Chiral Symmetry Restoration and Eigenvalue Density of Dirac Operator at Finite Temperature

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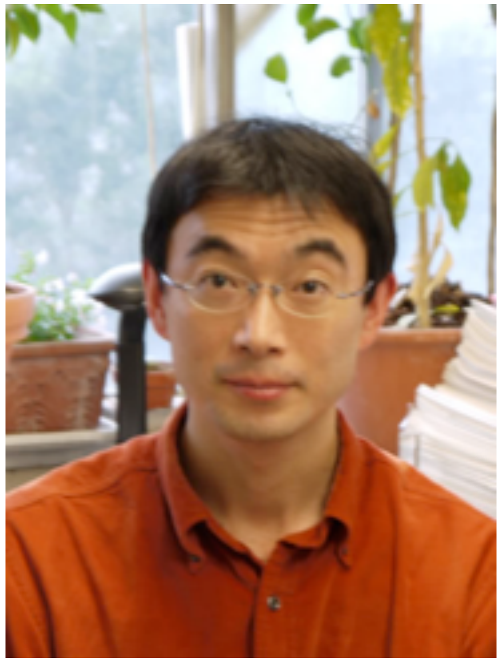


Lattice 2012, Cairns Convention Center,
Cairns, Australia, 2012.06.25

1. Introduction

Chiral symmetry of QCD

phase transition

low T $U(1)_B \otimes S(N_f)_V$ → high T $U(1)_B \otimes S(N_f)_L \otimes SU(N_f)_R$

restoration of chiral symmetry

Some questions

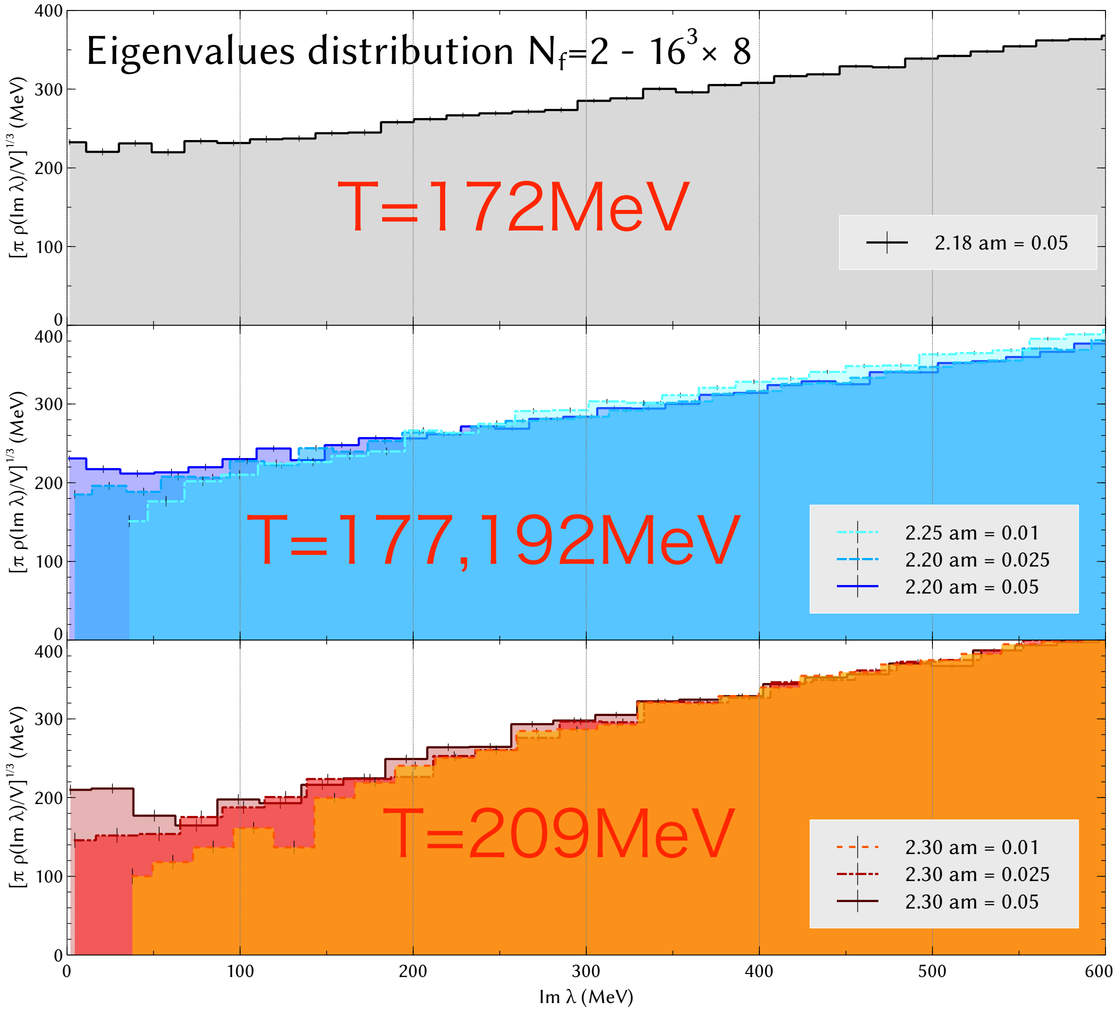
1. Eigenvalue distribution of Dirac operator

