B and B_s semileptonic decays with NRQCD/HISQ quarks

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Outline

- motivation
  - $B_s \to K\ell\nu$
  - $B \to \pi\ell\nu$
  - $B_s \to \eta_s$
  - $B \to K\ell^+\ell^-$
- calculation
  - simulation details
  - setup
  - correlator fits
  - extracting form factors
  - connecting with experiment
- preliminary results
- next steps

motivation

- $B_s \to K\ell\nu$ *(begun)*
  - not yet calculated on the lattice (Fermilab-MILC in progress)
  - B_s semileptonic decays becoming experimentally relevant: LHCb, next generation B-factories,…
  - warmup for $B \to \pi\ell\nu$
- $B \to \pi\ell\nu$
- $B_s \to \eta_s$
- $B \to K\ell^+\ell^-$

motivation

- $B_s \to K\ell\nu$ *(begun)*
- $B \to \pi\ell\nu$ *(begun)*
  - improve upon [HPQCD, PRD 73, 074502 (2006); Erratum-ibid D75, 119906 (2007)]
    - b-quark smearing
    - HISQ (vs. asqtad) light-quarks with random wall sources
    - better scale-determination and tuned quark masses
    - fitting advances (e.g. simultaneous fits to multiple separation times)
    - z-expansion
  - $|V_{ub}|$ *(inclusive vs. exclusive,* $B \to \tau\nu$*)*
- $B_s \to \eta_s$
- $B \to K\ell^+\ell^-$

motivation

- $B_s \to K\ell\nu$ *(begun)*
- $B \to \pi\ell\nu$ *(begun)*
- $B_s \to \eta_s$ *(begun)*
  - investigate ratios of form factors, e.g.

$$\left.\frac{f(B \to \pi\ell\nu)}{f(B_s \to \eta_s)}\right|_{\text{NRQCD } b} \times \left. f(B_s \to \eta_s)\right|_{\text{HISQ } b}$$

  - analogous to decay constants [HPQCD, arXiv:1202.4914]
    (H. Na's talk, Mon, 2:50pm, Weak Decays and ME's)
- $B \to K\ell^+\ell^-$

motivation

- $B_s \to K\ell\nu$ *(begun)*
- $B \to \pi\ell\nu$ *(begun)*
- $B_s \to \eta_s$ *(begun)*
- $B \to K\ell^+\ell^-$ *(planned)*
  - FCNC $b \to s$ transition probes NP
  - few unquenched lattice calculations (Liu et al., Fermilab-MILC in progress)
  - BaBar [PRL 102, 091803 (2009)] and Belle [PRL 102, 171801(2009)] results
  - promise of new results from LHCb, next generation B-factories

calculation

- simulation details
- setup
- correlator fits
- extracting form factors
- connecting with experiment

simulation details

• MILC 2+1, asqtad gauge configurations [MILC, A. Bazavov et al., RMP 82, 1349 (2010)]

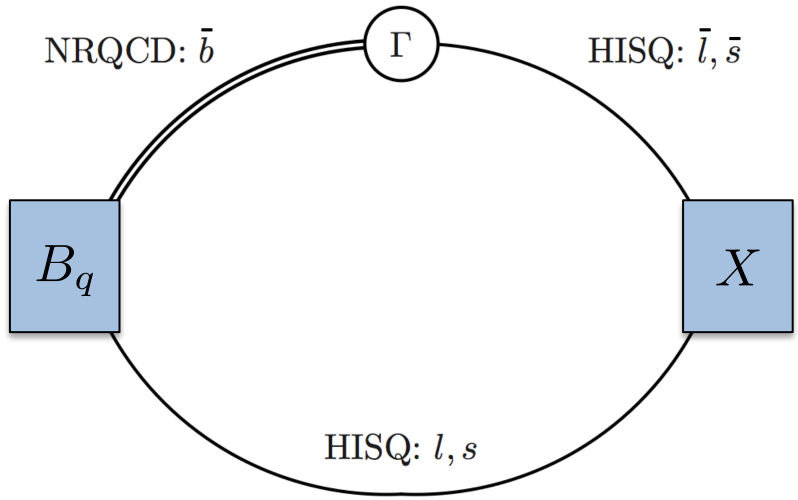
ensemble	$\approx a$ [fm]	$m_l(\text{sea})/m_s(\text{sea})$	N_{conf}	N_{tsrc}	$L^3 \times N_t$
C1	0.12	0.005/0.050	1200	2	$24^3 \times 64$
C2	0.12	0.010/0.050	1200	2	$20^3 \times 64$
C3	0.12	0.020/0.050	600	2	$20^3 \times 64$
F1	0.09	0.0062/0.031	1200	4	$28^3 \times 96$
F2	0.09	0.0124/0.031	600	4	$28^3 \times 96$

• NRQCD [HPQCD, G.P. Lepage et al., PRD 46, 4052 (1992)] bottom valence-quark, tuned in [HPQCD, H. Na et al., arXiv:1202.4914 [hep-lat]]

• HISQ [HPQCD, E. Follana et al., PRD 75, 054502 (2007)] light, strange valence-quark propagators generated for

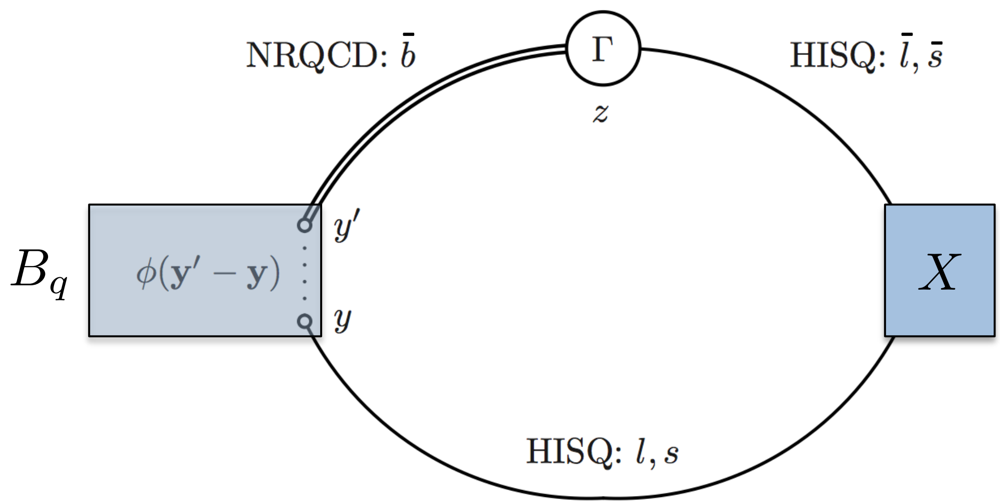
$D \to \pi \ell \nu$ [H. Na et al., PRD 84, 114505 (2011)]
$D \to K \ell \nu$ [H. Na et al., PRD 82, 14506 (2010)]

