$\bar{B} \to D$ decays at nonzero recoil with 2+1 flavors of improved staggered quarks

Si-Wei Qiu, C. DeTar
Daping Du, A.S. Kronfeld, J. Laiho, R. S. Van de Water
(Fermilab Lattice/MILC collaborations)

June 27, 2012

Motivation

- $|V_{cb}|$ normalizes the legs of the unitarity triangle.
- The dominant uncertainty comes from the hadronic form factors for $\bar{B} \to D + \dots$.
- The exclusive processes $\bar{B} \to D\ell\nu$ and $\bar{B} \to D^*\ell\nu$ can be studied in lattice gauge theory.
- Here we report on results for $\bar{B} \to D\ell\nu$.
- Lattice measurements at zero recoil have the smallest errors
- Because of the phase space suppression near zero recoil in $\bar{B} \to D\ell\nu$, experimental errors are largest there.
- So we need to work at nonzero recoil where the combined experimental and theoretical error is minimized.

Formalism

- The differential decay rate $\frac{d\Gamma(\bar{\mathcal{B}}\to\mathcal{D}\ell\overline{\nu})}{dq^2}$ is proportional to $|f_+|^2$ for $\ell = e, \mu$, where for $q = p_B - p_D$

$$\begin{aligned}\langle D(p_D)|\mathcal{V}^\mu|B(p_B)\rangle &= f_+(q^2)\left[(p_B+p_D)^\mu - \frac{M_B^2-M_D^2}{q^2}q^\mu\right] \\ &+ f_0(q^2)\frac{M_B^2-M_D^2}{q^2}q^\mu,\end{aligned}$$

- The alternative form factors h_+ and h_- are convenient:

$$\frac{\langle D(p_D)|\mathcal{V}^\mu|B(p_B)\rangle}{\sqrt{M_B M_D}} = h_+(w)(v+v')^\mu + h_-(w)(v-v')^\mu,$$

where $v = p_B/M_B$ and $v' = p_D/M_D$.

- They are related to f_+ and f_0 through

$$\begin{aligned}f_+(q^2) &= \frac{1}{2\sqrt{r}}\left[(1+r)h_+(w) - (1-r)h_-(w)\right], \\ f_0(q^2) &= \sqrt{r}\left[\frac{w+1}{1+r}h_+(w) - \frac{w-1}{1-r}h_-(w)\right],\end{aligned}$$

where $r = M_D/M_B$ and $q^2 = M_B^2 + M_D^2 - 2wM_BM_D$ or $w = v\cdot v'$.

Formalism

- In the $\bar{B}$ meson rest frame for any recoil D-momentum $\mathbf{p}$ we get h_+ and h_- from matrix elements of the current starting from R_+ and R_-

$$
\begin{aligned}
R_+(\mathbf{p}) &\equiv \langle D(\mathbf{p})|V_4|B(\mathbf{0})\rangle \\
R_-(\mathbf{p}) &\equiv \frac{\langle D(\mathbf{p})|V_1|B(\mathbf{0})\rangle}{\langle D(\mathbf{p})|V_4|B(\mathbf{0})\rangle} \\
x_f(\mathbf{p}) &\equiv \frac{\langle D(\mathbf{p})|V_1|D(\mathbf{0})\rangle}{\langle D(\mathbf{p})|V_4|D(\mathbf{0})\rangle} \\
w(\mathbf{p}) &= [1 + x_f(\mathbf{p})^2]/[1 - x_f(\mathbf{p})^2] \\
h_+(w) &= R_+(\mathbf{p})[1 - x_f(\mathbf{p})R_-(\mathbf{p})] \\
h_-(w) &= R_+(\mathbf{p})[1 - R_-(\mathbf{p})/x_f(\mathbf{p})]
\end{aligned}
$$

- At zero recoil, we can also use the double ratio of Hashimoto *et al*:

$$
|h_+(\mathbf{0})|^2 = \frac{\langle D(\mathbf{0})|V_1|B(\mathbf{0})\rangle \, \langle B(\mathbf{0})|V_1|D(\mathbf{0})\rangle}{\langle D(\mathbf{0})|V_4|D(\mathbf{0})\rangle \, \langle B(\mathbf{0})|V_4|B(\mathbf{0})\rangle}
$$

