Looking Beyond the SM with *B*-Meson Form Factors

Fermilab Lattice and MILC Collaborations

arXiv:1202.6346 [hep-lat]; arXiv:1206.4992 [hep-lat]

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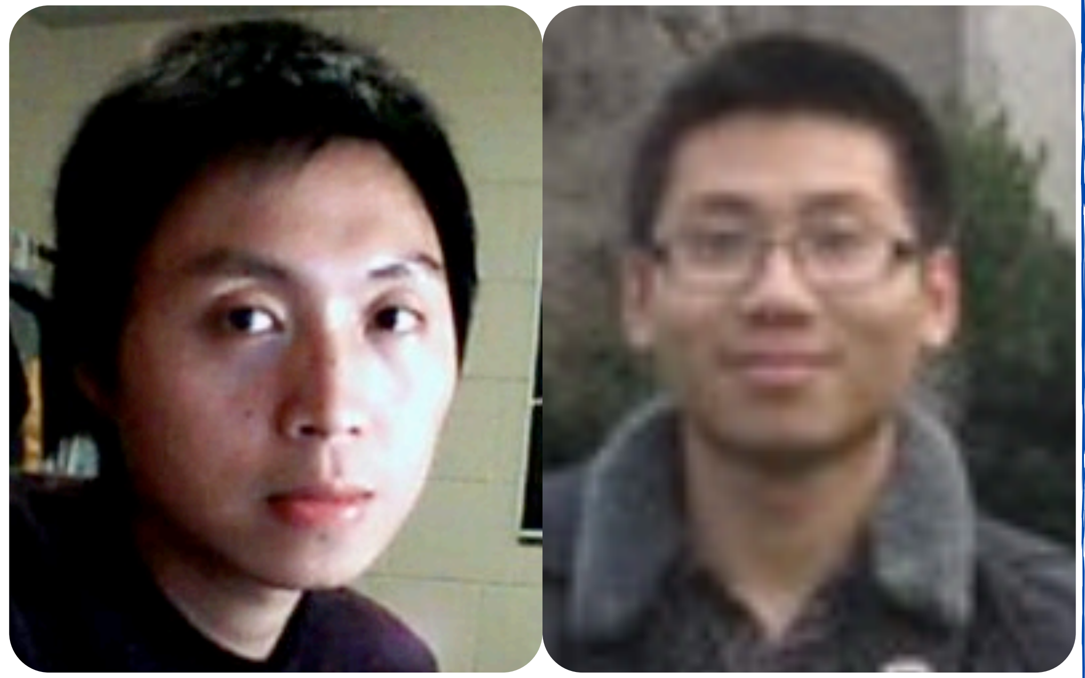
*speaker

Topics in Semileptonic *B* Decays from Fermilab-MILC

- New physics search with $B_s \to \mu^+\mu^-$ needs f_s/f_d:
  - Fleischer, Serra, Tuning [arXiv:1004.3982 [hep-ph], arXiv:1012.2784 [hep-ph]] suggest using hadronic modes such as $D_s\pi/DK$, $D_s\pi/D\pi$ to determine f_s/f_d;
  - arXiv:1202.6346 [hep-lat] computes form factor ratio.

