The disconnected contribution to the scalar pion form factor in chiral perturbation theory

Andreas Jüttner

University of Southampton

Cairns, 06/2012

- disconnected quark-contractions are everywhere

- expensive to compute, bad signal-to-noise ratio
- they can no longer be ignored, e.g. hadronic contributions to the vacuum polarisation, meson and baryon form factors, hadronic decays, iso-spin breaking

- disconnected quark-contractions are everywhere

- expensive to compute, bad signal-to-noise ratio
- they can no longer be ignored, e.g. hadronic contributions to the vacuum polarisation, meson and baryon form factors, hadronic decays, iso-spin breaking
- new techniques facilitate numerical evaluation but still costly

Foley et al. Comput. Phys. Commun. 172 (2005)
Bali, Collins, Schäfer, Comput. Phys. Commun. 181 (2010)

- disconnected quark-contractions are everywhere

- expensive to compute, bad signal-to-noise ratio
- they can no longer be ignored, e.g. hadronic contributions to the vacuum polarisation, meson and baryon form factors, hadronic decays, iso-spin breaking
- new techniques facilitate numerical evaluation but still costly

Foley et al. Comput. Phys. Commun. 172 (2005)
Bali, Collins, Schäfer, Comput. Phys. Commun. 181 (2010)

- HERE:
  - expressions for connected and disconnected contributions in chiral perturbation theory
  - spin-off: use of partially twisted boundary conditions also for flavour-diagonal quantities

A simple example - the flavour diagonal vector 2pt function

Sharpe, Shoresh, Phys. Rev. D 62 (2000)
Della Morte, AJ JHEP 1011 (2010) 154

say we were interested in the strange quark vector 2pt function

$$\langle \bar{s}\gamma_\mu s\, \bar{s}\gamma_\nu s \rangle \equiv \langle j_\mu^{ss} j_\nu^{ss} \rangle$$

A simple example - the flavour diagonal vector 2pt function

Sharpe, Shoresh, Phys. Rev. D 62 (2000)
Della Morte, AJ JHEP 1011 (2010) 154

say we were interested in the strange quark vector 2pt function

$$\langle \bar{s}\gamma_\mu s\,\bar{s}\gamma_\nu s\rangle \equiv \langle j_\mu^{ss} j_\nu^{ss}\rangle$$

Wick contraction leads to connected and disconnected piece:

$$\text{conn. piece} \rightarrow \left\langle \mathrm{Tr}\left\{ S_s(x,0)\gamma_\nu S_s(0,x)\gamma_\mu \right\}\right\rangle$$

$$= \left\langle \bar{s}(0)\gamma_\nu r(0)\bar{r}(x)\gamma_\mu s(x)\right\rangle \qquad = \langle j_\mu^{sr} j_\nu^{rs}\rangle$$

$$\text{disc. piece} \rightarrow \left\langle \mathrm{Tr}\left\{ S_s(x,x)\gamma_\nu \right\}\right\rangle$$

$$= \left\langle \bar{s}(0)\gamma_\nu s(0)\bar{r}(x)\gamma_\mu r(x)\right\rangle \qquad = \langle j_\mu^{ss} j_\nu^{rr}\rangle$$

A simple example - the flavour diagonal vector 2pt function

Sharpe, Shoresh, Phys. Rev. D 62 (2000)
Della Morte, AJ JHEP 1011 (2010) 154

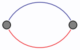
say we were interested in the strange quark vector 2pt function

$$\langle \bar{s}\gamma_\mu s\, \bar{s}\gamma_\nu s\rangle \equiv \langle j_\mu^{ss} j_\nu^{ss}\rangle$$

