Dirac-mode expansion analysis for Polyakov loop

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Collaborators: S.Gongyo and H.Suganuma (Kyoto Univ.)

- Reference: S.Gongyo, TI, and H.Suganuma, arXiv:1202.4130.

Chiral Symmetry Breaking and Confinement

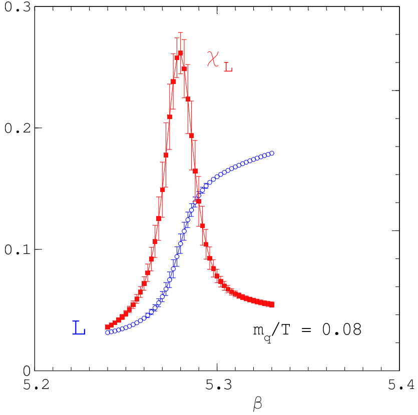


Figure: Polyakov loop

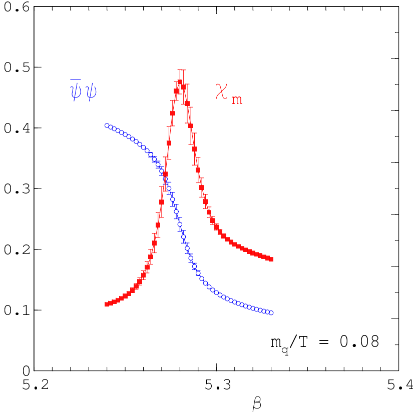


Figure: chiral condensate

Fig. F.Karsch ('02)

simultaneous chiral and deconfined phase transtion at finite temperature

Chiral Symmetry Breaking and Dirac Eigenmodes

- Dirac eigenmode $\Rightarrow$ chiral symmetry breaking

Banks-Casher Relation — Banks, Casher ('80)

$$\langle \bar{q}q \rangle = -\lim_{m\to 0}\lim_{V\to\infty}\frac{\pi}{V}\langle \rho(0)\rangle$$

$\rho(\lambda)$:Dirac spectrum

- What about the correspondence between Dirac eigenmode and confinement ?

Dirac Eigenmode and Confinement

Related Works

- Dirac spectrum sum and Polyakov loop

  — Gattringer ('06), Bruckmann, Gattringer, Hagen ('07), Synatschke, Wipf, Langfeld ('08), . . .

  Polyakov loop $\Longrightarrow$ Dirac spectrum sum with various the boundary condition
- Dirac eigenmode and meson mass — Lang, Schröck ('11)

  removing low-lying Dirac eigenmodes from quark propagator

  $\Longrightarrow$ meson remains as bound state
- Dirac eigenmode and interquark potential — Gongyo, TI, Suganuma ('11)

  expanding Wilson loop in terms of Dirac eigenmode

In this talk

Polyakov loop in terms of Dirac eigenmodes

Dirac Operator and Eigenmode

Dirac operator $\not{D} \equiv \gamma^\mu D_\mu$

- Dirac eigenstate $|n\rangle$ and Dirac eigenvalue λ_n

$$\not{D}|n\rangle = \lambda_n|n\rangle$$

- Dirac eigenfunction

$$\langle x|n\rangle = \psi_n(x)$$

$$\not{D}\psi_n(x) = \lambda_n\psi_n(x)$$

We calculate Dirac operator, eigenvalue λ_n and eigenfunction $\psi_n(x)$ numerically in lattice QCD.

Dirac Eigenmode Matrix Element

Link-variable operator $\hat{U}_\mu$

$$\langle x|\hat{U}_\mu|y\rangle = U_\mu(x)\delta_{x+\hat{\mu},y}$$

matrix element of the link-variable operator $\langle m|\hat{U}_\mu|n\rangle$

$$\langle m|\hat{U}_\mu|n\rangle = \sum_x \langle m|x\rangle\langle x|\hat{U}_\mu|x+\hat{\mu}\rangle\langle x+\hat{\mu}|n\rangle = \sum_x \psi_m^\dagger(x)U_\mu(x)\psi_n(x+\hat{\mu})$$

