Form factors for several semi-leptonic and radiative B decays

Ran Zhou, Steven Gottlieb, Jon Bailey, Daping Du,
Aida X. El-Khadra, R.D. Jain, Andreas Kronfeld, Yuzhi Liu,
Yannick Meurice, Ruth S. Van de Water

Indiana U., Seoul National U., U. Illinois, FNAL, U. Iowa, BNL

(FNAL Lattice/MILC Collaborations)

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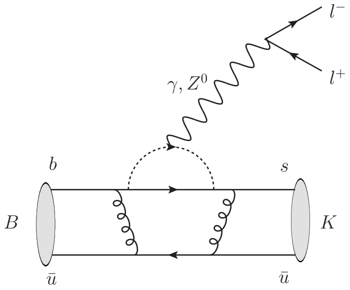
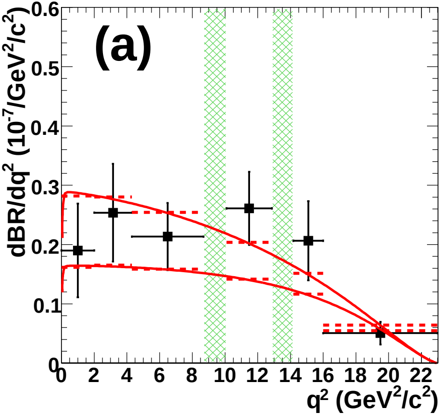
Motivation and theoretical background

$B \to Kll$ is a rare B decay mediated by a FCNC



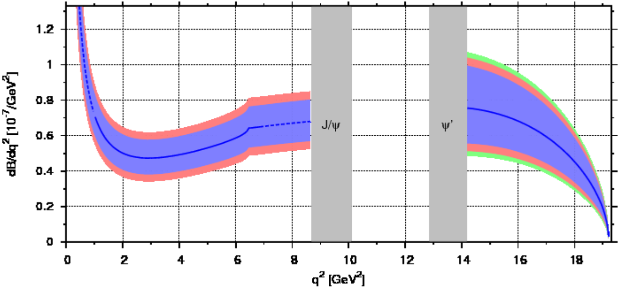
- Standard Model (SM) contribution occurs through penguin diagrams ($b \to sll$).
- SM contribution is small $\to$ opportunity to detect BSM physics
- Studied by experimental groups: BABAR, Belle, CDF, LHCb, etc.
- LHCb and SuperB will improve experimental precision.

Example of observable in $B \to Kll$ process



- $B^+ \to K^+\mu^+\mu^-$ differential branching ratio from CDF, PRL **106** 161801 (2011)
- Red lines are based on the maximum and minimum allowed form factors based on light cone sum rule (LCSR) calculations; Ali *et al.*, PRD 61, 074024 (2000).

Example of observable in $B \to K^* ll$ process



- $B^+ \to K^* l^+ l^-$ differential branching ratio from Bobeth, Hiller and Dyk, arXiv:1006.5013
- Blue band shows uncertainty due to form factor.
- Green band shows uncertainty due Λ/Q expansion of improved Isgur-Wise relations
- Red band comes from subleading terms of order $\alpha_s \Lambda/Q$ (low recoil); or Λ/m_b and Λ/E_{K^*} terms (high recoil).

Studies of $B \to Kll$ form factors from lattice QCD

Quenched lattice QCD:

- A. Abada et al. Phys. Lett. B 365, 275 (1996)
- L. Del Debbio et al. Phys. Lett. B 416, 392 (1998)
- D. Becirevic et al. Nucl. Phys. B 769, 31 (2007)
- A. Al-Haydari et al. (QCDSF) Eur. Phys. J. A 43, 107120 (2010)

Recent studies on dynamical Nf=2+1 flavors ensembles:

- FNAL/MILC: ($B \to Kll$), hep-lat/1111.0981
- Cambridge/W&M/Edinburgh group: ($B \to K/K^*ll$), hep-ph/1101.2726