$B_q \to X\ (\ell\nu)(\ell\ell)$ setup



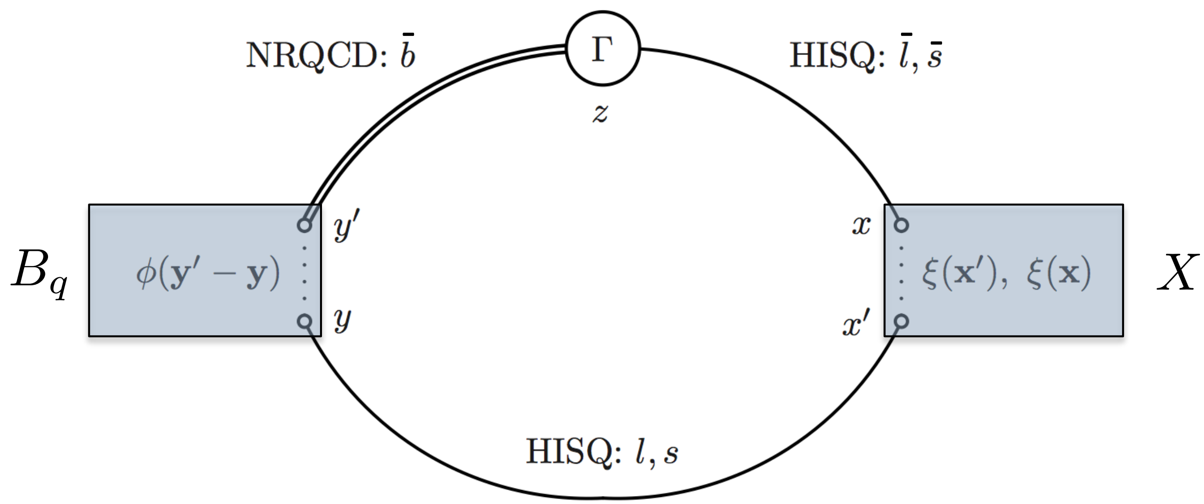
- Γ: spin-structure of flavor-changing current

$B_q \to X\ (\ell\nu)(\ell\ell)$ setup



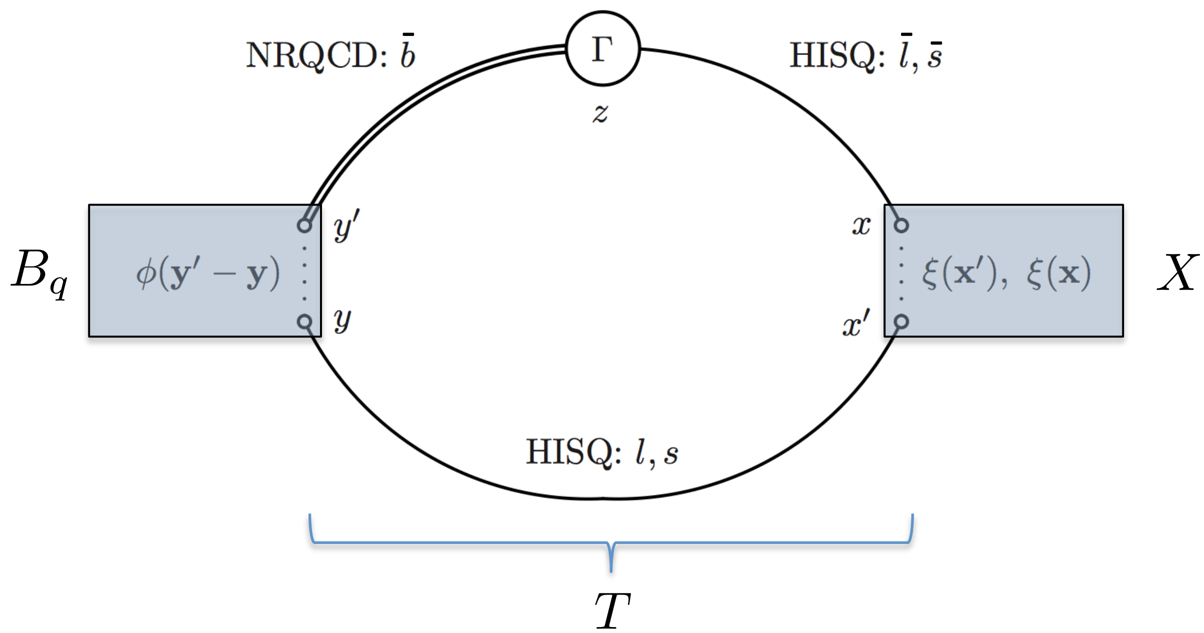
- Γ: spin-structure of flavor-changing current
- $\phi(\mathbf{y}' - \mathbf{y})$: Gaussian smearing of NRQCD heavy quark

$B_q \to X\ (\ell\nu)(\ell\ell)$ setup



- Γ: spin-structure of flavor-changing current
- $\phi(\mathbf{y}' - \mathbf{y})$: Gaussian smearing of NRQCD heavy quark
- $\xi(\mathbf{x}')$, $\xi(\mathbf{x})$: $U(1)$ phases for random-wall HISQ sources

$B_q \to X\ (\ell\nu)(\ell\ell)$ setup



- Γ: spin-structure of flavor-changing current
- $\phi(\mathbf{y}' - \mathbf{y})$: Gaussian smearing of NRQCD heavy quark
- $\xi(\mathbf{x}')$, $\xi(\mathbf{x})$: $U(1)$ phases for random-wall HISQ sources
- momentum inserted at x

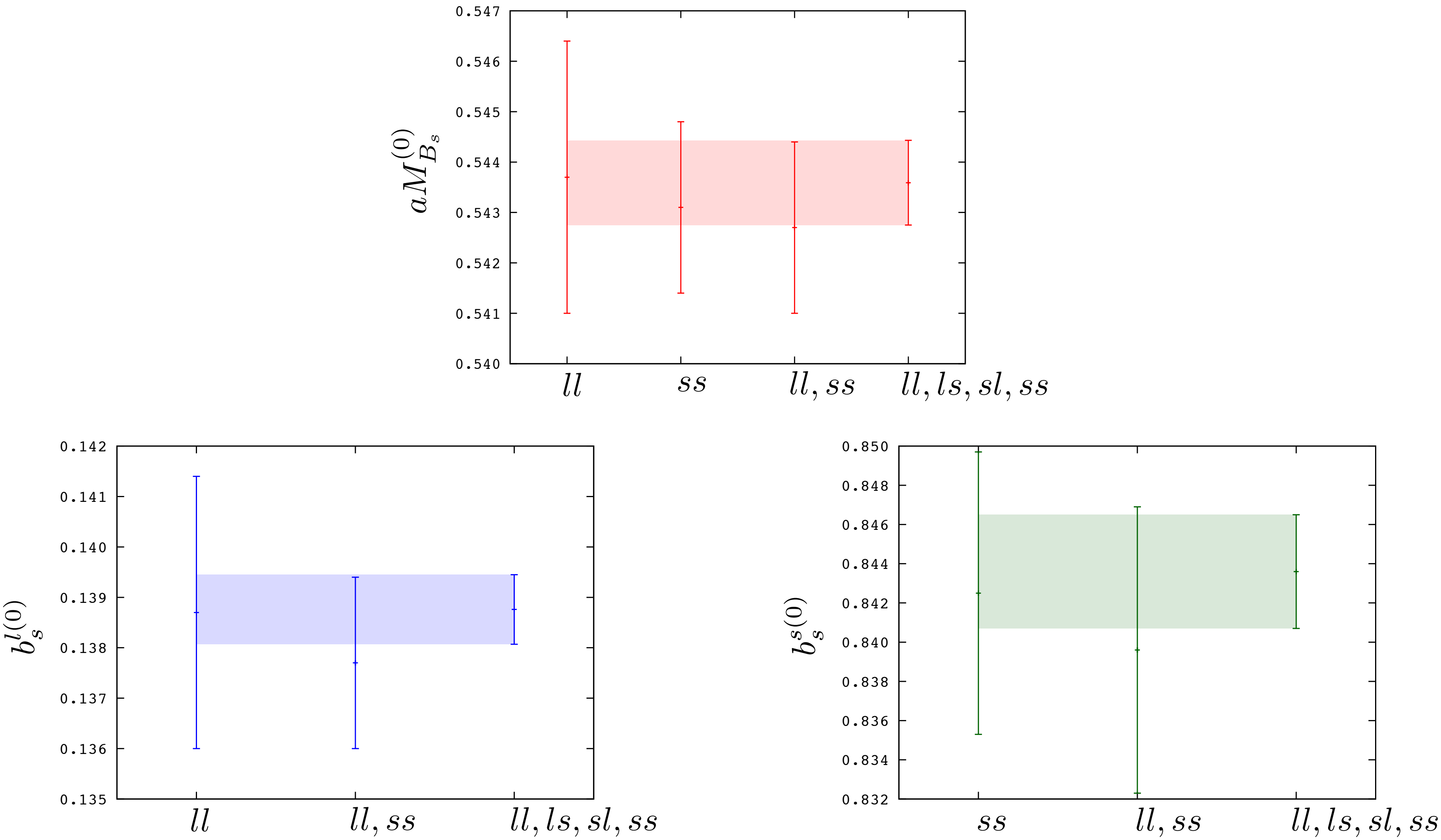
$B_q \to X\ (\ell\nu)(\ell\ell)$ parent 2pt fits