- $|n\rangle$: Dirac eigenstate, $\psi_n(x)$:Dirac eigenfunction

$$D|n\rangle = \lambda_n|n\rangle, \qquad D\psi_n = \lambda_n\psi_n$$

- matrix element : $\langle m|\hat{U}_\mu|n\rangle$ $\Longrightarrow$ gauge invariant

$$\begin{aligned}\langle m|\hat{U}_\mu|n\rangle &\to \sum \psi_m^\dagger(x)V^\dagger(x)V(x)U_\mu(x)V^\dagger(x+\hat{\mu})V(x+\hat{\mu})\psi_n(x+\hat{\mu}) \\ &= \sum \psi_m^\dagger(x)U_\mu(x)\psi_n(x+\hat{\mu}) \\ &= \langle m|\hat{U}_\mu|n\rangle\end{aligned}$$

Dirac Eigenmode Projection Operator

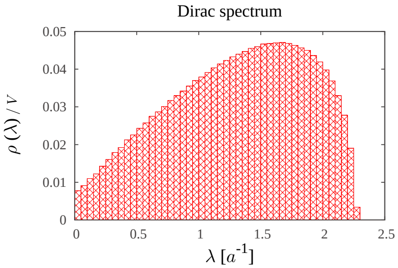
projection operator

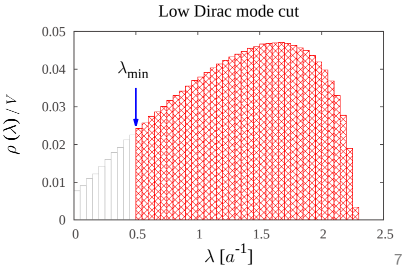
$$\hat{P} \equiv \sum_{n \in A} |n\rangle\langle n|$$

A is subset of full Dirac eigenmodes. ex. IR-mode cut $n > N_{\mathrm{IR}}$

Projected link-variable operator

$$\hat{U}_\mu^P \equiv \hat{P}\hat{U}_\mu\hat{P} = \sum_{n \in A}\sum_{m \in A} |m\rangle\langle m|\hat{U}_\mu|n\rangle\langle n|$$





Polyakov Loop Operator in Operator Formalism

Link-variable operator $\hat{U}_\mu$

$$\langle x|\hat{U}_\mu|y\rangle = U_\mu(x)\delta_{x+\hat{\mu},y}$$

Polyakov loop operator in operator formalism

$$L_P \equiv \frac{1}{3V}\mathrm{Tr}\prod^{N_t}\hat{U}_4$$

V:spatial volume, N_t:temporal size

coincides with standard definition

$$\begin{aligned}
L_P &= \frac{1}{3V}\mathrm{Tr}\hat{U}_4\hat{U}_4\cdots\hat{U}_4 \\
&= \frac{1}{3V}\mathrm{tr}\sum_x \langle\vec{x}|\hat{U}_4|\vec{x}+\hat{4}\rangle\langle\vec{x}+\hat{4}|\hat{U}_4|\vec{x}+2\cdot\hat{4}\rangle\cdots\langle\vec{x}+N_t\hat{4}|\hat{U}_4|\vec{x}\rangle \\
&= \frac{1}{3V}\mathrm{tr}\sum_x U_4(\vec{x},1)U_4(\vec{x},2)\cdots U_4(\vec{x},N_t)
\end{aligned}$$

Dirac-mode Projected Polyakov Loop

Dirac-mode projected link-variable operator

$$\hat{U}_4^P \equiv \hat{P}\hat{U}_4\hat{P} = \sum_{n\in A}\sum_{m\in A} |m\rangle\langle m|\hat{U}_4|n\rangle\langle n|$$

$|m\rangle$:Dirac-mode eigenstate

IR/UV Dirac-mode projected Polyakov loop

$$\langle L_P\rangle_{\mathrm{IR/UV}} \equiv \frac{1}{3V}\langle \mathrm{Tr}\prod^{N_t}\hat{U}_4^P\rangle$$
$$=\frac{1}{3V}\mathrm{tr}\sum_{n_1\in A}\sum_{n_2\in A}\cdots\sum_{n_{N_t}\in A}\langle n_1|\hat{U}_4|n_2\rangle\langle n_2|\hat{U}_4|n_3\rangle\cdots\langle n_{N_t}|\hat{U}_4|n_1\rangle$$

