Staggered Chiral Perturbation Theory for All-Staggered Heavy-Light Mesons

Javad Komijani, Claude Bernard

Washington University in St. Louis

June 24, 2012

Introduction

- Accurate calculations of heavy-light quantities can help to determine CKM matrix elements.
- Highly improved staggered quark (HISQ) action makes it possible to have staggered charm quarks as well as staggered light quarks.
  - Many advantages: avoiding renormalization for partially conserved currents, ...
- SχPT can help us extrapolate lattice data to the continuum limit and extrapolate or interpolate to the physical values of light quark masses.
- Aubin and Bernard derived SχPT for staggered light quarks and Fermilab heavy quarks.
  - The doubler states of a heavy quark are treated as integrated out.
  - The heavy quark fields have no degree of freedom corresponding to taste.
- HISQ charm: each staggered heavy quark has a taste-degree of freedom.

Introduction

- Accurate calculations of heavy-light quantities can help to determine CKM matrix elements.
- Highly improved staggered quark (HISQ) action makes it possible to have staggered charm quarks as well as staggered light quarks.
    - Many advantages: avoiding renormalization for partially conserved currents, ...
- SχPT can help us extrapolate lattice data to the continuum limit and extrapolate or interpolate to the physical values of light quark masses.
- Aubin and Bernard derived SχPT for staggered light quarks and Fermilab heavy quarks.
    - The doubler states of a heavy quark are treated as integrated out.
    - The heavy quark fields have no degree of freedom corresponding to taste.
- HISQ charm: each staggered heavy quark has a taste-degree of freedom.
- We need to extend the program to include staggered heavy quarks

Introduction

- SχPT in continuum limit:
  - Normal continuum heavy-light χPT but extra taste degree for both heavy and light quarks.
  - Separate $SU(4)$ taste symmetry for both heavy and light quarks.

Introduction

- SχPT in continuum limit:
  - Normal continuum heavy-light χPT but extra taste degree for both heavy and light quarks.
  - Separate $SU(4)$ taste symmetry for both heavy and light quarks.
- SχPT at non-zero lattice spacing:
  - Taste symmetries (heavy+light) are broken.
  - But splittings of D's is small (comparing to pion's)

$$\frac{1}{16}\sum_{tastes}(m_D - m_{D_5})$$
$$= \frac{1}{16}\sum_{tastes}\frac{m_D^2 - m_{D_5}^2}{m_D + m_{D_5}}$$
$$\approx 11\ MeV \quad (at\ a = .12\ fm)$$

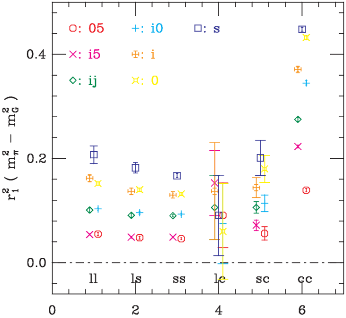


Figure: Coarse MILC ensemble

Introduction

- SχPT in continuum limit:
  - Normal continuum heavy-light χPT but extra taste degree for both heavy and light quarks.
  - Separate $SU(4)$ taste symmetry for both heavy and light quarks.
- SχPT at non-zero lattice spacing:
  - Taste symmetries (heavy+light) are broken.
  - But splittings of D's is small (comparing to pion's)

$$\frac{1}{16}\sum_{tastes}(m_D - m_{D_5})$$
$$= \frac{1}{16}\sum_{tastes}\frac{m_D^2 - m_{D_5}^2}{m_D + m_{D_5}}$$
$$\approx 11\ MeV \quad (at\ a = .12\ fm)$$

So leading order staggered version of heavy-light meson Lagrangian is same as continuum SχPT.

