Large-N string tension from rectangular Wilson loops

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Robert Lohmayer, Herbert Neuberger

Rutgers University

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Outline

Introduction

- We study Wilson loop operators $W(\mathscr{C})$ in 4D Euclidean $\mathrm{SU}(N)$ pure gauge theory.
  ($\mathscr{C}$ is a closed, non-selfintersecting, continuous curve in $\mathbb{R}^4$, with a finite number of kinks $\longrightarrow$ rectangles on the lattice)
- Perimeter and corner divergences of W require smearing.
- Extra (continuous) smearing parameter s represents effective thickness of $\mathscr{C}$ ($\sqrt{s}$: observer's resolution of localized objects).
- At large-N: sharp transition between weakly coupled short distance regime and qualitatively different strongly coupled long distance regime:
  - Small loops: insensitive to the compact nature of $\mathrm{SU}(N)$; large loops: full group is explored (key ingredient for confinement).
  - At the transition point: gap in the eigenvalue spectrum of the Wilson loop matrix closes;
    natural point where PT and a long distance description ($\longrightarrow$ effective string theory) could be matched.
  - Traces of Wilson loops remain smooth through the transition (even at $N = \infty$).
- Here: determine by lattice methods how an effective string description on the strong coupling side of the transition and close to it works in detail.

Introduction

Expectation from effective string theory for asymptotic expansion of large loops around minimal area configuration (Nambu-Goto + boundary terms):
for a dilated loop ($\mathscr{C} \to \rho\,\mathscr{C}$), asymptotically as $\rho \to \infty$:

$$\log(W(\rho\,\mathscr{C})) \sim -\sigma\rho^2 \mathrm{Area}_{\min}(\mathscr{C}) + \Gamma_0 \rho\, \mathrm{Length}(\mathscr{C}) + \Gamma_1(\mathscr{C}) \log(\rho) + \Gamma_2(\mathscr{C}) + \Gamma_3 + \Gamma_4(\mathscr{C})/\rho^2 + \mathscr{O}(1/\rho^3)\,.$$

- String tension $\sigma > 0$ used to set the scale from the outside in EST.
- EST cannot make predictions for perimeter and corner terms: Γ_0, Γ_3 are non-universal numbers, independent of $\mathscr{C}$, dependent on smearing (diverge like $1/\sqrt{s}$ and $\log^\kappa s$ for $s \to 0$).
- $\Gamma_1(\mathscr{C})$, $\Gamma_2(\mathscr{C})$, $\Gamma_4(\mathscr{C})$: universal scale-invariant functions of the shape of $\mathscr{C}$

Potential problem: interference between further smearing-dependent subleading terms and smearing-independent terms coming from EST expansion.

Simulation parameters

- Averages of smeared rectangular ($L \times L$, $L \times (L+1)$, $L \times 2L$) Wilson loops on a symmetric hypercubic lattice are obtained from 160 uncorrelated gauge fields.
- Gauge action is of the single-plaquette Wilson type.
- Wilson loop on the lattice:

$$W_N(L_1, L_2, b, S, V) = \frac{1}{N} \langle \mathrm{Tr} \prod_{l \in \mathscr{C}} U_l \rangle \,.$$

  (product over links l around rectangle of sides $L_{1,2}$).
- We mainly use couplings $0.359 \leq b = \frac{\beta}{2N^2} \leq 0.369$,
  N-values 7, 11, 13, 19, 29,
  smearing levels $0.2 \leq S \leq 0.4$.
- All fits are applied to

$$w_N(L_1, L_2, b, S, V) = -\log W_N(L_1, L_2, b, S, V)\,.$$

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Square loops: infinite-N,V limits

First step: $\lim_{N\to\infty}\left(\lim_{V\to\infty} w_N(L,b,S,V)\right)$ for square $L\times L$ loops.
Potential shortcut: large-N reduction (requires tests and fits; finite-volume effects depend on V, N, b, and loop size L)