Wick contraction leads to connected and disconnected piece:

conn. piece $\rightarrow \left\langle \mathrm{Tr}\left\{ S_s(x,0)\gamma_\nu S_r(0,x)\gamma_\mu \right\}\right\rangle$

$$= \left\langle \bar{s}(0)\gamma_\nu r(0)\bar{r}(x)\gamma_\mu s(x)\right\rangle \quad = \quad \langle j_\mu^{sr} j_\nu^{rs}\rangle$$



disc. piece $\rightarrow \left\langle \mathrm{Tr}\left\{ S_s(x,x)\gamma_\nu \right\}\right\rangle$

$$= \left\langle \bar{s}(0)\gamma_\nu s(0)\bar{r}(x)\gamma_\mu r(x)\right\rangle \quad = \quad \langle j_\mu^{ss} j_\nu^{rr}\rangle$$

A simple example - the flavour diagonal vector 2pt function

Sharpe, Shoresh, Phys. Rev. D 62 (2000)
Della Morte, AJ JHEP 1011 (2010) 154

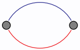
say we were interested in the strange quark vector 2pt function

$$\langle \bar{s}\gamma_\mu s\, \bar{s}\gamma_\nu s\rangle \equiv \langle j_\mu^{ss} j_\nu^{ss}\rangle$$

Wick contraction leads to connected and disconnected piece:

conn. piece $\rightarrow \left\langle \mathrm{Tr}\left\{ S_s(x,0)\gamma_\nu S_s(0,x)\gamma_\mu \right\}\right\rangle$

$$= \left\langle \bar{s}(0)\gamma_\nu r(0)\bar{r}(x)\gamma_\mu s(x)\right\rangle \quad = \quad \langle j_\mu^{sr} j_\nu^{rs}\rangle$$



disc. piece $\rightarrow \left\langle \mathrm{Tr}\left\{ S_s(x,x)\gamma_\nu \right\}\mathrm{Tr}\left\{ S_s(0,0)\gamma_\mu \right\}\right\rangle$

$$= \left\langle \bar{s}(0)\gamma_\nu s(0)\bar{r}(x)\gamma_\mu r(x)\right\rangle \quad = \quad \langle j_\mu^{ss} j_\nu^{rr}\rangle$$

A simple example - the flavour diagonal vector 2pt function

Sharpe, Shoresh, Phys. Rev. D 62 (2000)
Della Morte, AJ JHEP 1011 (2010) 154

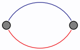
say we were interested in the strange quark vector 2pt function

$$\langle \bar{s}\gamma_\mu s\, \bar{s}\gamma_\nu s \rangle \equiv \langle j_\mu^{ss} j_\nu^{ss} \rangle$$

Wick contraction leads to connected and disconnected piece:

conn. piece $\rightarrow \left\langle \mathrm{Tr}\left\{ S_s(x,0)\gamma_\nu S_s(0,x)\gamma_\mu \right\} \right\rangle$

$$= \left\langle \bar{s}(0)\gamma_\nu r(0)\bar{r}(x)\gamma_\mu s(x) \right\rangle \qquad = \qquad \langle j_\mu^{sr} j_\nu^{rs} \rangle$$



disc. piece $\rightarrow \left\langle \mathrm{Tr}\left\{ S_s(x,x)\gamma_\nu \right\} \mathrm{Tr}\left\{ S_r(0,0)\gamma_\mu \right\} \right\rangle$

$$= \left\langle \bar{s}(0)\gamma_\nu s(0)\bar{r}(x)\gamma_\mu r(x) \right\rangle \qquad = \qquad \langle j_\mu^{ss} j_\nu^{rr} \rangle$$

adding a valence quark

Della Morte, AJ JHEP 1011 (2010) 154

$$\langle j_\mu^{ss} j_\nu^{ss} \rangle = \langle j_\mu^{sr} j_\nu^{rs} \rangle + \langle j_\mu^{ss} j_\nu^{rr} \rangle$$

