The string tension at large N from smeared Wilson loops

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Large N gauge theory

- Large N gauge theories are very interesting: Simpler, close to phenom., connected to string picture, etc.
- Interesting simplifying properties: planar PT, factorization, stable resonances, etc.
- LGT provides first-principles approach to gauge theories.
- Combining both is numerically challenging ($\times N^2$ degrees of freedom).
- An early (80's) look at the lattice loop eqs. at large N led to an interesting observation

Reduction (Eguchi-Kawai)

Schwinger-Dyson equations for Wilson loops adopt a simple form on the lattice
Expectation values of Wilson loops become volume independent in the large N limit (Assuming Z_N^4 symmetry is unbroken) $\Rightarrow$
Eguchi-Kawai model: LGT in one point

Continuum reduction?

However Reduction breaks down at weak coupling for EK model because the classical vaccuum breaks the symmetry

Is breaking an unavoidable feature of finite volume continuum theory? $\Leftrightarrow$ No continuum reduction

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If NOT it would be possible to modify lattice model.

- **Quenched EK model (Bhanot, Heller, Neuberger)**
- **Twisted EK model (A.G-A, Okawa)**
- **Double trace deformations (Shifman-Unsal-Yaffe)**
- **Cut-off scale adjoint fermions (Kovtun-Unsal-Yaffe)**

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Reduction could still work for other large N theories

TEK Model

Use twisted boundary conditions (magnetic flux):

$$S = -bN \sum_{\mu,\nu} z_{\mu\nu}^{(N)} \mathrm{Tr}(U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger)$$

With appropriate choice of $z_{\mu\nu}^{(N)}$ symmetry subgroup unbroken at weak coupling; $N = L^2 \quad z_{\mu\nu}^{(N)} = \exp\{i2\pi k \epsilon_{\mu\nu}/L\}$
$Z_N^4 \longrightarrow Z_N^2 = Z_L^4 \Leftrightarrow$ **Reduction** $(N^2 = L^4)$

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  Explanation: Other minima dominate (First order transition)
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- Azeyanagi et al showed $\Rightarrow$ No continuum reduction
- AGA-Okawa 2010: No breaking if k/L $\longrightarrow$ const$\neq 0$.

Checking our prescription

- Complicated semiclassical analysis. Ongoing 2+1 dim. analysis a first step (M. Garcia Perez talk)
- Check of symmetry breaking up to $N = 37^2$
- **Check the physics (This study arXiV:1206.0049)**
  Extract String tension from SU(N) LGT ⇒ Take continuum limit ⇒ Extrapolate to large N ⇒ Compare with TEK

$N = 3$	$b \in [0.3278, 0.3611]$ (7 values)	260 confs. 32^4
$N = 5$	$b \in [0.3513, 0.3772]$ (5 values)	260 confs. 32^4
$N = 6$	$b \in [0.3541, 0.3792]$ (5 values)	260 confs. 32^4
$N = 8$	$b \in [0.3569, 0.3815]$ (5 values)	260 confs. 32^4
$N = 4$	$b \in [0.3464, 0.3725]$ (5 values)	260 confs. 32^4
$N = 29^2$	$b \in [0.36, 0.385]$ (5+1 values)	5400 confs TEK

Observables

- Must use the same method for LGT and TEK (single loops, symmetric lattice, . . .)

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- Use rectangular Wilson loops $W(R,T)$ for R≈T
- To eliminate constant + perimeter dependence: **Creutz ratios**

$$\chi(T,R) = -\log \frac{W(T+0.5,R+0.5)W(T-0.5,R-0.5)}{W(T+0.5,R-0.5)W(T-0.5,R+0.5)}$$

- One must use 4d APE Smearing for noise reduction We eliminate the effect of smearing on $\chi(T,R)$ by extrapolating back. Click

Results

- For string tension we use square loops $R = T \in [3.5, 8.5]$
- At each N and b the 6 values are well-fitted to a 3-param formula

$$\chi(R,R) = \kappa + \frac{2\gamma}{R^2} + \frac{\eta}{R^4}$$

Good Fits ▸ Click

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Continuum extrapolation

- Data display scaling with 1% accuracy for top 5 b values and intermediate size loops
- To fix the scale we use 2 perturbative prescriptions with improved coupling definitions (Edwards et al and Allton et al.)
- Non-perturbative prescription (similar to Sommer scale)

$$-RT\chi(R,T)|_{R=T=r_0 a(b)} = 1.65$$

Large N limit

- Using Allton et al $a(b)$ we made a linear fit in $1/N^2$ Click

$$\Lambda_{\overline{\mathrm{MS}}}/\sqrt{\sigma} = 0.525(2) \quad \text{for TEK} \quad 0.523(5)$$