2. Recovery of U(1)_A symmetry at high T ?

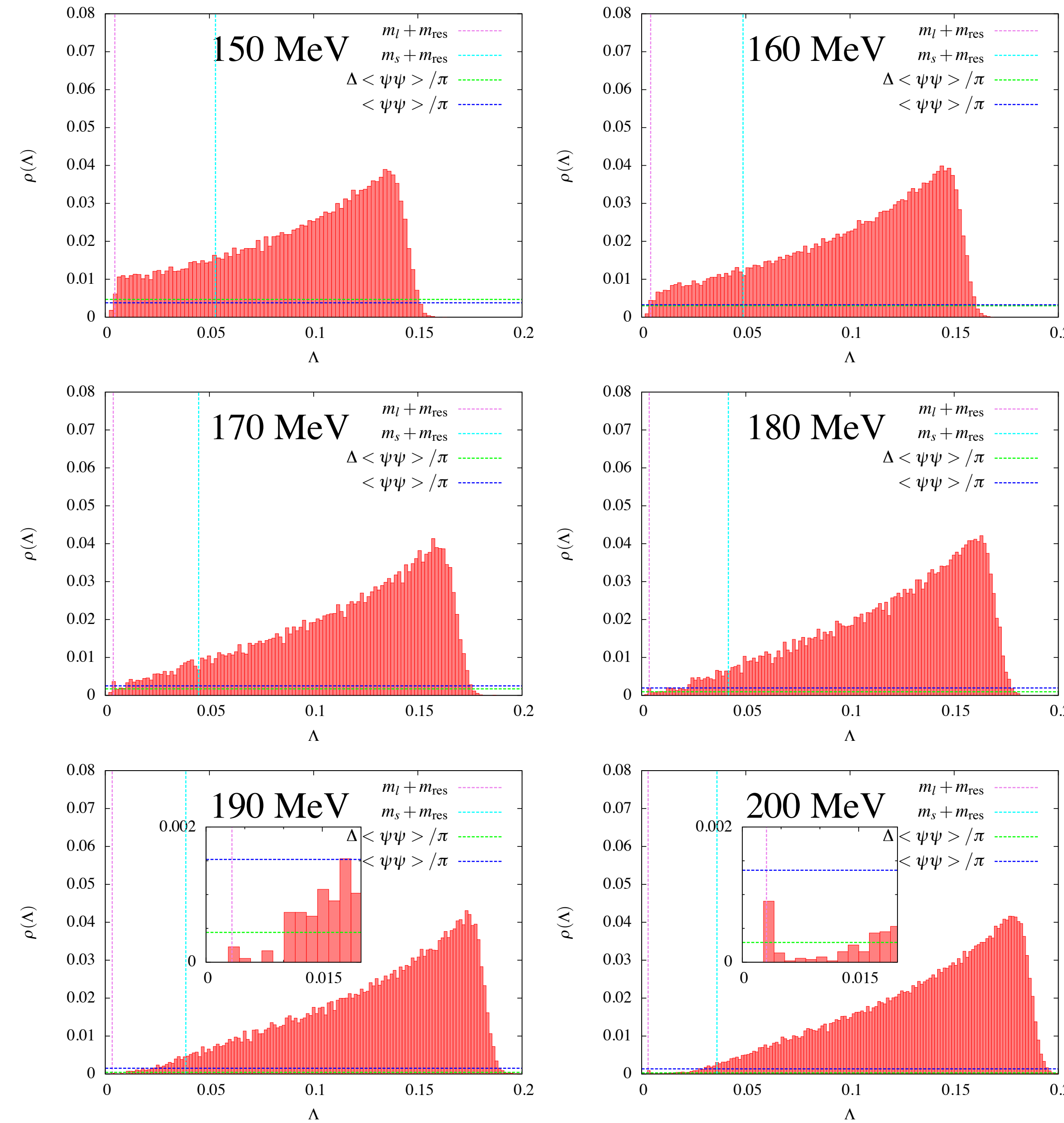
Previous studies on 1

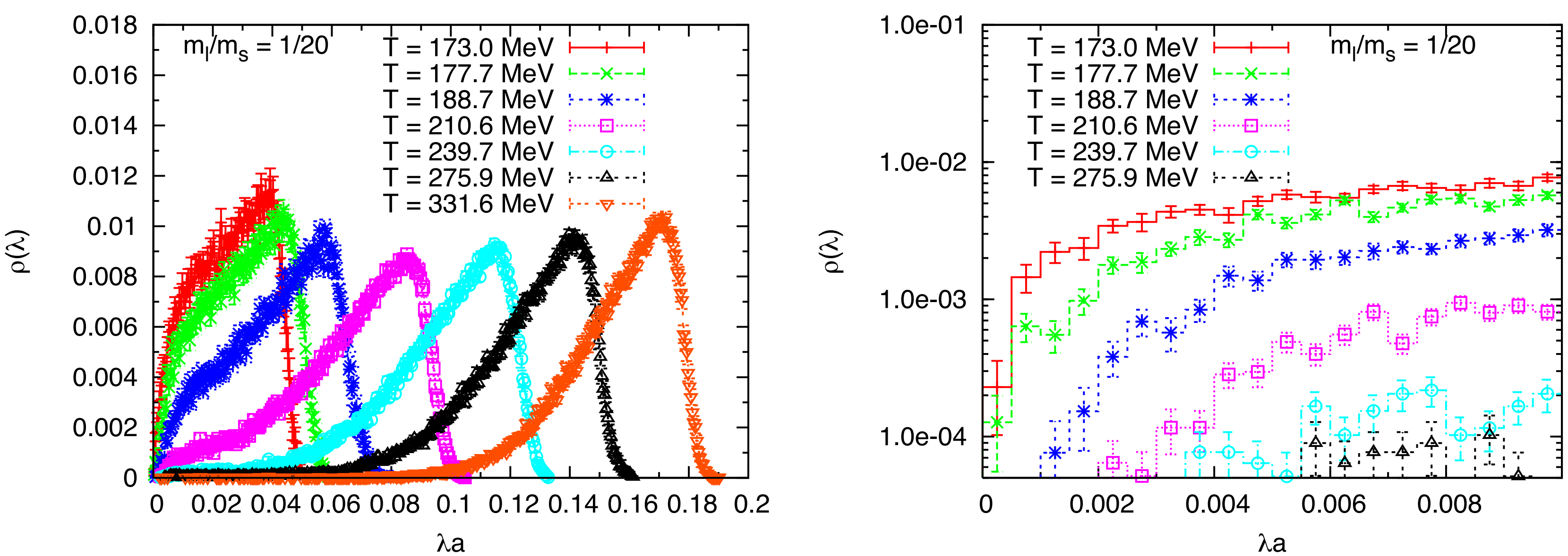
$$\rho(\lambda) = \lim_{V\to\infty} \frac{1}{V} \sum_n \delta(\lambda - \lambda_n)$$

