$B \to V$ form factors at low recoil

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WITH
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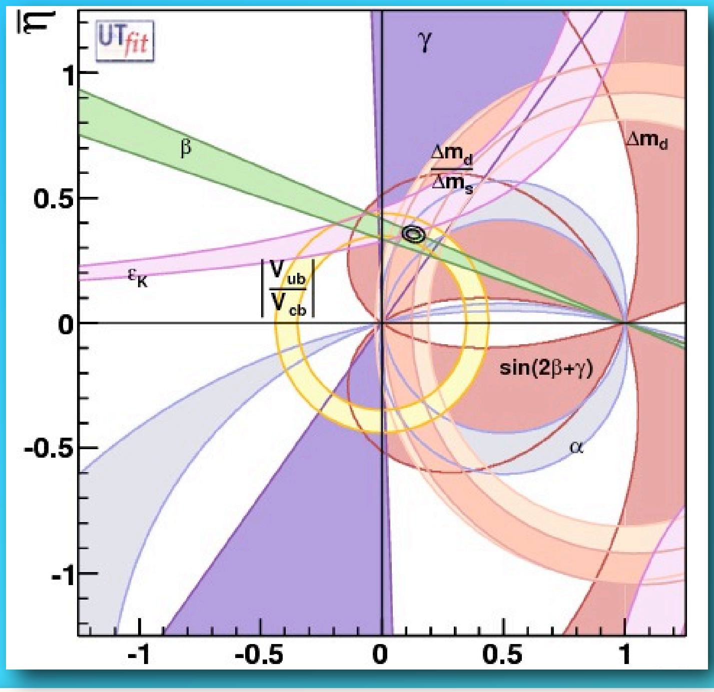
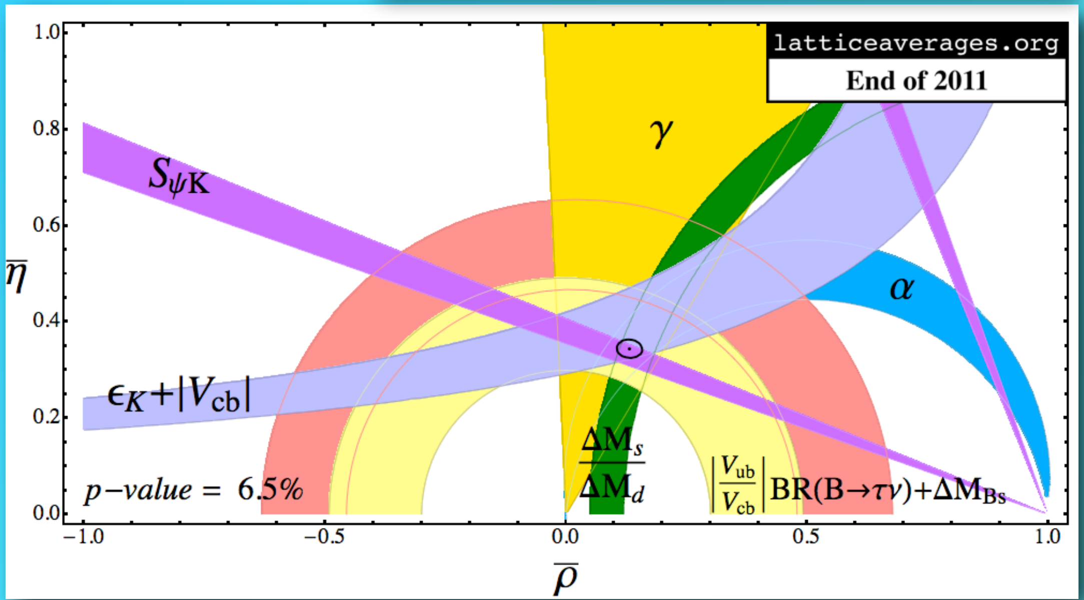
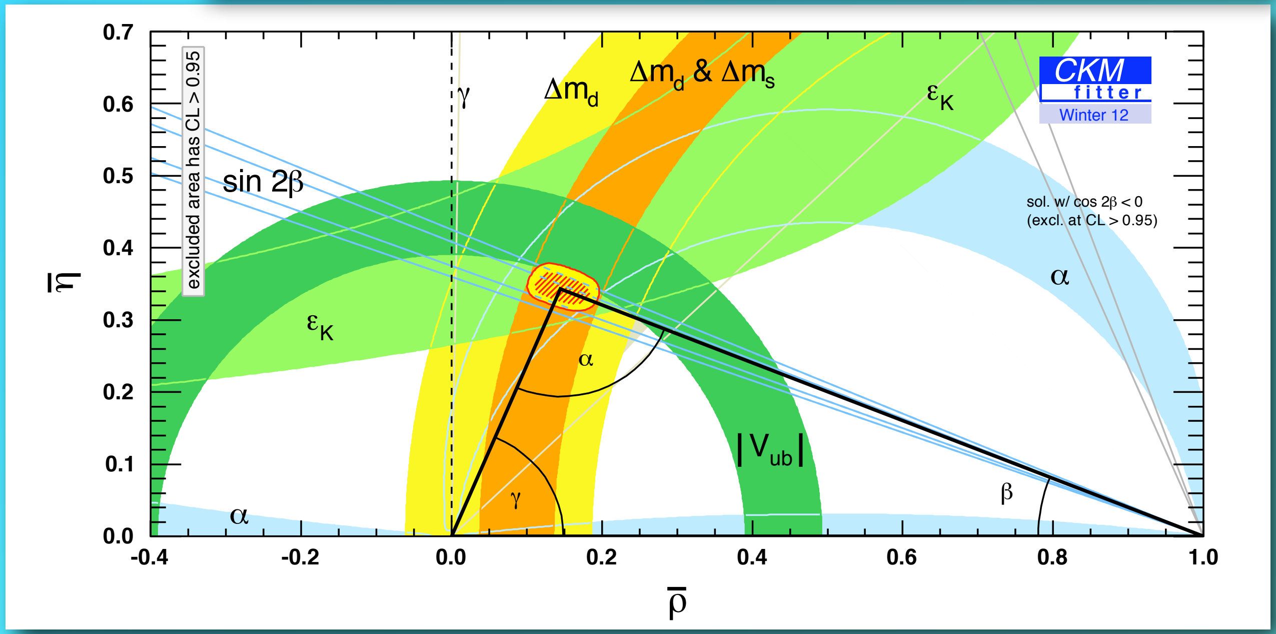
Since the nineties…

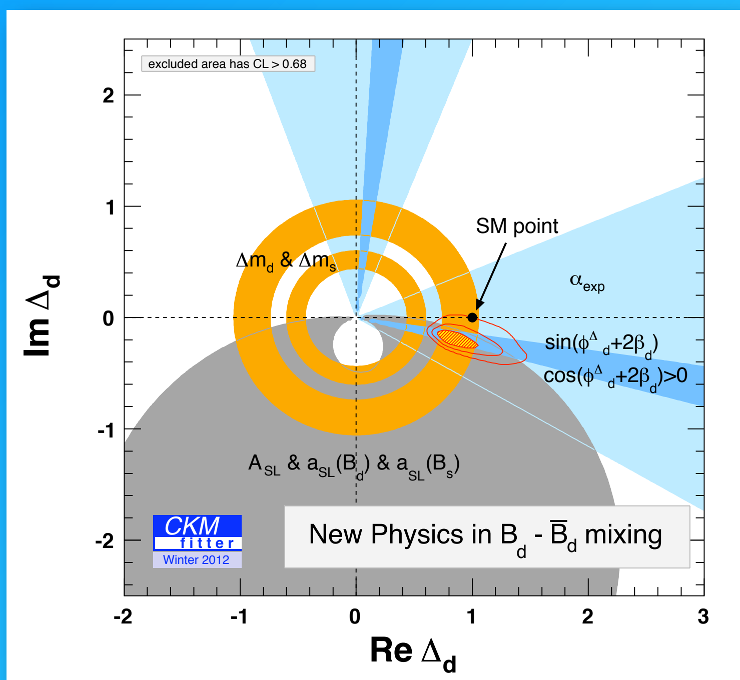




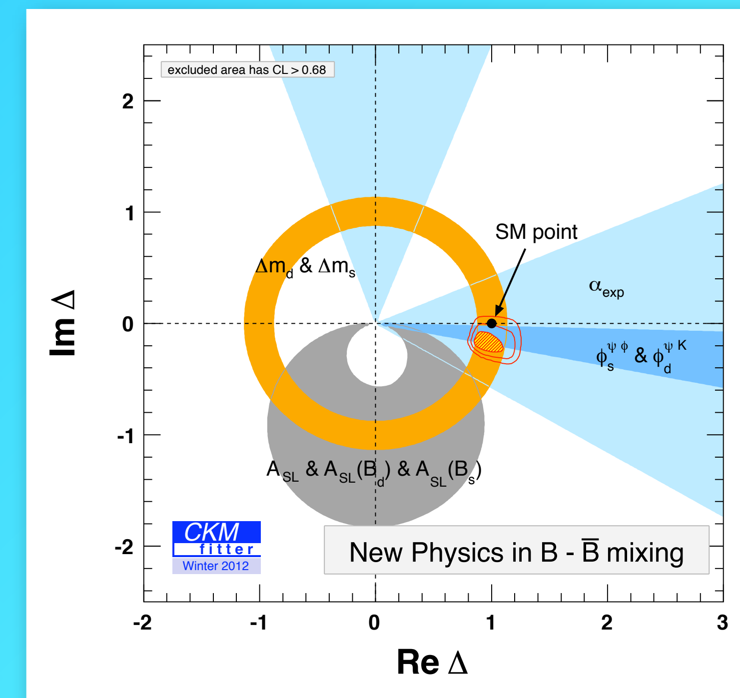


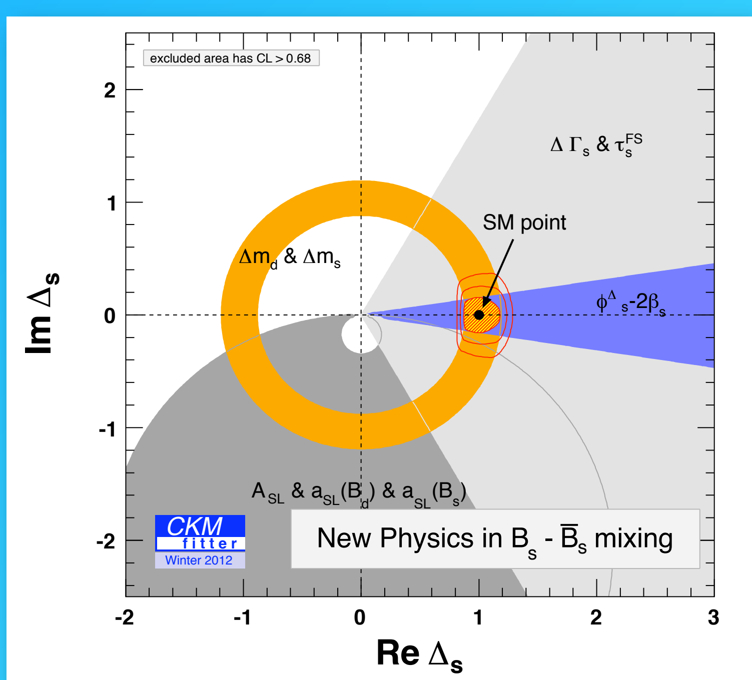
- ✤ [*Hat-tip* B-factories]
- ✤ Are the 4 CKM parameters enough to fit global data?
- ✤ High precision now
- ✤ Lattice QCD role
- ✤ No cracks have penetrated the foundation (yet)

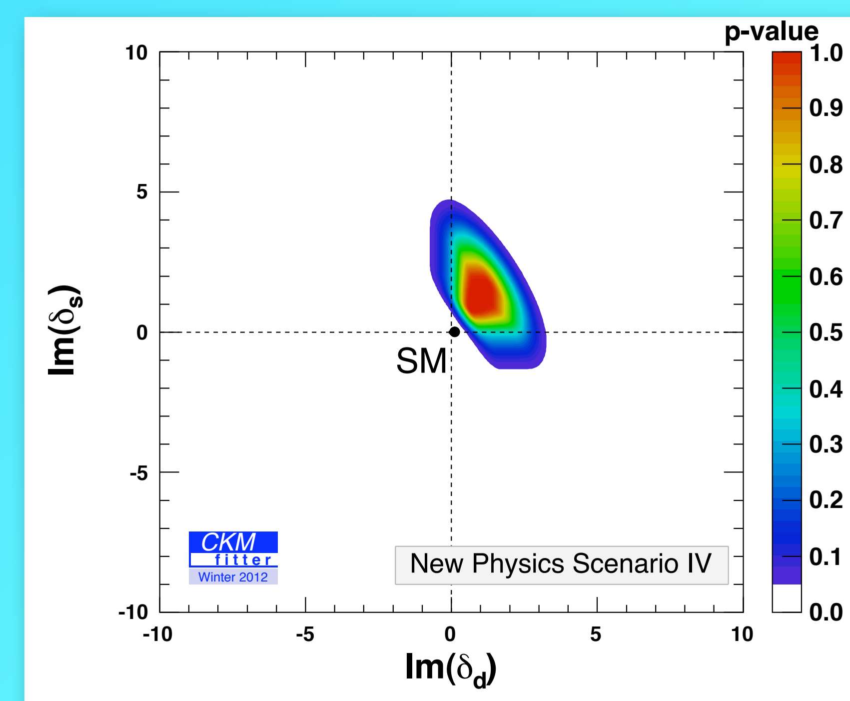
Since the noughties...