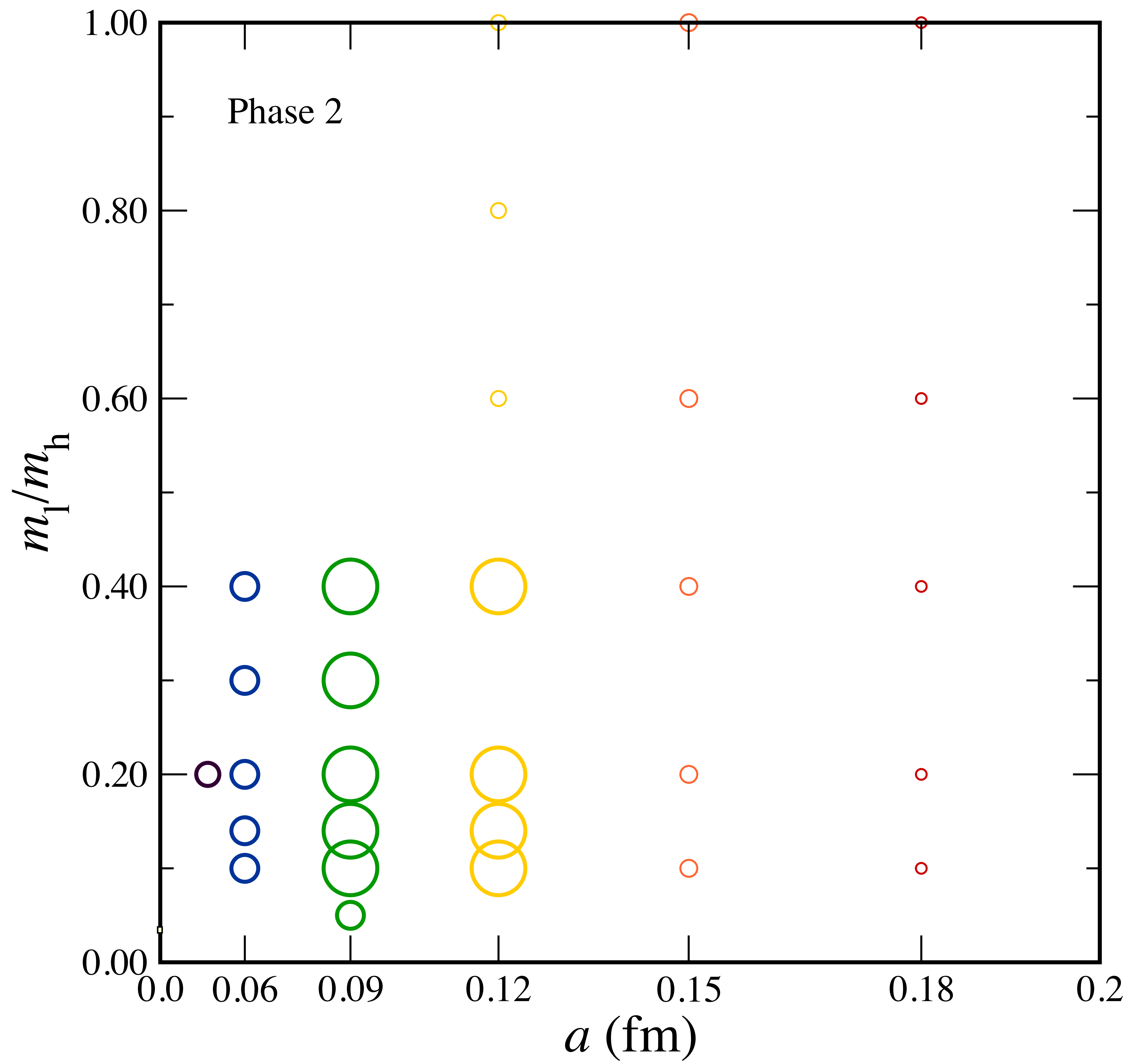


- New physics in $\mathrm{BR}(B \to D\tau\nu)/\mathrm{BR}(B \to Dl\nu)$:
  - BaBar [arXiv:1205.5442 [hep-ex]] quotes ~2σ deviation from SM;
  - arXiv:1206.4992 [hep-lat] asks if LQCD form factors relieve tension.

Actions, Definitions, & Calculations

MILC ensembles

- All MILC asqtad ensembles:
  - $L > 2$ fm, $Lm_\pi \sim 4$
- Lüscher-Weisz gluon action ...
  - … with $O(n_g\alpha_s)$ but not $O(n_f\alpha_s)$.
- Rooted asqtad staggered sea: 2+1.
- Asqtad staggered valence light quarks.
- Sheikholeslami-Wohlert ♣ with Fermilab interpretation for b and c quarks.