- we added an artificial valence quark r with $m_r = m_s$
- that's merely a book-keeping trick allowing to write the connected and disconnected part as individual correlation functions

adding a valence quark

Della Morte, AJ JHEP 1011 (2010) 154

$$\langle j_\mu^{ss} j_\nu^{ss} \rangle = \langle j_\mu^{sr} j_\nu^{rs} \rangle + \langle j_\mu^{ss} j_\nu^{rr} \rangle$$

- we added an artificial valence quark r with $m_r = m_s$
- that's merely a book-keeping trick allowing to write the connected and disconnected part as individual correlation functions
- field theoretically meaningful in the frame work of partially quenched QCD

$$L_{\rm QCD}^{2+1} \to L_{\rm QCD}^{2+1} + \bar{r}(\not{D} + m_s)r + \tilde{r}^\dagger(\not{D} + m_s)\tilde{r} = L_{\rm PQQCD}$$

adding a valence quark

Della Morte, AJ JHEP 1011 (2010) 154

$$\langle j_\mu^{ss} j_\nu^{ss} \rangle = \langle j_\mu^{sr} j_\nu^{rs} \rangle + \langle j_\mu^{ss} j_\nu^{rr} \rangle$$

- we added an artificial valence quark r with $m_r = m_s$
- that's merely a book-keeping trick allowing to write the connected and disconnected part as individual correlation functions
- field theoretically meaningful in the frame work of partially quenched QCD

$$L_{\rm QCD}^{2+1} \to L_{\rm QCD}^{2+1} + \bar{r}(\not{D} + m_s)r + \tilde{r}^\dagger(\not{D} + m_s)\tilde{r} = L_{\rm PQQCD}$$

- $\tilde{r}$ is a commuting spin 1/2 ghost field
- the PQQCD partition function is the one of QCD
- any quark-n-point correlation function can be decomposed in this way (one may have to add many bookkeeping quarks)

scalar pion form factor

AJ JHEP 1201 (2012) 007

$$\langle \pi^i(p')|\bar{u}u + \bar{d}d|\pi^j(p)\rangle = \delta^{ij} F_{S,2}(t)$$

- probing a pion with a scalar current
- interesting for understanding low-energy QCD, related to $I = 0$ $\pi\pi$ phase shift

scalar pion form factor

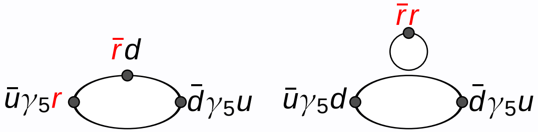
AJ JHEP 1201 (2012) 007

$$\langle \pi^i(p')|\bar{u}u + \bar{d}d|\pi^j(p)\rangle = \delta^{ij} F_{S,2}(t)$$

- probing a pion with a scalar current
- interesting for understanding low-energy QCD, related to $I = 0$ $\pi\pi$ phase shift

$$\langle O_{ud}\, S_{dd} O_{du}\rangle = \langle O_{ur}\, S_{rd} O_{du}\rangle + \langle O_{ud}\, S_{rr} O_{du}\rangle$$

full — connected — disconnected



In the effective theory

AJ JHEP 1201 (2012) 007

- at this point decomposition in PQQCD achieved but nothing gained in practice

In the effective theory

AJ JHEP 1201 (2012) 007

- at this point decomposition in PQQCD achieved but nothing gained in practice
- idea: compute expressions for connected and disconnected piece in PQχPT and find correlation between both parts that allow for predicting one through the other

In the effective theory

AJ JHEP 1201 (2012) 007

- at this point decomposition in PQQCD achieved but nothing gained in practice
- idea: compute expressions for connected and disconnected piece in PQχPT and find correlation between both parts that allow for predicting one through the other
- for the $N_f = 2$ form factor one additional r quark is sufficient $SU(2) \to$ PQ symmetry group is $SU(2+1|1)$

