Volume dependence in 2+1 Yang-Mills theory

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OBJECTIVE

* Investigate the volume and large N dependence of Yang-Mills theories.

* Tool - use the volume dependence to control non-perturbative effects

SET UP

Toy model - SU(N) pure gauge Yang-Mills in 2+1d

2-d torus x R of Size - L

With a magnetic flux - m

Dimensionful 't Hooft coupling - $\lambda = g^2 N$

We will see that the non-perturbative effects are controlled by the magnetic and electric fluxes and the dimensionless combination $\lambda L N$

A few basics

Introducing magnetic flux through twisted boundary conditions

$$A_i(x + L_j e_j) = \Gamma_j A_i(x) \Gamma_j^\dagger$$

$$\Gamma_1 \Gamma_2 = e^{\frac{2\pi i m}{N}} \Gamma_2 \Gamma_1$$

m integer mod N - magnetic flux

m and N co-prime

Boundary conditions imply momentum quantized in units of NL like on a box of size $(NL)^2$

$$\vec{p} = \frac{2\pi}{NL} \vec{n}, \quad n_i \in \mathbb{Z}, \quad \vec{n} \neq \vec{0} \ (\mathrm{mod}\ N)$$

One-gluon states characterized by the electric flux related to the gluon momentum through

$$e_i = -\bar{k}\epsilon_{ij}n_j, \quad \text{with} \quad m\bar{k} = 1 \quad (\text{mod} N)$$

Generated by Polyakov loop operators (winding)

N^2 electric flux sectors

Spectrum from Polyakov loop correlators

Tree level PT Confinement

$$\left(\frac{2\pi|\vec{n}|}{NL}\right)^2 \quad \Rightarrow \quad \sigma_{\vec{e}}\, l_{\vec{e}}$$

Discuss energy spectrum vs NL from PT to confinement

Perturbation theory

One-gluon states with electric flux $e_i = -\bar{k}\epsilon_{ij}n_j$

$$\frac{E^2}{\lambda^2} = \left(\frac{2\pi|\vec{n}|}{NL\lambda}\right)^2 - \frac{4\pi}{NL\lambda}G\left(\frac{\vec{e}}{N}\right)$$

Note $NL\lambda$ dependence

For fixed $\vec{n}$ and $\frac{\vec{e}}{N}$

$$N \leftrightarrow L$$

Gluon self-energy on-shell projected $\perp \vec{n}$

$$G\Big(\frac{\vec{e}}{N}\Big) = \frac{1}{16\pi^2}\int_0^\infty \frac{dx}{\sqrt{x}}\Big(\theta_3^2(0,x) - \prod_{i=1}^{2}\theta_3(\frac{e_i}{N}, ix) - \frac{1}{x}\Big)$$