Asqtad ensembles used in $B \to Kll$ work

$\approx a(fm)$	size	am_l/am_s	$N_{\rm meas}$
0.12	$20^3 \times 64$	0.02/0.05	2052
0.12	$20^3 \times 64$	0.01/0.05	2259
0.12	$20^3 \times 64$	0.007/0.05	2110
0.12	$20^3 \times 64$	0.005/0.05	2099
0.09	$28^3 \times 96$	0.0124/0.031	1996
0.09	$28^3 \times 96$	0.0062/0.031	1931
0.09	$32^3 \times 96$	0.00465/0.031	984
0.09	$40^3 \times 96$	0.0031/0.031	1015
0.09	$64^3 \times 96$	0.00155/0.031	791
0.06	$48^3 \times 144$	0.0036/0.018	673
0.06	$64^3 \times 144$	0.0018/0.018	827

Table: Ensembles of asqtad $N_f = 2+1$ configurations analyzed. Four time sources($0, \frac{N_t}{4}, \frac{N_t}{2}, \frac{3N_t}{4}$) are used for all measurements.

Form factors in $B \to Kll$ semileptonic decays: I

Two matrix elements are needed in $B \to Kll$ work:
$\langle B(p)|\bar{b}\gamma^\mu s|K(k)\rangle$, and $\langle B(p)|\bar{b}\sigma^{\mu\nu} s|K(k)\rangle$

Vector current:

$$\langle B(p)|\bar{b}\gamma^\mu s|K(k)\rangle = (p^\mu + k^\mu - \frac{m_B^2 - m_K^2}{q^2} q^\mu) f_+(q^2) + \frac{m_B^2 - m_K^2}{q^2} q^\mu f_0(q^2)$$

Tensor current:

$$\langle B(p)|\bar{b}\sigma^{\mu\nu} s|K(k)\rangle = \frac{i f_T}{m_B + m_K} [(p^\mu + k^\mu) q^\nu - (p^\nu + k^\nu) q^\mu]$$

Form factors in $B \to Kll$ semileptonic decays: II

For LQCD convenient to work in B rest frame. We define:

$$\langle B(p)|\bar{b}\gamma^\mu s|K(k)\rangle = \sqrt{2m_B}\left[f_\parallel \frac{p^\mu}{m_B} + f_\perp p_\perp^\mu\right].$$

The new form factors are considered to be functions of kaon energy:

$$\begin{cases} f_\parallel(E_K) = \dfrac{\langle B(p)|\bar{b}\gamma^0 s|K(k)\rangle}{\sqrt{2m_B}} \\ f_\perp(E_K) = \dfrac{\langle B(p)|\bar{b}\gamma^i s|K(k)\rangle}{2\sqrt{m_B}}\dfrac{1}{p_i} \end{cases}$$

and

$$f_T = \frac{m_B + m_K}{\sqrt{2m_B}} \frac{\langle B(p)|i\bar{(b)}\sigma^{0i} s|K(k)\rangle}{\sqrt{2m_B}k^i}$$

$B_x \to P_{xy} ll$ semileptonic decays in NLO SChPT

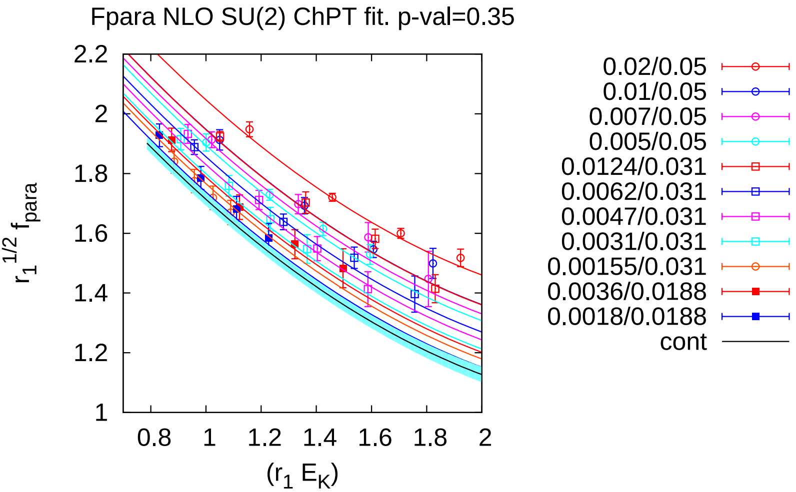
$$f_{\parallel} = \frac{C_0}{f}(1 + \mathrm{logs} + C_1 m_x + C_2 m_y + C_3 E + C_4 E^2 + C_5 a^2)$$