In the effective theory

AJ JHEP 1201 (2012) 007

- at this point decomposition in PQQCD achieved but nothing gained in practice
- idea: compute expressions for connected and disconnected piece in PQχPT and find correlation between both parts that allow for predicting one through the other
- for the $N_f = 2$ form factor one additional r quark is sufficient $SU(2) \to$ PQ symmetry group is $SU(2+1|1)$
  - $SU(2+1|1)$ has 15 generators T^i, $\psi^T = (u,d,r,\tilde{r})$
  - $$\begin{aligned} O_{ud} &= \bar{\psi} \quad T^1 \quad \gamma_5\, \psi \\ O_{ur} &= \bar{\psi} \quad T^4 \quad \gamma_5\, \psi \\ S_{rr} &= \bar{\psi} \left(T^0 - \frac{2S^8 + S^{15}}{\sqrt{3}} \right) \psi \end{aligned}$$

In the effective theory

Gasser, Leutwyler, Annl. of Phys. 1984, Nucl. Phys. B250 1985
Bernard, Golterman Phys. Rev. D 46 (1992), Phys. Rev. D 49 (1994)
Sharpe, Shoresh, Phys. Rev. D 62 (2000)

$$\mathcal{L}^{(2)} = \frac{F^2}{4}\mathrm{Str}\left\{\partial_\mu U \partial^\mu U^\dagger\right\} + \frac{F^2}{4}\mathrm{Str}\left\{\chi U^\dagger + U\chi^\dagger\right\}$$

In the effective theory

Gasser, Leutwyler, Annl. of Phys. 1984, Nucl. Phys. B250 1985
Bernard, Golterman Phys. Rev. D 46 (1992), Phys. Rev. D 49 (1994)
Sharpe, Shoresh, Phys. Rev. D 62 (2000)

$$
\begin{aligned}
\mathcal{L}^{(2)} &= \frac{F^2}{4}\mathrm{Str}\left\{\partial_\mu U \partial^\mu U^\dagger\right\} + \frac{F^2}{4}\mathrm{Str}\left\{\chi U^\dagger + U\chi^\dagger\right\} \\
\mathcal{L}^{(4)} &= L_4 \mathrm{Str}\left\{\partial_\mu U (\partial^\mu U)^\dagger\right\}\mathrm{Str}\left\{\chi U^\dagger + U\chi^\dagger\right\} \\
&\quad + L_5 \mathrm{Str}\left\{\left(\partial_\mu U (\partial^\mu U)^\dagger\right)\left(\chi U^\dagger + U\chi^\dagger\right)\right\} \\
&\quad + L_6 \mathrm{Str}\left\{\chi U^\dagger + U\chi^\dagger\right\}^2 + L_8 \mathrm{Str}\left\{U\chi^\dagger U\chi^\dagger + \chi U^\dagger \chi U^\dagger\right\}^2 .
\end{aligned}
$$

there are more terms than in the original $SU(2)$ Lagrangian because some trace-identities of $SU(2)$ don't hold in $SU(2+1|1)$

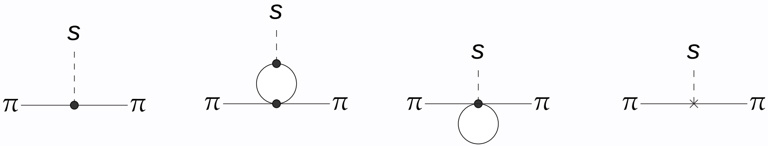
In the effective theory

Gasser, Leutwyler, Annl. of Phys. 1984, Nucl. Phys. B250 1985
Bernard, Golterman Phys. Rev. D 46 (1992), Phys. Rev. D 49 (1994)
Sharpe, Shoresh, Phys. Rev. D 62 (2000)

$$\mathcal{L}^{(2)} = \frac{F^2}{4}\mathrm{Str}\left\{\partial_\mu U \partial^\mu U^\dagger\right\} + \frac{F^2}{4}\mathrm{Str}\left\{\chi U^\dagger + U\chi^\dagger\right\}$$