- Method 1)
  At fixed N, use volumes sufficiently large for finite-volume effects to be negligible ($V=24^4$, 18^4, 14^4, 12^4 for $N=7$, 11, 19, 29, resp.), then fit

$$w_N(V=\infty)=w_\infty(V=\infty)+\frac{a_1(V=\infty)}{N^2}+\frac{a_2(V=\infty)}{N^4}.$$

- Method 2)
  First take $N\to\infty$ at fixed V (fitting $1/N^2$, $1/N^4$ corrections as above).
  No volume dependence in the inf.-N theory (as long as center sym. unbroken):
  $w_\infty(V)=w_\infty(V=\infty)$.
    - 2a) $w_\infty(V=12^4)$ from $N=11$, 13, 19, 29
    - 2b) $w_\infty(V=14^4)$ from $N=7$, 11, 13, 19

Square loops: infinite-N,V limits

- Agreement between the three results at their stat. accuracy of about 0.1%.
- Truncating the expansion at order $1/N^2$ results in very large χ^2/N_{dof} (i.e., we cannot set $a_2/N^4 = 0$)
- For $V = 12^4$ case: including $N = 29$ result is crucial; including $N = 7$ would require $1/N^6$ correction.
  When V gets close to critical size at which center symmetry breaks, we need to go to higher N's to determine $\lim_{N,V\to\infty} w_N(V)$.
- computation time $\sim N^3 V$:
  method 2a) [$V = 12^4$] is 1.75 times more expensive than method 2b) [$V = 14^4$];
  method 1) [large V's] is 2.5 times more expensive than method 2b).

  However, we became confident that we have correctly determined $\lim_{N,V\to\infty} w_N(L, b, S, V)$ only after having obtained agreeing results from 1), 2a) and 2b).

Square loops: lattice string tension (at infinite N)

- For square $L \times L$ loops (fixed b, S: $w_\infty(L) \equiv \lim_{N,V\to\infty} w_N(L,b,S,V)$):

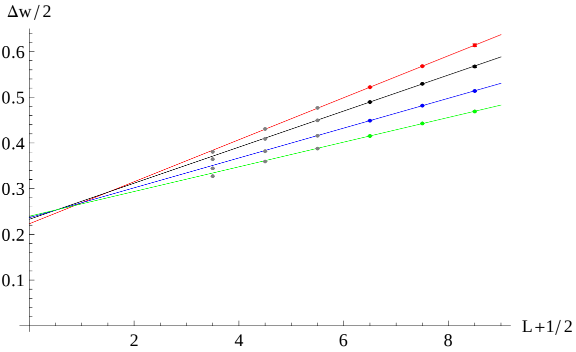
$$w_\infty(L) + \frac{1}{4}\log L^2 = c_1 + c_2 L + \sigma L^2 + \mathcal{O}\left(\frac{1}{\sigma L^2}\right).$$

- The log term comes from the effective string description (including it gives good fits while excluding it gives bad fits).
  We have also fitted its coefficient using $L \times L$, $L \times L+1$, and $L \times 2L$ loops and obtained agreement with the predicted value of $1/4$.
- Neglecting corrections of order $\frac{1}{\sigma L^3}$, we fit

$$\frac{1}{2}\left(w_\infty(L+1) - w_\infty(L) + \frac{1}{2}\log\left(1+\frac{1}{L}\right)\right) = \sigma\left(L+\frac{1}{2}\right) + \frac{c_2}{2} + \mathcal{O}\left(\frac{1}{\sigma L^3}\right)$$

  to a straight line as a function of $L + \frac{1}{2}$ to determine σ and the perimeter coefficient c_2.
- For the b and S values we use: 5×5 loops fall in the vicinity of the large-N transition (smaller loops have a gap in their eigenvalue distribution). For our fits we use square loops with $6 \le L \le 9$.