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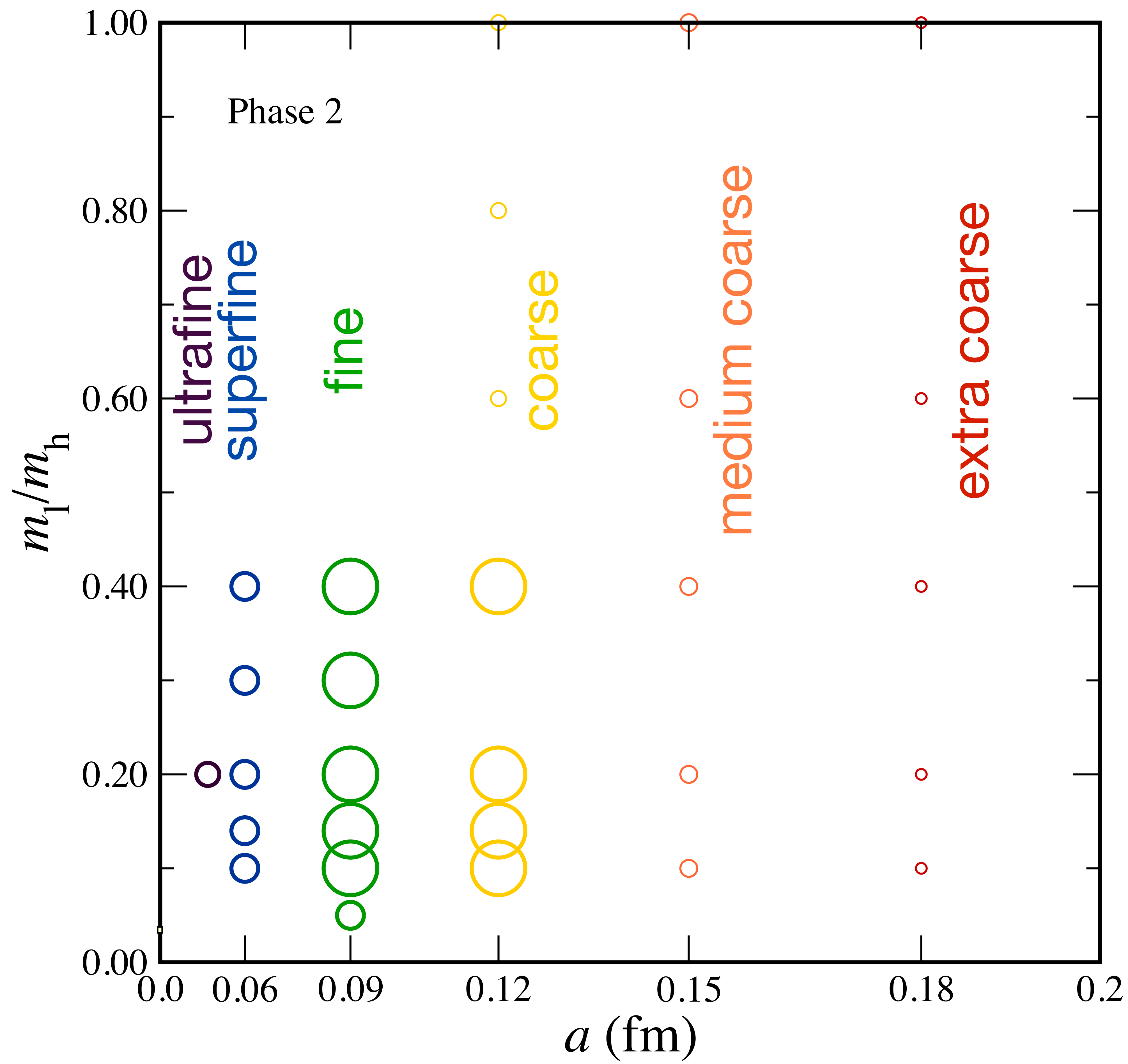