- Agrees with N dependence by Allton (slope ≈ 0.3)
- Value is off by 4-5% (compatible within systematic errors) (See Lohmayer talk)
- Most of the N-dependence is in $r_0\Lambda_{\overline{\mathrm{MS}}}$ Click

Approach to large sizes

The parameter γ defining approach to large R is very interesting:
Our value $\gamma = 0.272(5)$

- Slope is correlated with s.t. value

$$\delta\sigma \approx -\frac{\delta\gamma}{20a^2}$$

- These parameters are connected with the string picture of the chromoelectric flux-tube (Luscher-Symanzik-Weisz)
- This can be generalized to rectangular loops. We have

$$\chi(R,T) = \kappa + \phi(z)(\frac{1}{R^2} + \frac{1}{T^2}) + \dots$$

with $z = R/T$ (aspect ratio).

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Our values do not coincide with Nambu-Goto open string parameters.

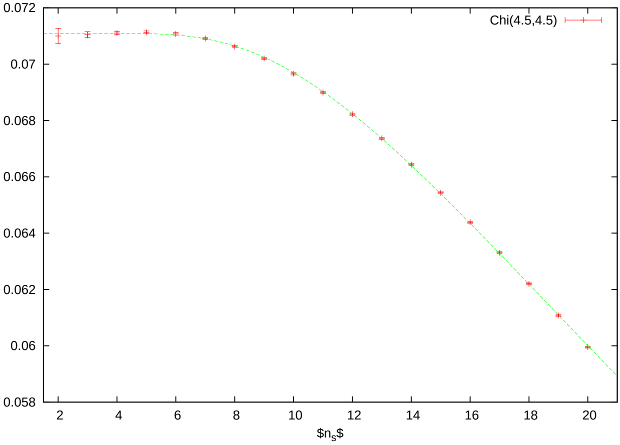
Aspect ratio dependence similar to l.o. perturbation theory ▸ PLOT

▸ FORMULAS

Conclusions

- TEK model works up to 1-2% precision in the physical range of params.
- We obtained a very precise determination of the large N string tension
- N-dependence is small and lies mostly in the scale
- String tension value is lower by 4-5% than Allton et al. (Systematics should be analysed further)
- Interesting results were obtained for the parameters describing approach to large sizes. Our results differ from the predictions of the Nambu-Goto string.

A more detailed account of our results including volume-dependence and additional data ($N = 4, 10$ and TEK at $N = 37^2$), is in progress.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0.07
0.068
0.066
0.064
0.062
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0.058
2
4
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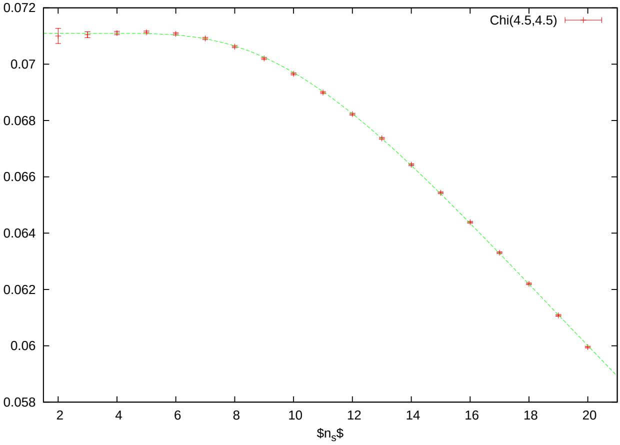
8
10
12
14
16
18
20
n_s



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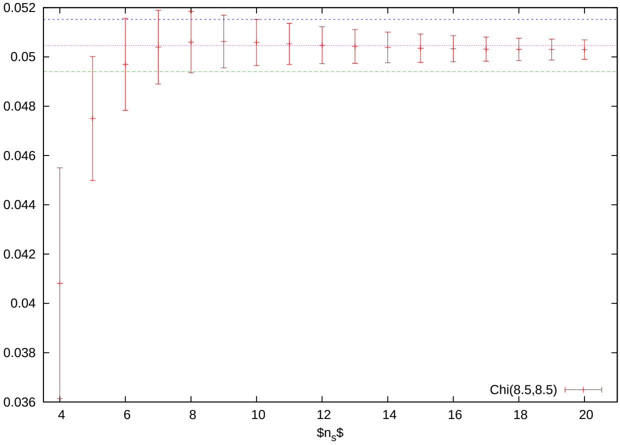

0.052
0.05
0.048
0.046
0.044
0.042
0.04
0.038
0.036
4
6
8
10
12
14
16
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20
n_s
Chi(8.5,8.5)