$$\begin{aligned}\mathcal{L}^{(4)} = \; & L_4 \mathrm{Str}\left\{\partial_\mu U (\partial^\mu U)^\dagger\right\}\mathrm{Str}\left\{\chi U^\dagger + U\chi^\dagger\right\} \\ & + L_5 \mathrm{Str}\left\{\left(\partial_\mu U (\partial^\mu U)^\dagger\right)\left(\chi U^\dagger + U\chi^\dagger\right)\right\} \\ & + L_6 \mathrm{Str}\left\{\chi U^\dagger + U\chi^\dagger\right\}^2 + L_8 \mathrm{Str}\left\{U\chi^\dagger U\chi^\dagger + \chi U^\dagger \chi U^\dagger\right\}^2 .\end{aligned}$$

there are more terms than in the original $SU(2)$ Lagrangian because some trace-identities of $SU(2)$ don't hold in $SU(2+1|1)$



Results: the form factor $N_f = 2$

AJ JHEP 1201 (2012) 007

At NLO:

$$F^{\mathrm{F}}_{S,2}(t) = 2B\left\{1 + \frac{1}{F^2}\left(-\frac{1}{2}\bar{A}(m_\pi^2) + \Lambda_2^{\mathrm{F}} + \frac{1}{2}(2t - \; m_\pi^2)\bar{B}(m_\pi^2, t)\right)\right\}$$

$$\Lambda_2^{\mathrm{F}} = t \; l_4^r + 4m_\pi^2 \; l_3^r \,,$$

Results: the form factor $N_f = 2$

AJ JHEP 1201 (2012) 007

At NLO:

$$F^{\rm F}_{S,2}(t) = 2B\left\{1 + \frac{1}{F^2}\left(-\frac{1}{2}\bar{A}(m_\pi^2) + \Lambda_2^{\rm F} + \frac{1}{2}(2t - \ m_\pi^2)\bar{B}(m_\pi^2, t)\right)\right\}$$

$$F^{\rm C}_{S,2}(t) = 2B\left\{1 + \frac{1}{F^2}\left(-\frac{1}{2}\bar{A}(m_\pi^2) + \Lambda_2^{\rm C} + \frac{1}{2}(\ t - 2m_\pi^2)\bar{B}(m_\pi^2, t)\right)\right\}$$

$$\Lambda_2^{\rm F} = t \ \ l_4^r \qquad\qquad + 4m_\pi^2 \ \ l_3^r \, ,$$

$$\Lambda_2^{\rm C} = t(l_4^r - 8\tilde{L}_4^r) + 4m_\pi^2(l_3^r \ \ + 4\tilde{L}_4^r - 8\tilde{L}_6^r)\, ,$$

Results: the form factor $N_f = 2$

AJ JHEP 1201 (2012) 007

At NLO:

$$F^{\mathrm{F}}_{S,2}(t) = 2B\Big\{1 + \tfrac{1}{F^2}\Big(-\tfrac{1}{2}\bar{A}(m_\pi^2) + \Lambda_2^{\mathrm{F}} + \tfrac{1}{2}(2t - m_\pi^2)\bar{B}(m_\pi^2, t)\Big)\Big\}$$

$$F^{\mathrm{C}}_{S,2}(t) = 2B\Big\{1 + \tfrac{1}{F^2}\Big(-\tfrac{1}{2}\bar{A}(m_\pi^2) + \Lambda_2^{\mathrm{C}} + \tfrac{1}{2}(t - 2m_\pi^2)\bar{B}(m_\pi^2, t)\Big)\Big\}$$

$$F^{\mathrm{D}}_{S,2}(t) = 2B\Big\{0 + \tfrac{1}{F^2}\Big(\Lambda_2^{\mathrm{D}} + \tfrac{1}{2}(t + m_\pi^2)\bar{B}(m_\pi^2, t)\Big)\Big\}$$

$$\Lambda_2^{\mathrm{F}} = t\; l_4^r + 4m_\pi^2\; l_3^r\,,$$

$$\Lambda_2^{\mathrm{C}} = t(l_4^r - 8\tilde{L}_4^r) + 4m_\pi^2(l_3^r + 4\tilde{L}_4^r - 8\tilde{L}_6^r)\,,$$

$$\Lambda_2^{\mathrm{D}} = t(+8\tilde{L}_4^r) + 4m_\pi^2(-4\tilde{L}_4^r + 8\tilde{L}_6^r)\,.$$