$$C^{\alpha\beta}_{B_q}(t) = \sum_{n=0}^{N_{B_q}-1} b_q^{\alpha(n)}\ b_q^{\beta(n)\dagger}\ (-1)^{nt}\ e^{-M^{(n)}_{B_q} t} \quad ;\ \alpha,\ \beta \text{ specify smearing}$$

$$b_q^{\alpha(n)} = \frac{\langle \Phi^{\alpha}_{B_q} | B_q^{(n)} \rangle}{\sqrt{2 M^{(n)}_{B_q}}}$$

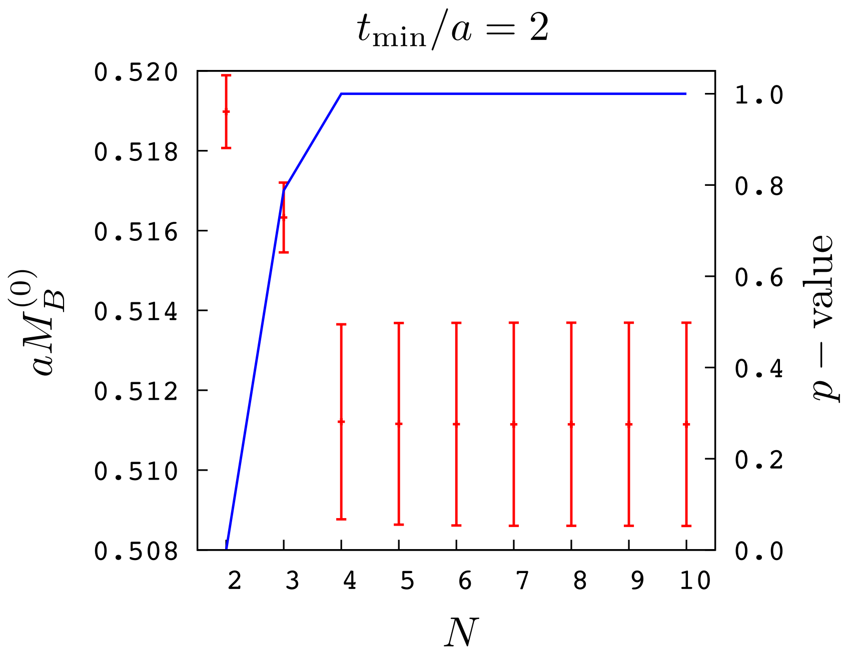
- analyze B and B_s
- work in parent rest frame
- bayesian fit to 2×2 matrix of local and gaussian smeared data

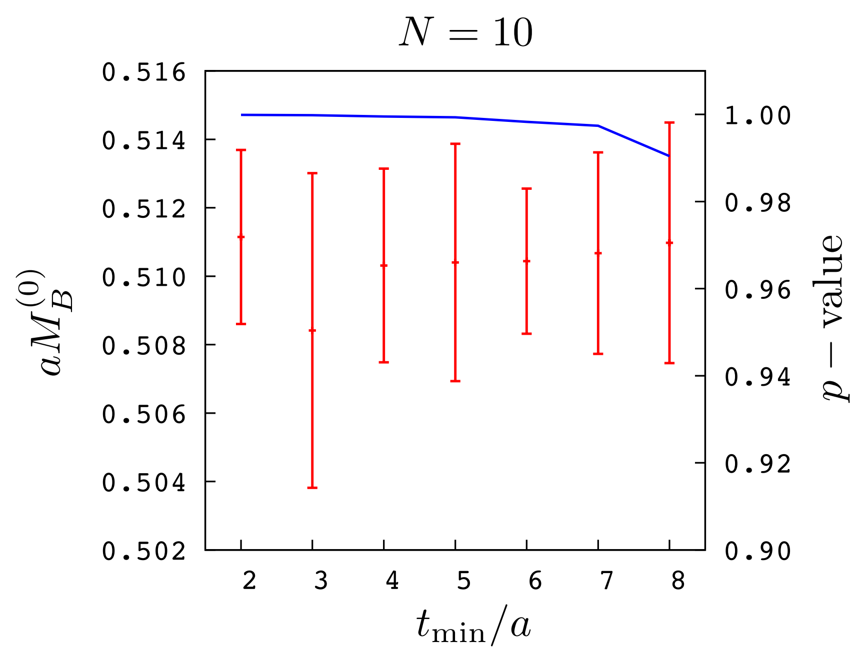
$B_q \to X\ (\ell\nu)(\ell\ell)$ parent 2pt fits



improvement from fitting local (l) and smeared (s) 2pt data (ensemble C2)

$B_q \to X\ (\ell\nu)(\ell\ell)$ parent 2pt fits





- B and B_s fits:
  - stable vs. N, t_{min} , t_{max}
  - use: $N = 10$, $t_{\mathrm{min}}/a = 2$, $t_{\mathrm{max}}/a = 19$
  - shown for B ground-state energy on ensemble C3

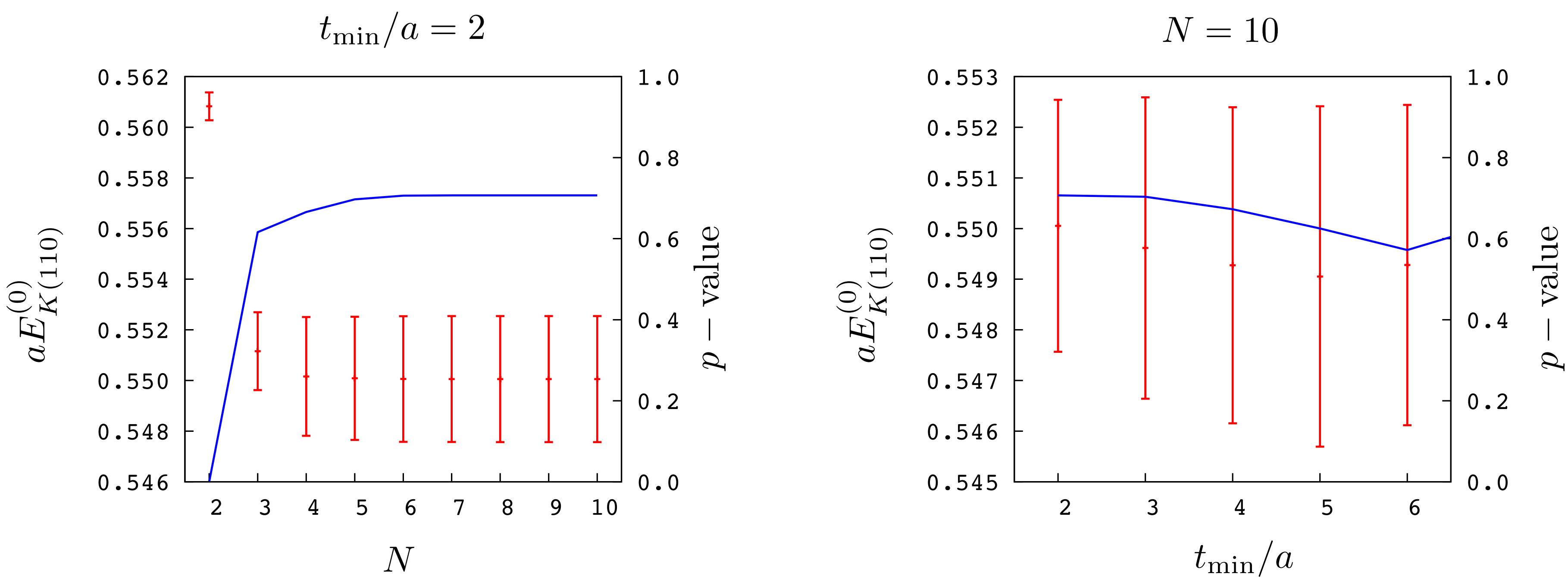
$B_q \to X (\ell\nu)(\ell\ell)$ daughter 2pt fits