Procedure of Polyakov Loop Analysis

1.Evaluate Dirac operator eigenfunction (KS-type fermion)

$$[D]_{x,y} = \frac{1}{2}\sum_{\mu=1}^{4}\eta_\mu(x)\left[U_\mu(x)\delta_{x+\mu,y} - U_\mu^\dagger(x-\hat{\mu})\delta_{x-\mu,y}\right]$$

$\eta_1(x) \equiv 1$, $\eta_\mu(x) = (-1)^{x_1+\cdots x_{\mu-1}}$ for $\mu \geq 2$

$$[D]_{x,y}\psi_n(y) = \lambda_n\psi_n(x)$$

2.Calculate Link-variable matrix element

$$\langle m|\hat{U}_\mu|n\rangle = \sum_x \psi_m^\dagger(x)U_\mu(x)\psi_n(x+\hat{\mu})$$

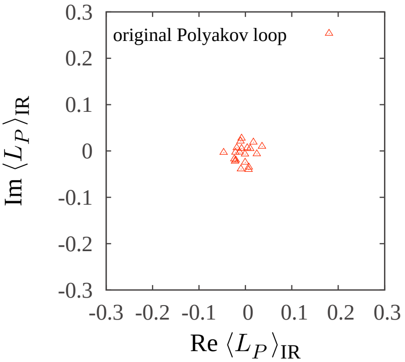
3.Calculate Polyakov Loop without IR/UV Dirac-mode

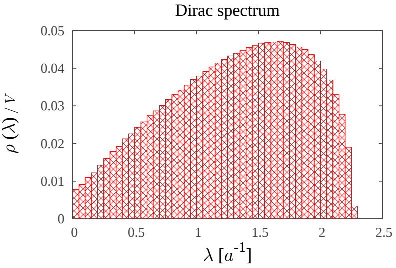
$$\langle\hat{L}_P\rangle_{\mathrm{IR/UV}} \equiv \frac{1}{3V}\langle\mathrm{Tr}\prod\hat{U}_4^P\rangle = \frac{1}{3V}\mathrm{tr}\sum_{n_1,\cdots,n_{N_t}\in A}\langle n_1|\hat{U}_4|n_2\rangle\cdots\langle n_{N_t}|\hat{U}_4|n_1\rangle$$

IR Dirac-mode Projected Polyakov Loop

IR Dirac eigenmode cut

$\langle L_P \rangle \simeq 0$





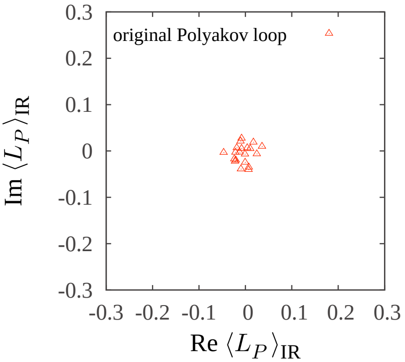
6^4 lattice with $\beta = 5.6$, $a = 0.25$fm at the quenched level

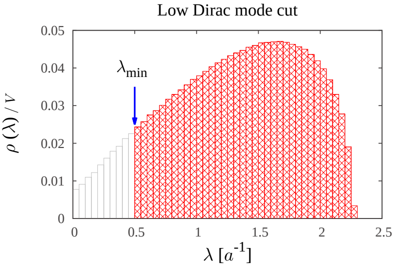
IR Dirac-mode Projected Polyakov Loop

IR Dirac eigenmode cut

- chiral condensate $|\langle \bar{q}q \rangle|$ $\Longrightarrow$ largely reduced

$\langle L_P \rangle \simeq 0$





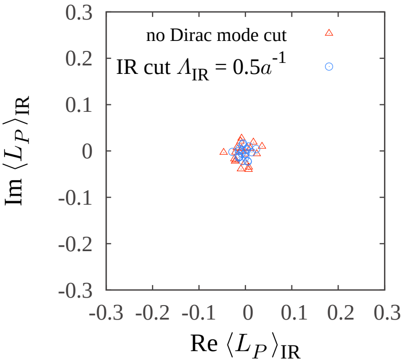
6^4 lattice with $\beta = 5.6$, $a = 0.25$fm at the quenched level