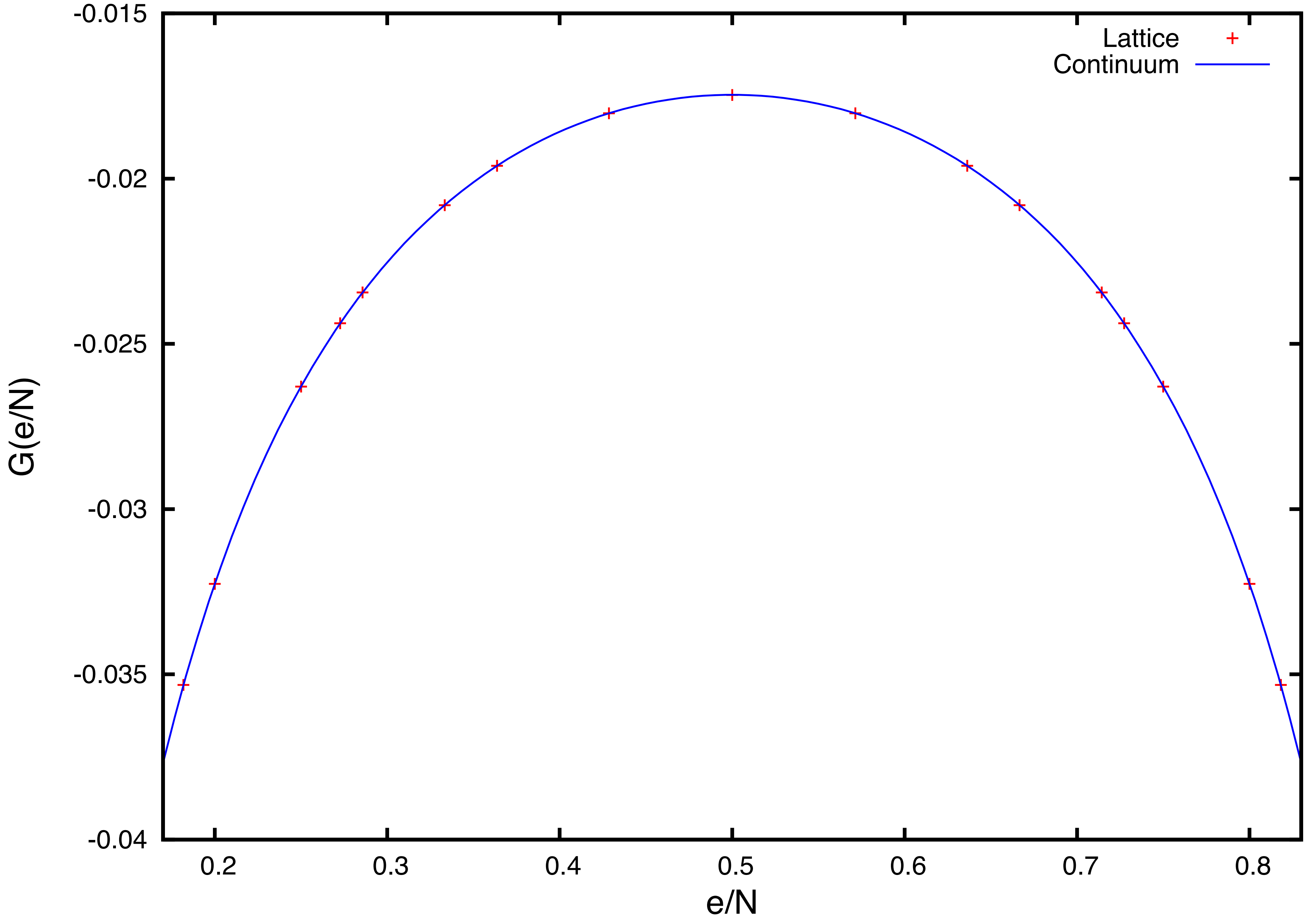
Jacobi Theta function

$$\theta_3(z, ix) = \sum_{n\in\mathbf{Z}} \exp\{-x\pi n^2 + 2\pi i n z\}$$

$$G\left(\frac{\vec{e}}{N}\right)$$

Lattice

Continuum



What about long-range dynamics?

Nambu-Goto string

$$\frac{E^2}{\lambda^2} = \left(\frac{\sigma_{\vec{e}}}{N\lambda^2}\right)^2 (\lambda N l_{\vec{e}})^2 \boxed{- \frac{\pi \sigma_{\vec{e}}}{3\lambda^2}}$$