Results: the form factor $N_f = 2$

AJ JHEP 1201 (2012) 007

At NLO:

$$F^{\rm F}_{S,2}(t) = 2B\Big\{1 + \tfrac{1}{F^2}\Big(- \tfrac{1}{2}\bar{A}(m_\pi^2) + \Lambda_2^{\rm F} + \tfrac{1}{2}(2t - m_\pi^2)\bar{B}(m_\pi^2, t)\Big)\Big\}$$

$$F^{\rm C}_{S,2}(t) = 2B\Big\{1 + \tfrac{1}{F^2}\Big(- \tfrac{1}{2}\bar{A}(m_\pi^2) + \Lambda_2^{\rm C} + \tfrac{1}{2}(t - 2m_\pi^2)\bar{B}(m_\pi^2, t)\Big)\Big\}$$

$$F^{\rm D}_{S,2}(t) = 2B\Big\{0 + \tfrac{1}{F^2}\Big(\Lambda_2^{\rm D} + \tfrac{1}{2}(t + m_\pi^2)\bar{B}(m_\pi^2, t)\Big)\Big\}$$

- kinematics of connected and disconnected contributions differ
- different LEC ($\Lambda_2^{\rm F,C,D}$) contribute to the respective parts
- $\Lambda_2^{\rm F} = \Lambda_2^{\rm C} + \Lambda_2^{\rm D}$

Results: the form factor $N_f = 2$

AJ JHEP 1201 (2012) 007

At NLO:

$$F^{\mathrm{F}}_{S,2}(t) = 2B\Big\{1 + \frac{1}{F^2}\Big(- \frac{1}{2}\bar{A}(m_\pi^2) + \Lambda_2^{\mathrm{F}} + \frac{1}{2}(2t - m_\pi^2)\bar{B}(m_\pi^2, t)\Big)\Big\}$$

$$F^{\mathrm{C}}_{S,2}(t) = 2B\Big\{1 + \frac{1}{F^2}\Big(- \frac{1}{2}\bar{A}(m_\pi^2) + \Lambda_2^{\mathrm{C}} + \frac{1}{2}(t - 2m_\pi^2)\bar{B}(m_\pi^2, t)\Big)\Big\}$$

$$F^{\mathrm{D}}_{S,2}(t) = 2B\Big\{0 + \frac{1}{F^2}\Big(\Lambda_2^{\mathrm{D}} + \frac{1}{2}(t + m_\pi^2)\bar{B}(m_\pi^2, t)\Big)\Big\}$$

$$\begin{aligned}
l_3^r &= 4(-8L_4^r - 4L_5^r + 16L_6^r + 8L_8^r) - \frac{1}{18}\tilde{L}(m_\eta^2)\,, \\
l_4^r &= 8L_4^r + 4L_5^r - \frac{1}{4}\tilde{L}(m_K^2)\,, \\
4\tilde{L}_4^r - \tilde{8}L_6^r &= 4L_4^r - 8L_6^r - \frac{5}{4}\frac{1}{18}\tilde{L}(m_\eta^2) - \frac{1}{12}\frac{1}{16\pi^2}\,, \\
\tilde{L}_4^r &= L_4^r\,,
\end{aligned}$$