- ✤ [*Hat-tip* Tevatron]
- ✤ B_d and B_s mixing
- ✤ New physics in ΔF=2 H_{eff}?
- ✤ SM seems OK, except for DØ like-sign dimuon CP asymmetry

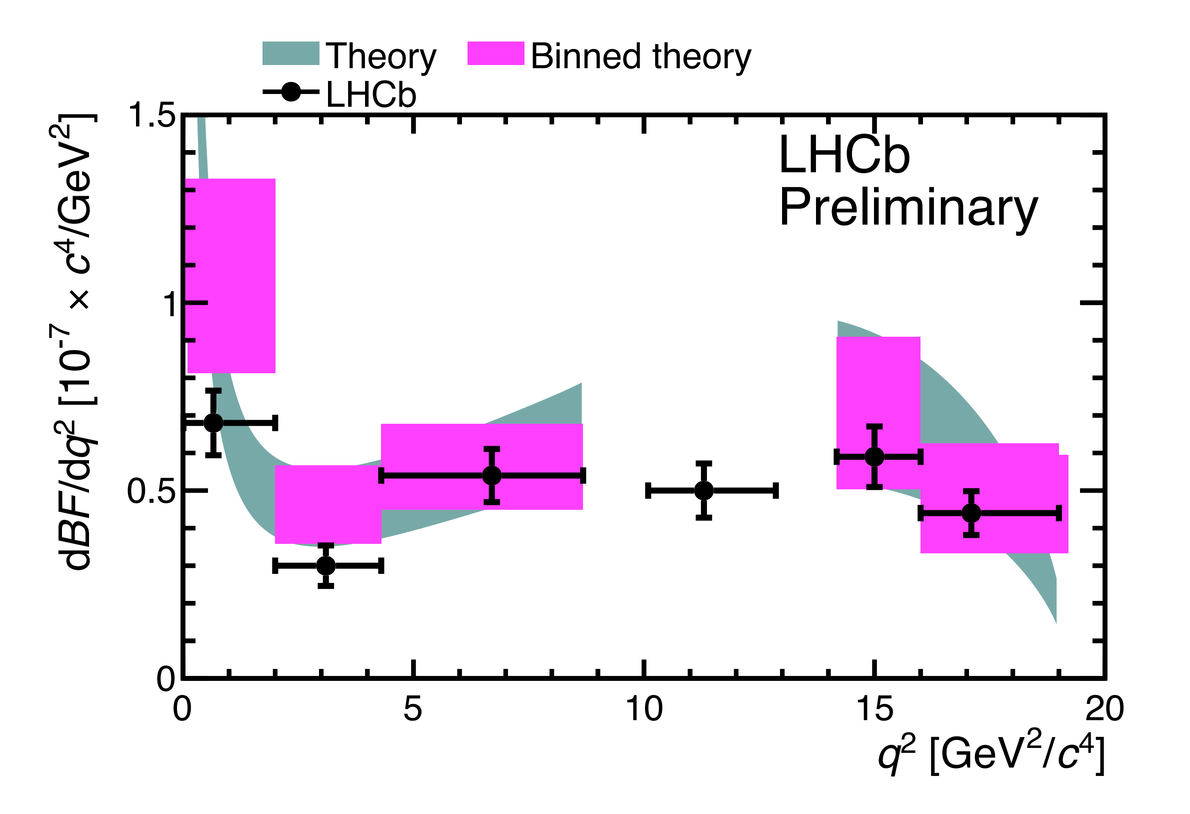






arXiv:1203.0283v2

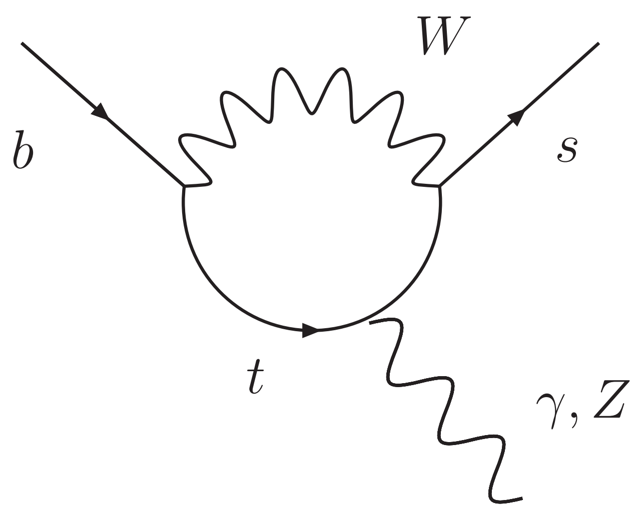
Now in the tweenies…

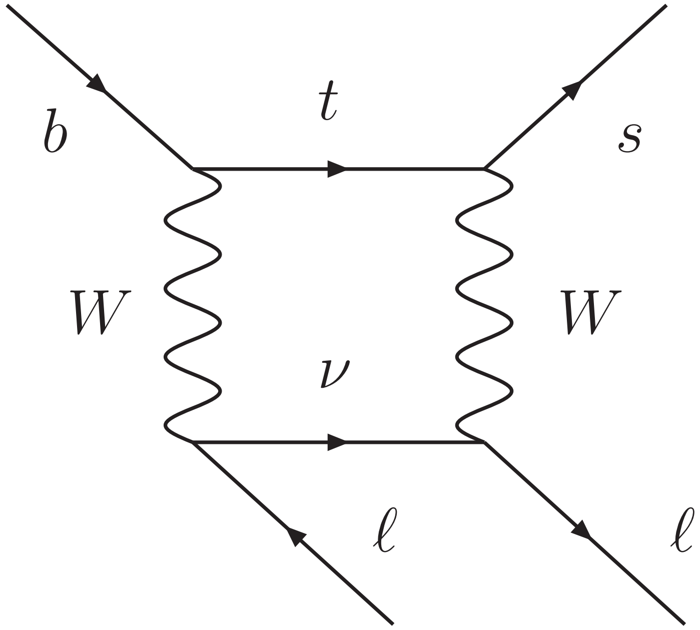


- [*Hat-tip* LHC]
- $b \rightarrow s$ semileptonic decays
- Data for $B \rightarrow K^{(*)} \mu^+\mu^-$ and $B_s \rightarrow \varphi\, \mu^+\mu^-$
- (see S. Meinel Thurs 2:50 for $\Lambda_b \rightarrow \Lambda\, \mu^+\mu^-$)
- Test $b \rightarrow s\, H_{eff}$

Parkinson, Moriond QCD, March 2012

$b \to s$ is rare in the SM

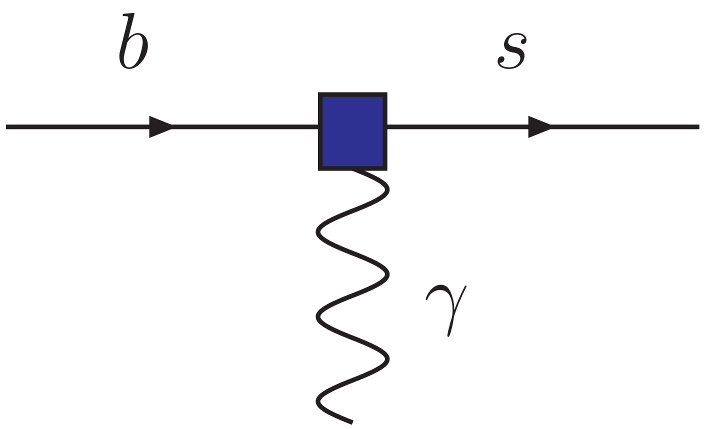




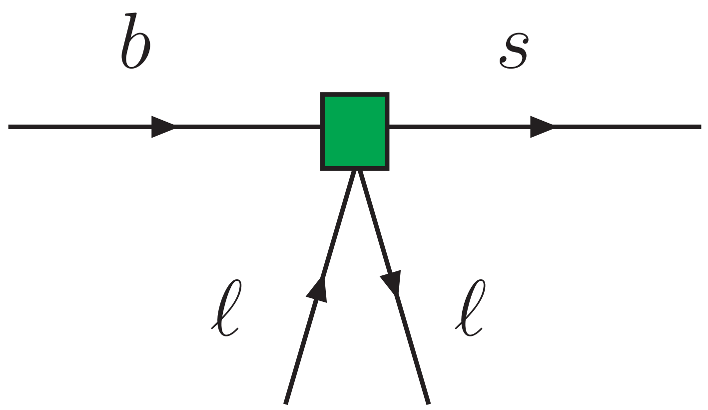
For energies $\ll m_W$

$$\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) Q_i(\mu)$$

Wilson coefficients   Local operators



$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}$$



Dominant operators

Decays

$$B \to K^* \gamma$$
$$B_s \to \phi \gamma$$
$$B \to (\rho/\omega)\gamma$$

$$B \to K^{(*)} \ell^+ \ell^-$$
$$B_s \to \phi\, \ell^+ \ell^-$$

$$\Lambda_b \to \Lambda\, \gamma$$
$$\Lambda_b \to \Lambda\, \ell^+ \ell^-$$

SM operators

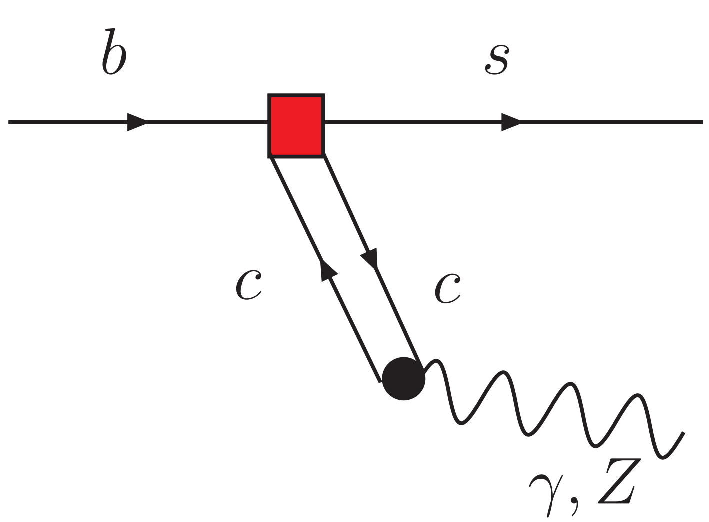
$$Q_{7\gamma} = \frac{e}{8\pi^2} m_b\, \bar{s}_i \sigma^{\mu\nu}(1+\gamma_5) b_i F_{\mu\nu}$$

$$Q_{9V} = \frac{e^2}{8\pi^2} (\bar{s}b)_{V-A} (\bar{\ell}\ell)_V$$

$$Q_{10A} = \frac{e^2}{8\pi^2} (\bar{s}b)_{V-A} (\bar{\ell}\ell)_A$$

$$Q_2 = (\bar{s}\, c)_{V-A}\, (\bar{c}\, b)_{V-A}$$

Charmonium resonances



Under control for some kinematics

Low q^2
Large recoil

Khodjamirian, et al, PLB **402** (1997)
Khodjamirian, et al, arXiv:1006.4945

High q^2
Low recoil

Buchalla & Isidori, NPB **525** (1998)
Grinstein & Pirjol, PRD **62** (2000), PRD **70** (2004)
Beylich, Buchalla, Feldmann, arXiv:1101.5118

Dominant operators

Decays

Goal: compute matrix elements of 2-quark operators (form factors)