From Luscher term

If $\sigma_{\vec{e}} = \sigma_0$

and $l_{\vec{e}} = |\vec{e}| L$

The formula is compatible with reduction

Testing the *reduction* conjecture

Lattice simulations

Ongoing project

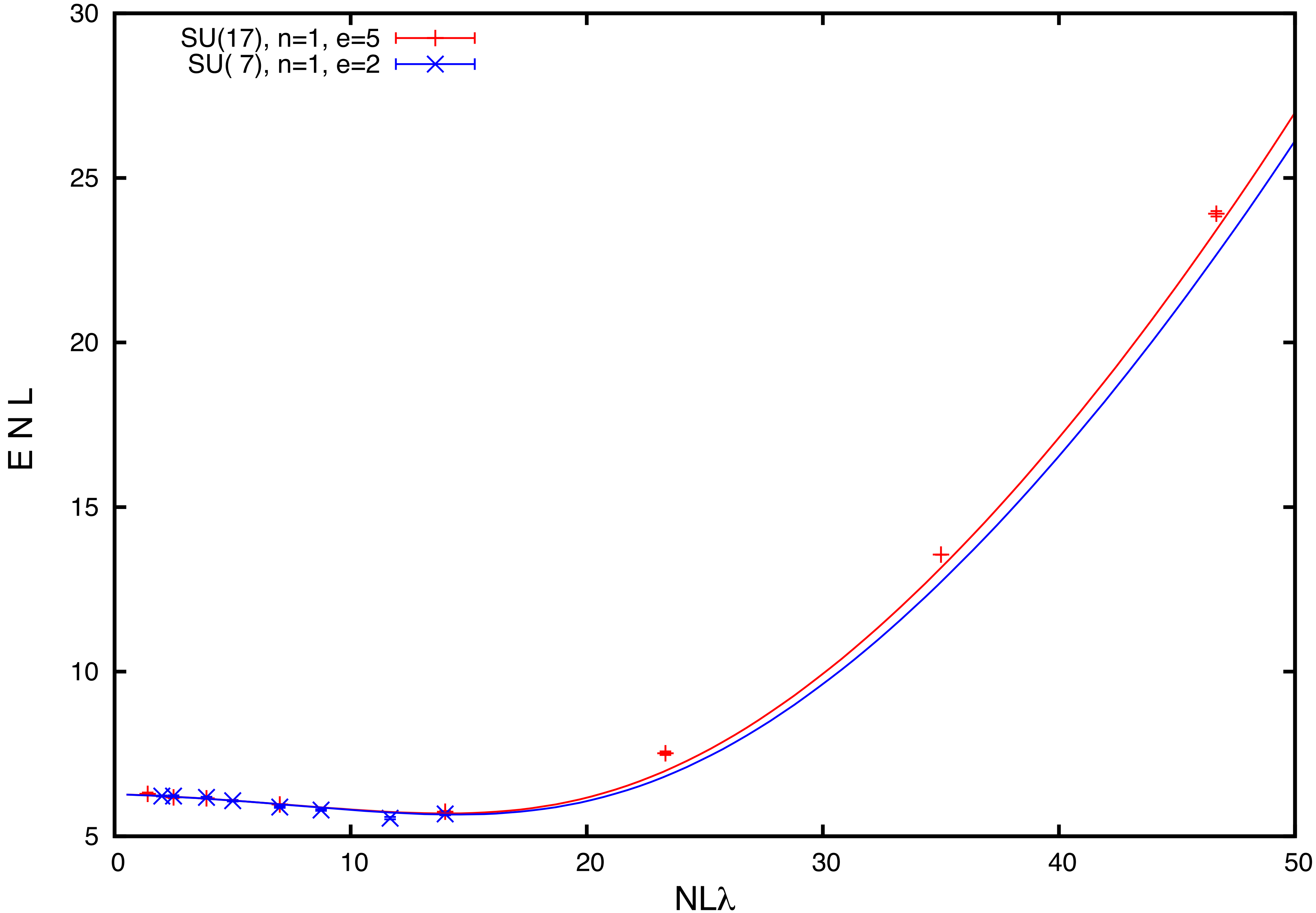
SU(5)	$14^2 \times 48$
	$22^2 \times 72$
SU(7)	$10^2 \times 32$
SU(17)	$4^2 \times 32$