$$C_X(t) = \sum_{n=0}^{N_X-1} \left|d_X^{(n)}\right|^2 (-1)^{nt} \left(e^{-E_X^{(n)} t} + e^{-E_X^{(n)}(T-t)}\right)$$

$$d_X^{(m)} = \frac{\langle \Phi_X | X^{(m)} \rangle}{\sqrt{2E_X^{(m)}}}$$

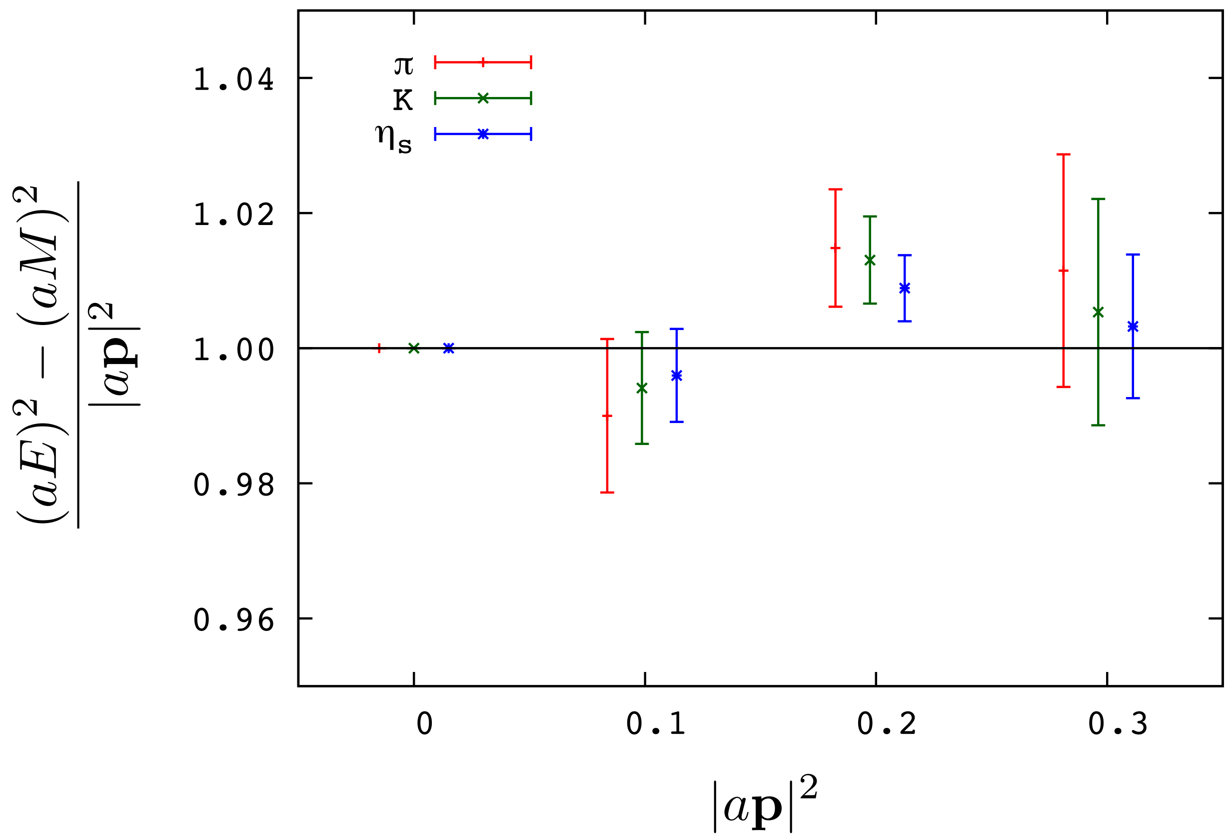
- analyze π , K , and η_s
- random-wall sources
- momenta: $\frac{2\pi}{L} \times \left\{(0,0,0),(1,0,0),(1,1,0),(1,1,1)\right\}$

$B_q \to X (\ell\nu)(\ell\ell)$ daughter 2pt fits



- $\pi(\mathbf{p})$, $K(\mathbf{p})$, $\eta_s(\mathbf{p})$ fits:
  - stable vs. N, $t_{\min}$, $t_{\max}$
  - use: $N = 10$, $t_{\min}/a = 2$, $t_{\max}/a \approx 30$
  - shown for $K(110)$ ground-state energy on ensemble C2