SM operators

$$Q_{7\gamma} = \frac{e}{8\pi^2} m_b \, \bar{s}_i \sigma^{\mu\nu}(1+\gamma_5) b_i F_{\mu\nu}$$

$$Q_{9V} = \frac{e^2}{8\pi^2} (\bar{s}b)_{V-A} (\bar{\ell}\ell)_V$$

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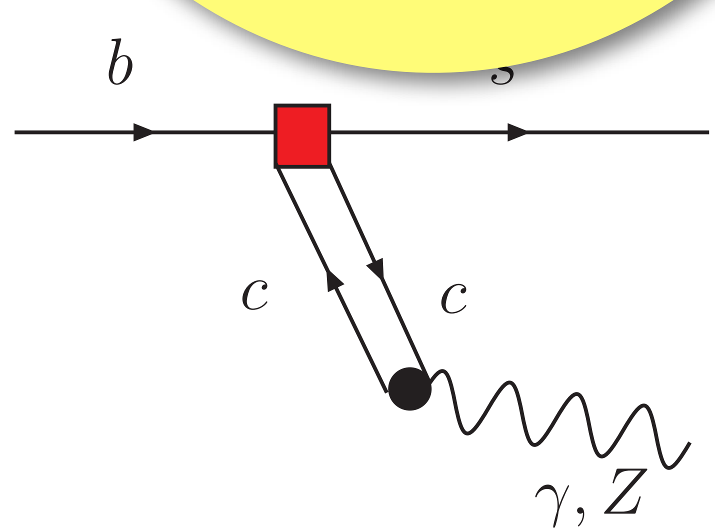
$$Q_2 = (\bar{s}\, c)_{V-A} \, (\bar{c}\, b)_{V-A}$$

harmonium resonances



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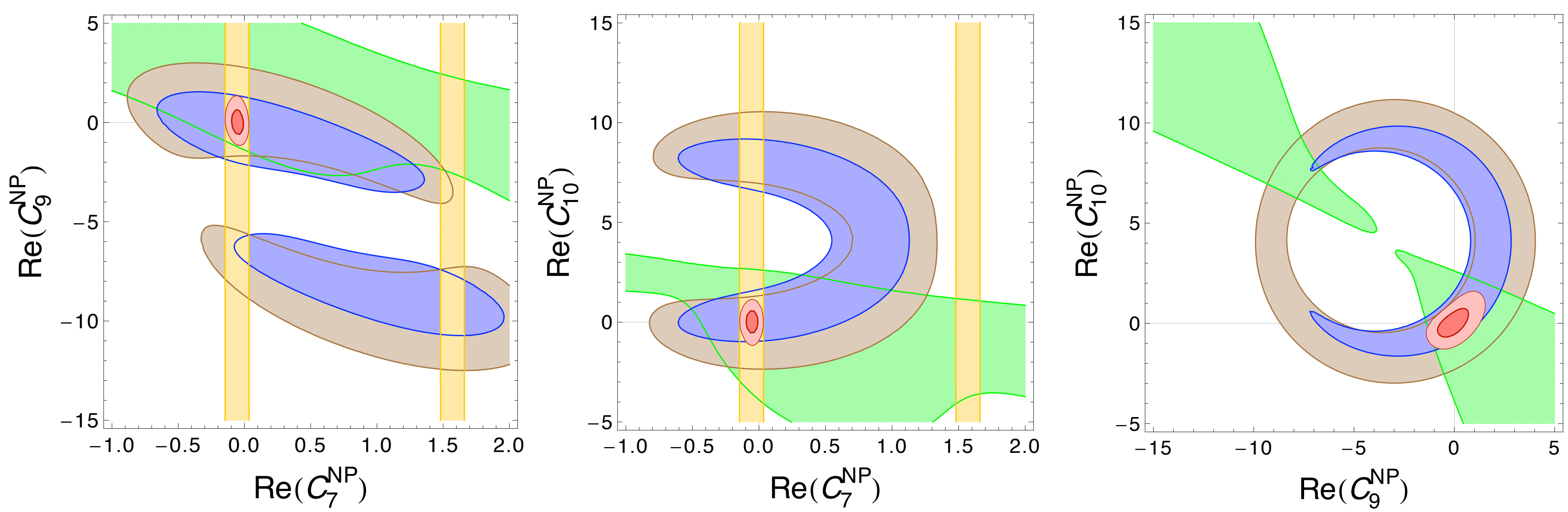
Data + QCD to constrain C's

Constraints on C_7 vs. C_9 vs. C_{10}

Now: correlations between 2 real Wilson coefficients

Std Model in red

LQCD form factors aim to reduce blue region

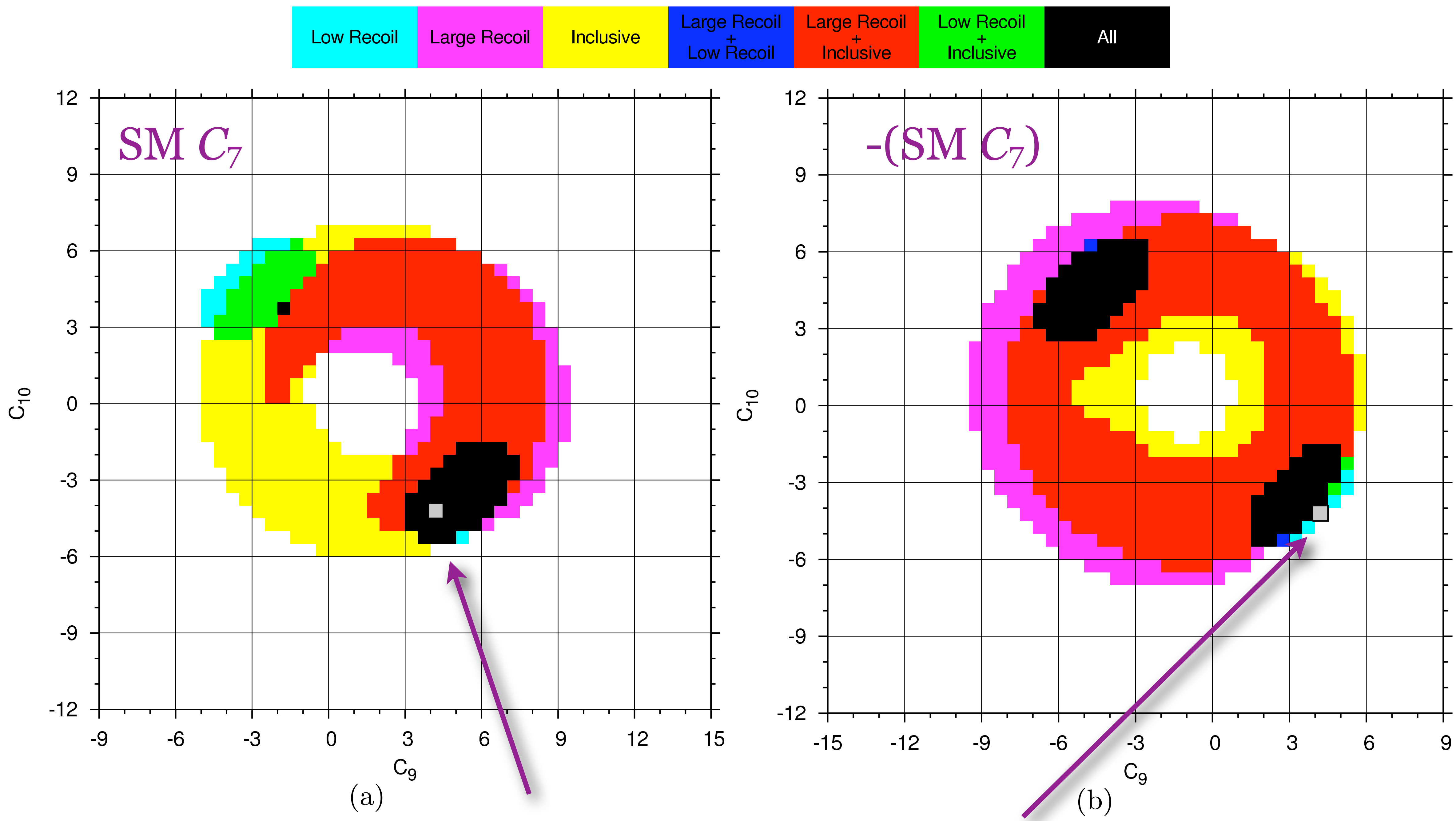


$\mathrm{BR}(B \to X_s \ell^+ \ell^-)$ $\mathrm{BR}(B \to X_s \gamma)$ $\mathrm{BR}(B \to K^* \mu^+ \mu^-)$ $A_{\mathrm{FB}}(B \to K^* \mu^+ \mu^-)$

- $B \to K^* \mu^+ \mu^-$ data exclude various "mirror solutions"

D Straub, Moriond EW, Mrach 2012

Data + QCD to constrain C's



Standard Model C_9, C_{10}

Bobeth, Hiller, van Dyk (2010)

Caveat

- The present LQCD calculation removes
  - Quenched approximation
  - Shape-dependent extrapolation of LCSR from low to high q^2
- LCSR & lattice both work in "narrow width approximation"
- Experiments must reconstruct K^* from $K\pi$
- Open question how to go beyond this theoretically

Lattice data

High statistics

MILC lattices (2+1 asqtad staggered)

	a(fm)	am_{sea}	Volume	$N_{conf} \times N_{src}$	am_{val}
coarse	$\sim$0.12	0.007/0.05	$20^3 \times 64$	2109×8	0.007/0.04
		0.02/0.05	$20^3 \times 64$	2052×8	0.02/0.04
fine	$\sim$0.09	0.0062/0.031	$28^3 \times 96$	1910×8	0.0062/0.031

m_π (MeV)
~300
~460
~320

Light meson momenta (units of $2\pi/L$)

- $(p_x, p_y, p_z) = (0,0,0)$.
- $(\tilde{q}$,0,0), (0,$\tilde{q}$,0), (0,0,$\tilde{q}$), where $\tilde{q}$=1 or 2.
- (1,1,0), (1,-1,0), (1,0,1), (1,0,-1), (0,1,1), (0,1,-1).
- (1,1,1), (1,1,-1), (1,-1,1), (1,-1,-1).