Scanning several values of $\lambda N L$ and magnetic flux m

Dependence of ENL on λNL

SU(17) n=1 e=5

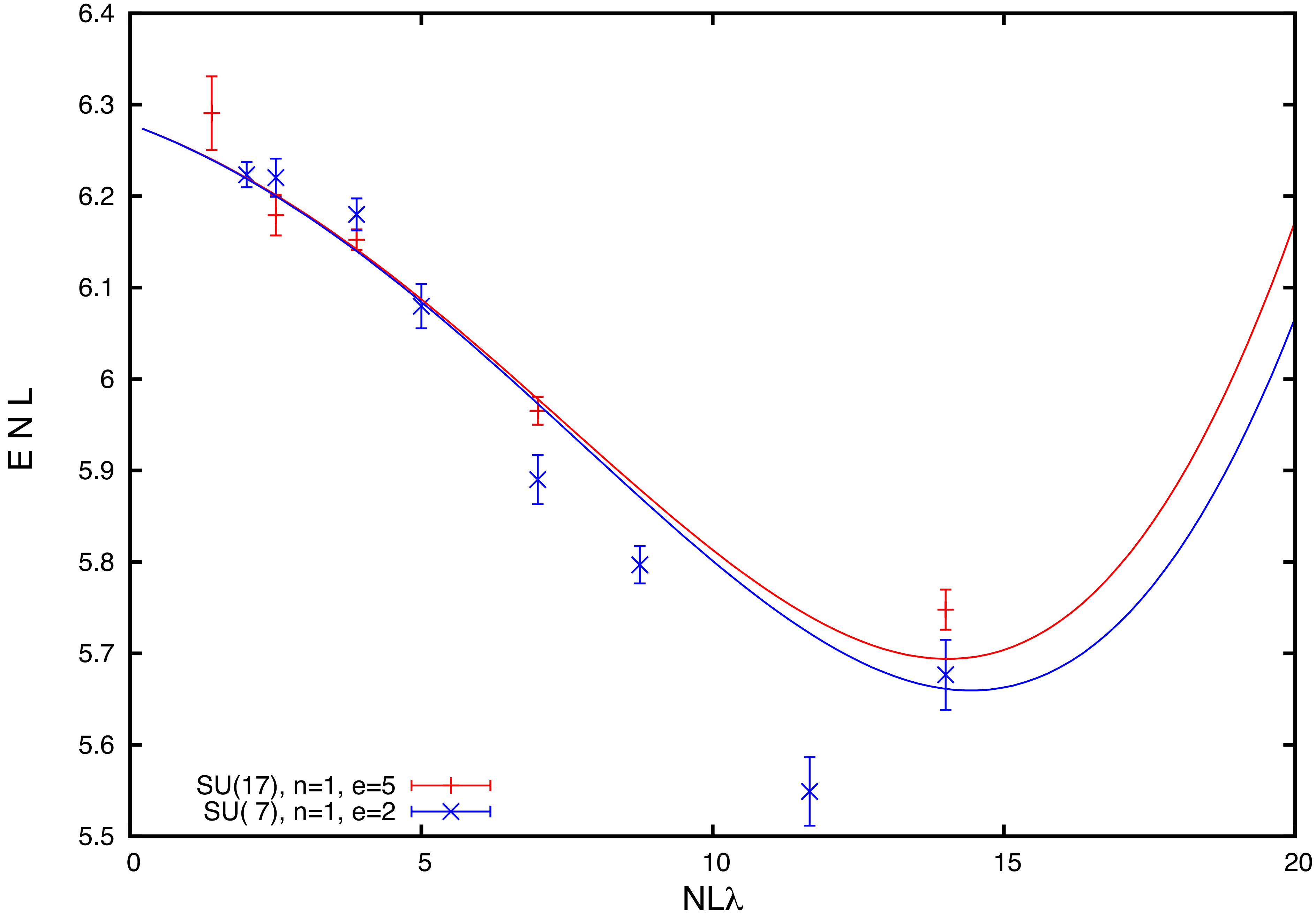
SU(7) n=1 e=2



Peturbative regime with Luscher term