“Mostly nonperturbative” current renormalization

- The continuum $\mathcal{V}^\mu$ and lattice V^μ currents are matched through

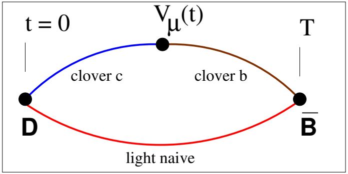
$$\mathcal{V}^\mu_{cb} = Z_{cb} V^\mu_{cb}$$

- We follow Hashimoto *et al*:

$$Z_{cb} = \rho_{cb} \sqrt{Z_{cc} Z_{bb}}$$

and determine ρ_{cb} from one loop lattice and continuum perturbation theory.

Lattice correlators needed



- We use naive light spectator quarks and clover heavy quarks.
- We need matrix elements $\langle Y(\mathbf{p})|V_\mu|X(0)\rangle$ for $X, Y \in \{B, D\}$.
- For interpolating operators $\mathcal{O}_X$, we measure two-point and three-point functions

$$
\begin{aligned}
C^{2pt,X}(\mathbf{p}, t) &= \left\langle \mathcal{O}_X^\dagger(0)\mathcal{O}_X(t)\right\rangle \\
C_\mu^{3pt,X\to Y}(\mathbf{p}; 0, t, T) &= \left\langle \mathcal{O}_Y^\dagger(0)V_\mu(t)\mathcal{O}_X(T)\right\rangle
\end{aligned}
$$

- We use both point and $1S$ smeared interpolators for the D meson and $1S$ smeared interpolators for the B

Example: $R_+(p) = \langle D(\mathbf{p})|V_4|B(\mathbf{0})\rangle$

- We include excited contributions but not doubly excited:

$$\begin{aligned} C_{V4}^{3pt,\bar{B}\to D}(\mathbf{p},t) &= \sqrt{Z_D(\mathbf{p})}\frac{e^{-E_D t}}{\sqrt{2E_D}}\langle D(\mathbf{p})|V_4|B(\mathbf{0})\rangle \frac{e^{-m_B(T-t)}}{\sqrt{2m_B}}\sqrt{Z_B(\mathbf{0})} \\ &+ \sqrt{Z_{D'}(\mathbf{p})}\frac{e^{-E_{D'} t}}{\sqrt{2E_{D'}}}\langle D'(\mathbf{p})|V_4|B(\mathbf{0})\rangle \frac{e^{-m_B(T-t)}}{\sqrt{2m_B}}\sqrt{Z_B(\mathbf{0})} \\ &+ \sqrt{Z_D(\mathbf{p})}\frac{e^{-E_D t}}{\sqrt{2E_D}}\langle D(\mathbf{p})|V_4|B'(\mathbf{p})\rangle \frac{e^{-m_{B'}(T-t)}}{\sqrt{2m_{B'}}}\sqrt{Z_{B'}(\mathbf{0})} \end{aligned}$$

- Or

$$\begin{aligned} C_{V_4}^{3pt,\bar{B}\to D}(\mathbf{p},t) &= C_0(\mathbf{p})\langle D(\mathbf{p})|V_4|B(\mathbf{0})\rangle e^{-E_D t}e^{-m_B(T-t)} \\ &\times \left[1 + C_1(\mathbf{p})e^{-\Delta E_D t} + C_2(\mathbf{p})e^{(t-T)\Delta m_B}\right] \end{aligned}$$

  where $C_0(\mathbf{p})$, $\Delta E_D = E_{D'} - E_D$, and $\Delta m_B = m_{B'} - m_B$ are determined in fits to two-point correlators.

- There are also oscillating terms from the naive light quark. Their contributions are suppressed by averaging over T, $T+1$ and t, $t+1$.