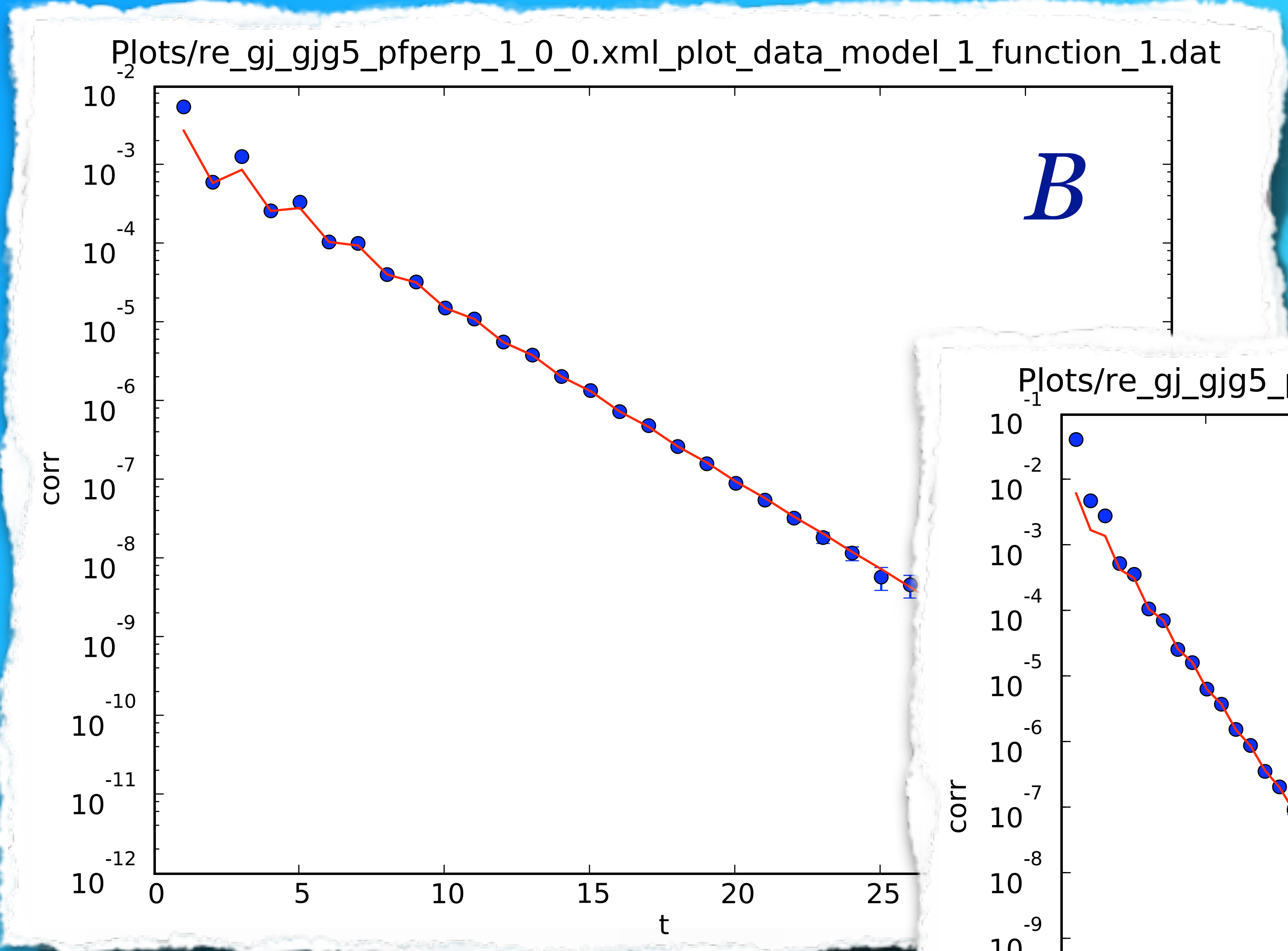
$p^2/(2\pi/L)^2$
0
1 or 4
2
3

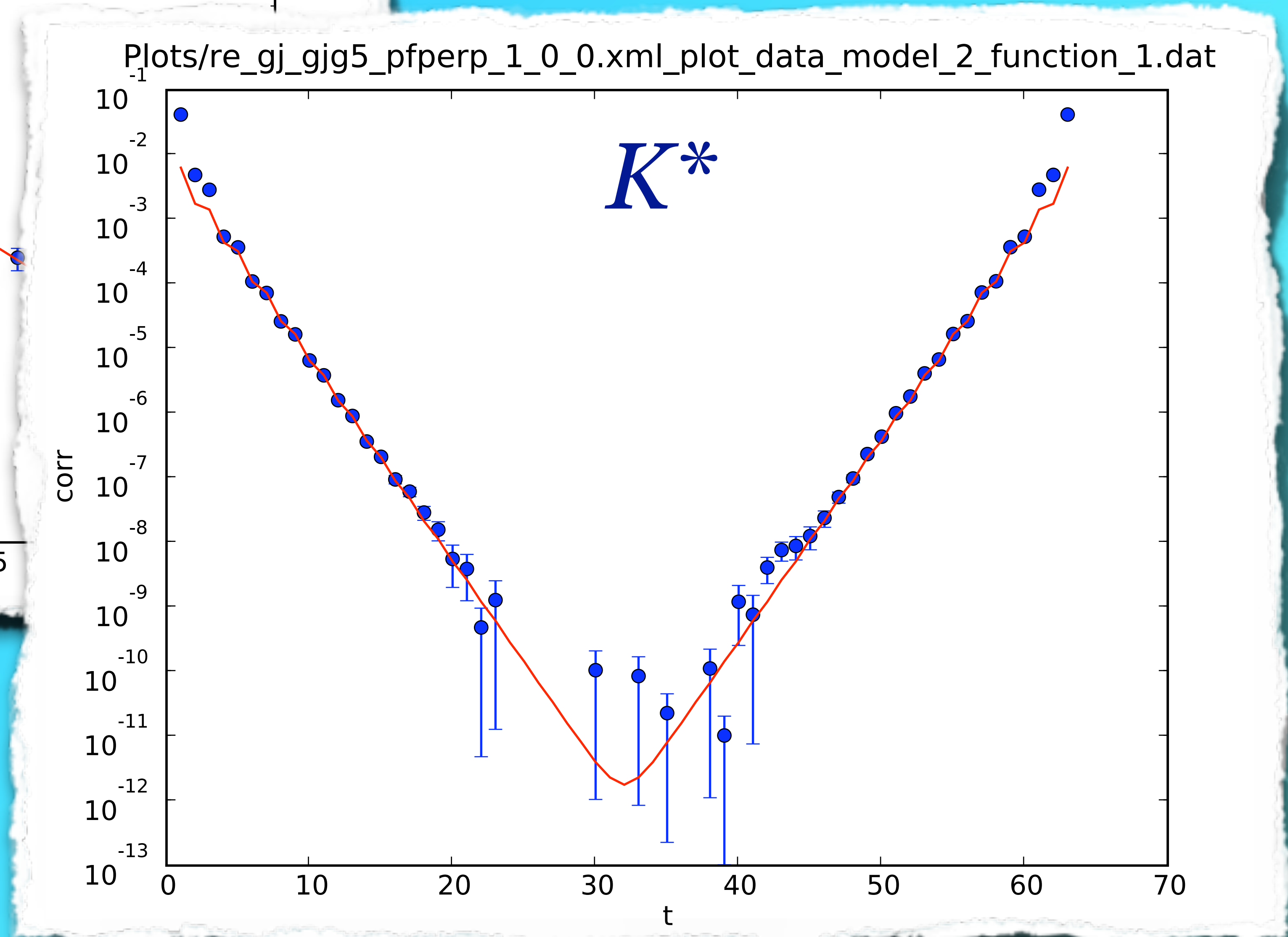
Many Source/Sink separations (16 coarse, 22 fine)

v=0 NRQCD used (B at rest).

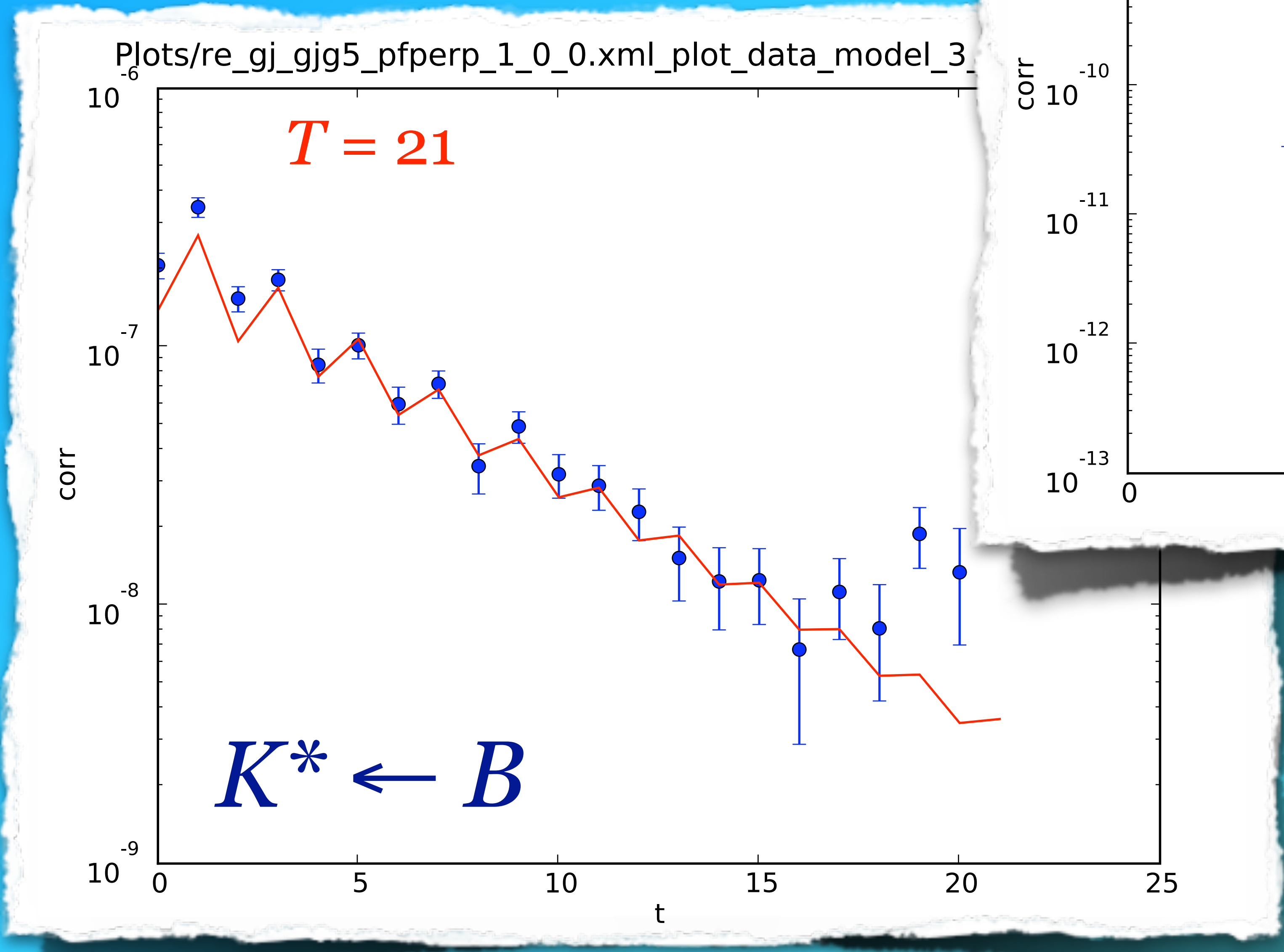
Leading order (HQET) current presently used.