SU(17) n=1 e=5

SU(7) n=1 e=2



Peturbative regime without Luscher term

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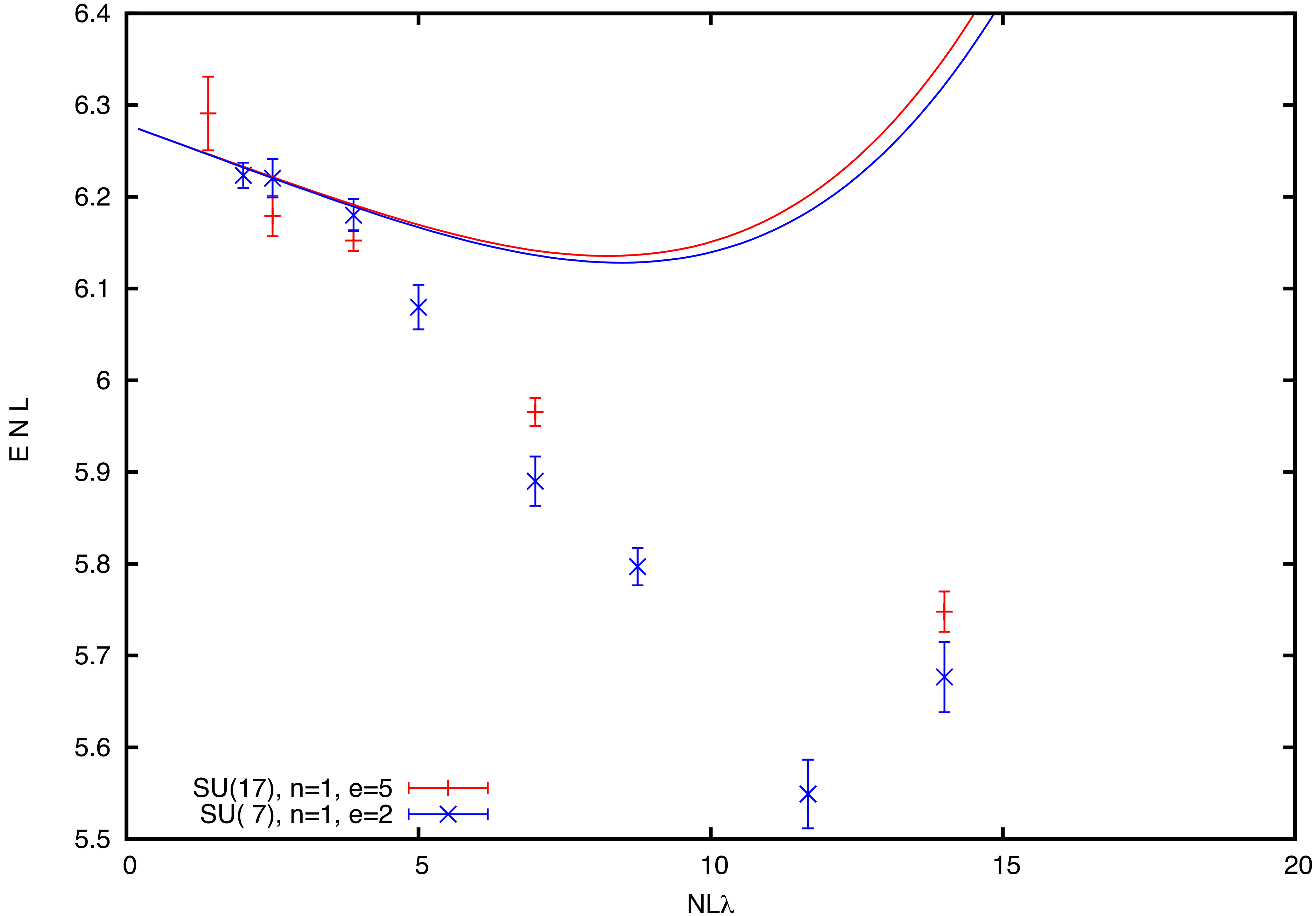
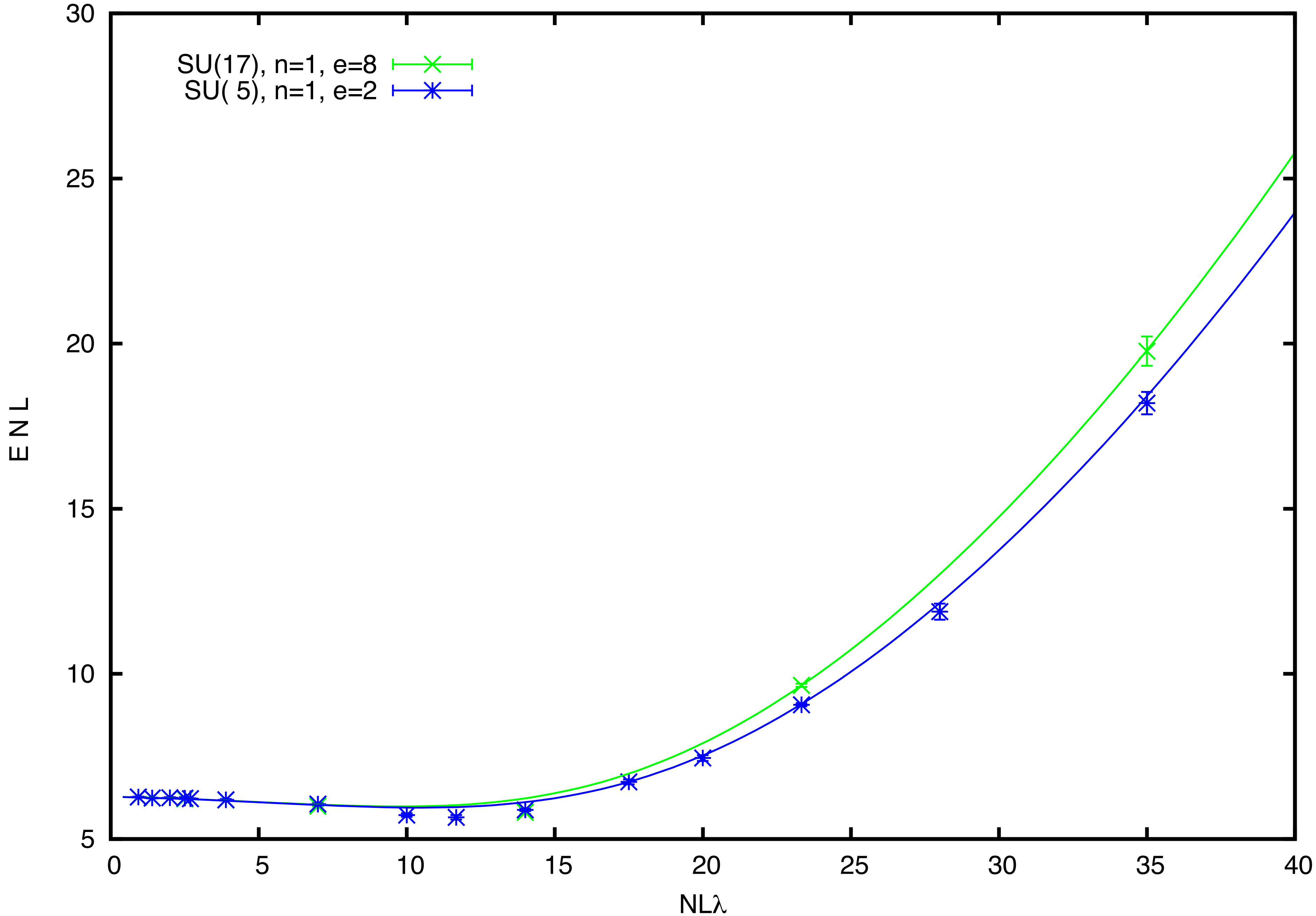