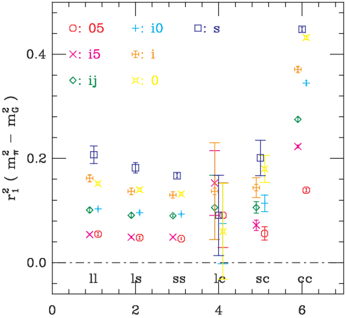


Figure: Coarse MILC ensemble

Chiral Lagrangian in the Continuum Limit (no taste)

- Consider a heavy quark with no taste degree of freedom:
- Due to the spin symmetry of heavy quarks, one can combine the spin zero and spin one eigenstates of a heavy-light meson.

The field destroying the heavy-light meson:

$$H_a = \frac{1+\not{v}}{2}\left[\gamma^\mu B^*_{\mu a} + i\gamma_5 B_a\right]$$

The creating field:

$$\overline{H}_a \equiv \gamma_0 H_a^\dagger \gamma_0$$

"a" is the combined flavor-taste index of the light quark and v is the meson's velocity.

- The H-field chiral Lagrangian at LO is:

$$\mathcal{L}_1 = -i\,\mathrm{Tr}(\overline{H}Hv\cdot\overleftarrow{D}) + g_\pi\,\mathrm{Tr}(\overline{H}H\gamma^\mu\gamma_5\mathbb{A}_\mu)\ ,$$

ChiPT for all-Staggered Heavy-Light Mesons

- Now, we assume the heavy quark is implemented by a staggered fermion.
- We generalize the definition of the creation and annihilation operators of a heavy-light meson.

The field destroying the heavy-light meson:

$$H_{\alpha a} = \frac{1+\not{v}}{2}\left[\gamma^\mu B^*_{\mu\alpha a} + i\gamma_5 B_{\alpha a}\right]$$

The field creating the heavy-light meson:

$$\overline{H}_{\alpha a} = \left[\gamma^\mu B^{\dagger *}_{\mu\alpha a} + i\gamma_5 B^\dagger_{\alpha a}\right]\frac{1+\not{v}}{2}$$

v: the meson's velocity
α: the heavy quark taste index
a: the light quark flavor-taste index

ChiPT for all-Staggered Heavy-Light Mesons

- Now, we assume the heavy quark is implemented by a staggered fermion.
- We generalize the definition of the creation and annihilation operators of a heavy-light meson.

The field destroying the heavy-light meson:

$$H_{\alpha a} = \frac{1+\not{v}}{2}\left[\gamma^\mu B^*_{\mu\alpha a} + i\gamma_5 B_{\alpha a}\right] \;\rightarrow\; H_{i\Xi}$$

The field creating the heavy-light meson:

$$\overline{H}_{\alpha a} = \left[\gamma^\mu B^{\dagger *}_{\mu\alpha a} + i\gamma_5 B^\dagger_{\alpha a}\right]\frac{1+\not{v}}{2} \;\rightarrow\; \overline{H}_{i\Xi}$$

v: the meson's velocity
α: the heavy quark taste index
a: the light quark flavor-taste index
i: the light quark flavor index
Ξ: the meson taste index
(16 indep. 4×4 taste matrices)

ChiPT for all-Staggered Heavy-Light Mesons

- Now, we assume the heavy quark is implemented by a staggered fermion.
- We generalize the definition of the creation and annihilation operators of a heavy-light meson.

The field destroying the heavy-light meson:

$$H_{\alpha a} = \frac{1+\not{v}}{2}\left[\gamma^\mu B^*_{\mu\alpha a} + i\gamma_5 B_{\alpha a}\right] \rightarrow H_{i\Xi}$$

The field creating the heavy-light meson:

$$\overline{H}_{\alpha a} = \left[\gamma^\mu B^{\dagger *}_{\mu\alpha a} + i\gamma_5 B^\dagger_{\alpha a}\right]\frac{1+\not{v}}{2} \rightarrow \overline{H}_{i\Xi}$$

v: the meson's velocity
α: the heavy quark taste index
a: the light quark flavor-taste index
i: the light quark flavor index
Ξ: the meson taste index
(16 indep. 4×4 taste matrices)