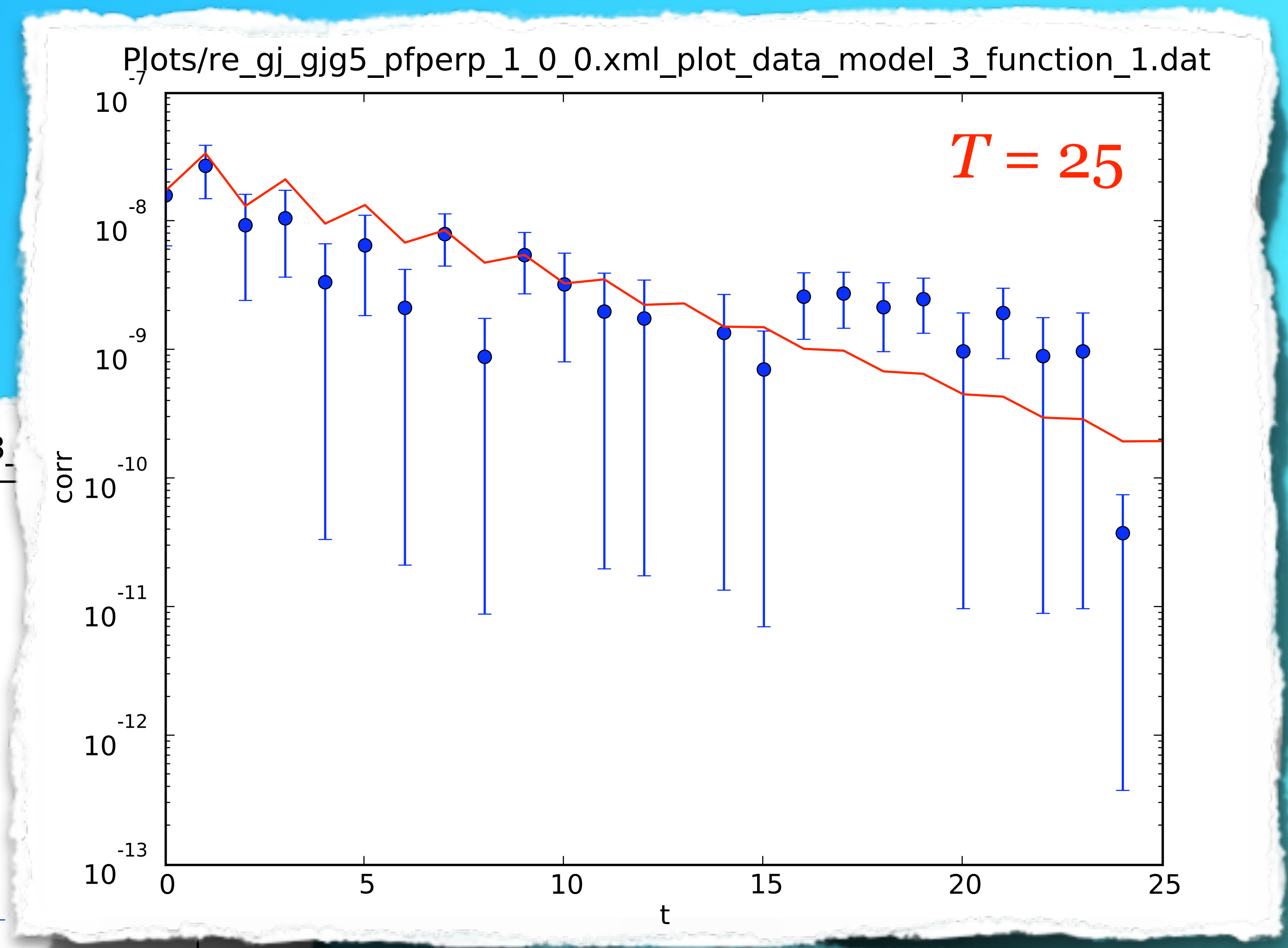
Meson correlators



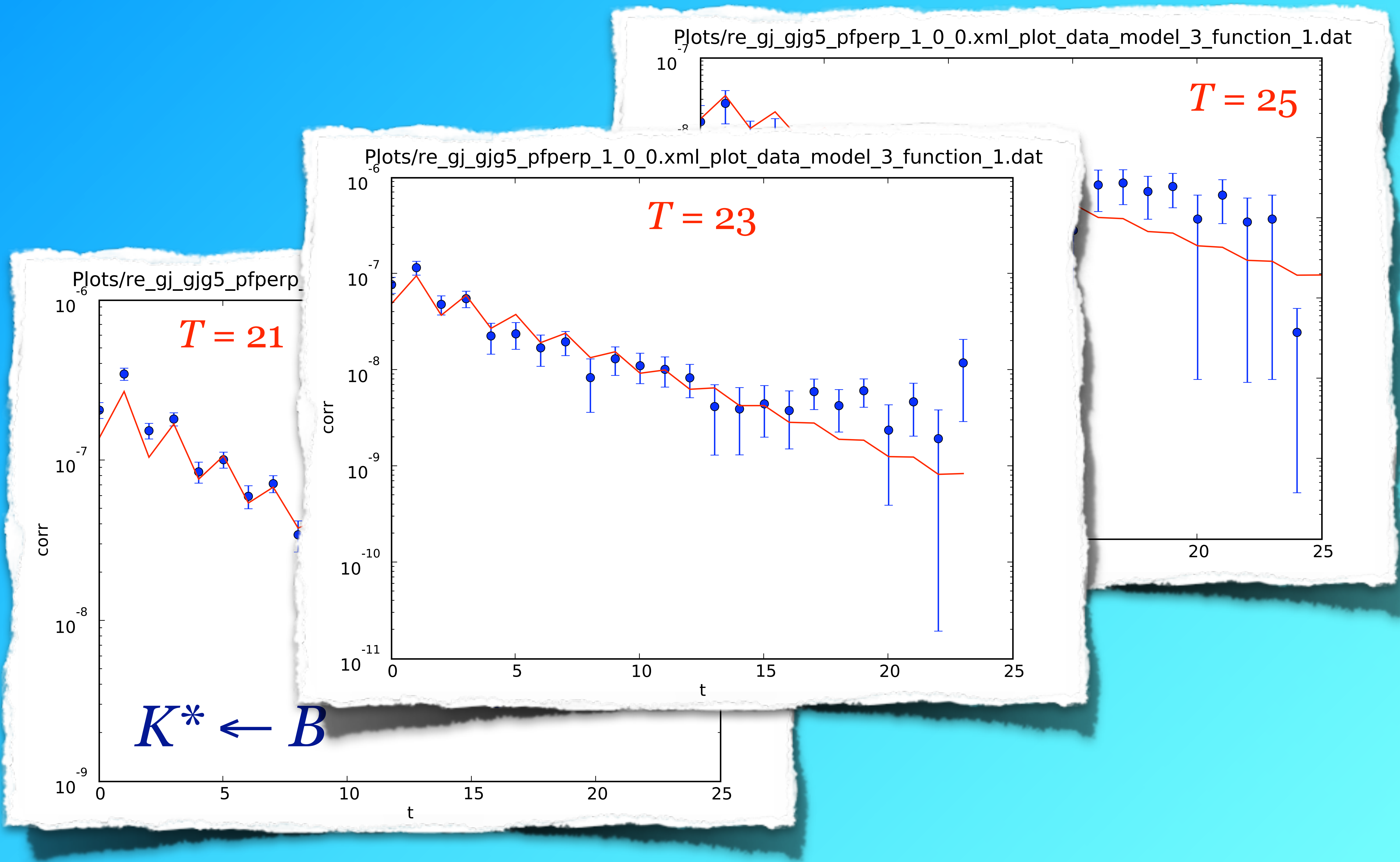


3-point correlators





3-point correlators



5-correlator fits

Matrix element from amplitudes

$$
\begin{aligned}
A^{(FJB)} &= \frac{\sqrt{Z_V}}{2E_V}\frac{\sqrt{Z_B}}{2E_B}\sum_s \varepsilon_j(p',s)\,\langle V\left(p',\varepsilon(p',s)\right)|\,J\,|B(p)\rangle, \\
A^{(BB)} &= \frac{Z_B}{2E_B}, \\
A^{(FF)} &= \sum_s \frac{Z_V}{2E_V}\,\varepsilon_j^*(p',s)\varepsilon_j(p',s)
\end{aligned}
$$

- One 3-point correlator whose amplitude gives matrix element
- Two 2-point correlators to divide out 2-pt amplitudes
- One 3-point correlator with precise *B* energy (*B* to *P* at |p'|=0)
- One 2-point correlator to further constrain *P* meson mass

2 analyses

- Bayesian:
  - Many-exponential fit function
  - Fit whole range of t (operator position)
  - Fit a couple values of T (source-sink separation)
- Frequentist:
  - Fewer-exponential fit functions
  - Randomly choose (plausible) t-ranges to fit
  - Fit all values of T
  - Rank "best" few fits, then use those t-ranges in bootstrap

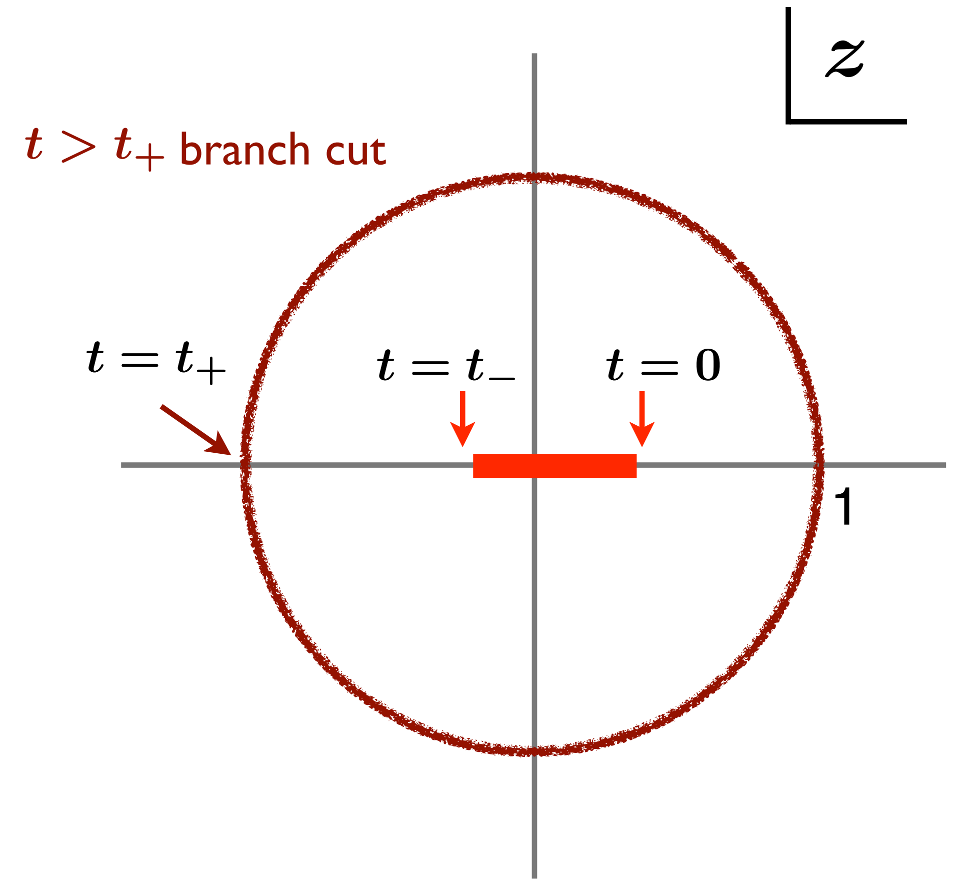
Form factor shape

Series (z) expansion

$$t = q^2 \qquad t_\pm = (m_B \pm m_F)^2$$

Choose, *e.g.* $\quad t_0 = 12\ \mathrm{GeV}^2$

$$z = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$



Simplified series expansion

$$F(t) = \frac{1}{1 - t/m_{\mathrm{res}}^2} \sum_n a_n z^n$$

Bourrely, Caprini, Lellouch PRD **79** (2009) following Okubo; Bourrely, Machet, de Rafael; Boyd, Grinstein, Lebed; Boyd & Savage; Arneson *et al.*; FNAL/MILC lattice collab; ...

Continuum-chiral-kinematic fits

HPQCD

$$F(t) = \frac{1}{1 - t/m_{\rm res}^2}\left[1 + b_1 (aE_F)^2 + \ldots\right] \sum_n a_n d_n z^n$$

discretization errors

$$d_n = 1 + c_{n1} \frac{m_P^2}{(4\pi f)^2} + \ldots$$

quark mass dependence

Form factor definitions

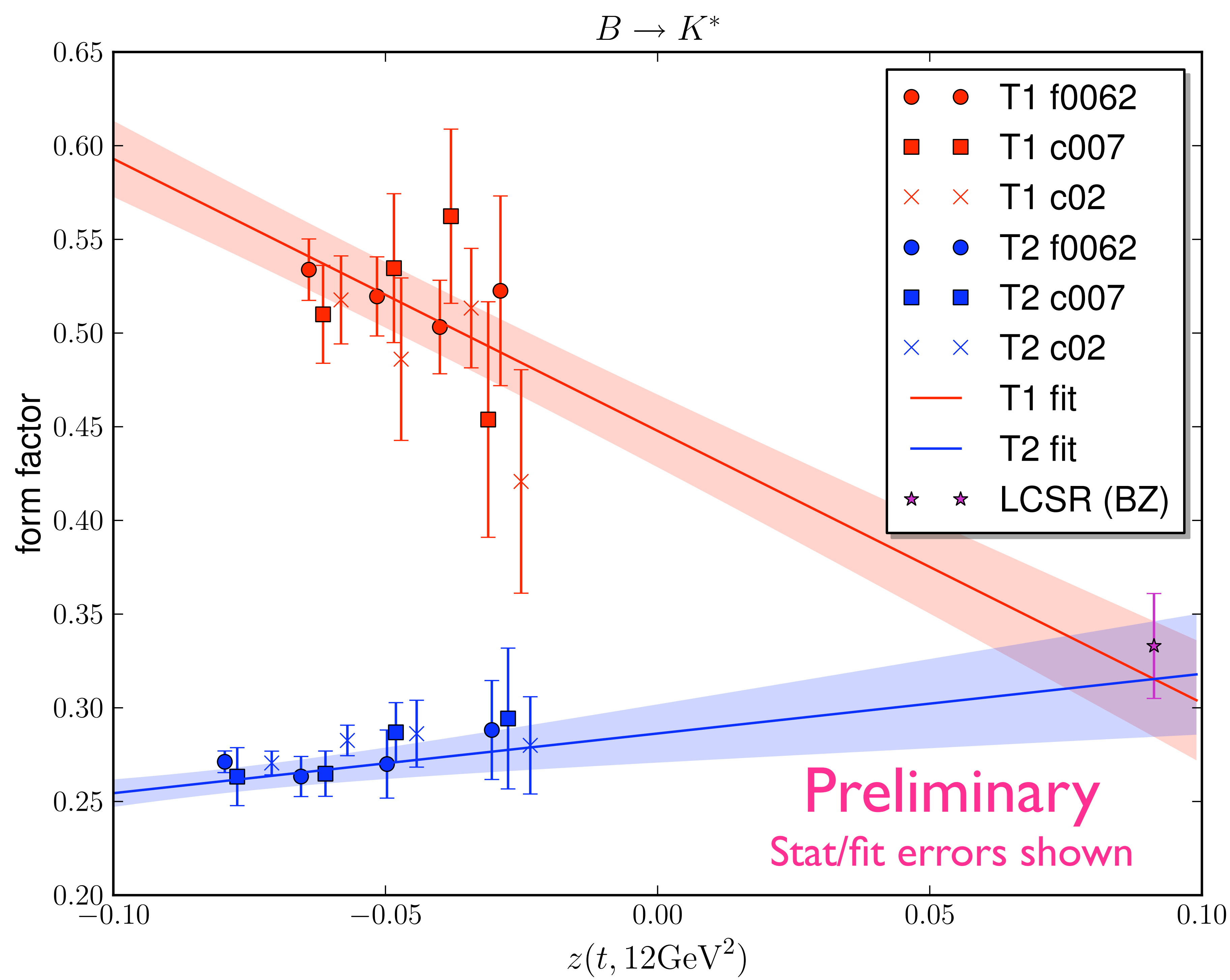
$$\langle V(p',\varepsilon)|\bar{q}\hat{\gamma}^{\mu}b|B(p)\rangle = \frac{2\mathrm{i}V(q^2)}{m_B+m_V}\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_{\nu}p'_{\rho}p_{\sigma}$$

$$\begin{aligned}\langle V(p',\varepsilon)|\bar{q}\hat{\gamma}^{\mu}\hat{\gamma}^5 b|B(p)\rangle &= 2m_V A_0(q^2)\frac{\varepsilon^*\cdot q}{q^2}q^{\mu} \\ &+(m_B+m_V)A_1(q^2)\left(\varepsilon^{*\mu}-\frac{\varepsilon^*\cdot q}{q^2}q^{\mu}\right) \\ &-A_2(q^2)\frac{\varepsilon^*\cdot q}{m_B+m_V}\left((p+p')^{\mu}-\frac{m_B^2-m_V^2}{q^2}q^{\mu}\right)\end{aligned}$$