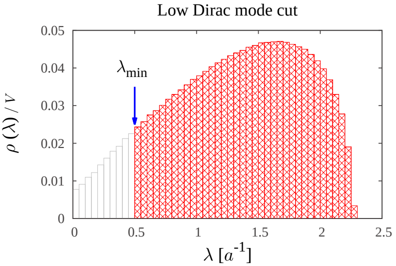
IR Dirac-mode Projected Polyakov Loop

IR Dirac eigenmode cut

- chiral condensate $|\langle \bar{q}q \rangle| \Longrightarrow$ largely reduced
- Polyakov loop $L \simeq 0 \Longrightarrow$ confining phase

$$\langle L_P \rangle \simeq 0 \Longrightarrow \langle L_P \rangle_{\rm IR} \simeq 0$$





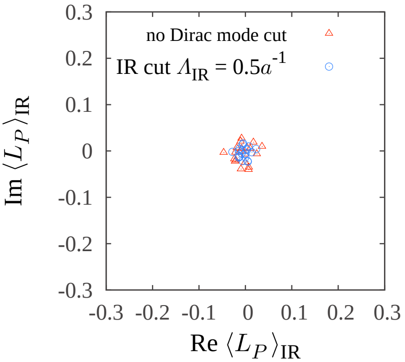
6^4 lattice with $\beta = 5.6$, $a = 0.25$fm at the quenched level

IR Dirac-mode Projected Polyakov Loop

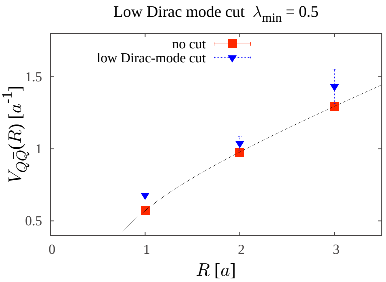
IR Dirac eigenmode cut

- chiral condensate $|\langle \bar{q}q \rangle|$ $\Longrightarrow$ largely reduced
- Polyakov loop $L \simeq 0$ $\Longrightarrow$ confining phase
- interquark potential $\Longrightarrow$ confining potential

$$\langle L_P \rangle \simeq 0 \Longrightarrow \langle L_P \rangle_{\mathrm{IR}} \simeq 0$$



$Q\bar{Q}$ potential — cf.arXiv:1202.4130



6^4 lattice with $\beta = 5.6$, $a = 0.25$fm at the quenched level

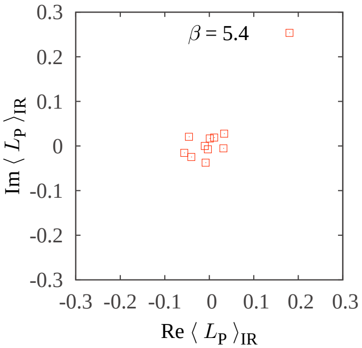
Polyakov Loop in Confined/Deconfined Phase

IR Dirac-mode cut

Polyakov loop without IR Dirac-mode

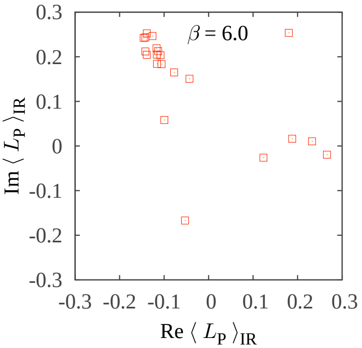
confined phase

$\langle L_P \rangle \simeq 0$



deconfined phase

$\langle L_P \rangle \neq 0$



6^4 lattice at the quenched level

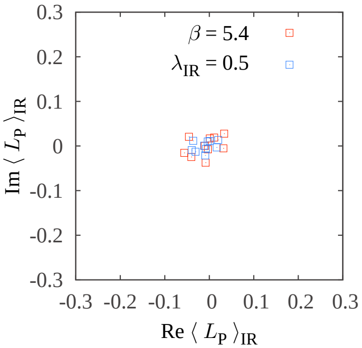
Polyakov Loop in Confined/Deconfined Phase