$B_q \to X (\ell\nu)(\ell\ell)$ daughter 2pt fits



dispersion relation on ensemble C3

$B_q \to X\ (\ell\nu)(\ell\ell)$ 3pt correlator fits

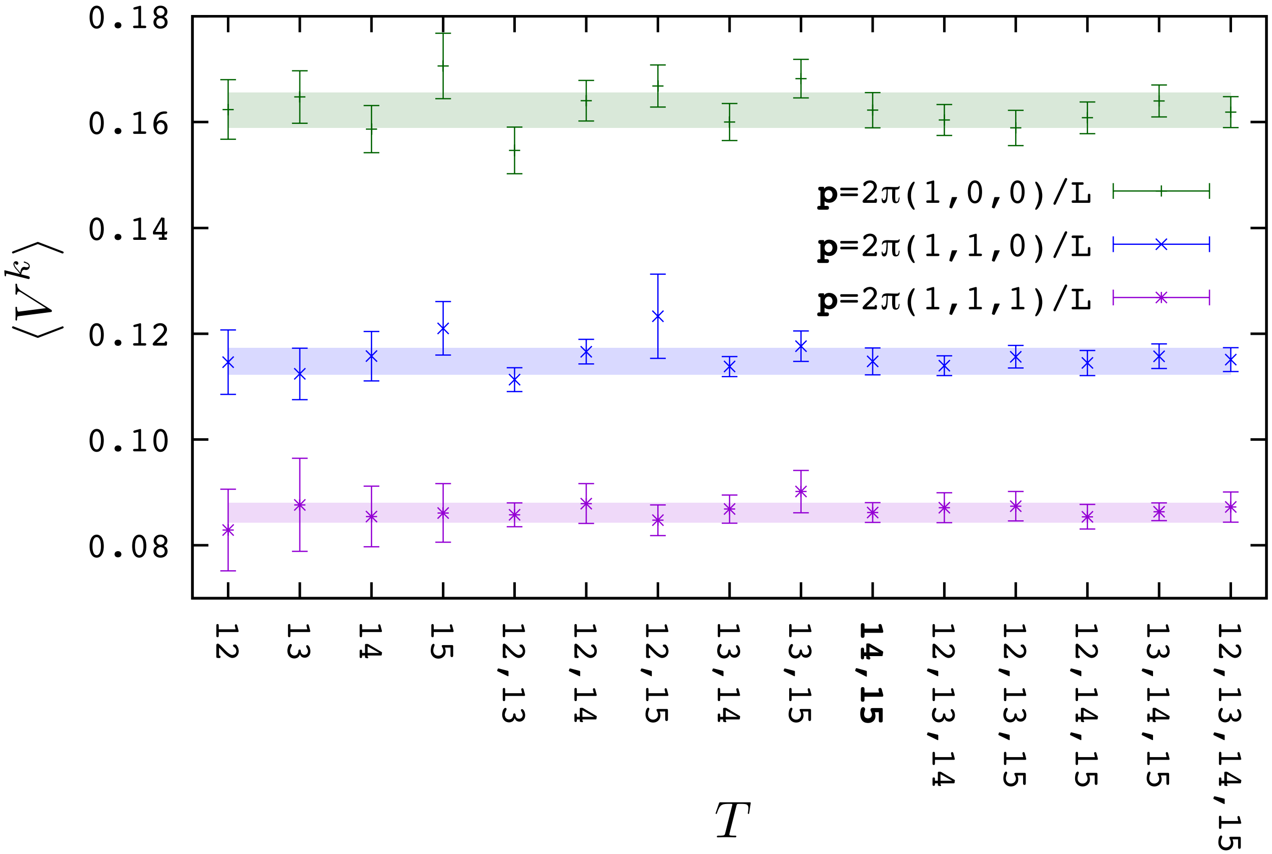
$$C^{\alpha}_{B_q X}(t,T) = \sum_{n,m=0}^{N-1} b_q^{\alpha(n)}\ A^{(nm)}_{B_q X}\ d_X^{(m)\dagger}\ (-1)^{mt+n(T-t)}\ e^{-E_X^{(m)} t}\ e^{-M^{(n)}_{B_q}(T-t)}$$

$$A^{(00)}_{B_q X} = \frac{\langle X|J|B_q\rangle}{2\sqrt{M^{(0)}_{B_q} E^{(0)}_X}} \quad ; \quad J = b\ \Gamma\ \bar{l}(\bar{s})$$

- choose current J to specify tree-level or $1/M$ correction
- fit parent 2pt, daughter 2pt, and 3pt data
  - simultaneous
  - bayesian
  - include 3pt data at multiple $T = 12, 13, 14, 15$

$B_q \to X\ (\ell\nu)(\ell\ell)$

3pt correlator fits



- improvement from simultaneous fit to multiple T's
- shown for $B \to \pi\ell\nu$ on ensemble C3
- use $T = 14, 15$

matching

- massless HISQ, one-loop PT matching to continuum ME's

  (C.J. Monahan's talk, Thurs, 3:30pm, Weak Decays and ME's)

$$\langle V_\mu \rangle = (1 + \alpha_s \tilde{\rho}_\mu^{(0)}) \langle J_\mu^{(0)} \rangle + \langle J_\mu^{(1),sub} \rangle$$

$$\langle J_\mu^{(1),sub} \rangle \equiv \langle J_\mu^{(1)} \rangle - \alpha_s \zeta_{10,\mu} \langle J_\mu^{(0)} \rangle$$

[HPQCD, PRD, 69, 074501 (2004)] and [HPQCD, PRD, 73, 074502 (2006)]

- currents that contribute through $\mathcal{O}(\alpha_s,\, \alpha_s/(aM))$:

$$\begin{aligned} J_\mu^{(0)} &= b\ \gamma_\mu\ \bar{q} \\ J_\mu^{(1)} &= -\frac{1}{2M}\ b\ \gamma_\mu \boldsymbol{\gamma} \cdot \boldsymbol{\nabla}\ \bar{q} \end{aligned}$$

extracting form factors

- SM $(V-A)^\mu$ interaction gives hadronic ME's $\langle X|V^\mu|B\rangle$
- parameterized via phenomenologically-relevant form factors

$$\langle X|V^\mu|B\rangle = f_+^{BX}(q^2)\left(p_B^\mu + p_X^\mu - \frac{M_B^2 - M_X^2}{q^2}q^\mu\right) + f_0^{BX}(q^2)\left(\frac{M_B^2 - M_X^2}{q^2}q^\mu\right)$$

$$q^\mu \equiv p_B^\mu - p_X^\mu$$

- recast in terms of lattice-convenient form factors

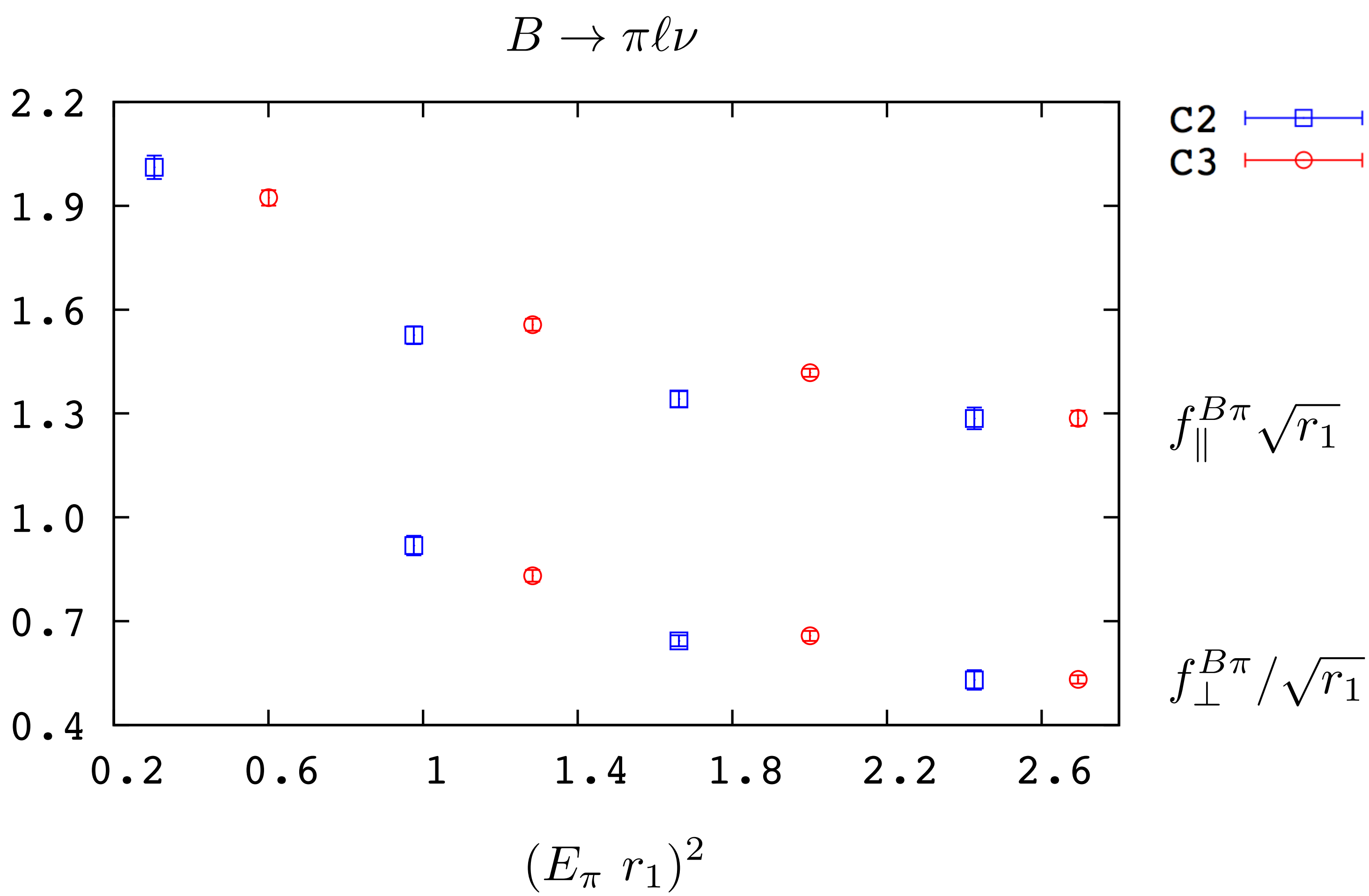
$$\langle X|V^\mu|B\rangle = \sqrt{2M_B}\ \left[v^\mu f_\parallel^{BX}(q^2)\ +\ p_\perp^\mu f_\perp^{BX}(q^2)\right]$$