Cossu *et al.* (JLQCD11), Overlap



Lin (HotQCD11), DW





Ohno *et al.* (11), HISQ

Is small λ suppressed ?

Previous studies on 2

Cohen(96), Theory Yes !

"Spectral" representation

$$\frac{1}{V}\int d^4x\langle\bar\psi(x)\psi(x)\rangle = -\frac{1}{V}\int D[G]\sum_j\left[\frac{1}{i\lambda_j - m}+\frac{1}{-i\lambda_j - m}\right]P_m(G) = \int D[G]P_m(G)\int d\lambda\rho_G(\lambda)\frac{2m}{\lambda^2+m^2}$$

eigenvalue of $\gamma^\mu D_\mu$ density of state $\rho_G(\lambda) = \frac{1}{V}\sum_j \delta(\lambda - \lambda_j(G))$

Chiral symmetry restoration

$$\rho(\lambda) = \int D[G]P_m(G)\rho_G(\lambda)$$

$$\langle\bar\psi\psi\rangle = 2\pi\rho(0) = 0, \quad m \to 0$$

$$P_m(\forall G) \geq 0$$

$$\rho_G(0) = 0 \text{ for } {}^\forall G$$

$$\chi_{U(1)_A}/V = \int D[G]P_m[G]\left\{\int d\lambda\rho_G(\lambda)\frac{2m}{\lambda^2+m^2}\right\}^2 = O(m^2) \to 0, \quad (m \to 0)$$

$$\chi_{U(1)_A}/V = 0, \quad (m \to 0)$$

$$\chi_{U(1)_A} = \int d^4x\,\langle\sigma(x)\sigma(0) - \delta(x)\delta(0)\rangle$$

$U(1)_A$ seems to be restored.

Lee-Hatsuda(96), Theory   No !

zero mode contributions are important.