- The tastes of a meson can be represented with the Hermitian taste generators:

$$T_\Xi = \{\xi_5, i\xi_{\mu 5}, i\xi_{\mu\nu}, \xi_\mu, \xi_I\}$$

ChiPT for all-Staggered Heavy-Light Mesons

- Leading order staggered version of heavy-light meson Lagrangian looks the same as continuum SχPT.
- The LO chiral Lagrangian involving the heavy-lights is:

$$\mathcal{L}_1 = -i\,\mathrm{Tr}(\overline{H}Hv\cdot\overleftarrow{D}) + g_\pi\,\mathrm{Tr}(\overline{H}H\gamma^\mu\gamma_5\mathbb{A}_\mu)$$

- $\overline{H}H$ is a $4n \times 4n$ matrix in the flavor-taste space of light quarks:

$$(\overline{H}H)_{ab} \equiv \overline{H}_{a\alpha}H_{\alpha b}$$

- This Lagrangian has the $SU(4)$ taste symmetry and the spin symmetry of the heavy quark.

ChiPT for all-Staggered Heavy-Light Mesons

- Leading order staggered version of heavy-light meson Lagrangian looks the same as continuum SχPT.
- The LO chiral Lagrangian involving the heavy-lights is:

$$\mathcal{L}_1 = -i\,\mathrm{Tr}(\overline{H}Hv\cdot\overleftarrow{D}) + g_\pi\,\mathrm{Tr}(\overline{H}H\gamma^\mu\gamma_5\mathbb{A}_\mu)$$

- $\overline{H}H$ is a $4n \times 4n$ matrix in the flavor-taste space of light quarks:

$$(\overline{H}H)_{ab} \equiv \overline{H}_{a\alpha}H_{\alpha b}$$

- This Lagrangian has the $SU(4)$ taste symmetry and the spin symmetry of the heavy quark.
- To improve the results, we are going to use Symanzik effective action for $m_Q a << 1$.

ChiPT for all-Staggered Heavy-Light Mesons

- We assume that the charm quark action is sufficiently improved that we may take $am_Q << 1$.

ChiPT for all-Staggered Heavy-Light Mesons

- We assume that the charm quark action is sufficiently improved that we may take $am_Q << 1$.
- This is the way we think about the theory:

Staggered Lattice Lagrangian

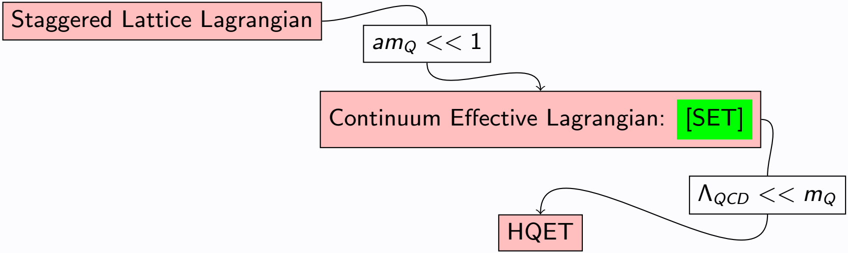
ChiPT for all-Staggered Heavy-Light Mesons

- We assume that the charm quark action is sufficiently improved that we may take $am_Q << 1$.
- This is the way we think about the theory:

$$\boxed{\text{Staggered Lattice Lagrangian}} \xrightarrow{\boxed{am_Q << 1}} \boxed{\text{Continuum Effective Lagrangian: [SET]}}$$