$$p_\perp^\mu \equiv p_X^\mu - (p_X \cdot p_B)\frac{p_B^\mu}{M_B^2}$$

  – work in B rest-frame

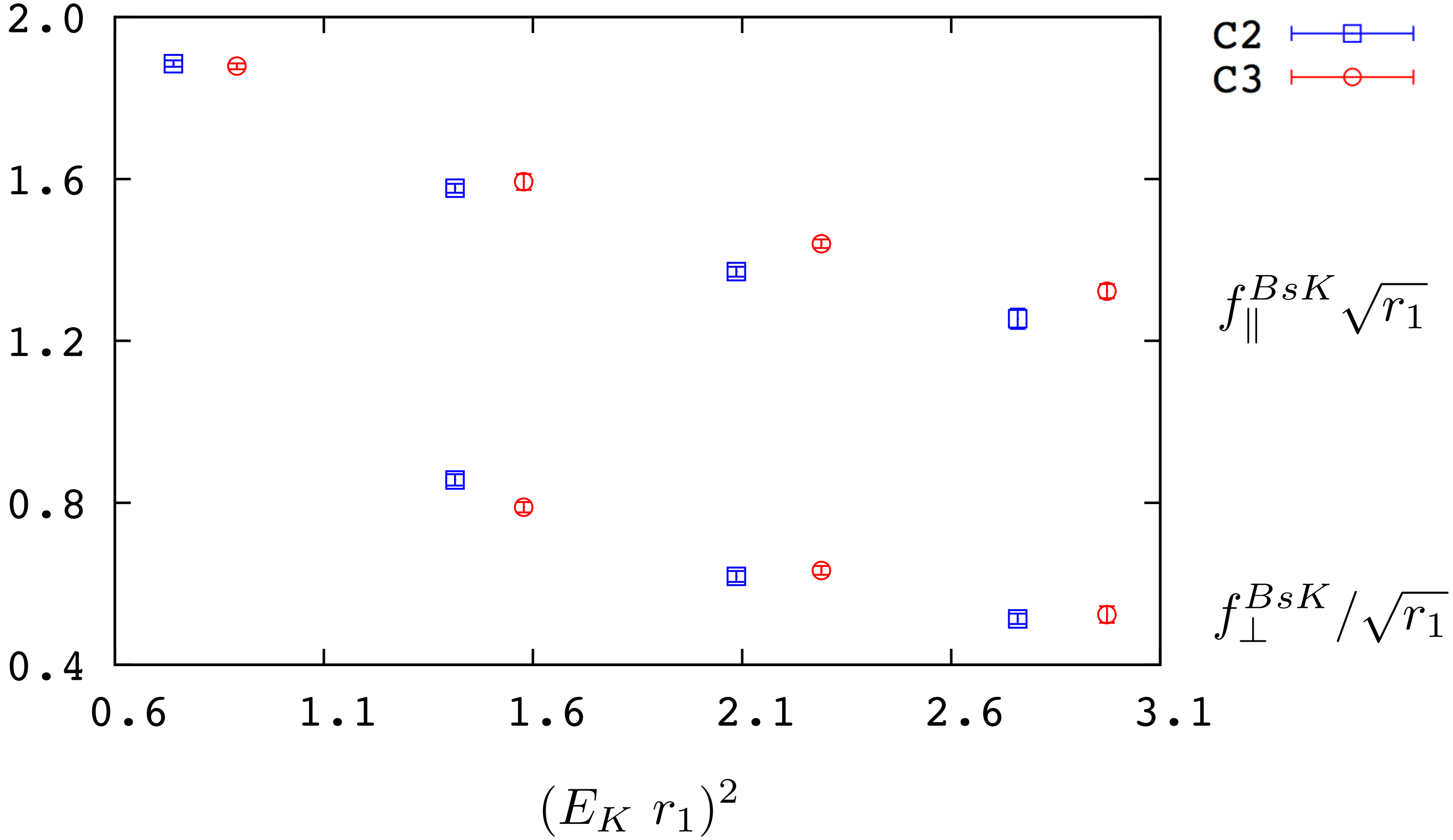
$$\langle X|V^0|B\rangle = \sqrt{2M_B}\ f_\parallel^{BX}$$