MILC ensembles used in these analyses

- All MILC asqtad ensembles:
  - $L > 2$ fm, $Lm_\pi \sim 4$
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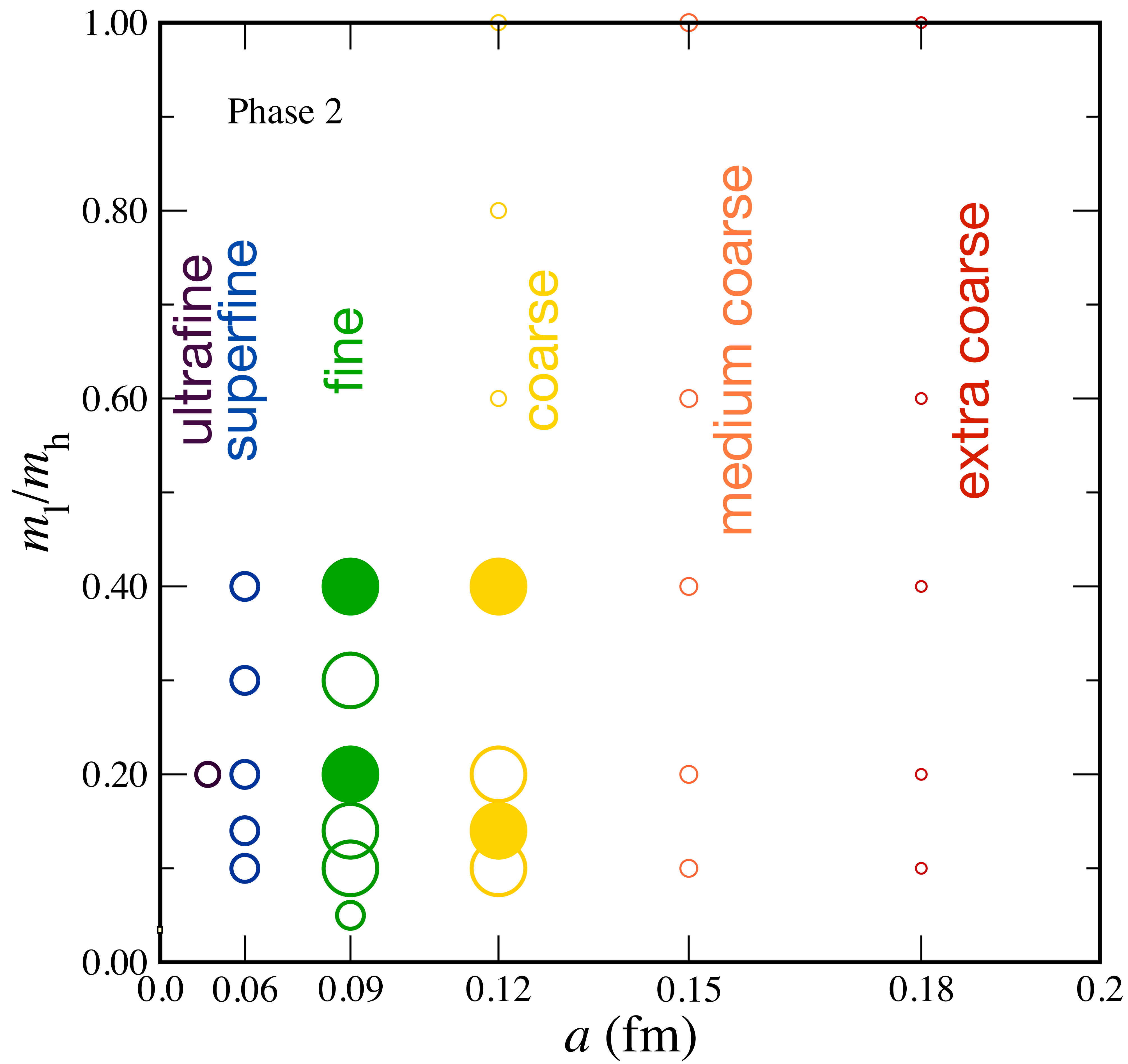