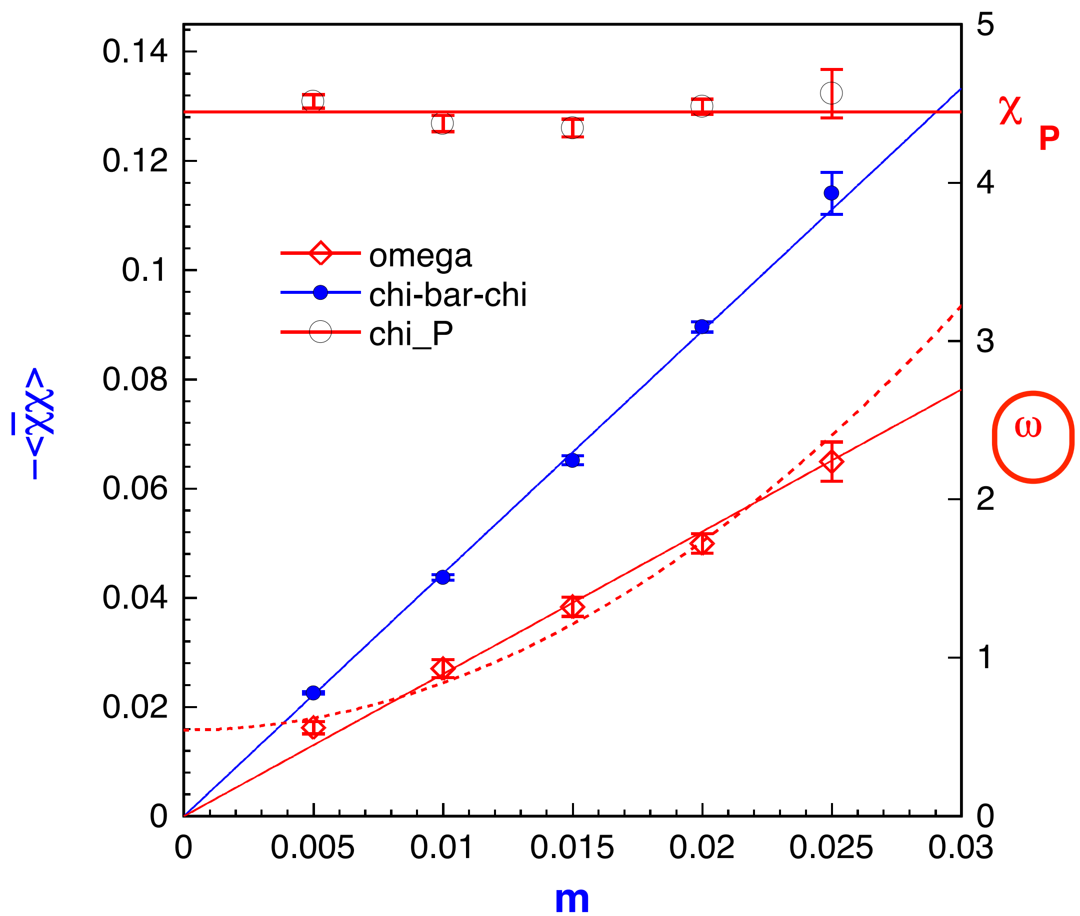
$$\chi_{U(1)_A} = O(m^2) + \Delta$$

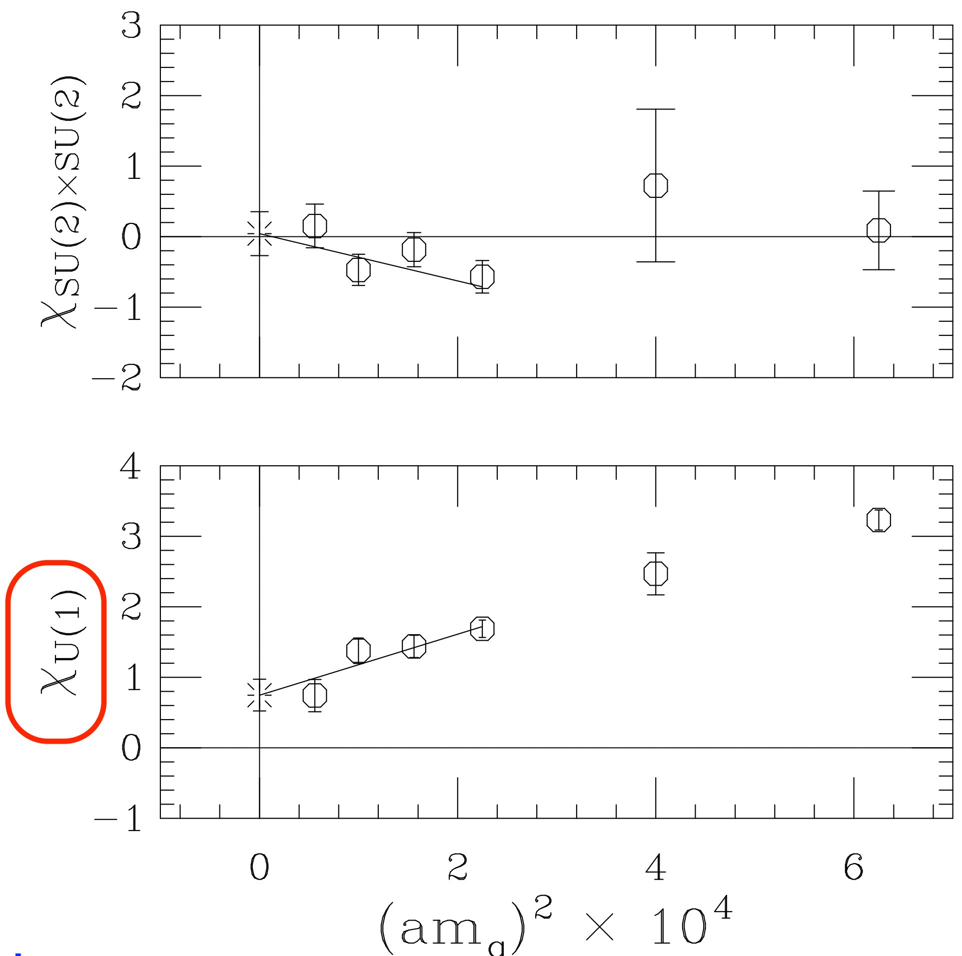
$\Delta = O(1)$ at $N_f = 2$: contributions from $Q = \pm 1$

Lattice results

Chandrasekharan *et al.*, (98), KS   No !

Bernald, *et al.* (96), KS   No !

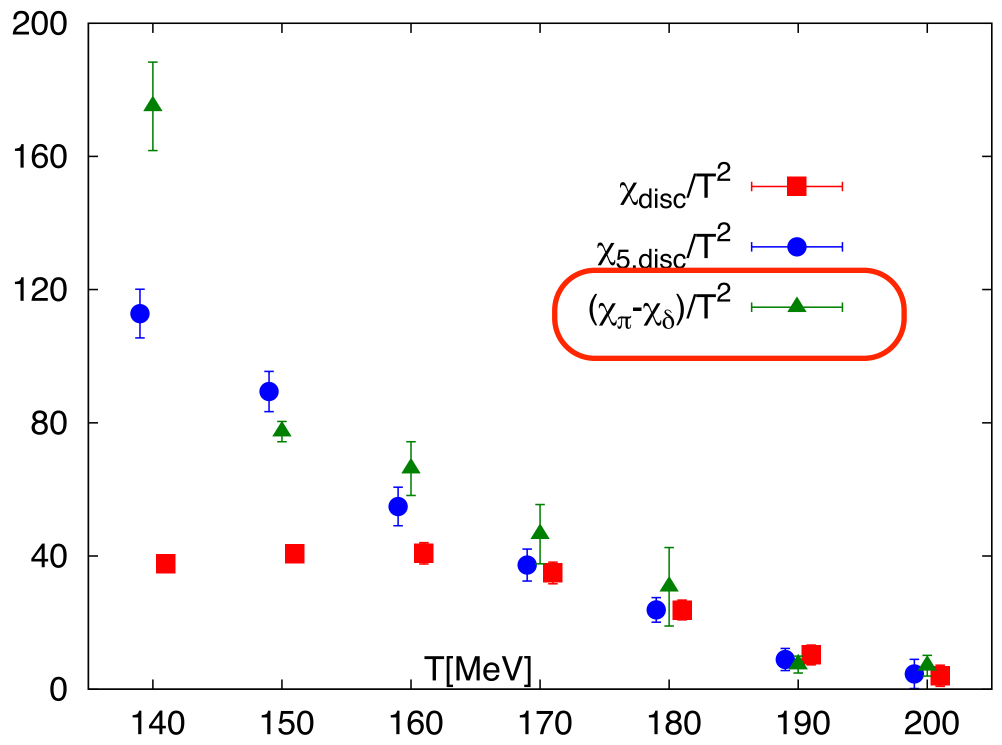




Chiral symmetry is restored.