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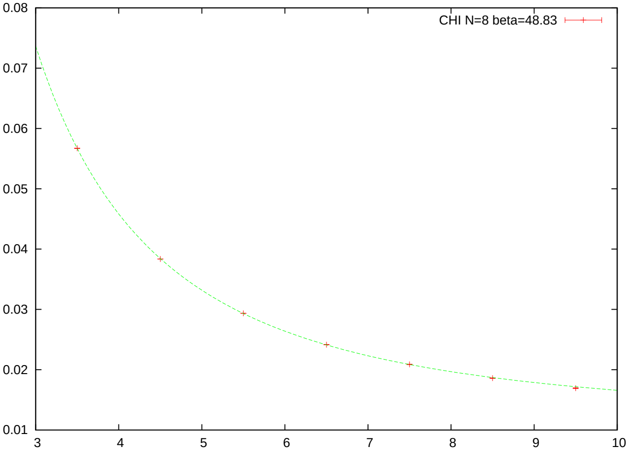

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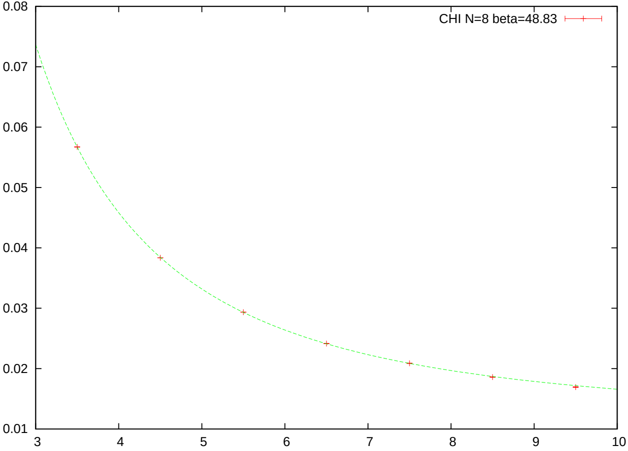

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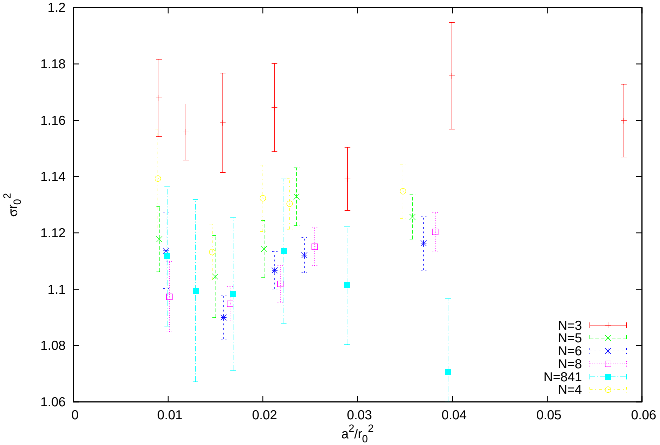

1.2
1.18
1.16
1.14
1.12
1.1
1.08
1.06
0
0.01
0.02
0.03
0.04
0.05
0.06
σr_0^2
a^2/r_0^2
N=3
N=5
N=6
N=8
N=841
N=4

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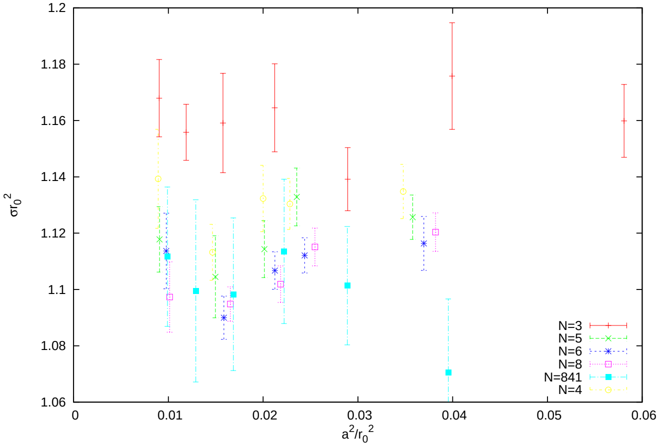

1.2
1.18
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0.01
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0.04
0.05
0.06
σr0 2
a2/r0 2
N=3
N=5
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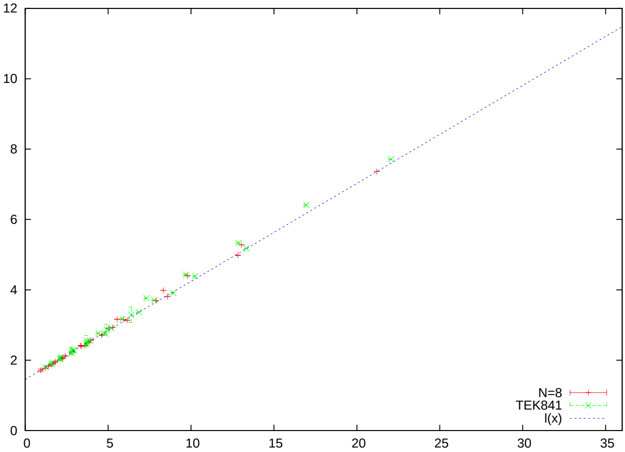