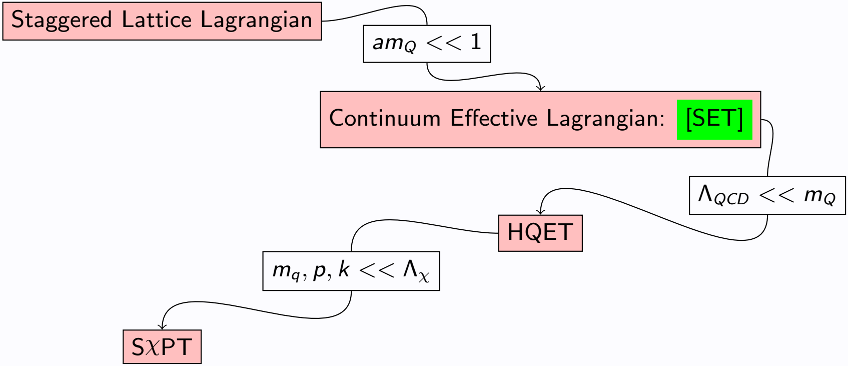
ChiPT for all-Staggered Heavy-Light Mesons

- We assume that the charm quark action is sufficiently improved that we may take $am_Q << 1$.
- This is the way we think about the theory:



ChiPT for all-Staggered Heavy-Light Mesons

- We assume that the charm quark action is sufficiently improved that we may take $am_Q << 1$.
- This is the way we think about the theory:



ChiPT for all-Staggered Heavy-Light Mesons

- We use Symanzik effective action to derive NLO terms for $m_Q a << 1$, which leads to heavy quark taste (or spin) symmetry violation.
- Three different types of operators in the SET at order a^2:

$$
\begin{aligned}
a^2\mathcal{O}_{ss'tt'} = & \quad c_1 \quad a^2\overline{q}_l(\gamma_s \otimes \xi_t)q_l\overline{q}_{l'}(\gamma_{s'} \otimes \xi_{t'})q_{l'} \\
+ & \quad c_2 \quad a^2\overline{q}_l(\gamma_s \otimes \xi_t)q_l\overline{q}_h(\gamma_{s'} \otimes \xi_{t'})q_h \\
+ & \quad c_3 \quad a^2\overline{q}_h(\gamma_s \otimes \xi_t)q_h\overline{q}_{h'}(\gamma_{s'} \otimes \xi_{t'})q_{h'}
\end{aligned}
$$

- The first type doesn't break the heavy taste symmetry.
- The third type doesn't lead to any new terms. (Since we don't consider heavy-light scatterings.)
- We now proceed to determine the chiral representatives of the second type.

ChiPT for all-Staggered Heavy-Light Mesons

- Using Spurion analysis, we obtain:

$$
\begin{aligned}
\mathcal{L}^{A2}_{2,a^2} \quad = \quad & K_{A1} a^2 \,\mathrm{Tr}\left(\overline{H}\xi_5 H \xi_5\right) + K_{A2} a^2 \,\mathrm{Tr}\left(\overline{H}\xi_\mu H \xi_\mu\right) + K_{A3} a^2 \,\mathrm{Tr}\left(\overline{H}\xi_{5\mu} H \xi_{\mu 5}\right) \\
& + K_{A4} a^2 \,\mathrm{Tr}\left(\overline{H}\xi_{\mu\nu} H \xi_{\nu\mu}\right) + K_{A5} a^2 \,\mathrm{Tr}\left(\overline{H}\gamma_{5\mu} H \gamma^{\mu 5}\right) \\
& + K_{A6} a^2 \,\mathrm{Tr}\left(\overline{H}\gamma_{5\mu}\xi_5 H \gamma^{\mu 5}\xi_5\right) + K_{A7} a^2 \,\mathrm{Tr}\left(\overline{H}\gamma_{\mu\nu}\xi_\lambda H \gamma^{\nu\mu}\xi_\lambda\right) \\
& + K_{A8} a^2 \,\mathrm{Tr}\left(\overline{H}\gamma_{\mu\nu}\xi_{5\lambda} H \gamma^{\nu\mu}\xi_{\lambda 5}\right) + K_{A9} a^2 \,\mathrm{Tr}\left(\overline{H}\gamma_{5\mu}\xi_{\nu\lambda} H \gamma^{\mu 5}\xi_{\lambda\nu}\right) \\
& + \text{double-trace terms}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}^{B2}_{2,a^2} \quad = \quad & \sum_\mu \Bigg(K_{B1} a^2 \,\mathrm{Tr}\left(\overline{H}\gamma_{\nu\mu}\xi_\mu H \gamma^{\mu\nu}\xi_\mu\right) + K_{B2} a^2 \,\mathrm{Tr}\left(\overline{H}\gamma_{\nu\mu}\xi_{5\mu} H \gamma^{\mu\nu}\xi_{\mu 5}\right) \\
& + K_{B3} a^2 v^\mu v_\mu \,\mathrm{Tr}\left(\overline{H}\xi_{\nu\mu} H \xi_{\mu\nu}\right) + K_{B4} a^2 \,\mathrm{Tr}\left(\overline{H}\gamma_{5\mu}\xi_{\nu\mu} H \gamma^{\mu 5}\xi_{\mu\nu}\right) \Bigg) \\
& + \text{double-trace terms}
\end{aligned}
$$