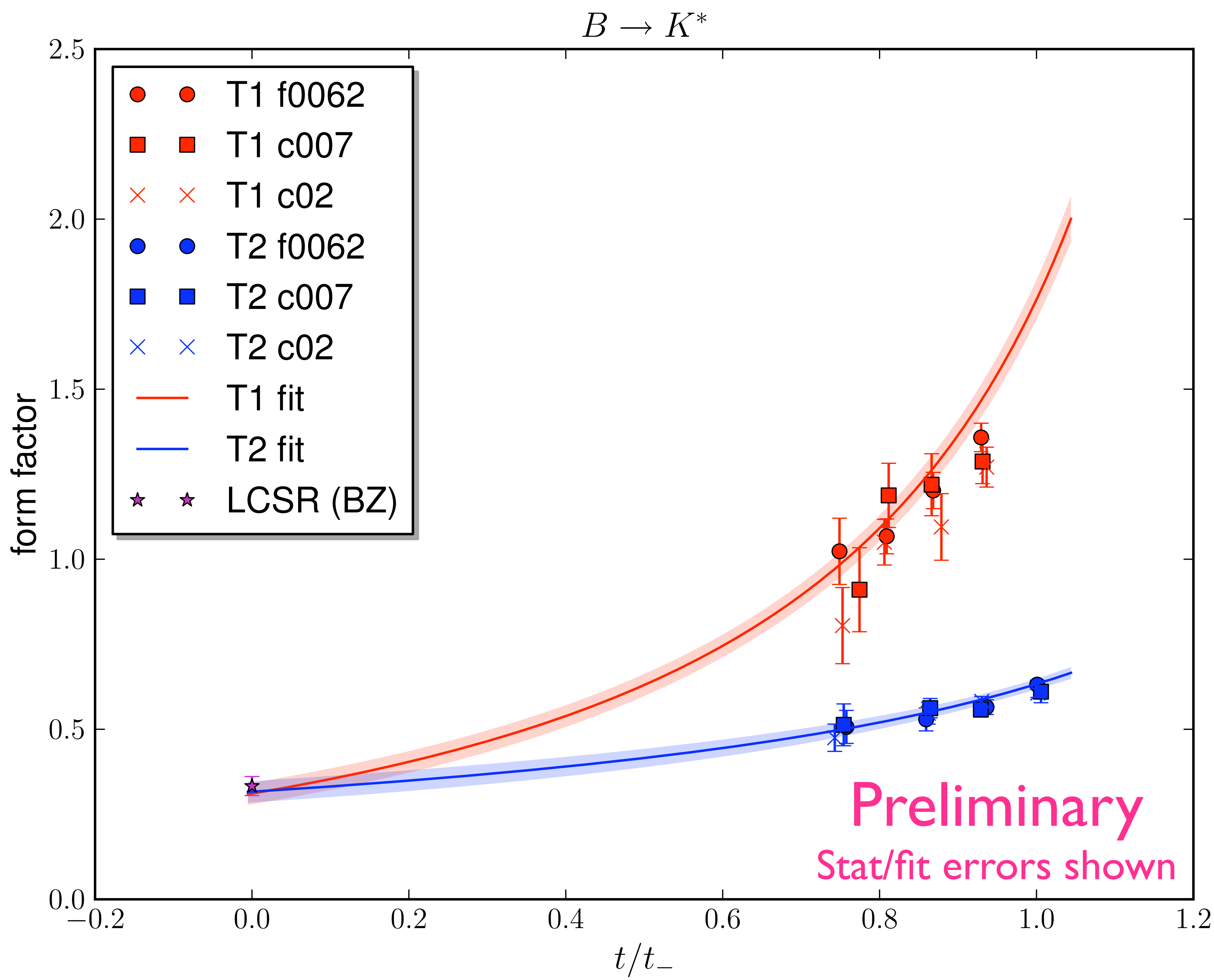
$$q^{\nu}\langle V(p',\varepsilon)|\bar{q}\hat{\sigma}_{\mu\nu}b|B(p)\rangle = 2\mathrm{i}T_1(q^2)\epsilon_{\mu\rho\tau\sigma}\varepsilon^{*\rho}p^{\tau}p'^{\sigma}$$

$$\begin{aligned}q^{\nu}\langle V(p',\varepsilon)|\bar{q}\hat{\sigma}_{\mu\nu}\hat{\gamma}^5 b|B(p)\rangle &= \mathrm{i}T_2(q^2)[\varepsilon^*_{\mu}(m_B^2-m_V^2)-(\varepsilon^*\cdot q)(p+p')_{\mu}] \\ &+\mathrm{i}T_3(q^2)(\varepsilon^*\cdot q)\left[q_{\mu}-\frac{q^2}{m_B^2-m_V^2}(p+p')_{\mu}\right]\end{aligned}$$

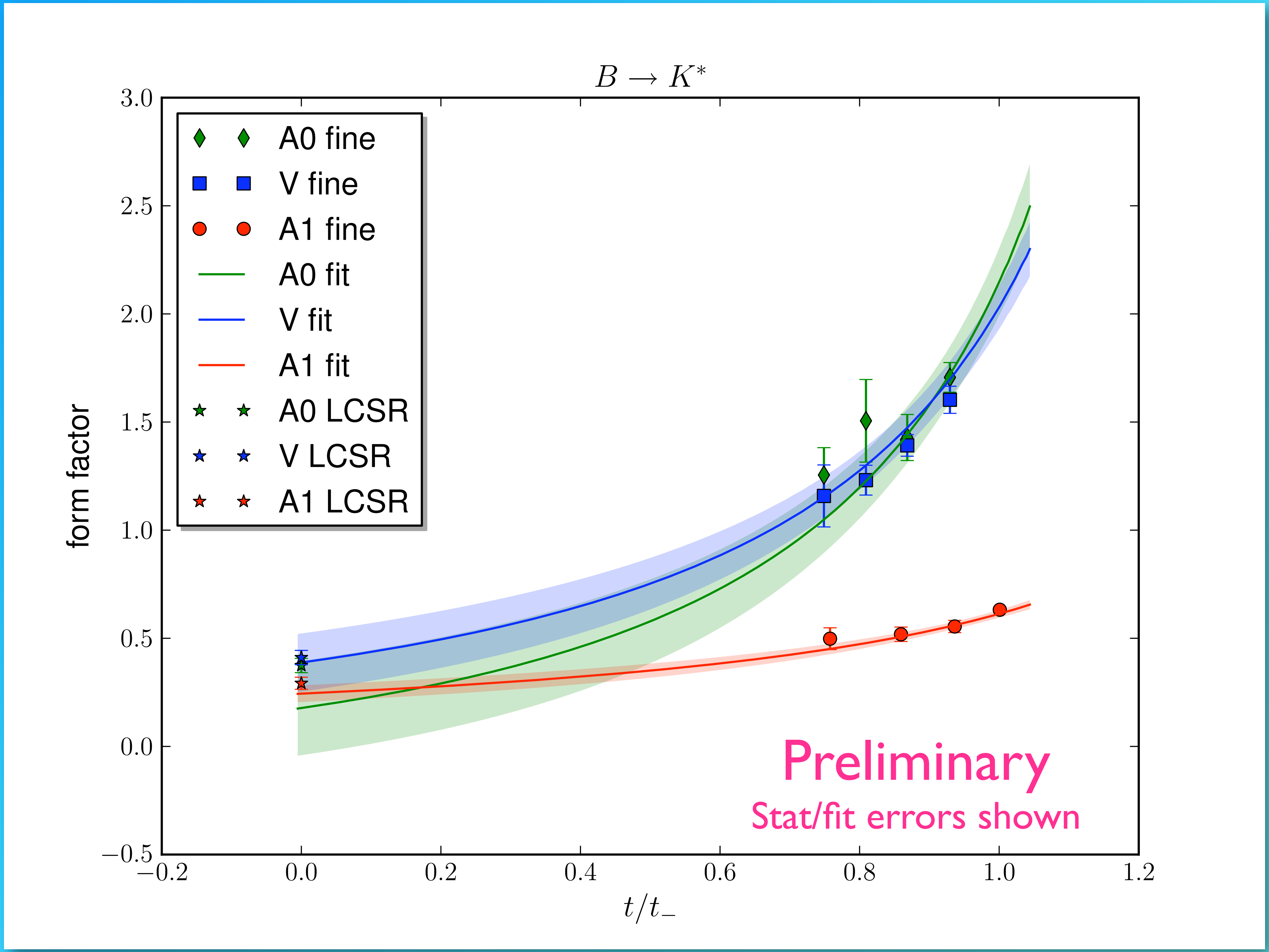
$B \to K^*$, $P(t)T_1$ & $P(t)T_2$, vs. z



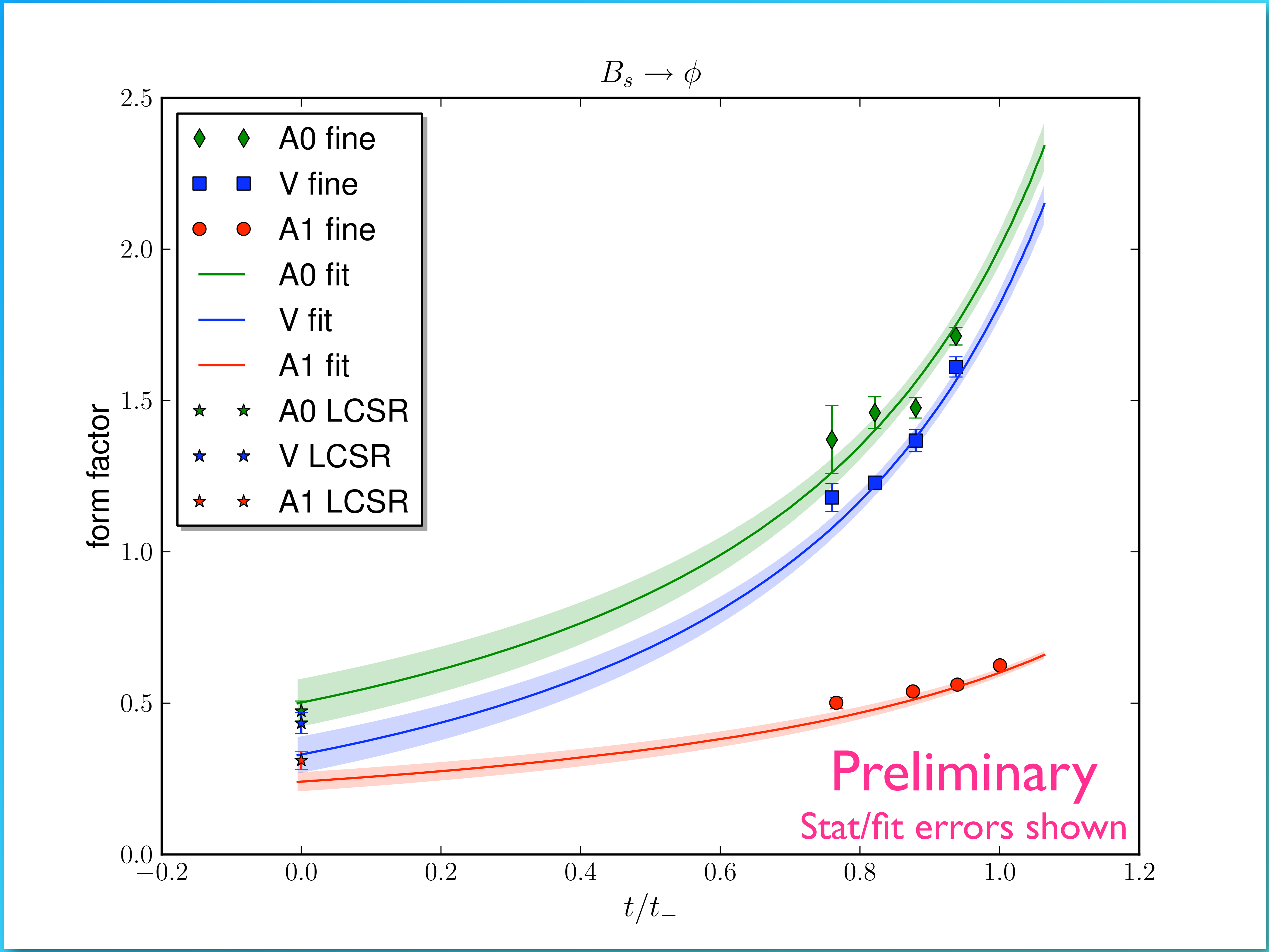
$B \to K^*$, T_1 & T_2, vs. $q^2/q^2{}_{max}$