12
10
8
6
4
2
0
0
5
10
15
20
25
30
35
N=8
TEK841
l(x)

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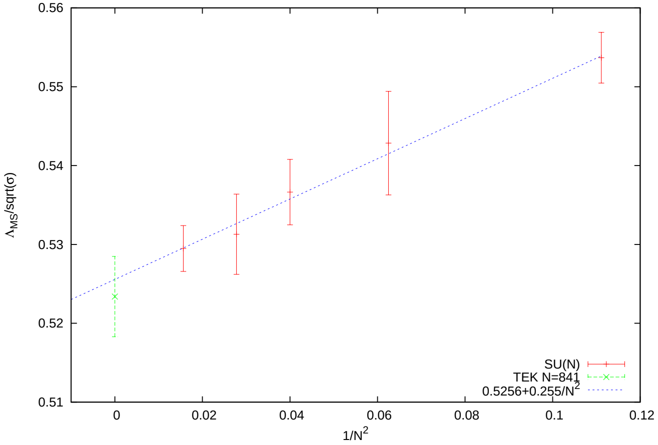

0.56
0.55
0.54
0.53
0.52
0.51
0
0.02
0.04
0.06
0.08
0.1
0.12
Λ_{MS}/sqrt(σ)
$1/N^2$
SU(N)
TEK N=841
$0.5256+0.255/N^2$

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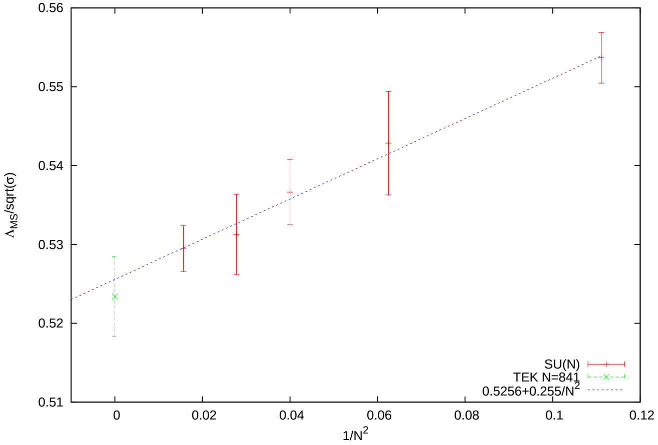

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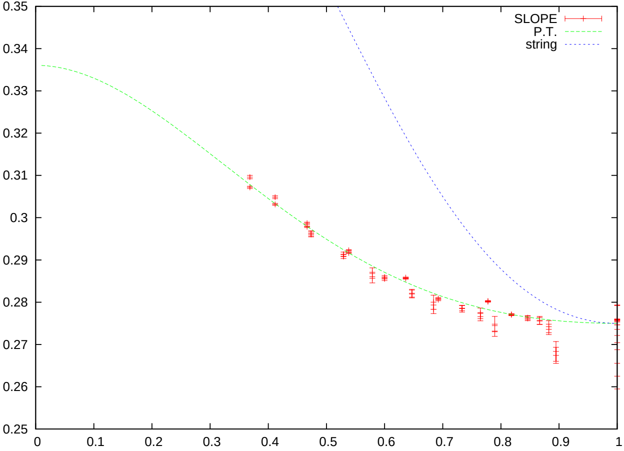
SLOPE
P.T.
string
0.35
0.34
0.33
0.32
0.31
0.3
0.29
0.28
0.27
0.26
0.25
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

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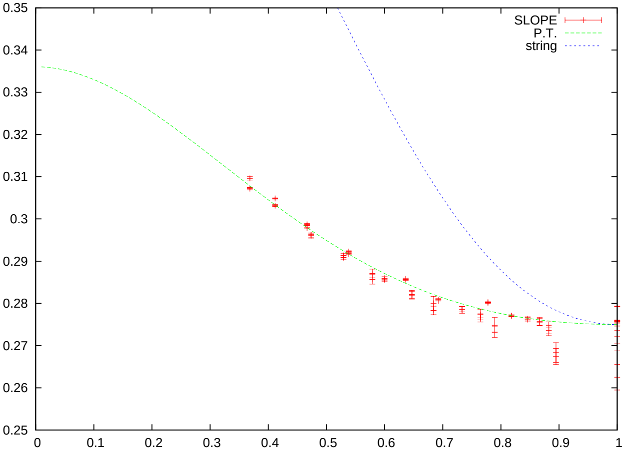

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0.6
0.7
0.8
0.9
1

BACK

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