Lattice string tension



Plots of $\frac{\Delta w}{2} = \frac{1}{2}\left(w_\infty(L+1) - w_\infty(L) + \frac{1}{2}\log\left(1+\frac{1}{L}\right)\right)$ [obtained with method 1)] at $S = 0.4$ and $b = 0.36$, 0.362, 0.365, 0.368

Error bars are not visible in the plot. Straight lines show linear fits (using $6 < L + \frac{1}{2} < 9$).

Continuum limit

- As expected, σ does not depend on smearing parameter S (within statistical errors; which decrease with increasing S).
- Extrapolations to continuum carried out with scale (roughly equal to L_c where center symmetry breaks)

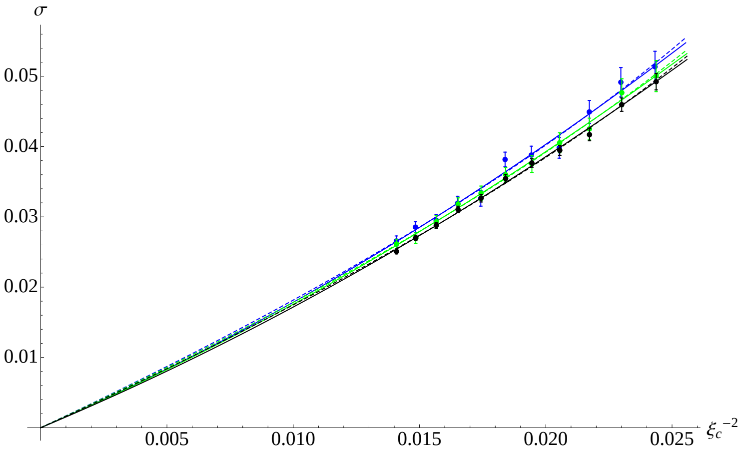
$$\xi_c(b) = 0.26 \left(\frac{\bar{\beta}_1}{\bar{\beta}_0^2} + \frac{b_I(b)}{\bar{\beta}_0} \right)^{-\frac{\bar{\beta}_1}{2\bar{\beta}_0^2}} \exp\left[\frac{b_I(b)}{2\bar{\beta}_0} \right] \exp\left[\frac{\bar{\beta}_2}{2\bar{\beta}_0^2 b_I(b)} \right]$$

  with tadpole improved coupling $b_I(b) = \lim_{N,V\to\infty} b\, W_N(L=1, b, S=0, V)$ and coefficients $\bar{\beta}_i = \beta_i / N^{i+1}$ for large N.
- Use two different two-parameter fits of the relation between $\sigma(b)$ and $\xi_c(b)$

$$\sigma(b) = \frac{d_0}{\xi_c(b)^2} + \frac{d_1}{\xi_c(b)^4}, \qquad \frac{1}{\xi_c(b)^2} = f_0^{-1}\sigma(b) + f_1 \sigma(b)^2$$

- Infinite-N continuum string tension: $\lim_{b\to\infty} \sigma(b)\xi_c^2(b) = 1.6(1)(3)$ (sys error: two fits d_0, f_0; ranges $0.359 \le b \le 0.369$ and $0.362 \le b \le 0.367$; different methods for $\lim_{N,V\to\infty}$).

Continuum limit



String tension from $\lim_{N,V\to\infty} w_N(L,b,S,V)$ obtained with:
method 1) [large V's], method 2a) [$V = 12^4$] method 2b) [$V = 14^4$].
Solid and dashed lines: different fit functions ($0.359 \le b \le 0.369$)

Continuum string tension: Wilson and Polyakov loops

- In terms of $\Lambda_{\overline{MS}}$, our result is $\sigma/\Lambda^2_{\overline{MS}} = 3.4(2)(6)$.
- From Polyakov loop correlators, the $N = \infty$ continuum result is [Allton, Teper, Trivini (2008)]: $\sigma/\Lambda^2_{\overline{MS}} = 3.95(3)(64)$.
- Independent study [Gonzalez-Arroyo, Okawa (2012) → next talk] of rectangular Wilson loops: $\sigma/\Lambda^2_{\overline{MS}} = 3.63(3)$ (with the same cont-extr. method).
- Disagreement at statistical level; systematic errors too large to claim evidence for a difference.