ChiPT for all-Staggered Heavy-Light Mesons

- Using Spurion analysis, we obtain:

$$
\begin{aligned}
\mathcal{L}^{A2}_{2,a^2} \quad = \quad & K_{A1} a^2 \,\mathrm{Tr}\left(\overline{H}\xi_5 H \xi_5\right) + K_{A2} a^2 \,\mathrm{Tr}\left(\overline{H}\xi_\mu H \xi_\mu\right) + K_{A3} a^2 \,\mathrm{Tr}\left(\overline{H}\xi_{5\mu} H \xi_{\mu 5}\right) \\
& + K_{A4} a^2 \,\mathrm{Tr}\left(\overline{H}\xi_{\mu\nu} H \xi_{\nu\mu}\right) + K_{A5} a^2 \,\mathrm{Tr}\left(\overline{H}\gamma_{5\mu} H \gamma^{\mu 5}\right) \\
& + K_{A6} a^2 \,\mathrm{Tr}\left(\overline{H}\gamma_{5\mu}\xi_5 H \gamma^{\mu 5}\xi_5\right) + K_{A7} a^2 \,\mathrm{Tr}\left(\overline{H}\gamma_{\mu\nu}\xi_\lambda H \gamma^{\nu\mu}\xi_\lambda\right) \\
& + K_{A8} a^2 \,\mathrm{Tr}\left(\overline{H}\gamma_{\mu\nu}\xi_{5\lambda} H \gamma^{\nu\mu}\xi_{\lambda 5}\right) + K_{A9} a^2 \,\mathrm{Tr}\left(\overline{H}\gamma_{5\mu}\xi_{\nu\lambda} H \gamma^{\mu 5}\xi_{\lambda\nu}\right) \\
& + \text{double-trace terms}
\end{aligned}
$$

Type-A operators are invariant over the full Euclidean space-time rotation group, $SO(4)$, as well as a corresponding $SO(4)$ of taste.

$$
\begin{aligned}
\mathcal{L}^{B2}_{2,a^2} \quad = \quad & \sum_\mu \Bigg(K_{B1} a^2 \,\mathrm{Tr}\left(\overline{H}\gamma_{\nu\mu}\xi_\mu H \gamma^{\mu\nu}\xi_\mu\right) + K_{B2} a^2 \,\mathrm{Tr}\left(\overline{H}\gamma_{\nu\mu}\xi_{5\mu} H \gamma^{\mu\nu}\xi_{\mu 5}\right) \\
& + K_{B3} a^2 v^\mu v_\mu \,\mathrm{Tr}\left(\overline{H}\xi_{\nu\mu} H \xi_{\mu\nu}\right) + K_{B4} a^2 \,\mathrm{Tr}\left(\overline{H}\gamma_{5\mu}\xi_{\nu\mu} H \gamma^{\mu 5}\xi_{\mu\nu}\right) \Bigg) \\
& + \text{double-trace terms}
\end{aligned}
$$