Dependence of ENL on λNL

SU(17) n=1 e=8

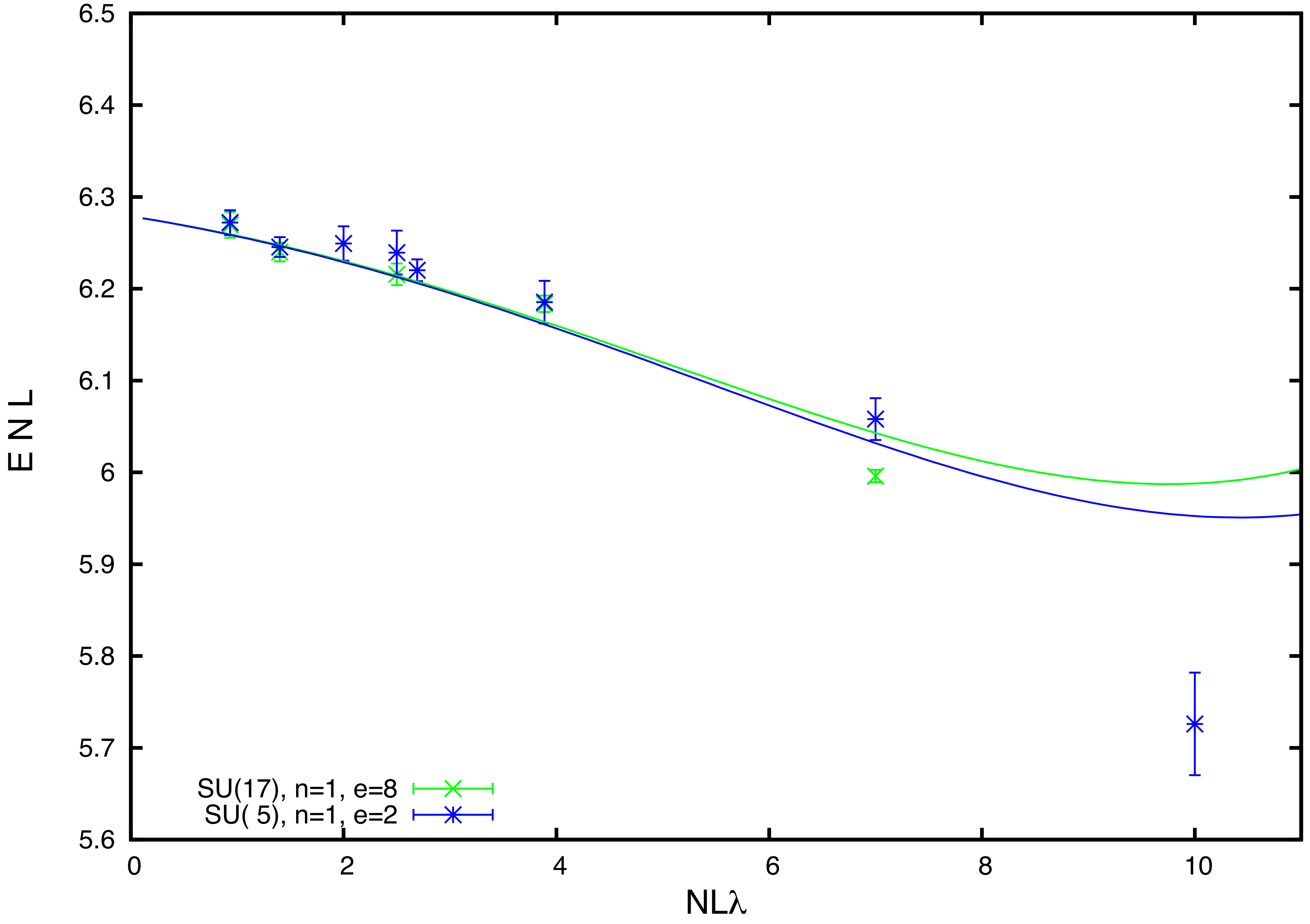
SU(5) n=1 e=3



Peturbative regime with Luscher term

SU(17) n=1 e=8

SU(5) n=1 e=3



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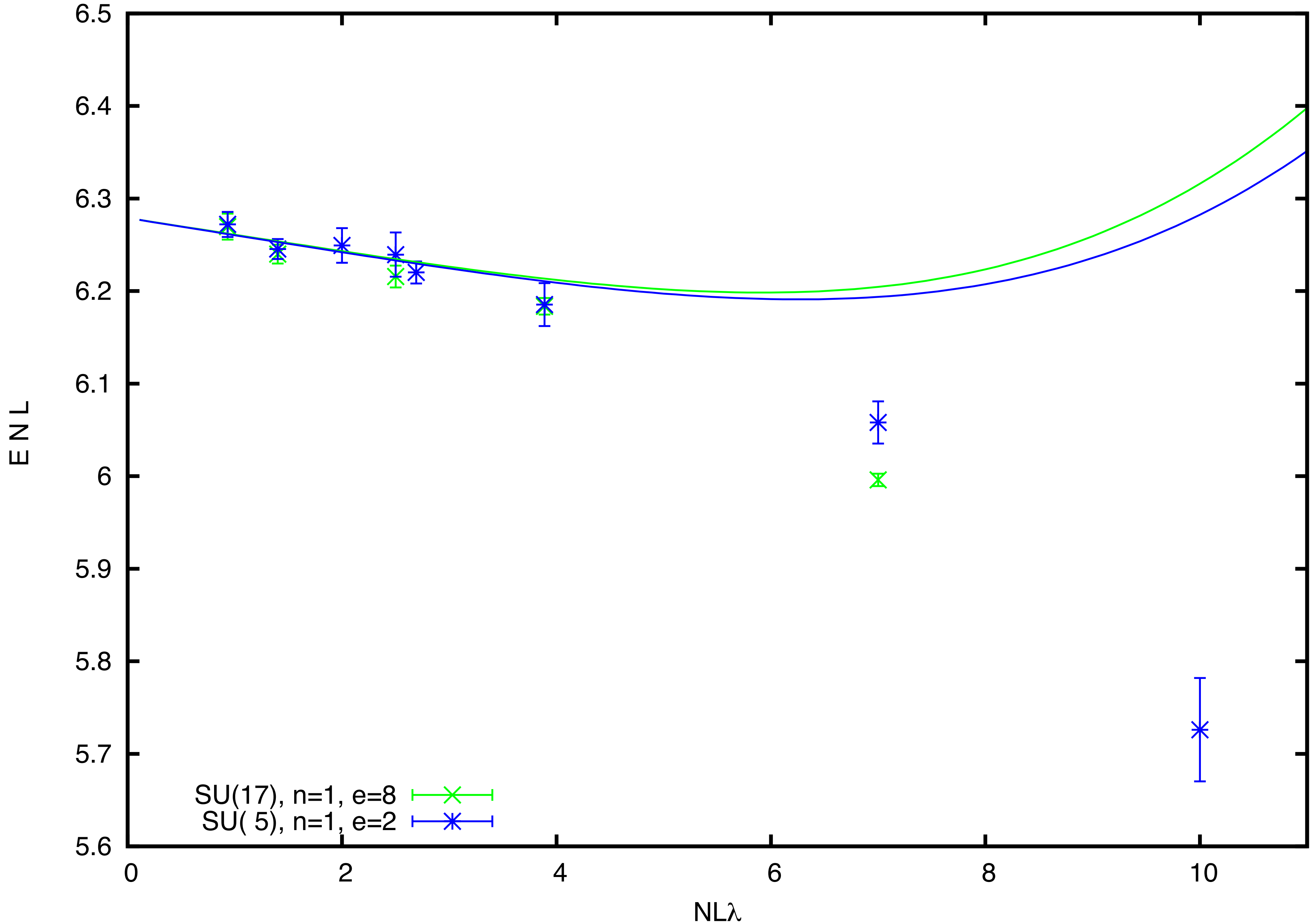