$$\langle X|V^k|B\rangle = \sqrt{2M_B}\ p_X^k\ f_\perp^{BX}$$

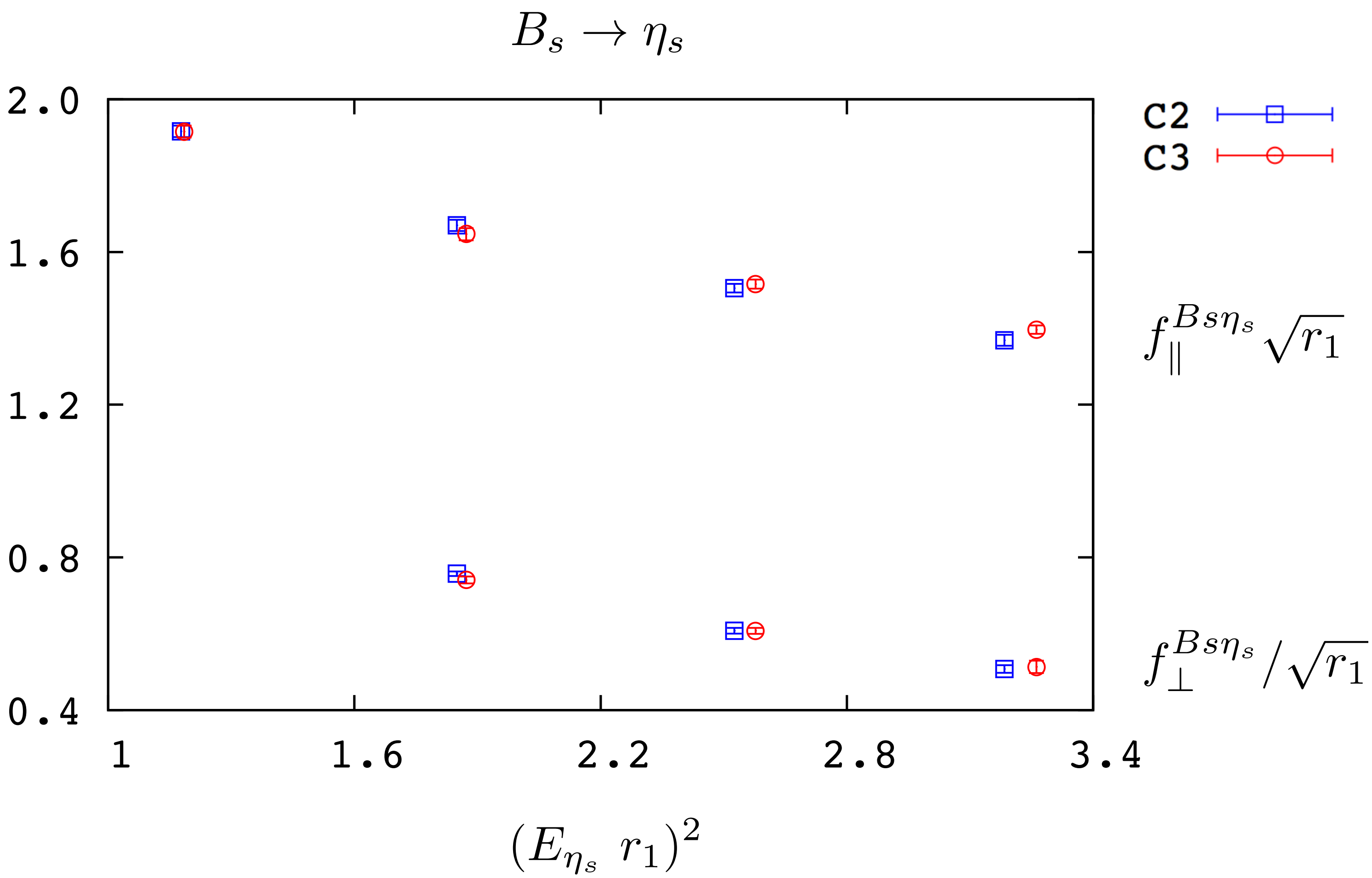


PRELIMINARY

$B_s \to K\ell\nu$



PRELIMINARY



PRELIMINARY

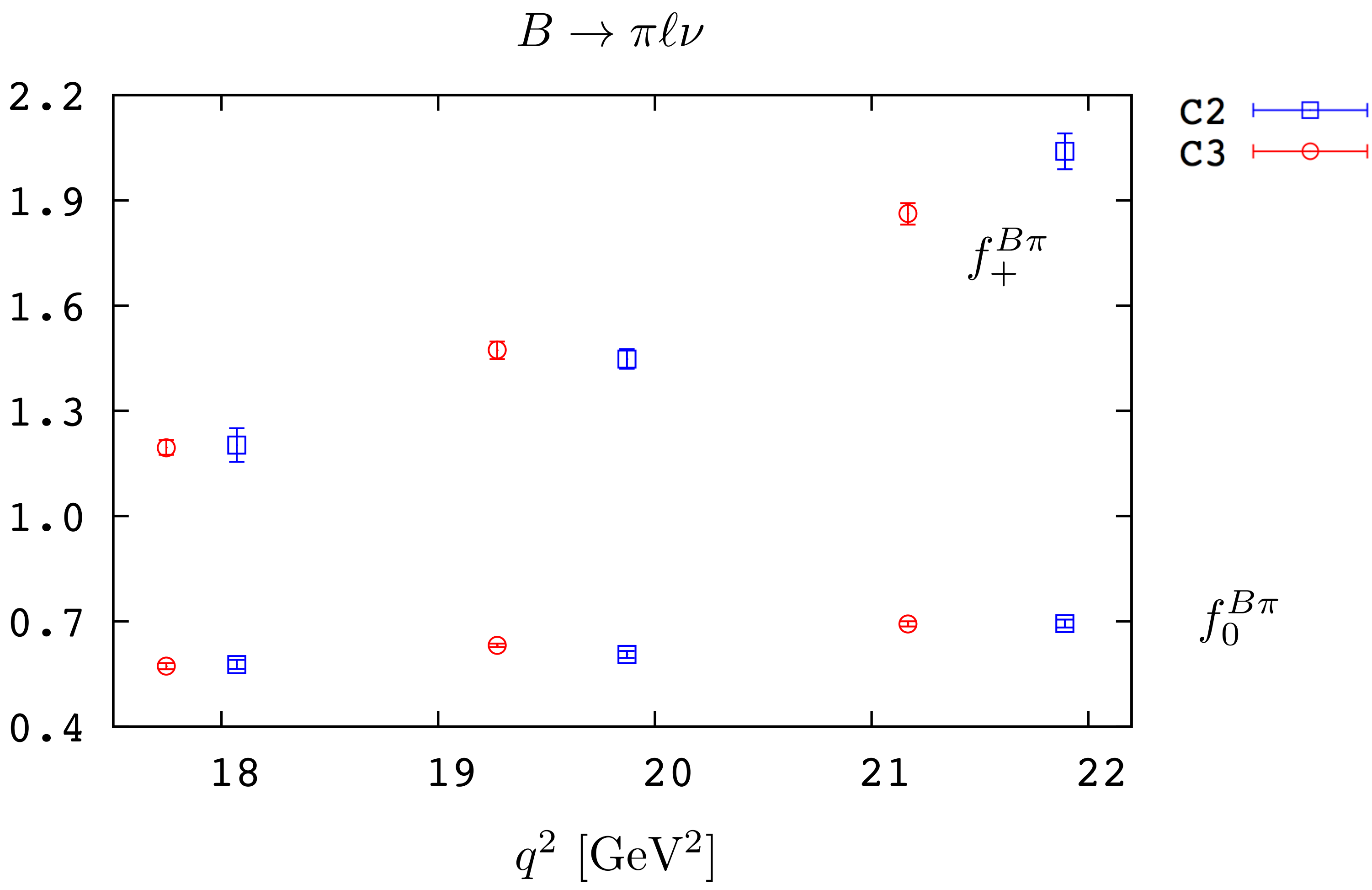
connecting with experiment

- f_+^{BX} related to differential decay width *(neglecting $m_{\ell,\nu}$)*

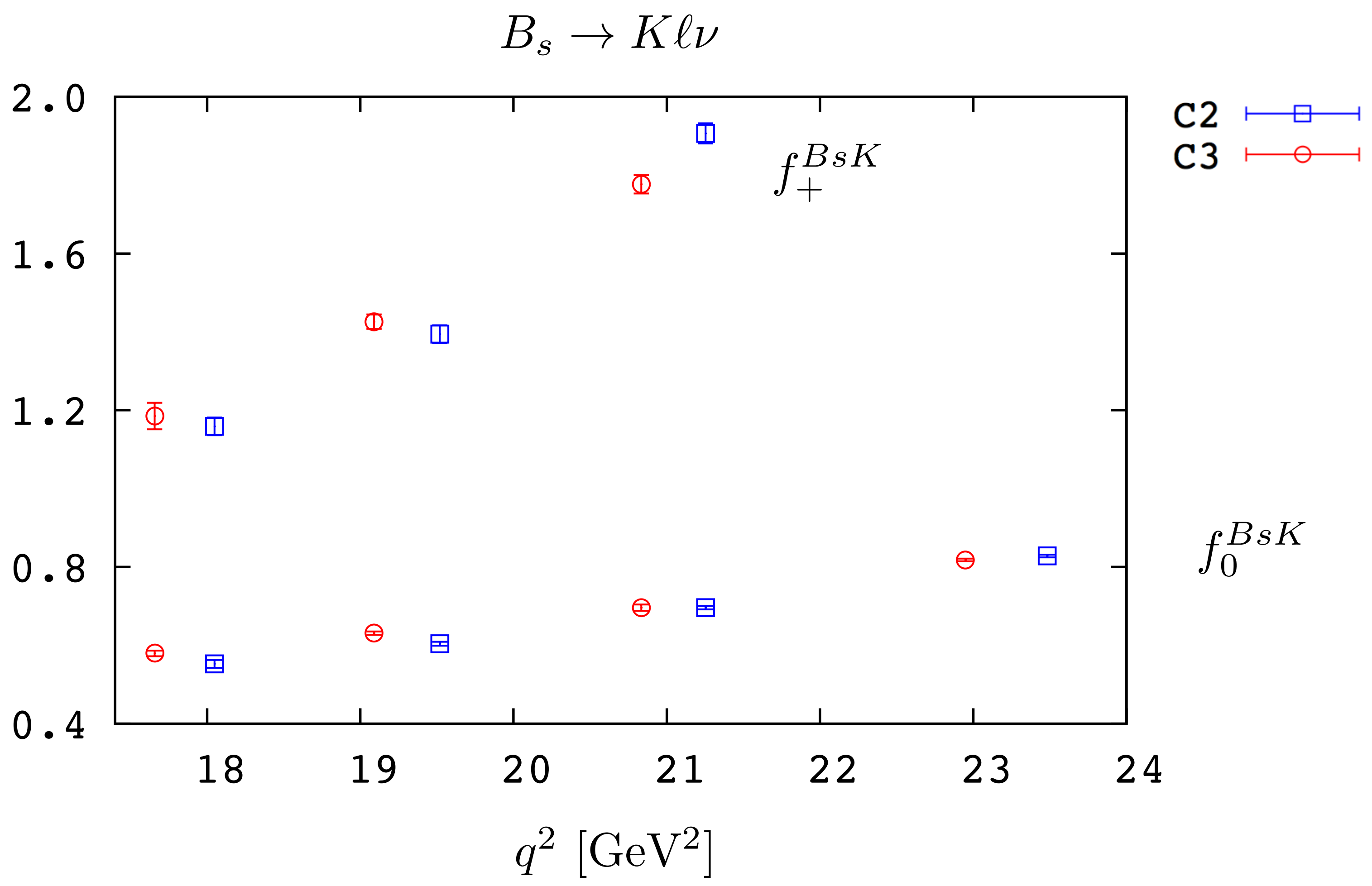
$$\frac{d\Gamma[B \to X(\ell\nu)(\ell\ell)]}{dq^2} = \frac{G_F^2 |V_{\rm CKM}|^2}{192\pi^3 M_B^3} \left[(M_B^2 + M_X^2 - q^2)^2 - 4M_B^2 M_X^2\right]^{3/2} \left[f_+^{BX}(q^2)\right]^2$$

  - comparison with differential decay rate "shape"

- f_0^{BX}
  - smaller errors than f_+^{BX}