Results

- Due to the terms of $\mathcal{L}^{A2}_{2,a^2}$ and $\mathcal{L}^{B2}_{2,a^2}$, different tastes of the D meson get different corrections to their mass.
  - Type-A: gives different masses to different representations of $SO(4)$ taste symmetry group, labeled by S, P, V, A, T.
  - Type-B: gives different masses to time and spatial components of a representation such as ξ_0 and ξ_i.

Results

- The following terms are corresponded to the dominant terms in taste splittings of light mesons (pions)

$$\mathcal{L}^{A2}_{2,a^2} = K_{A3} a^2 \, \mathrm{Tr}\left(\overline{H}\xi_{5\mu} H \xi_{\mu 5}\right) + K_{A8} a^2 \, \mathrm{Tr}\left(\overline{H}\gamma_{\mu\nu}\xi_{5\lambda} H \gamma^{\nu\mu}\xi_{\lambda 5}\right) + \ldots$$

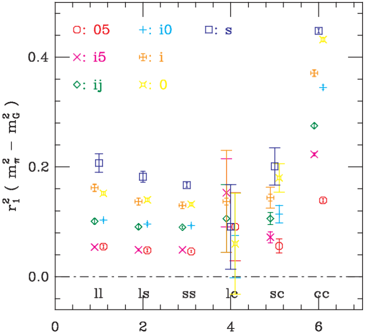
$$\mathcal{L}^{B2}_{2,a^2} = \sum_{\mu}\left(K_{B2} a^2 \, \mathrm{Tr}\left(\overline{H}\gamma_{\nu\mu}\xi_{5\mu} H \gamma^{\mu\nu}\xi_{\mu 5}\right) + \ldots \right)$$

The following pattern of mass splitting is common between the light and heavy-light mesons.

$\triangle M(.)$	ξ_5	$\xi_{5\mu}$	$\xi_{\mu\nu}$	ξ_μ	I
$8a^2(K_{A3} + 4K_{A8})$	-4	-2	0	+2	+4

In heavy-light mesons the following pattern of mass splitting appears at the same order as the previous one.

$\triangle M(.)$	ξ_5	ξ_{50}	ξ_{5i}	ξ_{ij}	ξ_{0i}	ξ_i	ξ_0	I
$8a^2 K_{B2}$	-6	-6	-2	-2	+2	+2	+6	+6



Results

We get the decay constant of the D meson for the partially quenched case as:

$$
\begin{aligned}
\left(\frac{f_{B_{x\Xi}}}{f^{\mathrm{LO}}_{B_x}}\right) \quad = \quad & 1+\frac{1}{16\pi^2 f^2}\frac{1+3g_\pi^2}{2}\left\{-\frac{1}{16}\sum_{f,\Xi'}\ell(m^2_{xf,\Xi'})\right. \\
& -\frac{1}{3}\sum_{j\in\mathcal{M}_I^{(3,x)}}\frac{\partial}{\partial m^2_{X,I}}\left[R_j^{[3,3]}(\mathcal{M}_I^{(3,x)};\mu_I^{(3)})\ell(m_j^2)\right] \\
& \left.-\left(a^2\delta'_V\sum_{j\in\mathcal{M}_V^{(4,x)}}\frac{\partial}{\partial m^2_{X,V}}\left[R_j^{[4,3]}(\mathcal{M}_V^{(4,x)};\mu_V^{(3)})\ell(m_j^2)\right]+[V\to A]\right)\right\} \\
& +C_s(m_u+m_d+m_s)+C_v m_x+C_{\Xi,a}a^2 \ ,
\end{aligned}
$$

where x is the valence flavor, Ξ is the valence taste, f runs over the three sea quarks u, d, and s, Ξ' runs over the 16 meson tastes.