Extracting $R_+(p) = \langle D(\mathbf{p})|V_4|B(\mathbf{0})\rangle$

- Putting information from three- and two-point functions together, we get

$$R_+(\mathbf{p}, t) \equiv \frac{C_{V_4}^{3pt,\bar{B}\to D}(\mathbf{p}, t) e^{(E_D - m_B)t + (m_B - m_D)T/2}}{\sqrt{C_{V_4}^{3pt,D\to D}(\mathbf{0}, t) C_{V_4}^{3pt,B\to B}(\mathbf{0}, t)}} \sqrt{\frac{Z_D(\mathbf{0}) E_D}{Z_D(\mathbf{p}) m_D}}$$

$$\approx R_+(\mathbf{p}) \left[1 + s_1(\mathbf{p}) e^{-\Delta E_D t} + s_2(\mathbf{p}) e^{(t-T)\Delta m_B}\right]$$

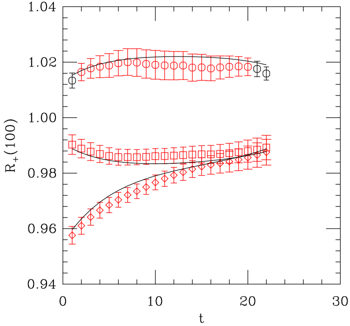
- The zero-recoil form factor $h_+(\mathbf{0}) = R_+(\mathbf{0})$ can be calculated very accurately from the double ratio. A good strategy is to use it to normalize the nonzero recoil values:

$$\frac{R_+(\mathbf{p}, t)}{R_+(\mathbf{0}, t)} = \frac{R_+(\mathbf{p})}{R_+(\mathbf{0})} \exp(\delta m\, t) + A(\mathbf{p}) \exp(-\Delta E_D t) + B(\mathbf{p}) \exp(\Delta m_B t)$$

- The parameters $\delta m = 0$, $\Delta E_D = E_{D'} - E_D$, and $\Delta m_B = m_{B'} - m_B$ are determined in fits to two-point functions. Their central values and errors become priors for the three-point fit.

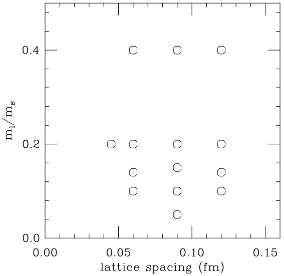
Sample fit

- Example from the $a = 0.06$ fm, $m_\ell/m_s = 0.15$ ensemble with $T = 24, 25$.



- We do a simultaneous fit to three three-point functions: the zero-recoil double ratio (upper) and the ratio $R_+(\mathbf{p}, t)/R_+(\mathbf{0}, t)$ for both 1S smeared (middle) and local D-meson interpolators (lower).
- Red points are included in the fit.

Asqtad ensembles used



- 14 ensembles in this study
- Valence bottom and charm quark masses tuned to the "kinetic" B_s and D_s masses.
- The light valence quark mass is always set equal to the light sea quark mass.

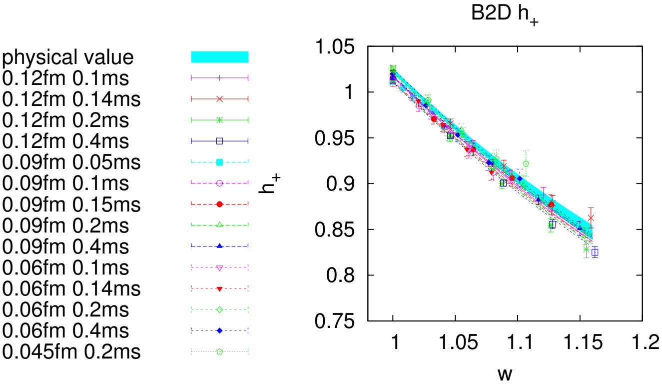
h_+ chiral model

- For light spectator quark mass m_ℓ, lattice spacing a, and $w = v \cdot v'$ we use the chiral/continuum model

$$
\begin{aligned}
h_+(a, m_\ell, w) &= 1 - \rho_+^2(w-1) + k_+(w-1)^2 + \frac{X_+(\Lambda_\chi)}{m_c^2} \\
&+ \frac{g_{D^* D\pi}^2}{16\pi^2 f^2} \mathrm{logs}_{1-\mathrm{loop}}(\Lambda_\chi, w, m_\ell, a) \\
&+ c_{1,+} m_\ell + c_{a,+} a^2 + c_{a,w,+} a^2 (w-1)
\end{aligned}
$$

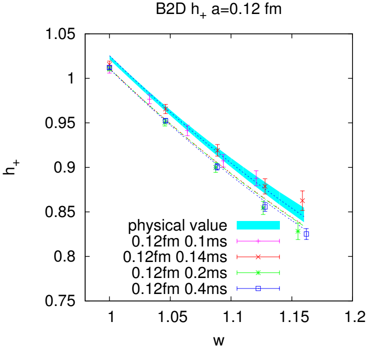
- For the one-loop chiral logs we use a staggered fermion version of Chow and Wise [hep-ph/9305229].