Targets

- For f_s/f_d (and, hence, $B_s \to \mu^+\mu^-$), we need ratios of form factors:

$$\frac{f_0^{B_s\to D_s}(M_\pi^2)}{f_0^{B\to D}(M_K^2)}, \qquad \frac{f_0^{B_s\to D_s}(M_\pi^2)}{f_0^{B\to D}(M_\pi^2)}$$

- For $\mathrm{BR}(B \to D^{(*)}\tau\nu)/\mathrm{BR}(B \to D^{(*)}\mu\nu)$, we need ratios of branching ratios, amounting to ratios of form factors, with an intricate integration over kinematics, e.g., for $B \to D\tau\nu$:

$$\frac{\int dq^2 \left\{M_B^2|f_+^{B\to D}(q^2)|^2 + m_\tau^2|f_0^{B\to D}(q^2)|^2\right\}}{\int dq^2 \left\{M_B^2|f_+^{B\to D}(q^2)|^2 + m_\mu^2|f_0^{B\to D}(q^2)|^2\right\}}$$

Semileptonic Form Factors

- Customary parametrization ($q = p - k$): $f_0(0) = f_+(0)$

$$\langle D(k)|\bar{c}\gamma^\mu b|\bar{B}(p)\rangle = \left[(p+k)^\mu - \frac{M_B^2 - M_D^2}{q^2} q^\mu\right] f_+(q^2) + \frac{M_B^2 - M_D^2}{q^2} q^\mu f_0(q^2),$$

$$\langle D(k)|\bar{c}b|\bar{B}(p)\rangle = \frac{M_B^2 - M_D^2}{m_b - m_c} f_0(q^2),$$

VWI

$$\langle D(k)|\bar{c}\sigma^{\mu\nu}b|\bar{B}(p)\rangle = iM_B^{-1}\left(p^\mu k^\nu - p^\nu k^\mu\right) f_2(q^2)$$

- Heavy-quark parametrization ($v = p/M_B$, $v' = k/M_D$, $q^2 = M_B^2 + M_D^2 - 2wM_BM_D$):

$$\langle D(k)|\bar{c}\gamma^\mu b|\bar{B}(p)\rangle = \sqrt{M_BM_D}\left[(v+v')^\mu h_+(w) - (v'-v)^\mu h_-(w)\right],$$

$$\langle D(k)|\bar{c}b|\bar{B}(p)\rangle = \sqrt{M_BM_D}\frac{M_B - M_D}{m_b - m_c}\left[(w+1)h_+(w) - (w-1)\frac{M_B + M_D}{M_B - M_D}h_-(w)\right],$$

$$\langle D(k)|\bar{c}\sigma^{\mu\nu}b|\bar{B}(p)\rangle = iM_D\left(v^\mu v'^\nu - v^\nu v'^\mu\right) h_2(w)$$

Relations

- Simple algebra ($q^2 = M_B^2 + M_D^2 - 2wM_BM_D$):

$$f_+(q^2) = \frac{1}{2\sqrt{M_BM_D}}\left[(M_B+M_D)h_+(w) - (M_B-M_D)h_-(w)\right],$$
$$f_0(q^2) = \sqrt{M_BM_D}\left[\frac{w+1}{M_B+M_D}h_+(w) - \frac{w-1}{M_B-M_D}h_-(w)\right],$$
$$f_2(q^2) = h_2(w),$$