$U(1)_A$ is NOT.

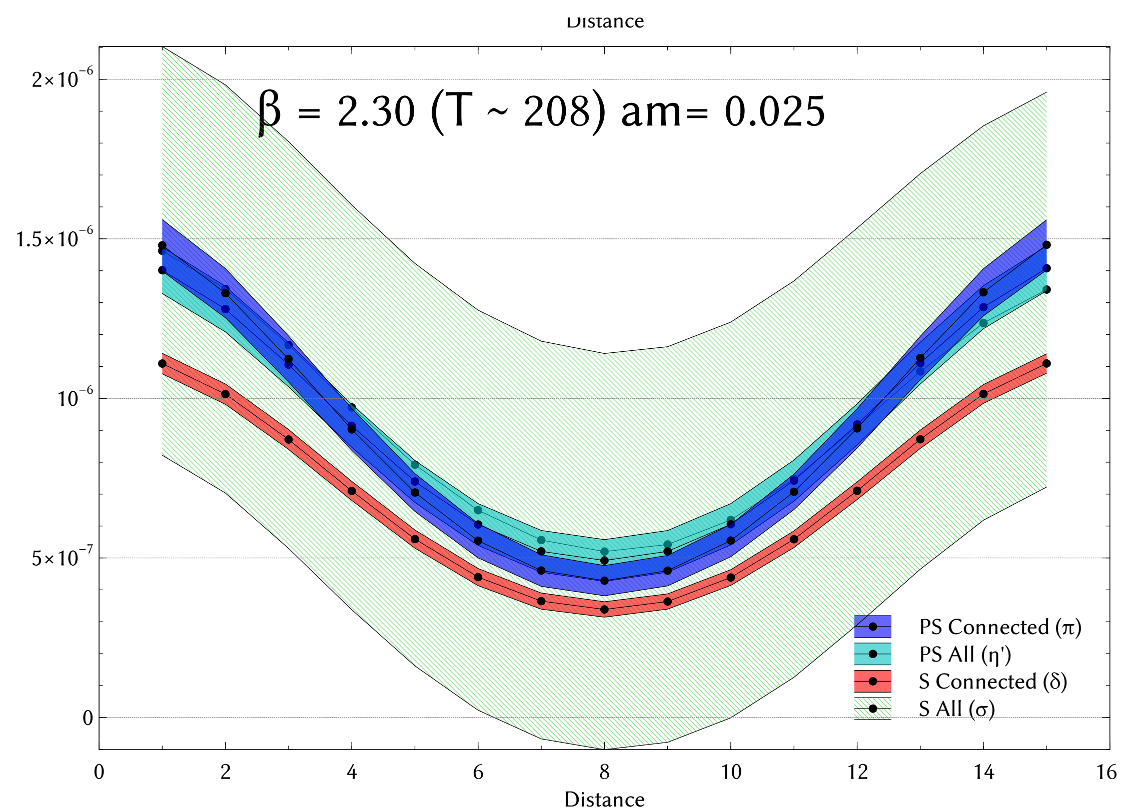
Recent lattice results

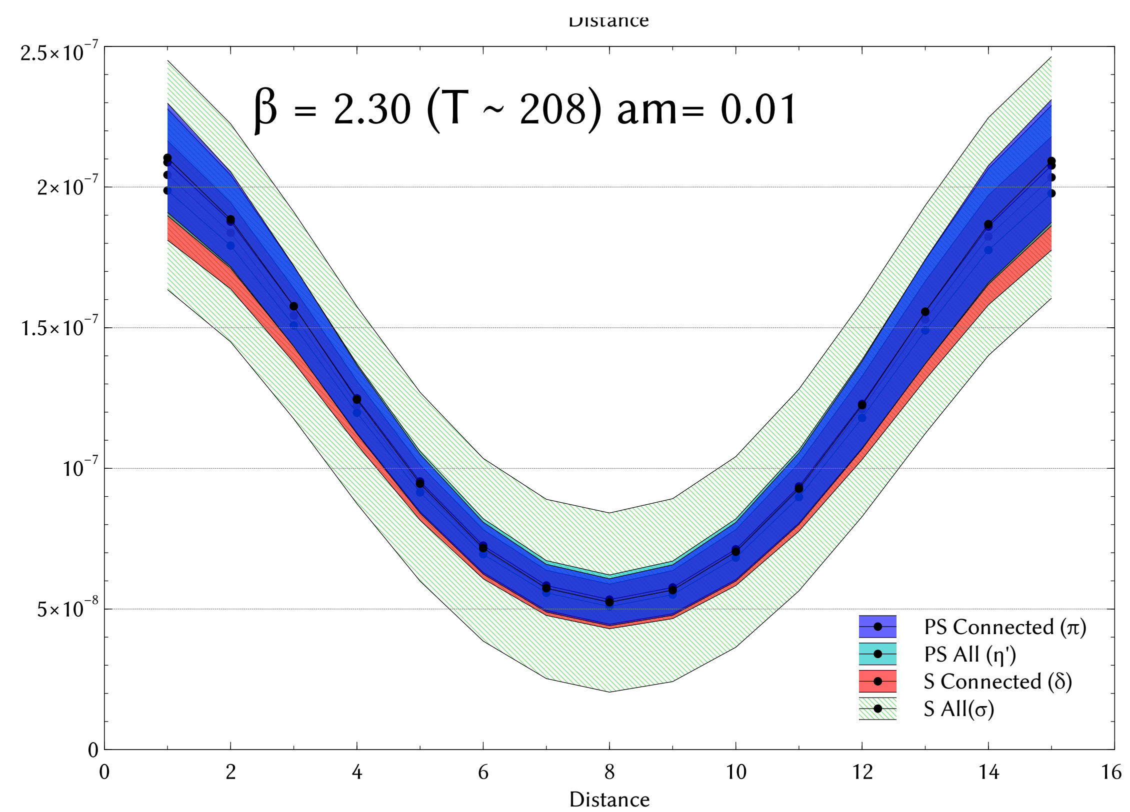


Hegde (HotQCD11), DW No ?!

$$\chi_{U(1)_A} = 0 \text{ or not ?}$$

Cossu *et al.* (JLQCD11), Overlap Yes ?! meson correlators





This talk

give constraints on eigenvalue densities of 2-flavor overlap fermions, if chiral symmetry in QCD is restored at finite temperature.
discuss a behavior of singlet susceptibility using the constraints.