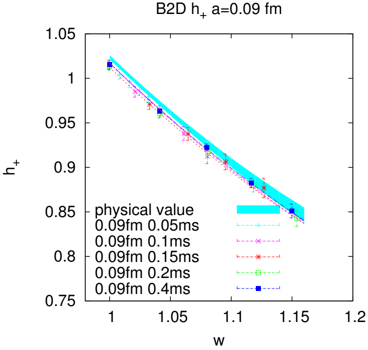
h_+ fit result full dataset



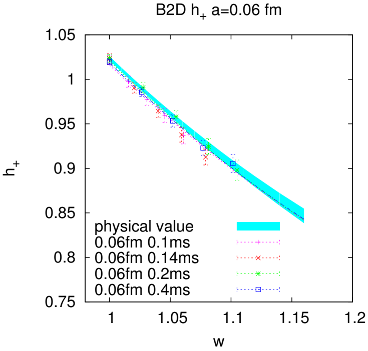
- Mild dependence on a and m_{ud}/m_s.

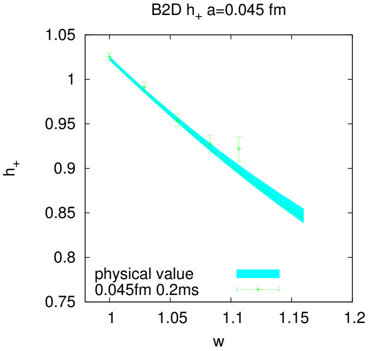
h_+ 0.12 fm and 0.09 fm





h_+ 0.06 fm and 0.045 fm



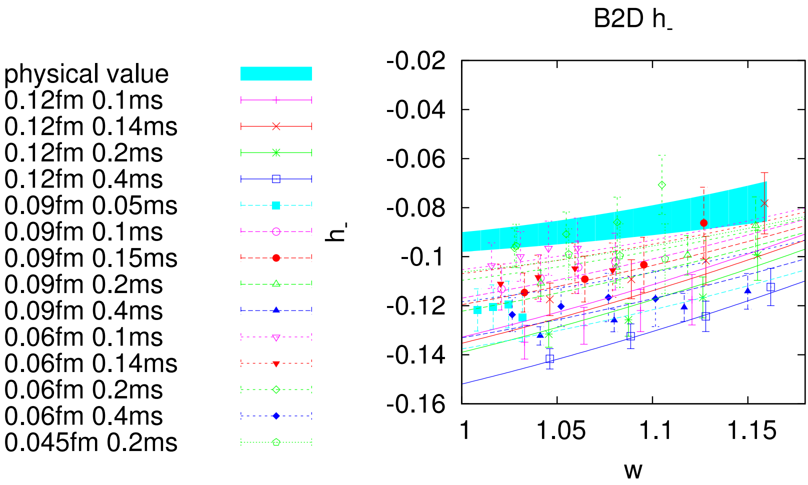


h_- chiral fit formula

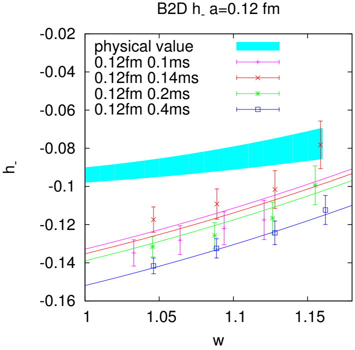
- For light spectator quark mass m_ℓ, lattice spacing a, and recoil factor w we fit