- Heavy-quark relations (heavy b; heavy b&c):

$$f_2(q^2) = -f_+(q^2) - \frac{M_B^2-M_D^2}{q^2}\left(f_+(q^2) - f_0(q^2)\right) + \mathrm{O}(1/m_b),$$

$$h_+(w) \sim 1 + \mathrm{O}(\alpha_s) + \mathrm{O}(1/m_Q),$$
$$h_-(w) \sim 0 + \mathrm{O}(\alpha_s) + \mathrm{O}(1/m_Q),$$

Double ratio and single ratios

$$R_+ \equiv \frac{\langle D|\bar{c}\gamma^0 b|B\rangle\langle B|\bar{b}\gamma^0 c|D\rangle}{\langle D|\bar{c}\gamma^0 c|D\rangle\langle B|\bar{b}\gamma^0 b|B\rangle} = |h_+(1)|^2, \quad \text{all } \boldsymbol{p} = \boldsymbol{0},$$

$$\boldsymbol{a} \equiv \frac{\langle D(\boldsymbol{p})|\bar{c}\boldsymbol{\gamma} b|B(\boldsymbol{0})\rangle}{\langle D(\boldsymbol{0})|\bar{c}\gamma^0 b|B(\boldsymbol{0})\rangle} = \frac{h_+(w) - h_-(w)}{2h_+(1)}\boldsymbol{v},$$

$$c \equiv \frac{\langle D(\boldsymbol{p})|\bar{c}\gamma^0 b|B(\boldsymbol{0})\rangle}{\langle D(\boldsymbol{0})|\bar{c}\gamma^0 b|B(\boldsymbol{0})\rangle} = \frac{(w+1)h_+(w) - (w-1)h_-(w)}{2h_+(1)}, \qquad \boldsymbol{b} = \boldsymbol{a}/c,$$

$$\boldsymbol{d} \equiv \frac{\langle D(\boldsymbol{p})|\bar{c}\boldsymbol{\gamma} c|D(\boldsymbol{0})\rangle}{\langle D(\boldsymbol{p})|\bar{c}\gamma^0 c|D(\boldsymbol{0})\rangle} = \frac{\boldsymbol{v}}{1+w}, \quad w^2 = 1 + |\boldsymbol{v}|^2 \Rightarrow w = \frac{1+\boldsymbol{d}\cdot\boldsymbol{d}}{1-\boldsymbol{d}\cdot\boldsymbol{d}},$$

$$f_0(q^2) = \sqrt{M_B M_D}\sqrt{R_+}\left[\frac{w+1}{M_B+M_D}(c - \boldsymbol{a}\cdot\boldsymbol{d}) - \frac{w-1}{M_B-M_D}\left(c - \frac{\boldsymbol{a}\cdot\boldsymbol{d}}{\boldsymbol{d}\cdot\boldsymbol{d}}\right)\right]$$

- All ensembles have very high statistics (~2000 configs and 4 sources), enabling a combined fit to 2- and 3-point correlators to extract R_+, $\boldsymbol{a}$, $\boldsymbol{b} = c\boldsymbol{a}$, $\boldsymbol{d}$.

Double ratio and single ratios

$$R_+ \equiv \frac{\langle D|\bar{c}\gamma^0 b|B\rangle\langle B|\bar{b}\gamma^0 c|D\rangle}{\langle D|\bar{c}\gamma^0 c|D\rangle\langle B|\bar{b}\gamma^0 b|B\rangle} = |h_+(1)|^2, \quad \text{all } \boldsymbol{p} = \boldsymbol{0},$$

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$$f_0(q^2) = \sqrt{M_B M_D}\, h_+(1)\left[\frac{w+1}{M_B+M_D}\;\frac{h_+(w)}{h_+(1)} - \frac{w-1}{M_B-M_D}\;\frac{h_-(w)}{h_+(1)}\right]$$