IR Dirac-mode cut

Polyakov loop without IR Dirac-mode $\Longrightarrow$ almost unchanged

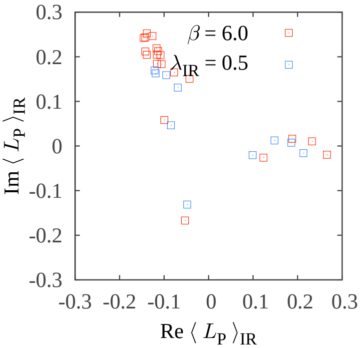
confined phase

$\langle L_P \rangle \simeq 0 \;\Longrightarrow\; \langle L_P \rangle_{\rm IR} \simeq 0$



deconfined phase

$\langle L_P \rangle \neq 0 \;\Longrightarrow\; \langle L_P \rangle_{\rm IR} \neq 0$



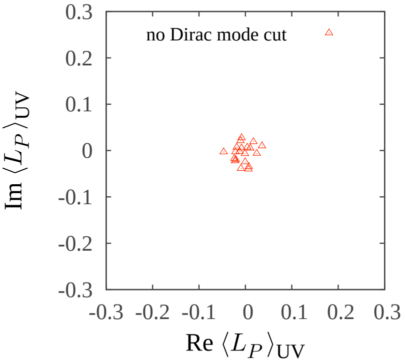
6^4 lattice at the quenched level

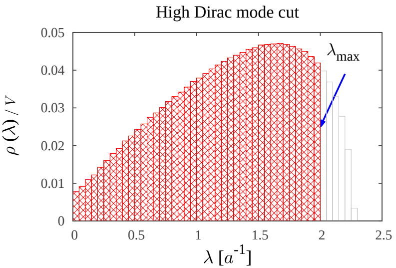
UV Dirac-mode Projected Polyakov Loop

UV Dirac eigenmode cut

- chiral condensate $|\langle\bar{\psi}\psi\rangle| \Longrightarrow$ unchanged
- Polyakov loop $L \simeq 0$

$\langle L_P \rangle \simeq 0$





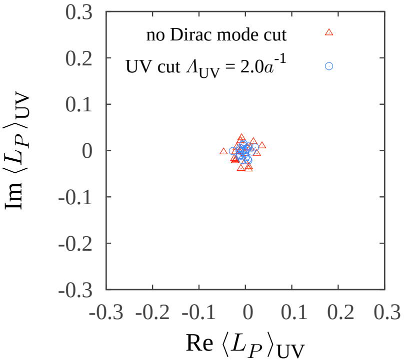
6^4 lattice with $\beta = 5.6$, $a = 0.25$fm at the quenched level

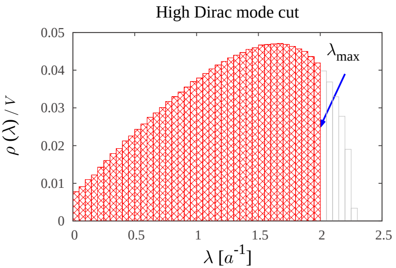
UV Dirac-mode Projected Polyakov Loop

UV Dirac eigenmode cut

- chiral condensate $|\langle\bar{\psi}\psi\rangle| \Longrightarrow$ unchanged
- Polyakov loop $L \simeq 0 \Longrightarrow$ confining phase

$$\langle L_P\rangle \simeq 0 \Longrightarrow \langle L_P\rangle_{\rm UV} \simeq 0$$

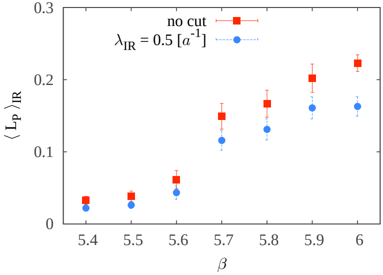




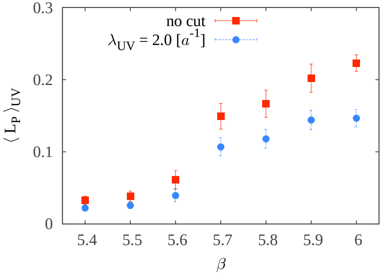
6^4 lattice with $\beta = 5.6$, $a = 0.25$fm at the quenched level