Outline

Shape-dependence: non-square loops

- Study the shape dependence of the size-independent term (c_1) in $w_N = -\log W_N$.
- Scaling-invariant shape parameter for rectangular $L_1 \times L_2$ loops:

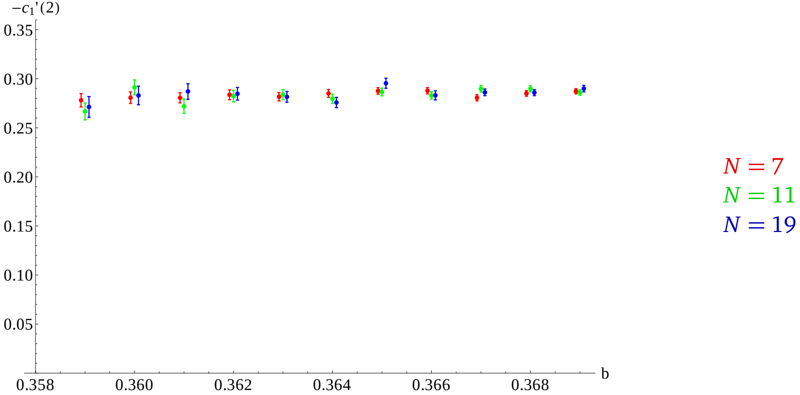
$$\zeta = \frac{L_1}{L_2} + \frac{L_2}{L_1}.$$

- At fixed b, S, V, and fixed finite N, we expect

$$w_N(L_1, L_2) + \frac{1}{4}\log L_1 L_2 = c_{1,N}(\zeta) + c_{2,N}\frac{L_1 + L_2}{2} + \sigma_N L_1 L_2 + \mathcal{O}\left(\frac{1}{\sigma_N L_1 L_2}\right).$$

- Using σ_N, $c_{2,N}$ from square loops, we determine $c_{1,N}\left(\zeta = \frac{5}{2}\right)$ from $L \times 2L$ loops, then compare it with $c_{1,N}(2)$
- From square and almost square $L \times L \pm 1$ loops, we obtain $c'_{1,N}(2)$.
- Allowing the coefficient of the $\log L_1 L_2$ term to become a fit parameter and expanding $c_1(\zeta)$ around $\zeta = 2$, we simultaneously fit $L \times L$, $L \times L + 1$, and $L \times 2L$ loops: confirmation of the expected value of $1/4$ and previous results for ζ-dependence of c_1.

Example for $-c'_{1,N}(2)$ as a function of b at $S = 0.4$:



- No significant dependence on b, N, S
- The effective string prediction is $c'_1(2) \approx -0.162276$.
- Similar deviation from effective string theory observed by Gonzalez-Arroyo and Okawa [→ next talk]

Shape dependence in PT

For a rectangular loop in tree-level continuum perturbation theory:

$$w_N^{\mathrm{PT}}(l_1,l_2,s) = \frac{g^2 C_2}{2}\left[\frac{1}{(2\pi)^{\frac{3}{2}}}\left(\frac{l_1+l_2}{\sqrt{s}}\right) + \frac{1}{\pi^2}\log\left(\frac{s}{l_1 l_2}\right) + h_0\left(\frac{l_2}{l_1}\right) + \mathcal{O}\left(\frac{s}{l_i^2}\right)\right]$$

(h_0 has an integral representation in terms of error functions).