$$f_{\perp} = \frac{C_0}{f}\left[\frac{g}{E + \Delta_B^* + D}\right] + \frac{(C_0/f)g}{E + \Delta_B^*}(\mathrm{logs} + C_1 m_x + C_2 m_y + C_3 E + C_4 E^2 + C_5 a^2)$$

where $\Delta_B^* = m_{B_s^*} - m_B$, D and logs are chiral log terms.

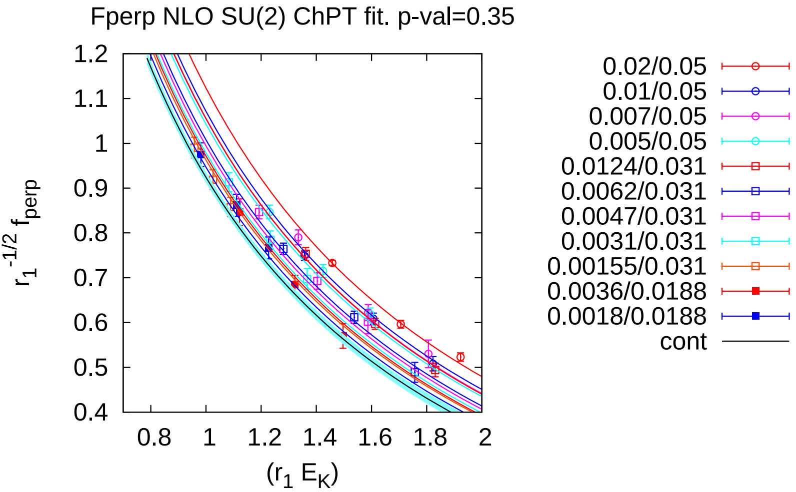
- We use SU(2) chiral logs in the chiral fit.
- The expressions for f_T and $f_{\perp}$ are the same at this order in the $1/m_B$ expansion (Becirevic *et al.*, PRD **68** 074003 (2003)).

$f_\parallel$ chiral-continuum extrapolation

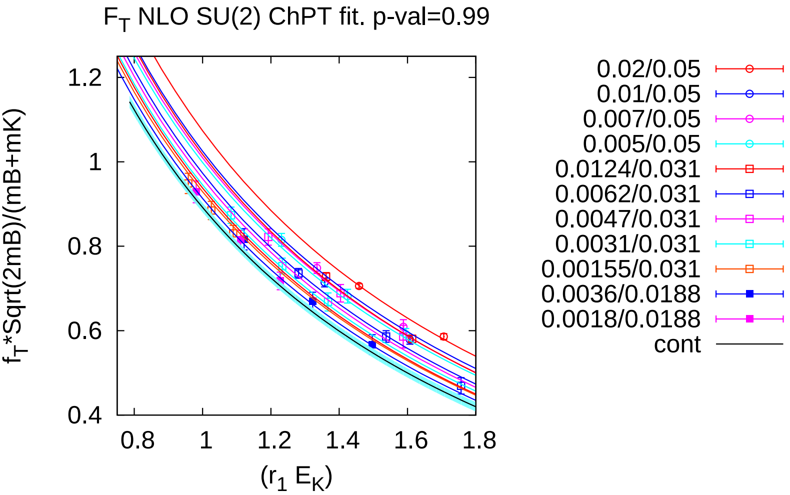


- Chiral-Continuum extrapolations give FFs at small E_K (large q^2).
- $q^2 = (p_B - p_K)^2 = m_B^2 + m_K^2 - 2m_B E_K$

$f_\perp$ chiral-continuum extrapolation



f_T chiral-continuum extrapolation



z-expansion on $B \to Kll$ form factors

z-expansion is based on field theoretic principles: analyticity, crossing symmetry, unitarity. It is systematically improvable by adding more orders.