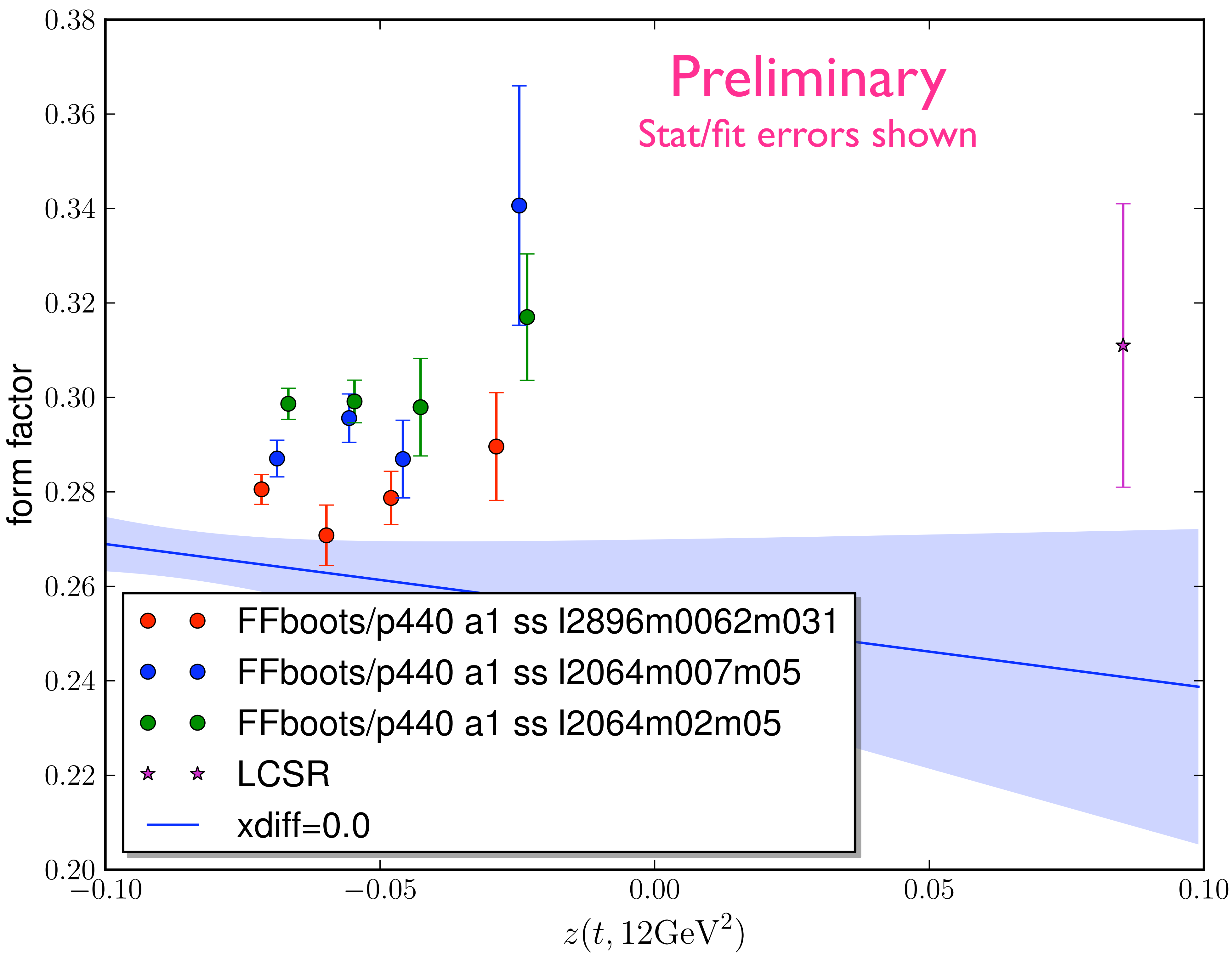
$B \to K^*$, V, A_0, A_1, vs. q^2/q^2_{max}



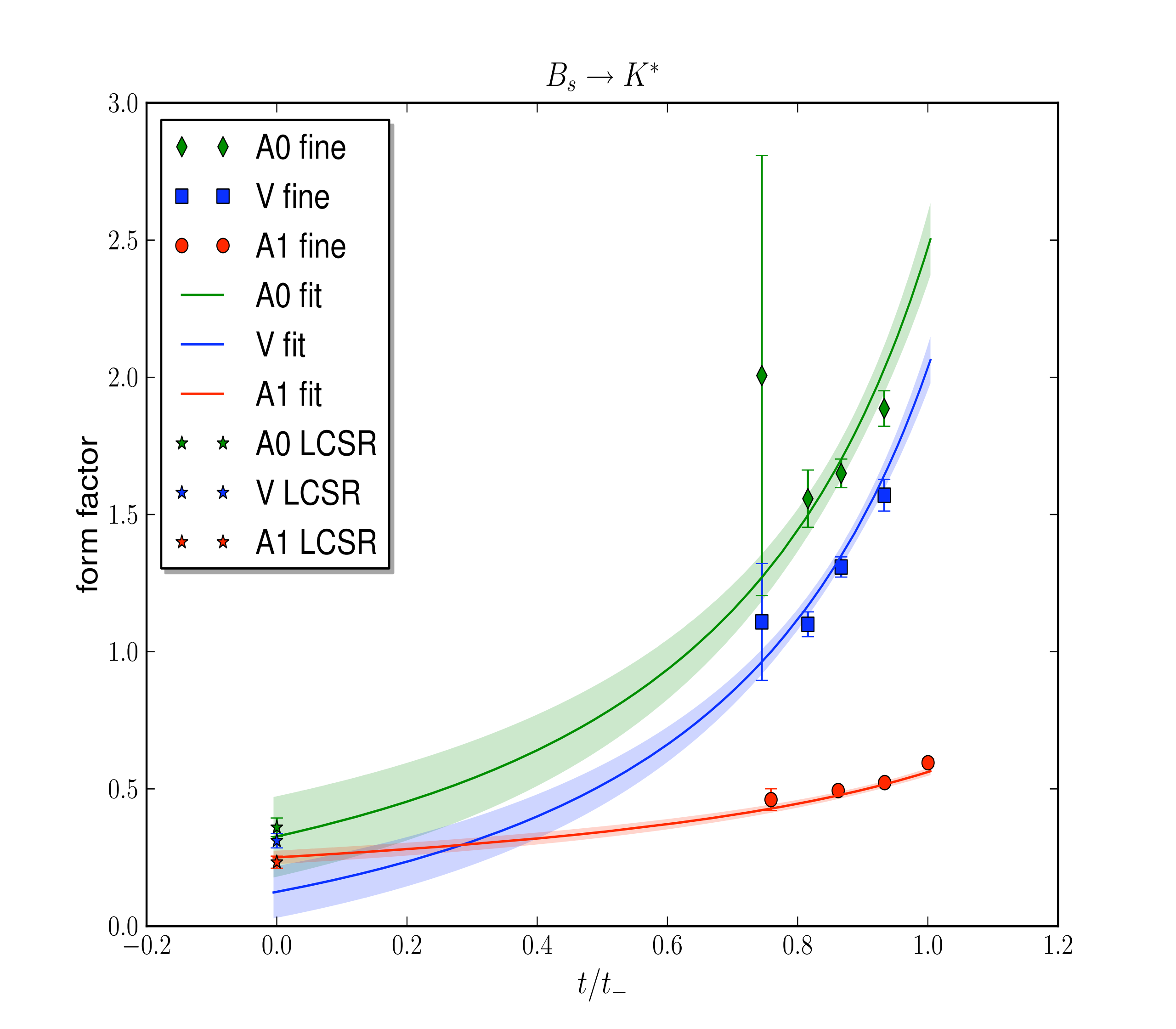
$B_s \rightarrow \varphi$, V, A_0, A_1, vs. q^2/q^2_{max}