Content

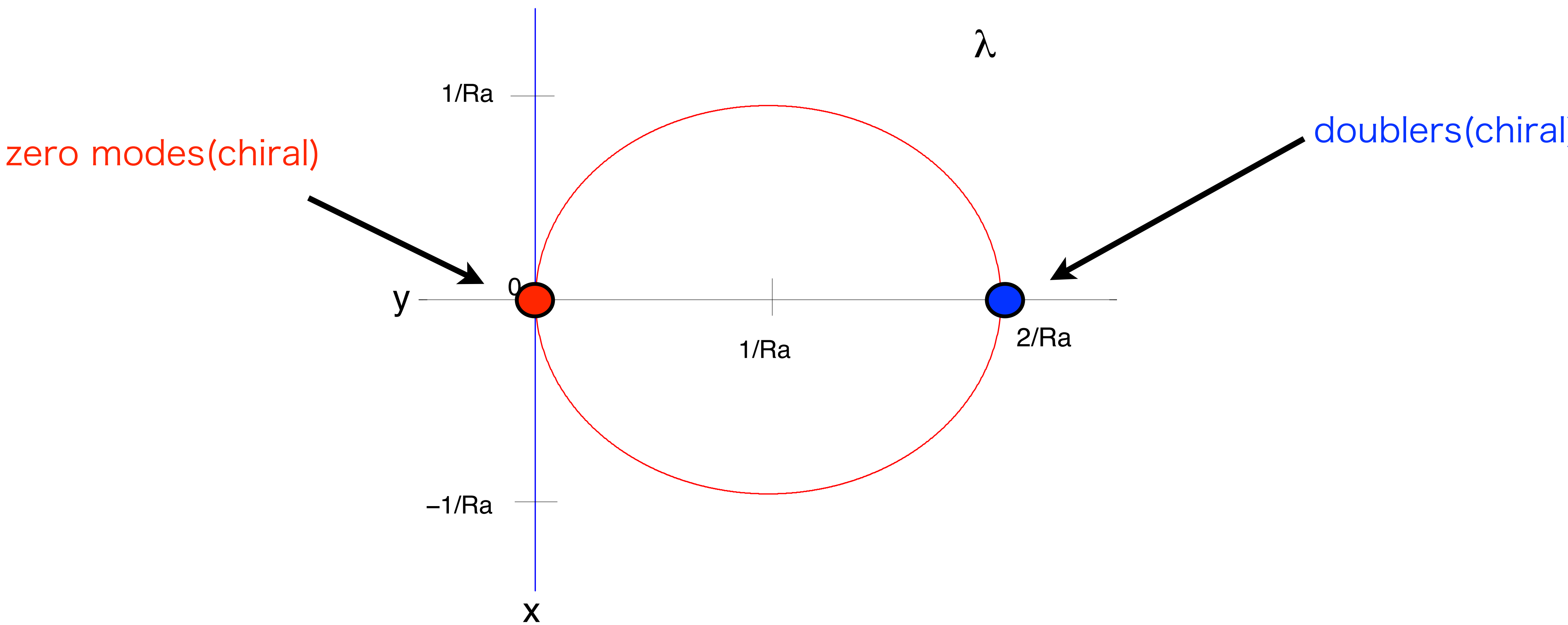
1. Introduction
2. Overlap fermions
3. Constraints on eigenvalue densities
4. Discussions: singlet susceptibility

2. Overlap fermions

Action $$S = \bar{\psi}[D - mF(D)]\psi, \quad F(D) = 1 - \frac{Ra}{2}D$$

Ginsparg-Wilson relation $$D\gamma_5 + \gamma_5 D = aDR\gamma_5 D$$

Eigenvalue spectrum $$\lambda_n^A + \bar{\lambda}_n^A = aR\bar{\lambda}_n^A\lambda_n^A$$



$$D(A)\gamma_5\phi_n^A = \bar{\lambda}_n^A\gamma_5\phi_n^A \quad \Rightarrow \quad D(A)\phi_n^A = \lambda_n^A\phi_n^A$$

Propagator

$$S(x,y) = \sum_n \left[\frac{\phi_n(x)\phi_n^\dagger(y)}{f_m\lambda_n - m} + \frac{\gamma_5\phi_n(x)\phi_n^\dagger(y)\gamma_5}{f_m\bar{\lambda}_n - m} \right] - \sum_{k=1}^{N_{R+L}} \frac{1}{m}\phi_k(x)\phi_k^\dagger(y) + \sum_{K=1}^{N_D} \frac{Ra}{2}\phi_K(x)\phi_K^\dagger(y)$$

bulk modes(non-chiral) zero modes(chiral) doublers(chiral)

$$f_m = 1 + \frac{Rma}{2}$$

Measure

\# of doublers

$$P_m(A) = e^{-S_{YM}(A)}(-m)^{N_f N_{R+L}^A} \left(\frac{2}{Ra}\right)^{N_f N_D^A} \prod_{\Im\lambda_n^A > 0} (Z_m^2 \bar{\lambda}_n^A \lambda_n^A + m^2)$$

\# of zero modes

$$Z_m^2 = 1 - (ma)^2 \frac{R^2}{4}$$

positive definite and even function of $m \neq 0$ for even N_f

N_f=2 in this talk.