- z-expansion maps q^2 to z by:

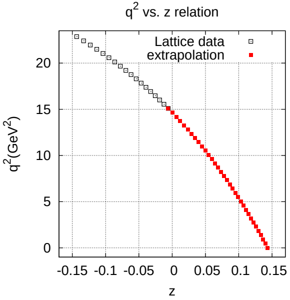
$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \qquad t_\pm = (m_B \pm m_K)^2$$

- Choose $t_0 = t_+ \left(1 - \sqrt{1 - \frac{t_-}{t_+}}\right)$ such that $z << 1$
- Expand form factors as a function of z.

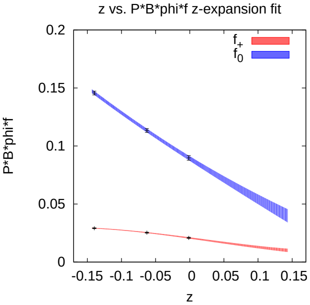
$$f(q^2) = \frac{1}{B(z)\phi(z)} \sum_{k=0}^{\infty} a_k z^k,$$

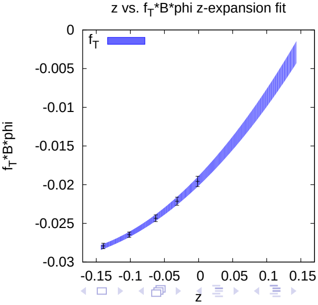
where $B(z) = z(q^2, m_R^2)$ is used to account for pole structure and $\phi(z)$ is selected such that $\sum_{k=0}^{\infty} a_k^2 \leq 1$

z-expansion on $B \to Kll$ form factors



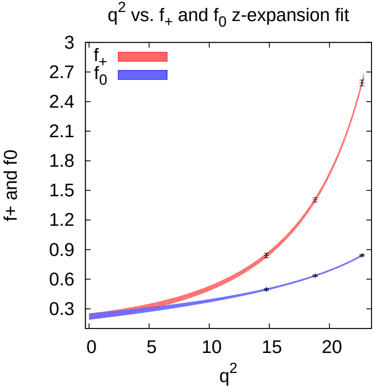
- $f(q^2) = \frac{1}{B(z)\phi(z)} \sum_{k=0}^{\infty} a_k z^k$
- $q^2 \in (0, 23) \to z \in (-0.15, 0.15)$
- B_s^* pole corresponds to $z = -0.367$
- Fit $f(q^2)B(z)\phi(z)$ as a polynomial of z

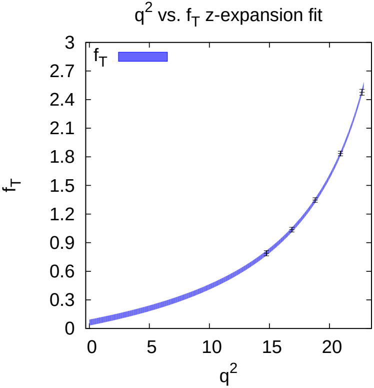




z-expansion on $B \to Kll$ form factors

(Statistical error only.)

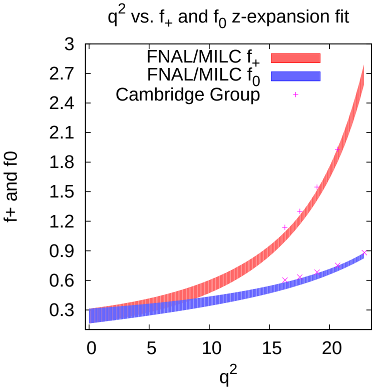


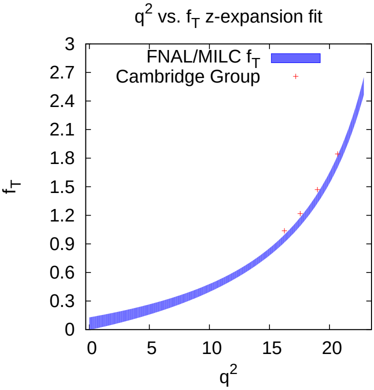


Kinematic constraint, $f_+(q^2=0) = f_0(q^2=0)$, is applied to z-expansion fit.