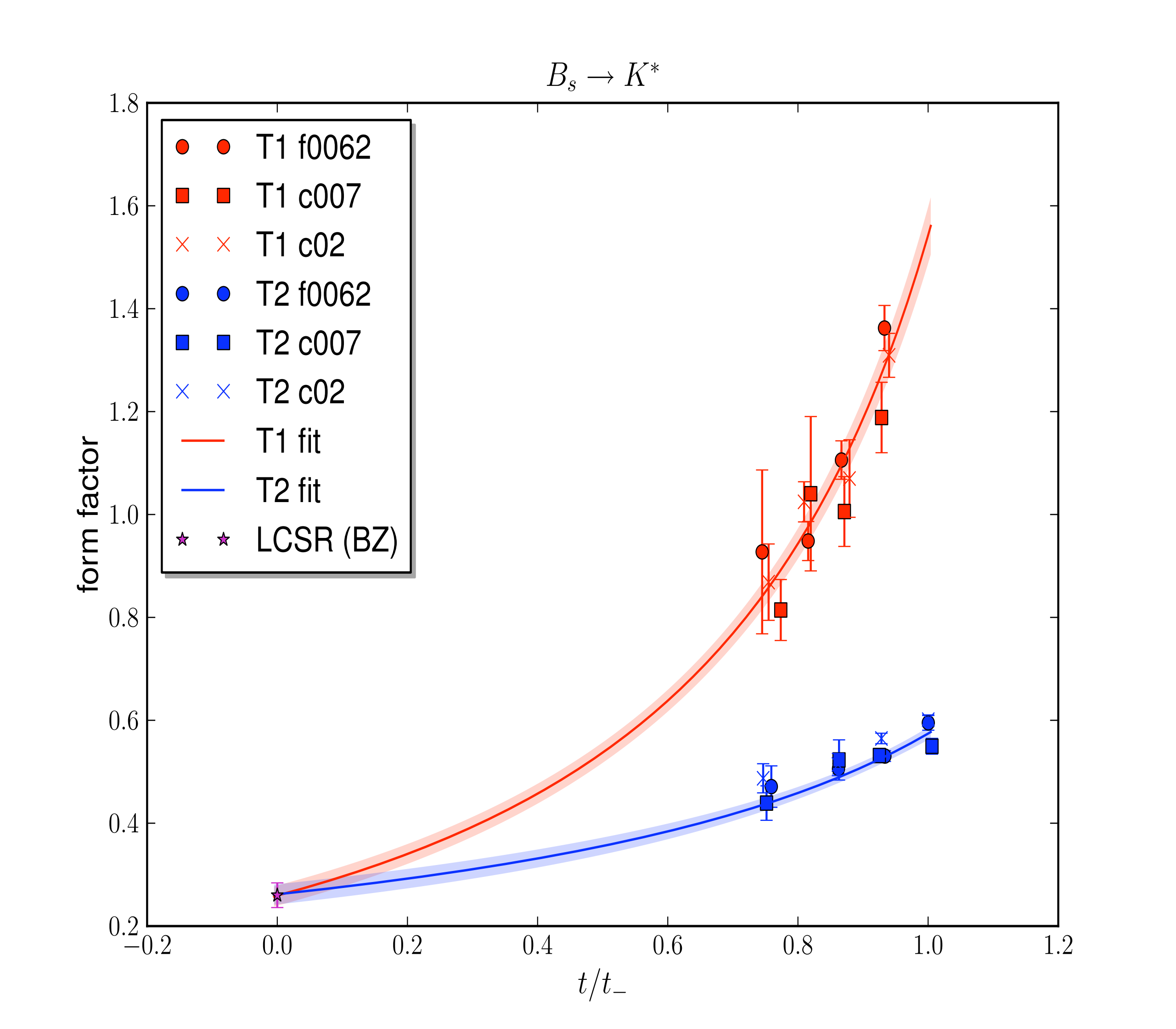


Discretization errors



$B_s \to K^*$





Preliminary

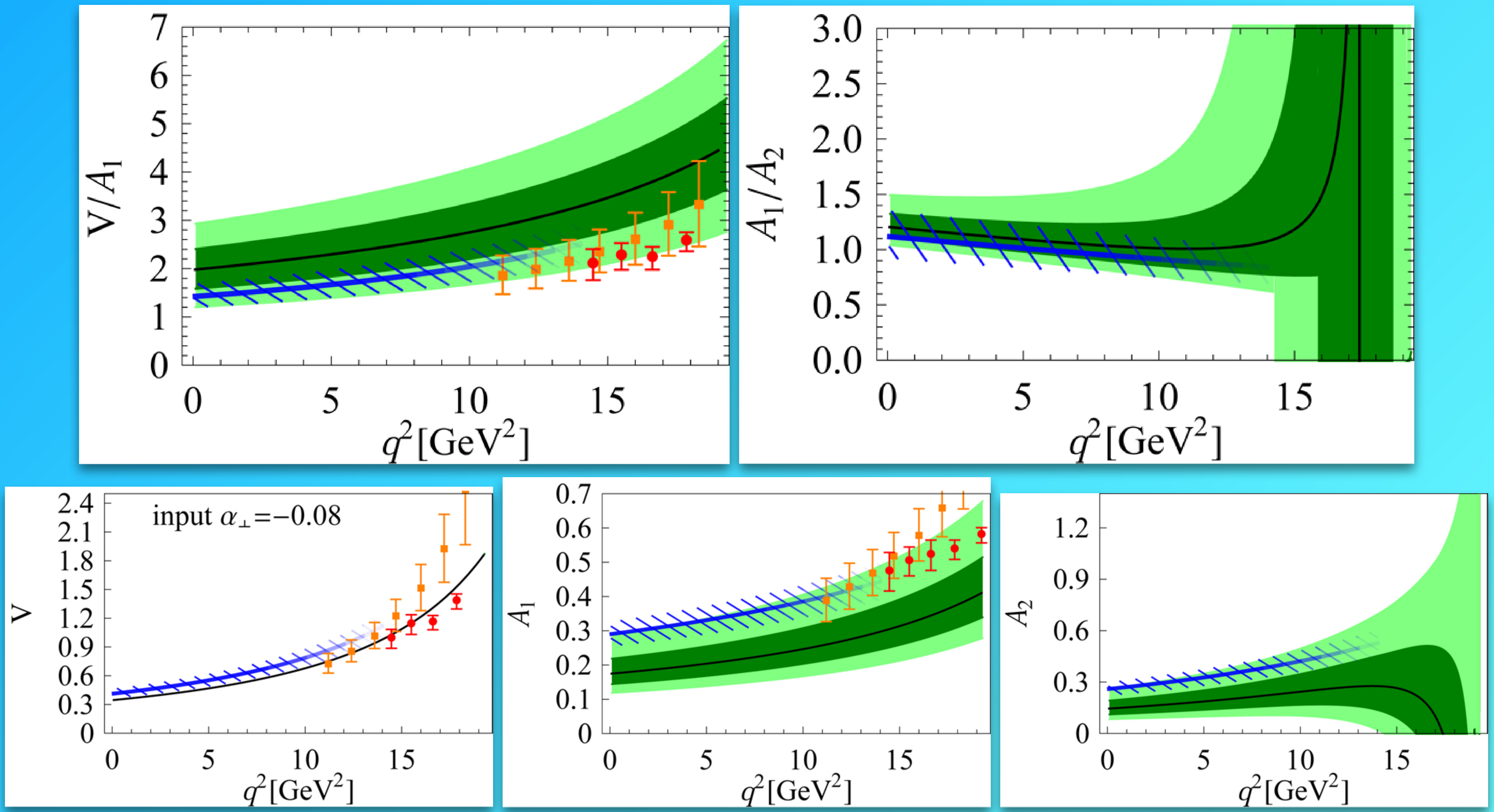
Stat/fit errors shown

Conclusions

- Rare decays provide new tests of weak-scale physics
- Low-recoil semi-leptonic decays:
  - Measurements + theory constrain Wilson coefficients
- Unquenched LQCD results
  - $B \to K^*$
  - $B_s \to \varphi$
  - $B_s \to K^*$
  - $B \to \rho$ (noisy!)

Comparison

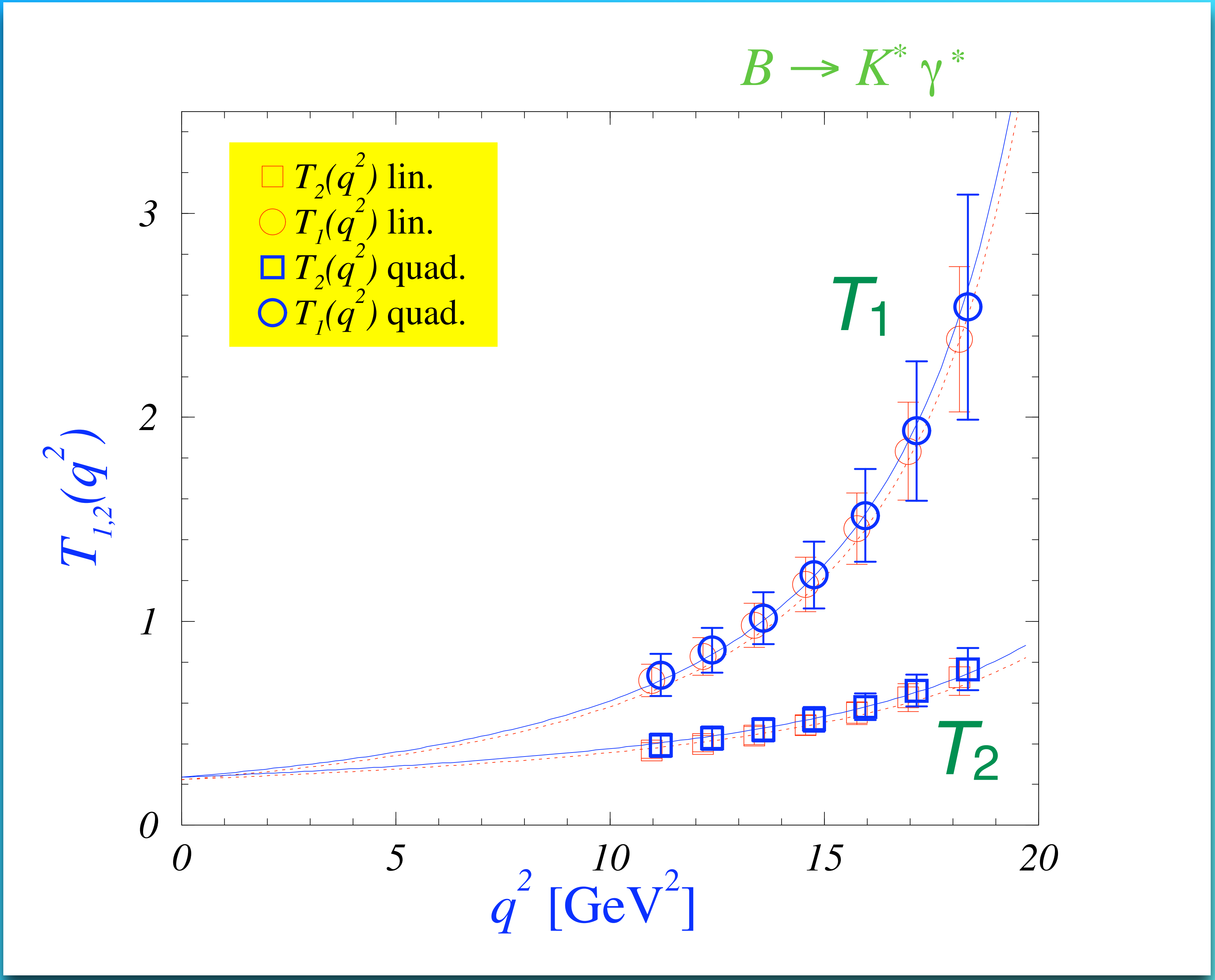
Green: Expt. (CDF/BaBar/LHCb + Hambrock & Hiller)
Blue: Light cone sum rules (Ball & Zwicky)
Red: Our preliminary lattice data (before extrapolation)
Orange: Quenched lattice (Becirevic, Lubicz, Mescia)



Plots from Hambrock & Hiller, 1204.4444

Quenched T_1 & T_2

Bećirević-Lubicz-Mescia, Nucl. Phys. B769, 31 (2007)



Quenched V, A_0, A_1, A_2

Bowler, Gill, Maynard, Flynn, JHEP 05 (2004) 035

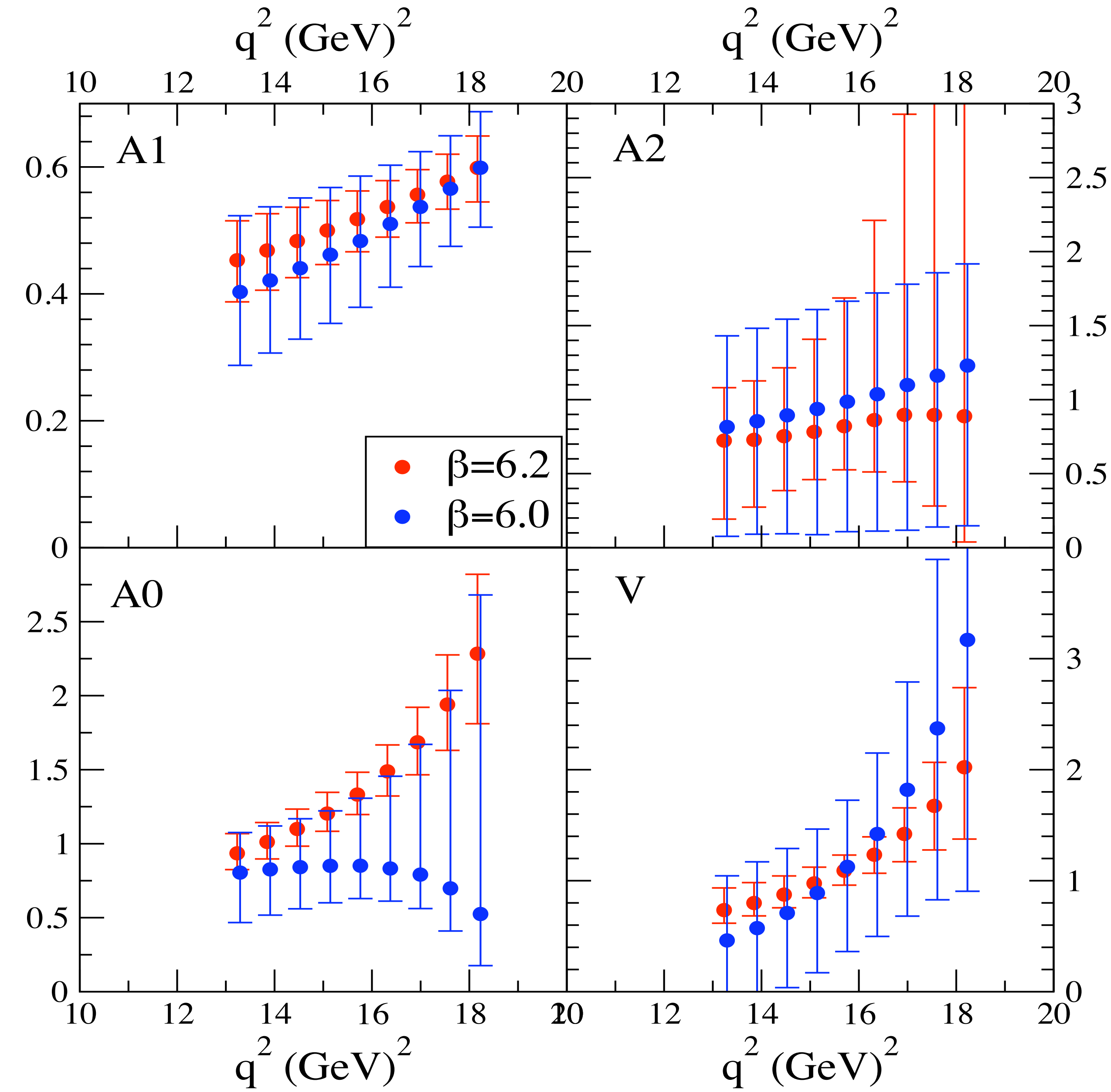


Figure 4: The form factors on both lattices. The vertical scale is different for each form factor.