- Terms divergent as $s \to 0$ (outside the reach of eff. string theory) enter additively in $w_N = -\log W_N$.
- Discrepancy with effective-string prediction for shape-dependent term might be explained if we assume that measured w_N is simply given by a sum of separate effective-string and tree-level PT contributions (would require $\frac{g^2 N}{4\pi} \approx 0.49$; about half the value obtained by matching s-dep. terms).
- Open question: Could even asymptotically large Wilson loops show elements of shape dependence determined by field theory at short distances?
- For arbitrary angles between the tangents at the kinks: Does effective-string framework allow inclusion of specific kink terms which could reproduce the perturbative angle dependence?

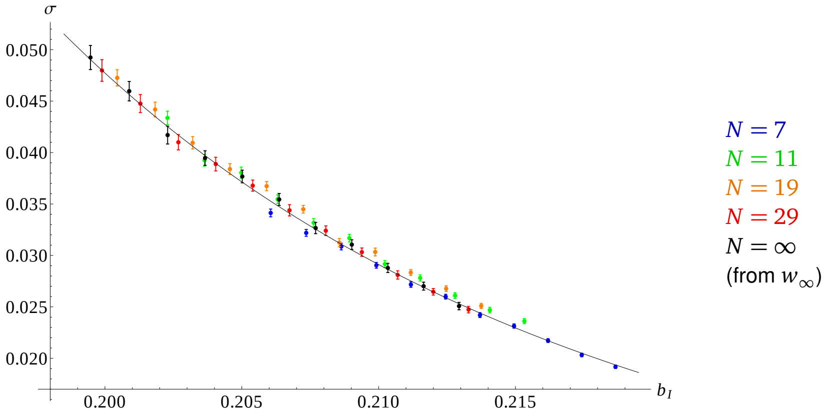
Outline

Conclusions

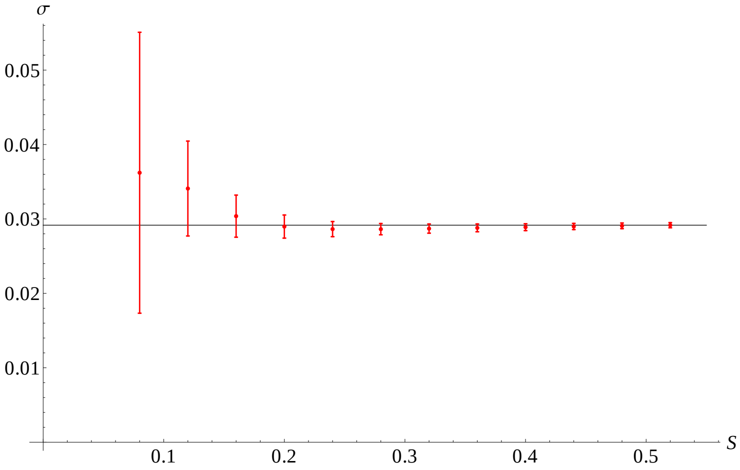
- Within estimates for systematic errors, results for large-N string tension from Wilson loops agree with those obtained from Polyakov loop correlators.
- Stringy parametrization for Wilson loops holds relatively well all the way down to the large-N transition point.
- Situation is less clear, even for large loops, at the level of purely shape-dependent parameters.
- Recently [Billo et al., 2012], effective string theory predictions have been confirmed numerically for 3d $\mathbb{Z}_2$ gauge theory.
- Situation for smeared SU(N) Wilson loops (with kinks) might be different. Shape dependence of planar Wilson loops presents interesting case for testing the limitations of the effective string approach. Further numerical checks are required (loops with different corner angles, self-intersections,...).