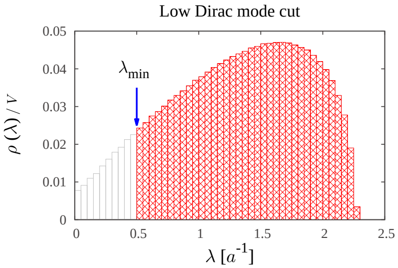
Dirac-mode Projected Polyakov Loop at Finite-T

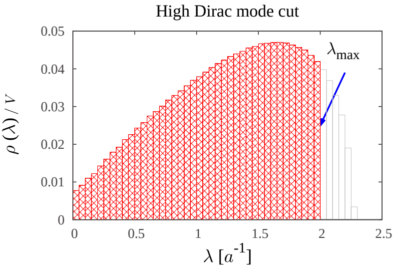
IR Dirac-mode cut



UV Dirac-mode cut







Summary

we study the Dirac-mode dependence of the Polyakov loop
using Dirac-mode expansion method in lattice QCD

Dirac-mode dependence of Polyakov loop

- no specific Dirac-mode region for zero/non-zero Polyakov loop
  - $\langle L_P \rangle \simeq 0 \Longrightarrow \langle L_P \rangle_{\rm IR} \simeq 0$ and $\langle L_P \rangle_{\rm UV} \simeq 0$
  - $\langle L_P \rangle \neq 0 \Longrightarrow \langle L_P \rangle_{\rm IR} \neq 0$ and $\langle L_P \rangle_{\rm UV} \neq 0$
- consistent with interquark potential analysis
  - confining potential persists without IR/UV Dirac eigenmode

Outlook

- Application of the Dirac-mode expansion analysis
  for other quantities.

Backup

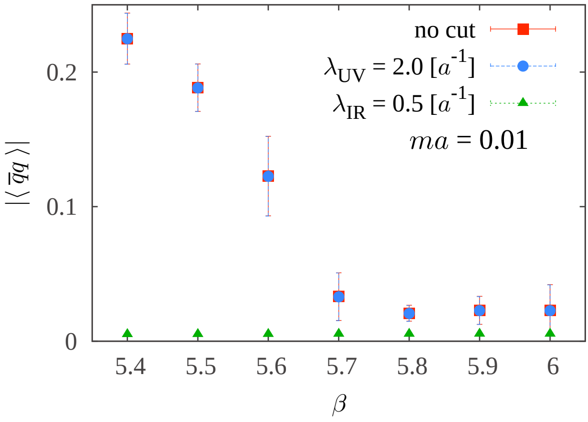
Lattice QCD Simulation Parameters

- SU(3) lattice QCD at the quenched level
- lattice size : $6^3 \times 6$ and $6^3 \times 4$
- The total number of the KS-fermion Dirac eigenmode
  $6^3 \times 6 \Longrightarrow$ 3888
  $6^3 \times 4 \Longrightarrow$ 2592
- $\beta = 5.4 \sim 6.0$

Chiral Condensate With Dirac-mode Projection

chiral condensate

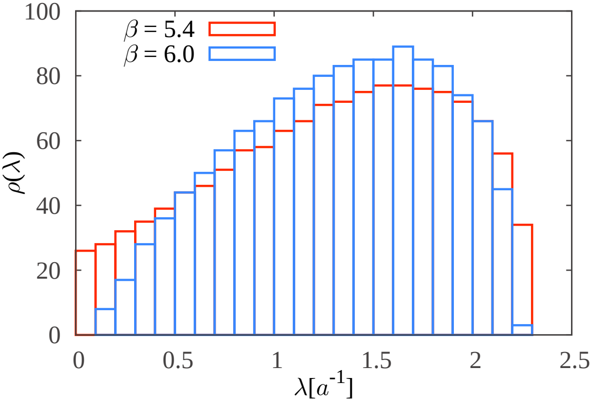
$$\langle \bar{q}q \rangle = -\frac{1}{4}\frac{1}{V}\sum_{\lambda>0}\frac{2m}{\lambda^2+m^2}$$