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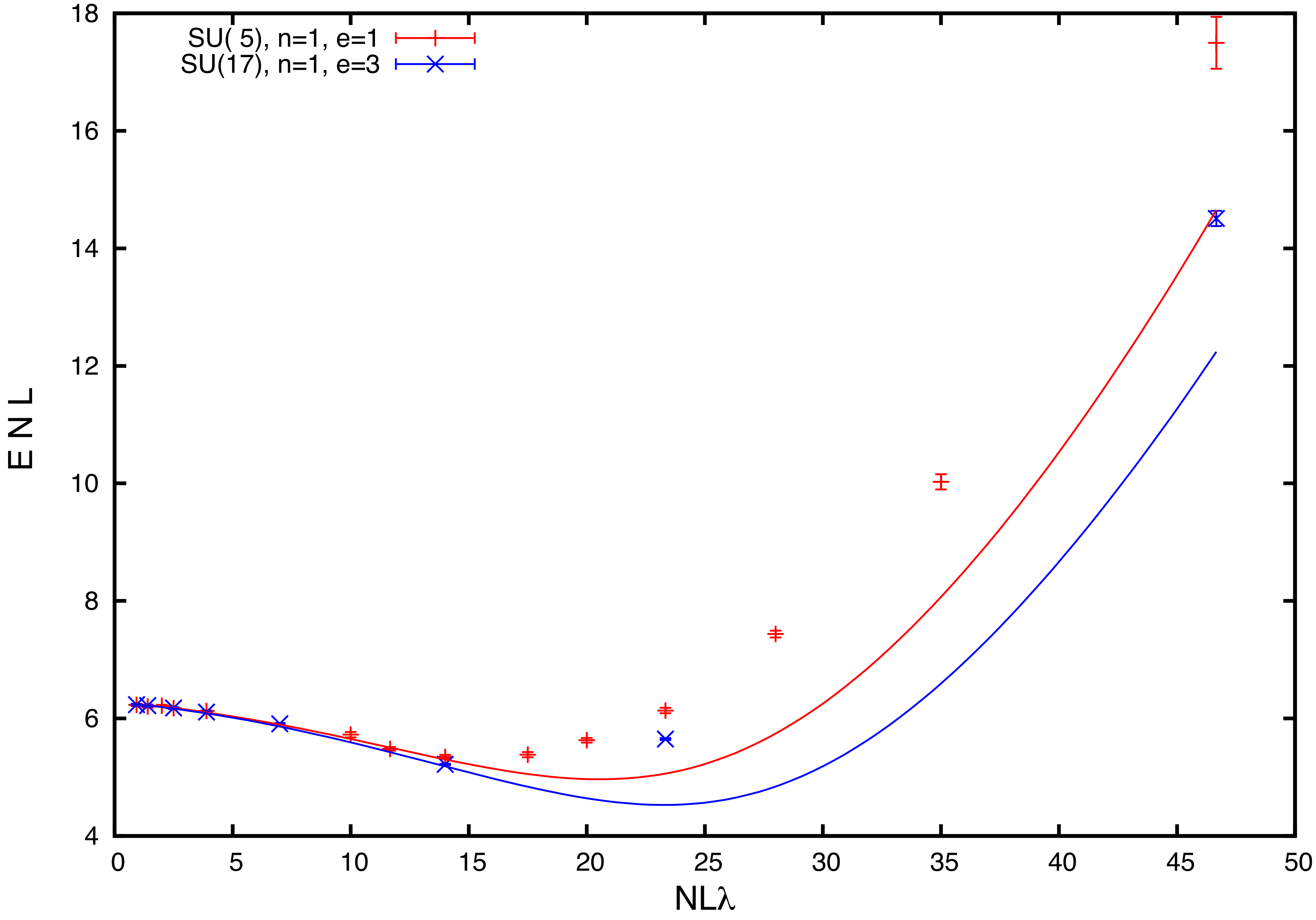

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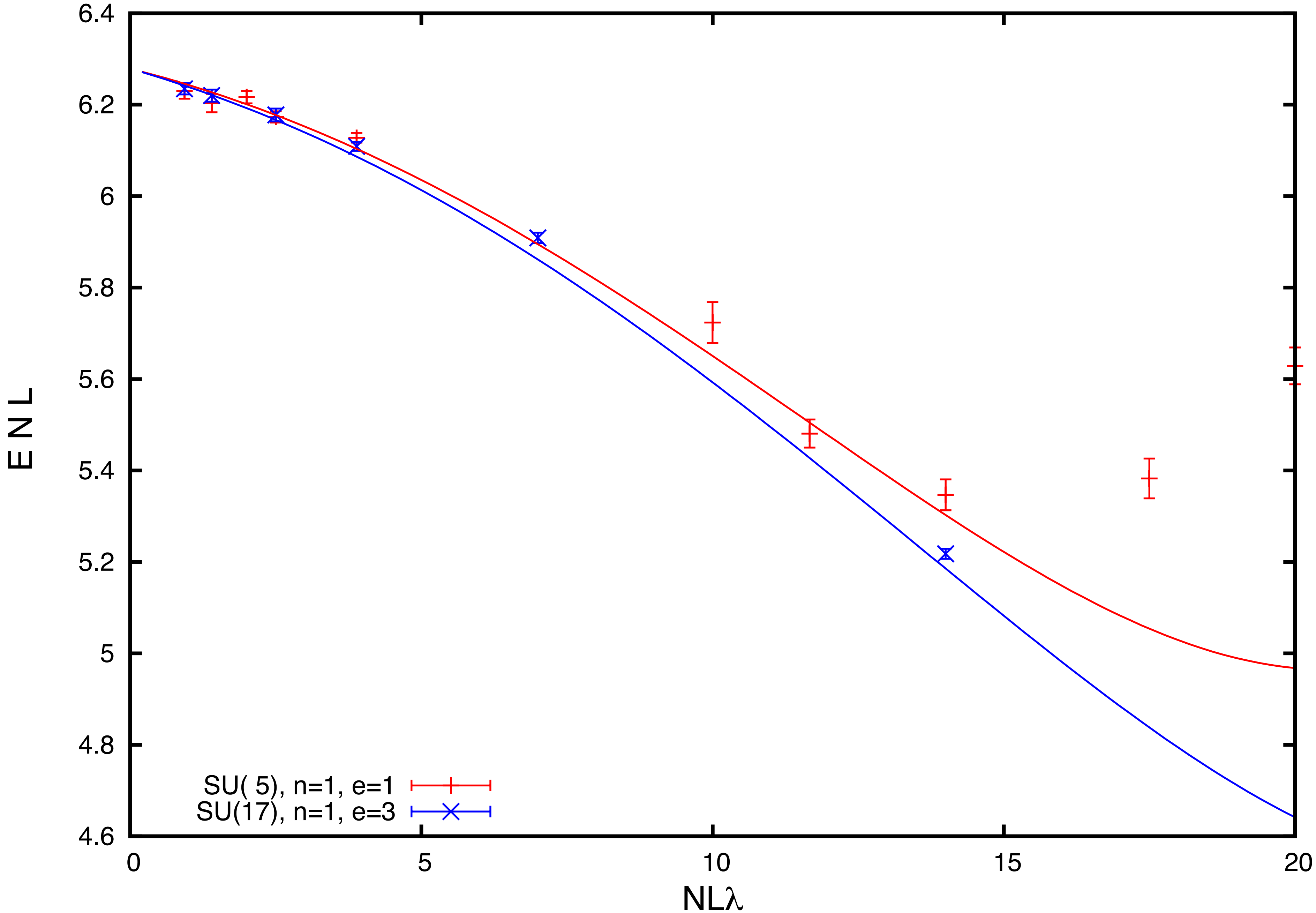