Results: the form factor $N_f = 2$

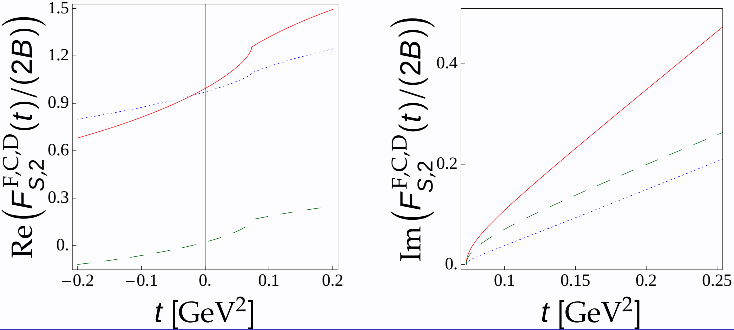
AJ JHEP 1201 (2012) 007

At NLO:

$$F^{\rm F}_{S,2}(t) = 2B\Big\{1 + \tfrac{1}{F^2}\Big(-\tfrac{1}{2}\bar{A}(m_\pi^2) + \Lambda_2^{\rm F} + \tfrac{1}{2}(2t - m_\pi^2)\bar{B}(m_\pi^2,t)\Big)\Big\}$$

$$F^{\rm C}_{S,2}(t) = 2B\Big\{1 + \tfrac{1}{F^2}\Big(-\tfrac{1}{2}\bar{A}(m_\pi^2) + \Lambda_2^{\rm C} + \tfrac{1}{2}(t - 2m_\pi^2)\bar{B}(m_\pi^2,t)\Big)\Big\}$$

$$F^{\rm D}_{S,2}(t) = 2B\Big\{0 + \tfrac{1}{F^2}\Big(\Lambda_2^{\rm D} + \tfrac{1}{2}(t + m_\pi^2)\bar{B}(m_\pi^2,t)\Big)\Big\}$$

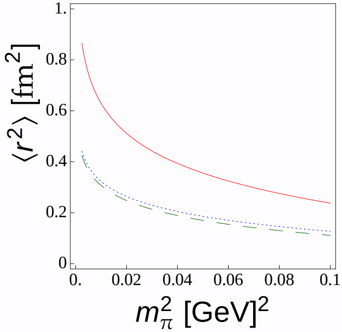


Results: the scalar radius $N_f = 2$

$$F_S(t) = F_S(0)\left(1 + \frac{1}{6}\langle r^2\rangle t + O(t^2)\right)$$

Results: the scalar radius $N_f = 2$

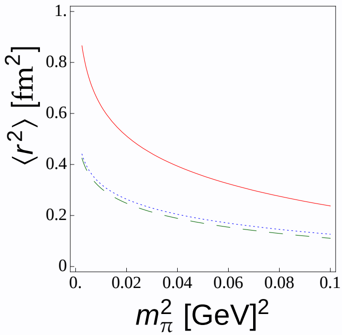
$$F_S(t) = F_S(0)\left(1 + \frac{1}{6}\langle r^2\rangle t + O(t^2)\right)$$



- disconnected and connected contributions are of similar size in the radius

Results: the scalar radius $N_f = 2$

$$F_S(t) = F_S(0)\left(1 + \frac{1}{6}\langle r^2\rangle t + O(t^2)\right)$$



- disconnected and connected contributions are of similar size in the radius!

Results: the form factor $N_f = 2 + 1$

$$\begin{aligned}
\langle \pi^i | \bar{u}u + \bar{d}d | \pi^j \rangle &= \delta^{ij} F_{S,2+1}(t) \\
\langle \pi^i | \bar{u}u + \bar{d}d - 2\bar{s}s | \pi^j \rangle &= \delta^{ij} F^8_{S,2+1}(t) \\
\langle \pi^i | \bar{u}u + \bar{d}d + \bar{s}s | \pi^j \rangle &= \delta^{ij} F^0_{S,2+1}(t)
\end{aligned}$$