β-dependence of the Dirac Spectrum

Dirac spectra

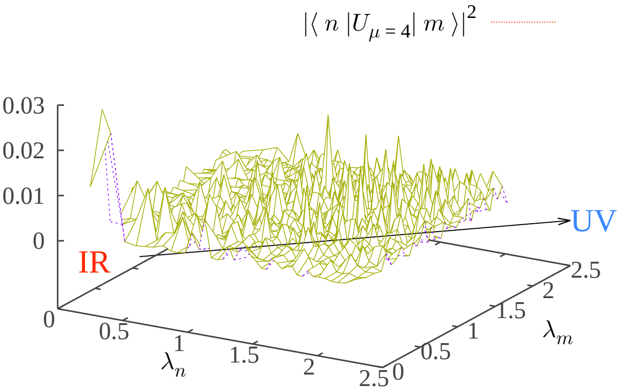
Low-lying Dirac eigenmodes vanishes for $T > T_c$



Dirac-mode Matrix Element

Absolute value of Matrix element $|\langle n|\hat{U}_4|m\rangle|$

- No characteristic behavior
- IR-mode enhancement ?



Dirac Spectrum Sum and Polyakov Loop

Gattringer ('06)

$$\langle L_P \rangle = \frac{1}{8V}\left(2\sum_{\lambda}\lambda^{N_t} - (1+i)\sum_{\lambda_+}\lambda_+^{N_t} - (1-i)\sum_{\lambda_-}\lambda_-^{N_t}\right)$$

N_t:temporal size $V = L^3 \times N_t$:lattice volume

Dirac spectra with

- λ: periodic boundary condition
- λ_+: boundary condition — phase factor $+i$
- λ_-: boundary condition — phase factor $-i$

Operator Formalism of Lattice QCD

Link-variable operator $\hat{U}_\mu$

$$\langle x|\hat{U}_\mu|y\rangle = U_\mu(x)\delta_{x+\hat{\mu},y}$$

Wilson loop operator $\hat{W}$

$$\hat{W} \equiv \prod_{k=1}^{L} \hat{U}_{\mu_k}$$

$$\begin{aligned}
\mathrm{Tr}\hat{W} &= \mathrm{tr}\sum_x \langle x|\hat{W}|x\rangle \\
&= \mathrm{tr}\sum_{x,x_2,\ldots,x_L} \langle x|\hat{U}_{\mu_1}|x_2\rangle\langle x_2|\hat{U}_{\mu_2}|x_3\rangle\cdots\langle x_L|\hat{U}_{\mu_L}|x\rangle \\
&= \mathrm{tr}\sum_x U_{\mu_1}(x)U_{\mu_2}(x+\mu_1)\cdots U_{\mu_L}(x+\sum \mu_k) \\
&= \langle W\rangle \cdot \mathrm{tr}\mathbf{1}
\end{aligned}$$

Dirac Operator and Eigenmode

Dirac operator $\not{D} \equiv \gamma^\mu D_\mu$

- eigenstate $|n\rangle$ and eigenvalue λ_n

$$\not{D}|n\rangle = \lambda_n|n\rangle$$

- eigenfunction

$$\langle x|n\rangle = \psi_n(x)$$

$$\not{D}\psi_n(x) = \lambda_n\psi_n(x)$$

Dirac Eigenmode Projected Wilson loop

Wilson loop operator

$$\hat{W} \equiv \prod \hat{U}_{\mu_k} = \hat{U}_{\mu_1} \hat{U}_{\mu_2} \cdots \hat{U}_{\mu_L}$$

Dirac eigenmode projected Wilson loop operator $\hat{W}^P$

$$\hat{W}^P \equiv \prod \hat{U}^P_{\mu_k} = \hat{U}^P_{\mu_1} \hat{U}^P_{\mu_2} \cdots \hat{U}^P_{\mu_L}$$
$$= \sum_{n_1, n_2, \cdots, n_{L+1} \in A} |n_1\rangle \langle n_1 | \hat{U}_{\mu_1} | n_2 \rangle \langle n_2 | \hat{U}_{\mu_2} | n_3 \rangle \cdots \langle n_L | \hat{U}_{\mu_L} | n_{L+1} \rangle \langle n_{L+1} |$$

