
$4\|4$	10.8	
$3\|3$		121
$3\|4$	$\overline{11.8}$	108
$3\|5$		106

β–function from $L = 4$

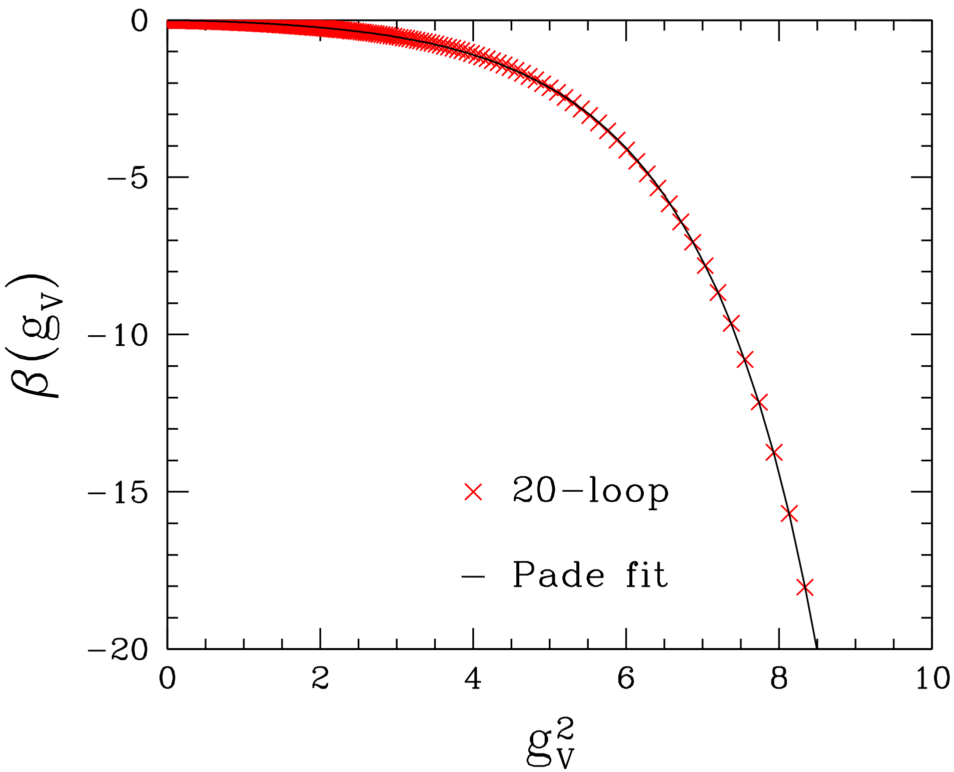


Supersymmetric Yang-Mills $N = 3$

$$\beta(g) = -\frac{9}{16\pi^2}\frac{g^3}{1 - 3g^2/8\pi^2}$$



Padé fit $g^3\,[2,1]$ to convergent sum $\Leftarrow$ Nonperturbative contribution negligible (!?)

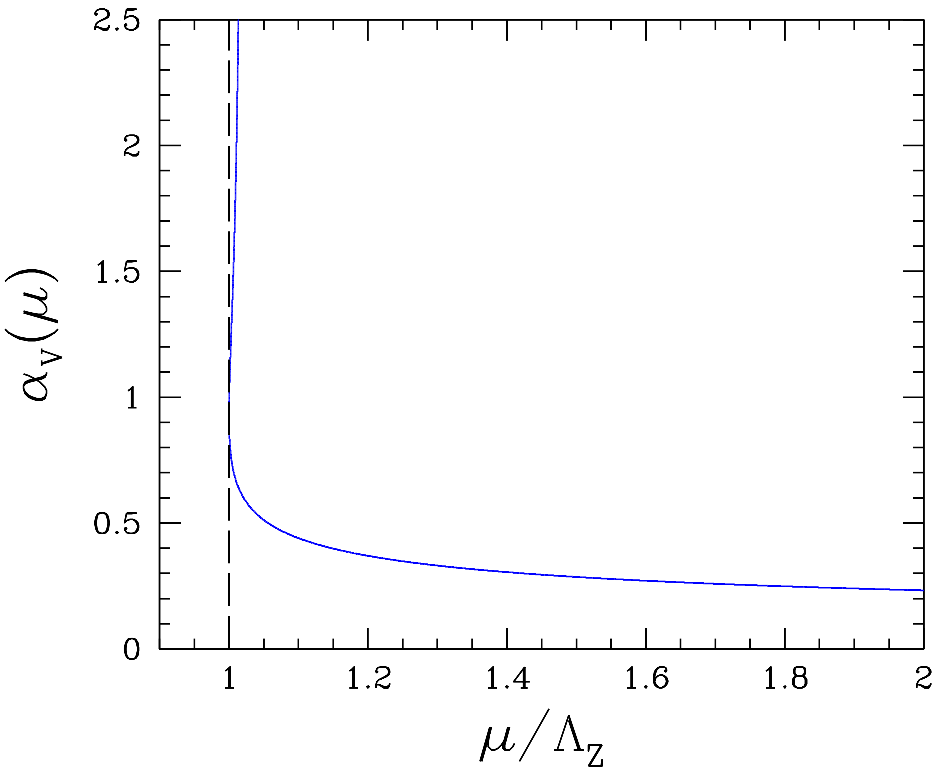


$$\beta(g) = -\frac{\beta_0}{16\pi^2}\,\frac{g^3\left(1+b_2 g^2+b_4 g^4\right)}{1+d_2 g^2}$$

$g^3\,[2,3] \Leftrightarrow$ SUSY type RG flow

$d_2 < 0$

Running coupling $\quad g^3[2,1]$



Conclusions

- Wilson loops from NSPT lend themselves to a calculation of the running coupling to twenty loops (and possibly higher)

- This enables us to map out the renormalization group flow of the running coupling over a wide range of scales from the ultraviolet to the infrared

- No sign of walking and of a conformal window is found

- β–function appears to change sign (like in the case of the SUSY Yang-Mills theory) at $g_V^2 \approx 12$ not through zero, as it would happen at a regular fixed point, but rather through pole

- Hope to repeat the calculations on rectangular $N_S^3 \times N_T$ lattices with time extent $N_T \gg N_S$ using boosted NSPT