  - $f_0^{BX}(0) = f_+^{BX}(0)$ can aid extrapolation to $q^2 = 0$ for simultaneous expansion in f_0^{BX} and f_+^{BX}

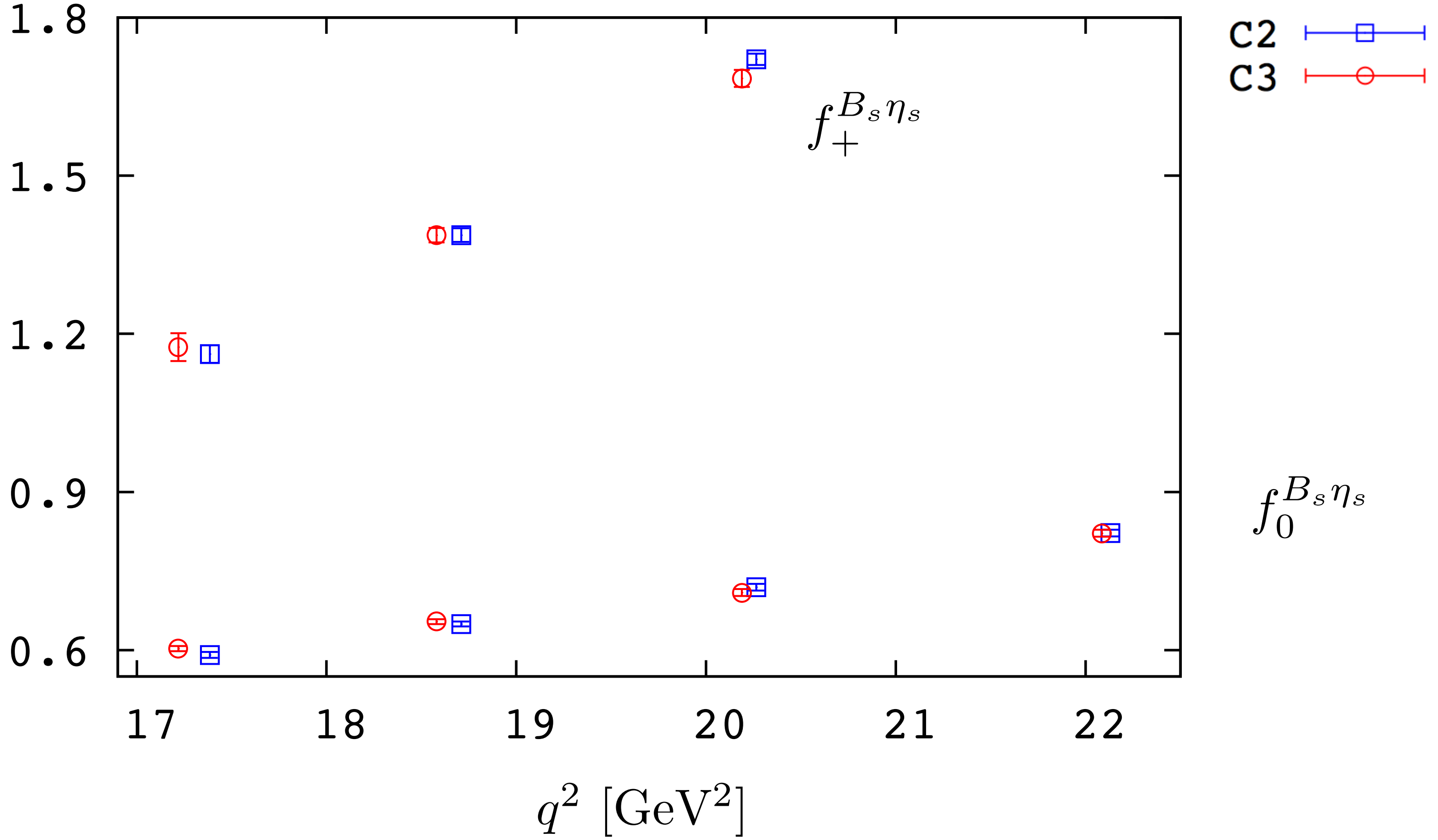


PRELIMINARY



PRELIMINARY

$B_s \to \eta_s$



PRELIMINARY

next steps

- additional ensembles
- "modified z-expansion"

  $D \to \pi \ell \nu$ [H. Na et al., PRD 84, 114505 (2011)]

  $D \to K \ell \nu$ [H. Na et al., PRD 82, 14506 (2010)]

  - incorporates q^2 kinematics
  - allows chiral/continuum extrapolation over full range of daughter momenta
- decide if improvements needed
  - more statistics, more chiral light-quarks, and/or finer lattice(s)
- form factor ratios
- $B \to K \ell^+ \ell^-$ (tensor and vector)