Trace of Dirac eigenmode projected Wilson loop

$$\mathrm{Tr} \hat{W}^P \equiv \mathrm{Tr} \prod \hat{U}^P_{\mu_k}$$
$$= \sum_{n_1, n_2, \cdots, n_L \in A} \mathrm{tr} \langle n_1 | \hat{U}_{\mu_1} | n_2 \rangle \langle n_2 | \hat{U}_{\mu_2} | n_3 \rangle \cdots \langle n_L | \hat{U}_{\mu_L} | n_1 \rangle$$

Dirac Eigenmode Projected Inter-Quark Potential

Trace of Dirac eigenmode projected Wilson loop

$$
\begin{aligned}
\mathrm{Tr}\hat{W}^P &\equiv \mathrm{Tr}\prod \hat{U}^P_{\mu_k} \\
&= \sum_{n_1, n_2, \cdots, n_L \in A} \mathrm{tr}\langle n_1|\hat{U}_{\mu_1}|n_2\rangle\langle n_2|\hat{U}_{\mu_2}|n_3\rangle \cdots \langle n_L|\hat{U}_{\mu_L}|n_1\rangle
\end{aligned}
$$

Dirac eigenmode projected interquark potential

$$
V^P(R) = -\lim_{T\to\infty}\frac{1}{T}\ln\left\{\mathrm{Tr}\hat{W}^P(R,T)\right\}
$$

Procedure of Wilson Loop Analysis from Dirac-mode

1. Evaluate Dirac operator eigenfunction $\psi_n(x)$

$$[D]_{x,y} = \frac{1}{2}\sum_{\mu=1}^{4}\eta_\mu(x)\left[U_\mu(x)\delta_{x+\mu,y} - U_\mu^\dagger(x-\hat{\mu})\delta_{x-\mu,y}\right]$$

$\eta_1(x) \equiv 1$, $\eta_\mu(x) = (-1)^{x_1+\cdots x_{\mu-1}}$ for $\mu \geq 2$

2.Calulate link-variable matrix element

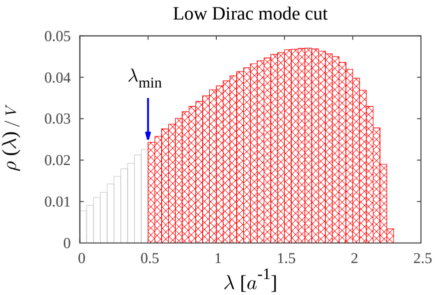
$$\langle m|\hat{U}_\mu|n\rangle = \sum_x \psi_m^\dagger(x)U_\mu(x)\psi_n(x+\hat{\mu})$$

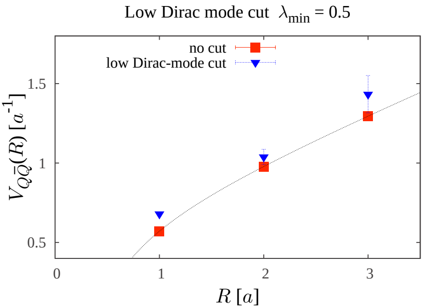
3.Calculate Wilson loop with IR/UV Dirac mode cut

$$\mathrm{Tr}\hat{W}^P = \sum_{n_1,n_2,\cdots,n_L\in A} \mathrm{tr}\langle n_1|\hat{U}_{\mu_1}|n_2\rangle\langle n_2|\hat{U}_{\mu_2}|n_3\rangle\cdots\langle n_L|\hat{U}_{\mu_L}|n_1\rangle$$

Lattice QCD Result

6^4 with $\beta = 5.6$ at quenched level
lattice spacing $a \simeq 0.25$fm, $a^{-1} \simeq 0.8$GeV





Low-lying Dirac mode cut — chiral condensate $\Rightarrow$ largely reduced

But $\Rightarrow$ confining potential
confinement does not relate to χSB via Dirac eigenmodes.