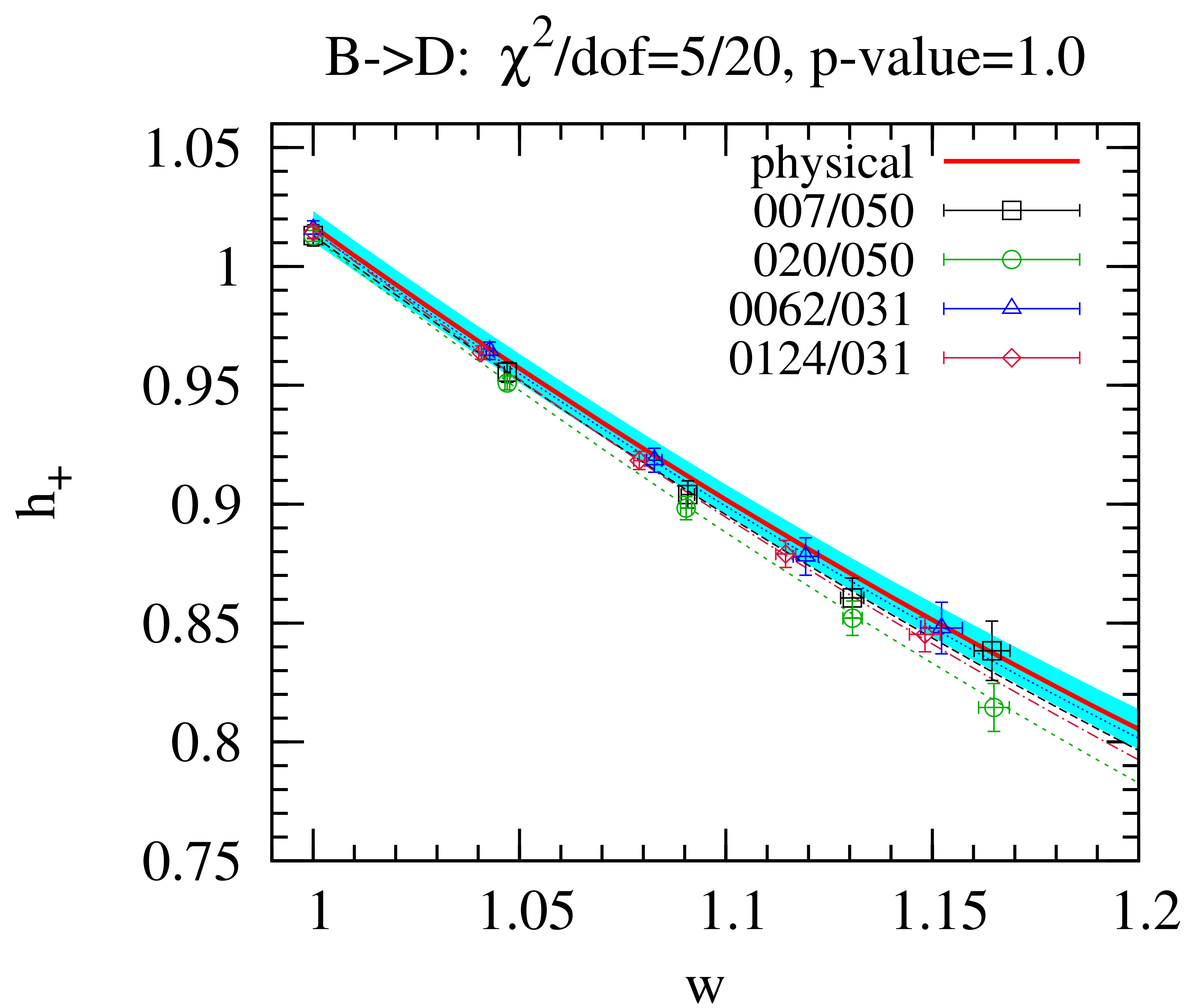
- All ensembles have very high statistics (~2000 configs and 4 sources), enabling a combined fit to 2- and 3-point correlators to extract R_+, $\boldsymbol{a}$, $\boldsymbol{b} = c\boldsymbol{a}$, $\boldsymbol{d}$.