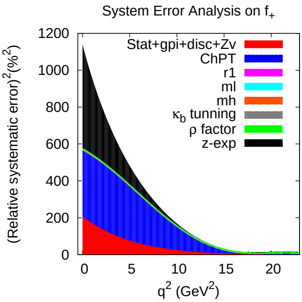
z-expansion on $B \to Kll$ form factors

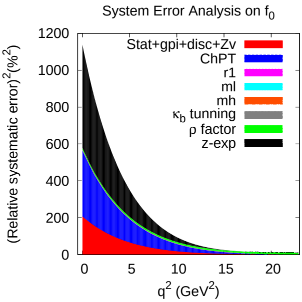
(Statistical and systematic error.)

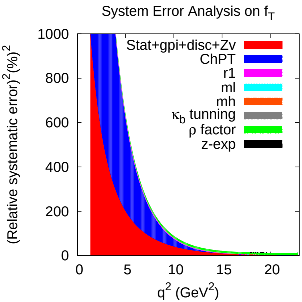




Systematic error budget

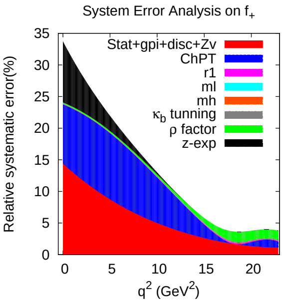


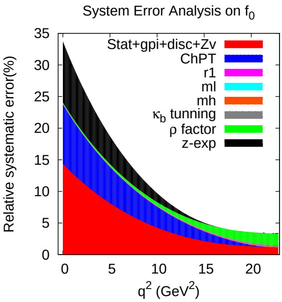


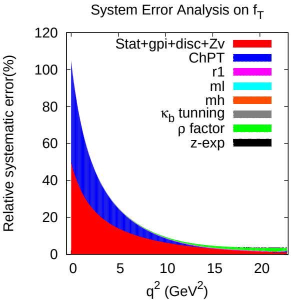


- Renormalization error could be smaller in the final result.
- ChPT error is important.
- Direct measurement of form factors at small q^2 is valuable.

Systematic error budget

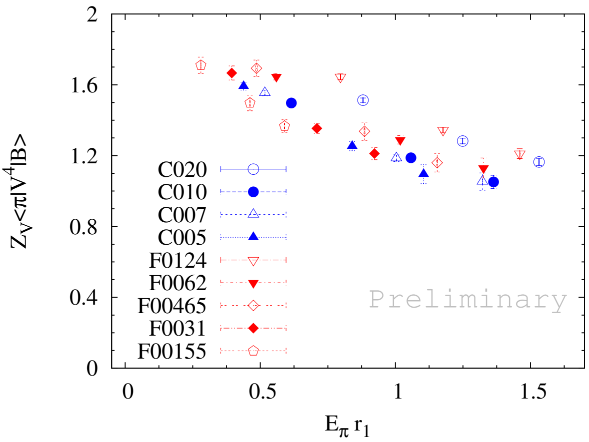






- 5% error in high q^2 range.
- larger relative error at small q^2

$B \to \pi l \nu$ decay



- Preliminary results for $<\pi|V^4|B>$ on four coarse and five fine ensembles fit by Daping Du.
- Higher statistics than in FNAL/MILC PRD **79** 054507 (2009)

Conclusions

- We are nearing completion of the study of form factors for the rare decay $B \to Kll$.
  - We still need to complete addition of renormalization factor ρ.
  - We have results for $f_{\parallel}$, $f_{\perp}$ and f_T which are needed for SM calculation.
  - BSM contributions can change the Wilson coefficients of the operators and the weighting of the form factors.
- $B \to \pi l \nu$ is also under investigation (Daping Du).
- $B_s \to K l \nu$ is also under investigation (Yuzhi Liu).
- $B \to K^* ll$ or $B \to \phi ll$ might be studied by us in the future.