A1. String tension at finite N

- We determine the string tension σ_N at $N = 7, 11, 19, 29$ to get a feel for the commutativity of $N \to \infty$ and $b \to \infty$.
- At fixed b: $\sigma_N(b) = \sigma_\infty(b) + \frac{h(b)}{N^2}$ (with $h \approx 10\sigma_\infty$).
- Finite-N corrections are absorbed by the improved coupling $b_I(b, N)$; no significant N-dependence for the continuum limits

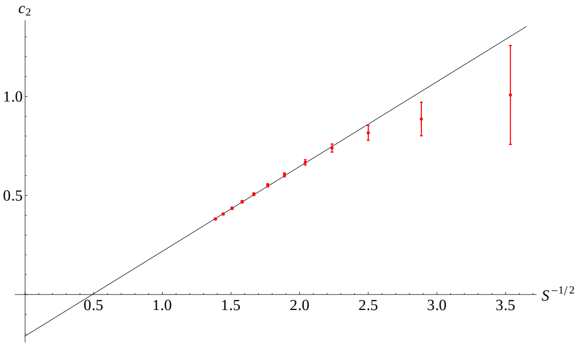


A2. Smearing dependence: string tension



Example for $N = 11$, $b = 0.365$, $V = 18^4$.
σ determined using square loops with $6 \leq L \leq 9$.

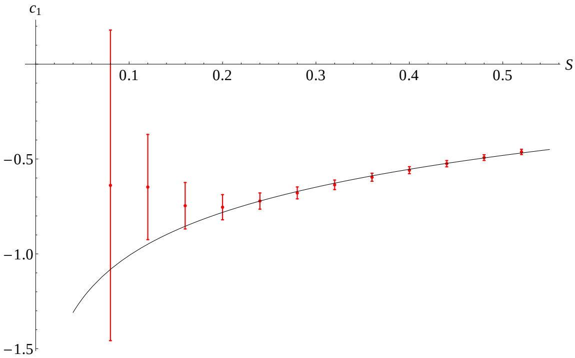
A3. Smearing dependence: perimeter term



Perimeter coeff. c_2 (for $N = 11$, $b = 0.365$). Fit: $c_2 = -0.2097 + 0.4279/\sqrt{S}$.

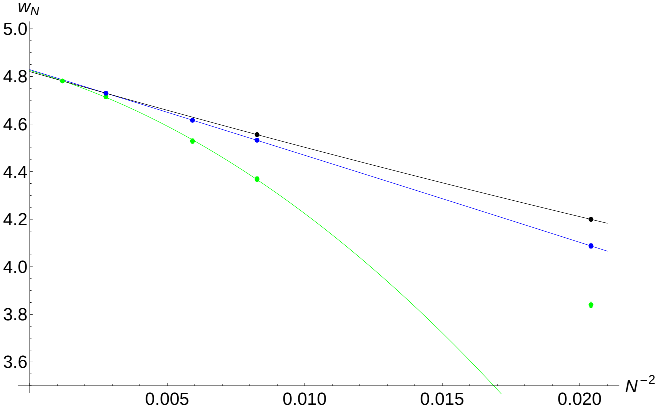
- No divergence as $S \to 0$ on the lattice; window where we see the behavior that would cause a divergence in the continuum.
- Tree-level PT perimeter term: $\frac{g^2 C_F}{2} \frac{1}{(2\pi)^{\frac{3}{2}}} \frac{2l}{\sqrt{s}}$ (above example: $\to \frac{g^2 N}{4\pi} \approx 1.08$)

A4. Smearing dependence: constant term



- S-dependence of the L-independent term c_1 is consistent with a $\log(S)$, $S \to 0$, divergence (example for $N = 11$, $b = 0.365$: $c_1 = -0.2538 + 0.3278 \log S$).
- Corner div. at tree-level: $\frac{g^2 C_F}{2} \frac{1}{\pi^2} \log s$ (example: $\to \frac{g^2 N}{4\pi} \approx 1.03$).

A5. Square loops: infinite-N,V limits



Plots of $w_N(L = 9, b = 0.368, S = 0.4, V)$ as a function of $1/N^2$:

$V = 12^4$, $V = 14^4$, $V = 24^4$ (at $N = 7$), $V = 18^4$ (at $N = 11$).