Ward-Takahashi identities under "chiral" rotation

$$\theta^a(x)\delta^a_x\psi(x) = i\theta^a(x)T^a\gamma_5(1-RaD)$$
$$\theta^a(x)\delta^a_x\bar\psi(x) = i\bar\psi(x)\theta^a(x)T^a\gamma_5,$$

Integrated operators

$$S^a = \int d^4x\, S^a(x), \quad P^a = \int d^4x\, P^a(x)$$

$$S^a(x) = \bar\psi(x)T^aF(D)\psi(x),$$ scalar
$$P^a(x) = \bar\psi(x)T^a i\gamma_5 F(D)\psi(x),$$ pseudo-scalar

chiral rotation at N_f=2

$$\delta^b S^a = 2\delta^{ab}P^a, \quad \delta^b P^a = -2\delta^{ab}S^a$$
$$\delta^0 S^a = \delta^a S^0 = 2P^a, \quad \delta^0 P^a = \delta^a P^0 = -2S^a$$

If the chiral symmetry is restored,

$$\lim_{m\to 0}\langle \delta^a \mathcal{O}_{n_1,n_2,n_3,n_4}\rangle_m = 0$$

WT identities

$$\mathcal{O}_{n_1,n_2,n_3,n_4} = (P^a)^{n_1}(S^a)^{n_2}(P^0)^{n_3}(S^0)^{n_4}$$

$$N = \sum_i n_i, \quad n_1+n_2 = \text{odd}, \; n_1+n_3 = \text{odd}$$

explicit from

$$\frac{\delta^a}{2}\mathcal{O}_{n_1,n_2,n_3,n_4} = -n_1\mathcal{O}_{n_1-1,n_2,n_3,n_4+1} + n_2\mathcal{O}_{n_1,n_2-1,n_3+1,n_4} - n_3\mathcal{O}_{n_1,n_2+1,n_3-1,n_4} + n_4\mathcal{O}_{n_1+1,n_2,n_3,n_4-1}$$

3. Constraints on eigenvalue densities

Assumption

eigenvalues density can be expanded as

$$\rho^A(\lambda) \equiv \lim_{V\to\infty} \frac{1}{V} \sum_n \delta\left(\lambda - \sqrt{\bar{\lambda}_n^A \lambda_n^A}\right) = \sum_{k=0}^{\infty} \rho_k^A \frac{|\lambda|^k}{k!} = \rho_0^A + \rho_1^A |\lambda| + \cdots ,$$

More precisely, configurations which can not be expandable are "measure zero" in the configuration space.

Method if $f(A)$: m independent, positive

$$\langle f(A) \rangle_m = O(m^k) \quad \Longleftrightarrow \quad \langle f(A)^n \rangle_m = O(m^k)$$

n: arbitrary integer

$$\langle f(A) \rangle_m = \frac{1}{Z} \int \mathcal{D}A \, \underline{P_m(A)} \, f(A)$$

positive

m dependence only from $P_m(A)$

$Z = O(1)$

general N(odd)

$m \to 0$

$$\frac{1}{V^N}\langle (S_0)^N\rangle_m = N_f^N \left\langle \left\{ \frac{N_{R+L}^A}{mV} + I_1 \right\}^N \right\rangle_m + O(V^{-1}) \to 0$$

$\Lambda_R = \frac{2}{Ra}$: cut-off

$$I_1 = \frac{1}{Z_m}\int_0^{\Lambda_R} d\lambda\, \rho^A(\lambda) g_0(\lambda^2) \frac{2m_R}{\lambda^2 + m_R^2} = \pi\rho_0^A + O(m)$$

Both ρ_0^A and N_{R+L}^A are positive.

$g_0(\lambda^2) = 1 - \frac{\lambda^2}{\Lambda_R^2},\ m_R = m/Z_m$

$\Rightarrow$ $\langle \rho_0^A \rangle_m = O(m^2)$ 1st constraint

$$\lim_{V\to\infty}\left\langle \frac{N_{R+L}}{V}\right\rangle_m = O(m^{N+1}) \Rightarrow \lim_{V\to\infty}\left\langle \frac{N_{R+L}}{V}\right\rangle_m = 0$$

$^\forall N$

for small but non-zero m

N=2

$$\chi^{\sigma-\pi} = \frac{1}{V^2}\langle S_0^2 - P_a^2\rangle_m, \qquad \chi^{\eta-\delta} = \frac{1}{V}\langle P_0^2 - S_a^2\rangle_m$$

=0

topological charge

$$\chi^{\eta-\delta} = N_f \left\langle \frac{1}{m^2 V}\{2N_{R+L} - N_f Q(A)^2\} + \frac{1}{Z_m}\left(\frac{I_1}{m_R} + I_2\right)\right\rangle_m \qquad Q(A) = N_R^A - N_L^A$$

=0

$$I_2 = \frac{2}{Z_m}\int_0^{\Lambda_R} d\lambda\, \rho^A(\lambda)\, \frac{m_R^2 - \lambda^2 g_0(\lambda^2) g_m}{(\lambda^2 + m_R^2)^2}, \quad g_m = \frac{1}{Z_m^2}\left(1 + \frac{m^2}{2\Lambda_R^2}\right)$$

$$\frac{I_1}{m_R} + I_2 = \rho_0^A\left(\frac{\pi_m}{m} + \frac{2}{\Lambda_R}\right) + 2\rho_1^A + O(m),$$

$$\lim_{m\to 0}\chi^{\eta-\delta} = 0 \quad \Rightarrow \quad \lim_{m\to 0}\frac{N_f^2\langle Q(A)^2\rangle_m}{m^2 V} = 2\lim_{m\to 0}\langle \rho_1^A\rangle_m$$