- the qualitative picture does not change with resp. to $N_f = 2$
- disconnected piece depends on L_4 and L_6 while connected piece depends on L_4, L_5, L_6 and L_8
- $SU(3)$-symmetry forces the prediction for the disconnected contribution to the octet form factor to be parameter-free

$$F^{\mathrm{D},8}_{S,3}(t) = \frac{2B}{F^2}\left\{ -\frac{m_\pi^2}{6}\bar{B}(m_\eta^2, t) + \frac{1}{2}(t + m_\pi^2)\bar{B}(m_\pi^2, t) - \frac{t}{2}\bar{B}(m_K^2, t) - \frac{m_\pi^2}{3}\bar{B}(m_\eta^2, m_\pi^2, t)\right\}$$

An aside - twisted boundary conditions

Bedaque PLB 593 2004, Bedaque, Chen PLB 616 2005, Sachrajda, Villadoro PLB 609 2005

- flavour twisted boundary conditions allow for inducing arbitrary momentum in hadrons in a finite volume
  $\psi(x + L\hat{e}_i) = e^{i\theta_i/L}\psi(x)$
- for a charged pion $E_\pi = \sqrt{m_\pi^2 + (\vec{\theta}_u - \vec{\theta}_d)^2/L^2}$
- for a neutral pion the twists cancel
- similarly the net-effect of twisting in the scalar form factor calculation cancels (forward matrix element)

An aside - twisted boundary conditions

- flavour twisted boundary conditions allow for inducing arbitrary momentum in hadrons in a finite volume
  $\psi(x + L\hat{e}_i) = e^{i\theta_i/L}\psi(x)$

Bedaque PLB 593 2004, Bedaque, Chen PLB 616 2005, Sachrajda, Villadoro PLB 609 2005

- for a charged pion $E_\pi = \sqrt{m_\pi^2 + (\vec{\theta}_u - \vec{\theta}_d)^2/L^2}$
- for a neutral pion the twists cancel
- similarly the net-effect of twisting in the scalar form factor calculation cancels (forward matrix element)
- after the decomposition

$$\langle O_{ud}\, S_{dd} O_{du}\rangle = \langle O_{ur}\, S_{rd} O_{du}\rangle + \langle O_{ud}\, S_{rr} O_{du}\rangle$$

  twisting has a net effect for the connected part

- *at least the connected part can be computed with a very fine momentum resolution*

Conclusions

- quark disconnected Wick contractions can be parameterised in chiral perturbation theory
- the method works for all low energy processes where chiral perturbation theory is applicable
- scenarios:
  - simulate connected only and extract physics (LECs)
  - simulate connected and predict disconnected
  - simulate disconnected for subset of parameters and predict (extrapolate) disconnected for others
  - . . .

Conclusions

- quark disconnected Wick contractions can be parameterised in chiral perturbation theory
- the method works for all low energy processes where chiral perturbation theory is applicable
- scenarios:
  - simulate connected only and extract physics (LECs)
  - simulate connected and predict disconnected
  - simulate disconnected for subset of parameters and predict (extrapolate) disconnected for others
  - ...
- large disconnected contribution to the scalar radius - disconnected diagrams need not be small!
- twisted boundary conditions applicable for connected part
  compute connected, predict disconnected

Conclusions

- quark disconnected Wick contractions can be parameterised in chiral perturbation theory
- the method works for all low energy processes where chiral perturbation theory is applicable
- scenarios:
  - simulate connected only and extract physics (LECs)
  - simulate connected and predict disconnected
  - simulate disconnected for subset of parameters and predict (extrapolate) disconnected for others
  - . . .
- large disconnected contribution to the scalar radius - disconnected diagrams need not be small!
- twisted boundary conditions applicable for connected part
  compute connected, predict disconnected
- a numerical check? See Vera Gülper's talk

Thursday 3:50 - 4:10pm
(Hadron Structure, meeting room 6)

The research leading to these results has received funding from the European Research Council under the European Community's Seventh Framework Programme (FP7/2007-2013) ERC grant agreement No 279757