Results

We get the decay constant of the D meson for the partially quenched case as:

$$
\begin{aligned}
\left(\frac{f_{B_{x\Xi}}}{f^{\mathrm{LO}}_{B_x}}\right) &= 1 + \frac{1}{16\pi^2 f^2}\frac{1+3g_\pi^2}{2}\Bigg\{-\frac{1}{16}\sum_{f,\Xi'}\ell(m^2_{xf,\Xi'}) \\
&\quad -\frac{1}{3}\sum_{j\in\mathcal{M}_I^{(3,x)}}\frac{\partial}{\partial m^2_{X,I}}\left[R_j^{[3,3]}(\mathcal{M}_I^{(3,x)};\mu_I^{(3)})\ell(m_j^2)\right] \\
&\quad -\left(a^2\delta'_V\sum_{j\in\mathcal{M}_V^{(4,x)}}\frac{\partial}{\partial m^2_{X,V}}\left[R_j^{[4,3]}(\mathcal{M}_V^{(4,x)};\mu_V^{(3)})\ell(m_j^2)\right]+[V\to A]\right)\Bigg\} \\
&\quad + C_s(m_u+m_d+m_s)+C_v m_x + C_{\Xi,a}a^2\ ,
\end{aligned}
$$

where x is the valence flavor, Ξ is the valence taste, f runs over the three sea quarks u, d, and s, Ξ' runs over the 16 meson tastes.

- $C_{\Xi,a}$ is the only coefficient depending on the taste of the meson.
- 1-loop chiral logs are the same in a paper by Aubin and Bernard, where heavy quarks have no degree of freedom corresponding to taste.

Results

We get the decay constant of the D meson for the partially quenched case as:

$$
\begin{aligned}
\left(\frac{f_{B_{x\Xi}}}{f^{\rm LO}_{B_x}}\right) \quad = \quad & 1+\frac{1}{16\pi^2 f^2}\frac{1+3g_\pi^2}{2}\Bigg\{-\frac{1}{16}\sum_{f,\Xi'}\ell(m^2_{xf,\Xi'}) \\
& -\frac{1}{3}\sum_{j\in\mathcal{M}_I^{(3,x)}}\frac{\partial}{\partial m^2_{X,I}}\left[R_j^{[3,3]}(\mathcal{M}_I^{(3,x)};\mu_I^{(3)})\ell(m_j^2)\right] \\
& -\left(a^2\delta'_V\sum_{j\in\mathcal{M}_V^{(4,x)}}\frac{\partial}{\partial m^2_{X,V}}\left[R_j^{[4,3]}(\mathcal{M}_V^{(4,x)};\mu_V^{(3)})\ell(m_j^2)\right]+[V\to A]\right)\Bigg\} \\
& + C_s(m_u+m_d+m_s)+C_v m_x + C_{\Xi,a}a^2 \ ,
\end{aligned}
$$

where x is the valence flavor, Ξ is the valence taste, f runs over the three sea quarks u, d, and s, Ξ' runs over the 16 meson tastes.

- Taste-independence of 1-loop chiral logs follows from the residual discrete taste symmetry of the light quarks and $SU(4)$ taste symmetry of the heavy quarks at LO.

Conclusion

In summary:

- We generalized the chiral Lagrangian of a heavy-light meson so that each heavy quark has four tastes.
- We derived the NLO chiral Lagrangian which breaks the $SU(4)$ heavy taste symmetry.
- We derived the NLO decay constants and the taste splittings in the heavy-light masses.

Our future work:

- to fit MILC HISQ data for heavy-light decay constants to the NLO chiral form we derived
    - Even though HISQ ensembles have physical light quark masses, the chiral fits should help control the extrapolation to the continuum, as well as allowing us to use additional ensembles with heavier masses.

Thanks for your attention