N=3

WT identities

$$\langle \mathcal{O}_{2001} \rangle_m \to 0, \quad \langle -\mathcal{O}_{0201} + 2\mathcal{O}_{1110} \rangle_m \to 0, \quad \langle \mathcal{O}_{0021} + 2\mathcal{O}_{1110} \rangle_m = 0$$

$$\langle -\mathcal{O}_{0003} + 2\mathcal{O}_{2001} \rangle_m \to 0, \quad \langle \mathcal{O}_{0021} - \mathcal{O}_{0201} + \mathcal{O}_{1110} \rangle_m \to 0,$$

$$\langle \rho_0^A \rangle_m = -\frac{m^2}{2} \langle \rho_2^A \rangle_m + O(m^4)$$

$$\lim_{V \to \infty} \frac{\langle Q(A)^2 \rho_0^A \rangle_m}{V} = O(m^4)$$

N=4

$$\langle \mathcal{O}_{4000}-\mathcal{O}_{0004}\rangle_m \to 0, \quad \langle \mathcal{O}_{4000}-3\mathcal{O}_{2002}\rangle_m \to 0,$$
$$\langle \mathcal{O}_{0400}-\mathcal{O}_{0040}\rangle_m \to 0, \quad \langle \mathcal{O}_{0400}-3\mathcal{O}_{0220}\rangle_m \to 0,$$
$$\langle \mathcal{O}_{2020}-\mathcal{O}_{0202}\rangle_m \to 0, \quad \langle \mathcal{O}_{2200}-\mathcal{O}_{0022}\rangle_m \to 0,$$
$$\langle 2\mathcal{O}_{1111}-\mathcal{O}_{0202}+\mathcal{O}_{0022}\rangle_m \to 0.$$

$$3N_f^2\langle (I_2+I_1/m)(I_1-I_2/m)\rangle_m + \frac{6N_f^3}{m^3V}\langle Q(A)^2 I_1\rangle_m - \frac{N_f^4}{m^4V^2}\langle Q(A)^4\rangle_m \to 0.$$

$\sim \log m$ $\quad \sim \frac{1}{m}$ $\quad \sim \frac{1}{m^2}$

$$\lim_{V\to\infty}\frac{\langle Q(A)^2\rangle_m}{V} = O(m^4)$$

$$\langle \rho_1^A\rangle_m = O(m^2)$$

$$\lim_{m\to 0}\frac{N_f^2\langle Q(A)^2\rangle_m}{m^2V} = 2\lim_{m\to 0}\langle \rho_1^A\rangle_m$$

2nd constraint

$$-3N_f^2\frac{\pi^2}{m^2}\langle (\rho_0^A)^2\rangle_m - \frac{N_f^4}{m^4V^2}\langle Q(A)^4\rangle_m \to 0.$$

negative semi-definite

$$\lim_{V\to\infty}\frac{\langle Q(A)^2\rangle_m}{V} = O(m^6)$$

$$\langle \rho_0^A\rangle_m = O(m^4)$$

$$\langle \rho_2^A\rangle_m = O(m^2)$$

$$\langle \rho_0^A\rangle_m = -\frac{m^2}{2}\langle \rho_2^A\rangle_m$$

3rd constraint

+ result from N=4k (general)

Final results

$$\lim_{m\to 0}\langle \rho^A(\lambda)\rangle_m = \lim_{m\to 0}\langle \rho_3^A\rangle_m \frac{|\lambda|^3}{3!} + O(\lambda^4)$$

No constraints to higher $\langle \rho_n^A\rangle_m$

$\langle \rho_3^A\rangle_m \neq 0$ even for "free" theory.

$$\langle \rho_0^A\rangle_m = 0$$

$$\lim_{V\to\infty}\frac{1}{V^k}\langle (N_{R+L}^A)^k\rangle_m = 0, \quad \lim_{V\to\infty}\frac{1}{V^k}\langle Q(A)^{2k}\rangle_m = 0$$

4. Discussion: Singlet susceptibility

Singlet susceptibility at high T

$$\lim_{m\to 0}\chi^{\pi-\eta} = \lim_{m\to 0}\lim_{V\to\infty}\frac{N_f^2}{m^2V^2}\langle Q(A)^2\rangle_m = 0$$

Both Cohen and Lee-Hatsuda are inaccurate.

This, however, does not mean U(1)_A symmetry is recovered at high T.

$$\lim_{m\to 0}\chi^{\pi-\eta} = 0$$

is necessary but NOT "sufficient" for the recovery of U(1)_A .

More general Singlet WT identities

$$\langle J^0 \mathcal{O} + \delta^0 \mathcal{O} \rangle_m = O(m)$$

anomaly(measure)　　singlet rotation

We can show for $\mathcal{O} = \mathcal{O}_{n_1,n_2,n_3,n_4} = (P^a)^{n_1}(S^a)^{n_2}(P^0)^{n_3}(S^0)^{n_4}$

$$\lim_{V\to\infty} \frac{1}{V^k} \langle J^0 \mathcal{O} \rangle_m = \lim_{V\to\infty} \left\langle \frac{Q(A)^2}{mV} \times O(V^0) \right\rangle_m = 0$$

where k is the smallest integer which makes the $V \to \infty$ limit finite.

$$\Rightarrow \quad \lim_{m\to 0} \lim_{V\to\infty} \frac{1}{V^k} \langle \delta^0 \mathcal{O} \rangle_m = 0$$

Breaking of U(1)_A symmetry is absent for these “bulk quantities”.