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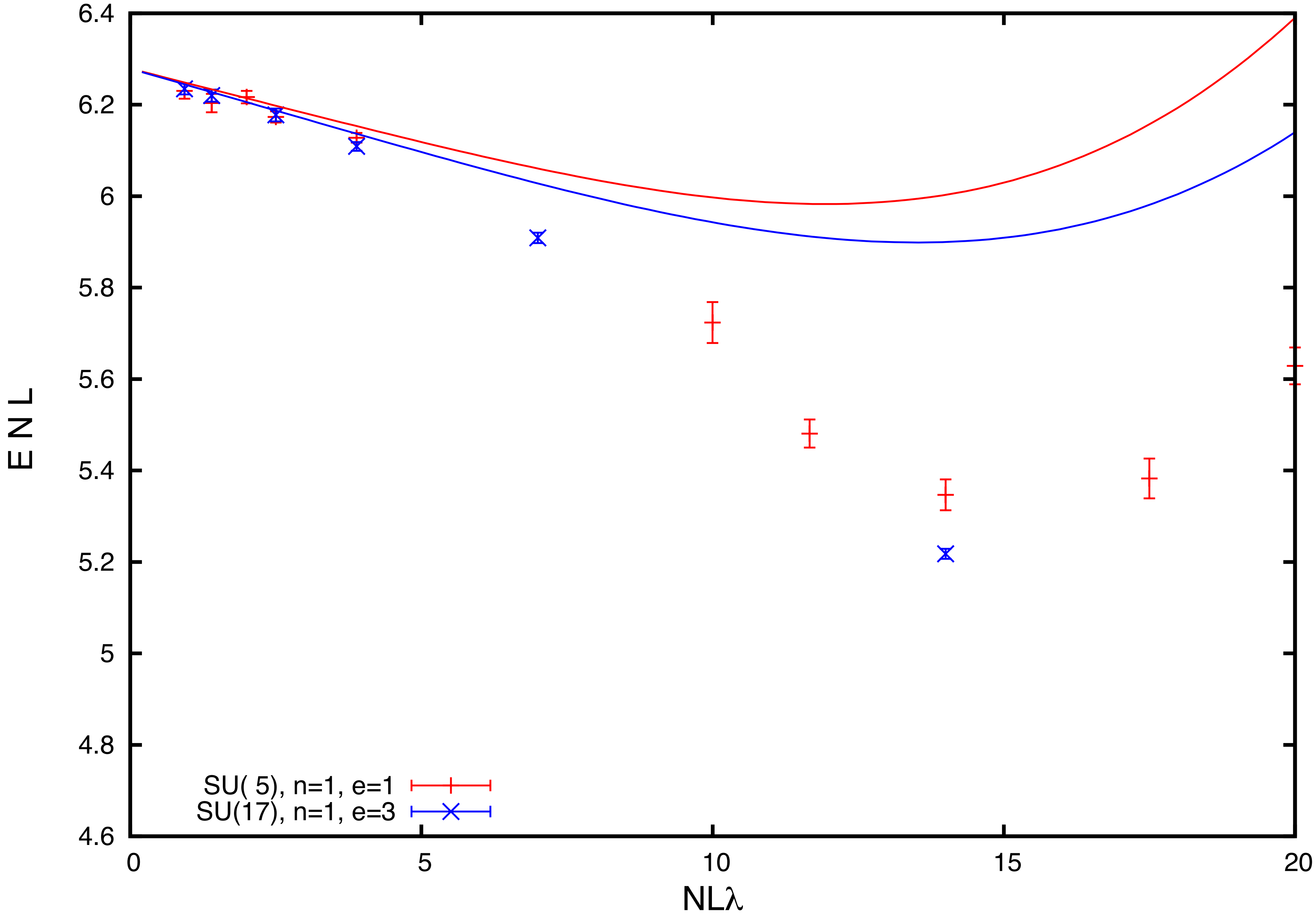

Combined formula

$$\frac{E^2}{\lambda^2} = \Big(\frac{|\vec{e}|\ \sigma}{N\lambda^2}\Big)^2 (\lambda N\ L)^2 - \frac{\pi\sigma}{3\lambda^2} - \frac{4\pi}{NL\lambda} G(\frac{\vec{e}}{N}) + \Big(\frac{2\pi|\vec{m}|}{NL\lambda}\Big)^2$$

Good qualitative description of the data with only one parameter

Taken here to $\sqrt{\sigma} = 0.19638\,\lambda$ (Teper et al.)

Take $N \to \infty$

Scale $m\ \&\ \bar{k}$ like N to avoid tachyonic energy

(see talk by A. González-Arroyo)

Summary

- We have analyzed the combined N and L dependence of electrix flux energies
- For fixed colour momentum and $\frac{\vec{e}}{N}$ the relevant parameter turns out to be $\lambda N L$
- Suggests *reduction* $N \leftrightarrow L$ which holds in PT and under certain assumptions in the confined regime
- Luscher term essential also in the small $\lambda N L$ regime
- Ongoing simulations towards more quantitative tests (continuum extrapolation)