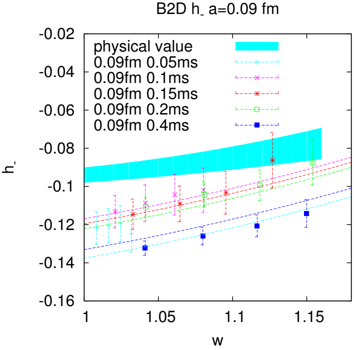
$$\begin{aligned} h_-(a, m_\ell, w) &= \frac{X_-}{m_c} - \rho_-^2(w-1) + k_-(w-1)^2 \\ &+ c_{1,-} m_\ell + c_{a,-} a^2 + c_{a,w,-} a^2 (w-1) \end{aligned}$$

h_- fit result full dataset

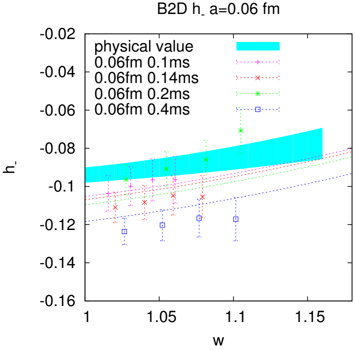


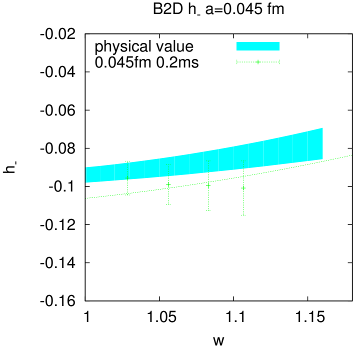
h_- 0.12 fm and 0.09 fm





h_- 0.06 fm and 0.045 fm





z expansion

- Model-independent parameterization of the q^2 (or w) dependence of f_+ and f_0.
- Build in constraints from analyticity and unitarity.
- Becher and Hill [hep-ph/0509090] propose the conformal map

$$z(w) = \frac{\sqrt{1+w} - \sqrt{2}}{\sqrt{1+w} + \sqrt{2}}$$

  - Maps the physical region $w \in [1, 1.59]$ to $z \in [0, 0.0644]$.
  - Pushes poles and branch cuts far away at $|z| \approx 1$.
- Use

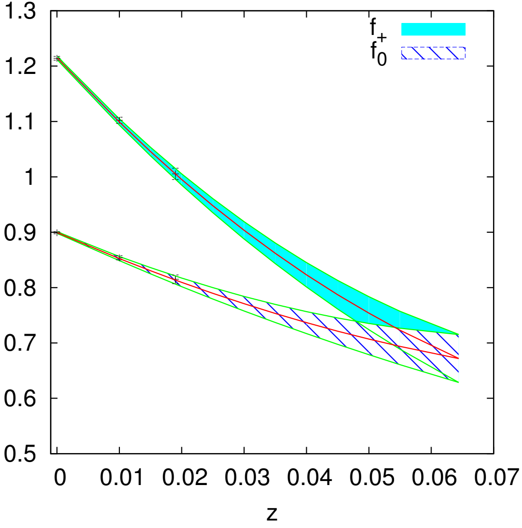
$$f_i(z) = \frac{1}{P_i(z)\phi_i(z)} \sum_{n=0}^{\infty} a_{i,n} z^n$$

  - The Blaschke factor $P_+(z)$ could include the pole at $q^2 = M^2_{B_c^*}$.
  - The "outer functions" ϕ_i are chosen to simplify the unitarity bound:

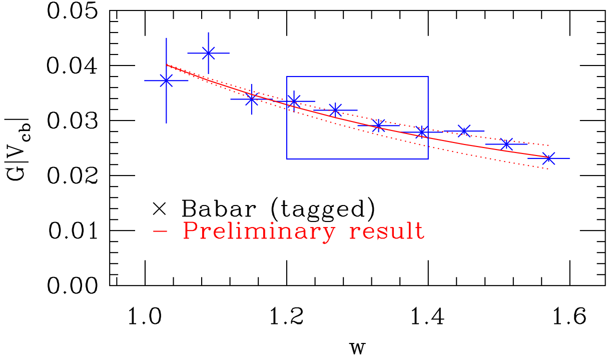
$$\sum_n |a_{i,n}|^2 \leq 1.$$

- Note, also, the kinematic constraint $f_+ = f_0$ at $q^2 = 0$ or $z \approx 0.0644$.

z expansion preliminary result with full data set and $w = 1$, 1.08, and 1.16



Comparison with experiment



For the sake of this comparison we take $|V_{cb}|$ from $B \to D^*$ at zero recoil (Fermilab/MILC, CKM2010) and show statistical errors only. The boxed region appears to have the smallest combined error.

To do list

- Yet to complete
  - Charm, bottom quark mass tuning corrections.
  - Current renormalization factors.
  - Full error analysis.