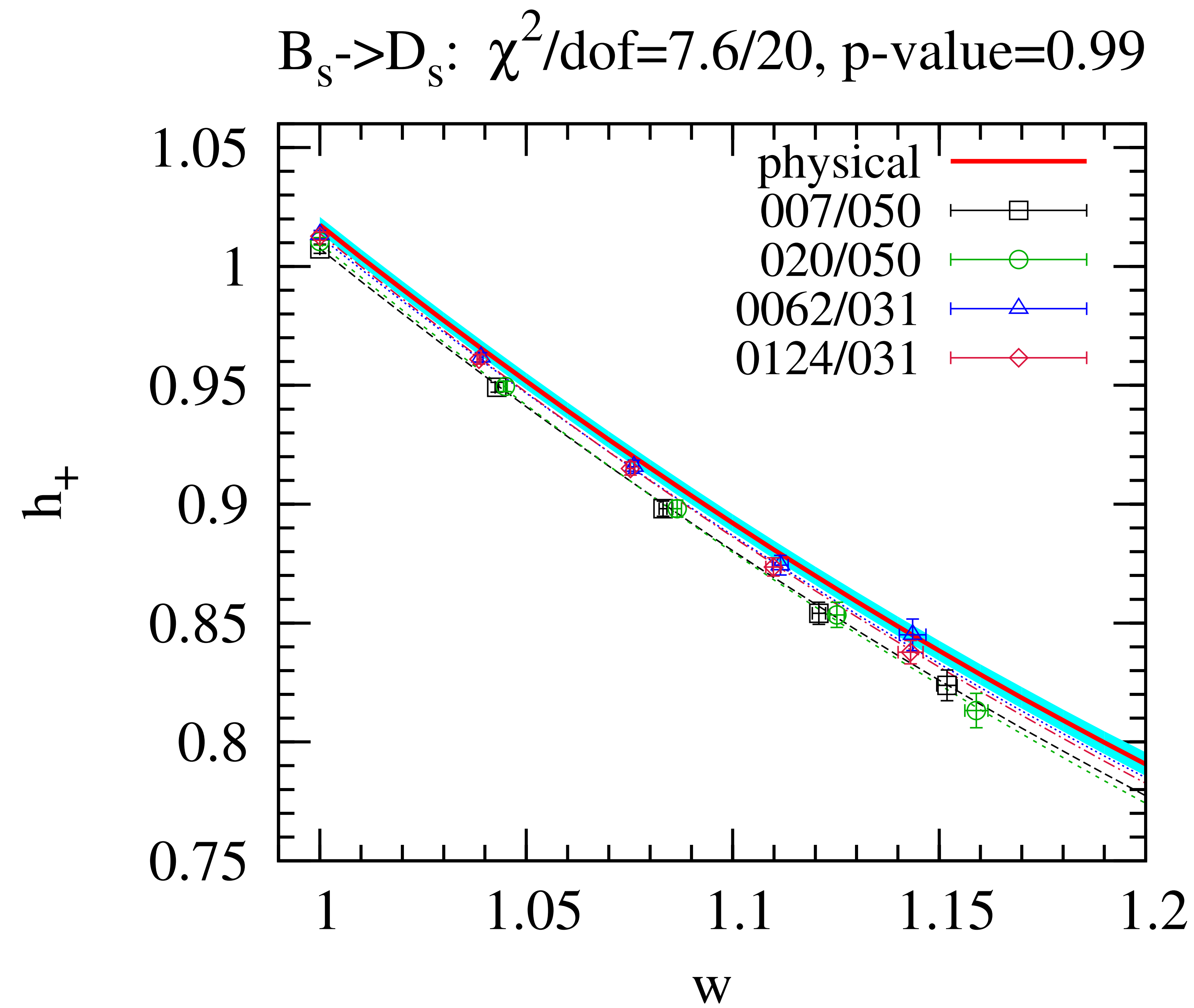
Our strategy

- Targets are ratios, so some compromises are acceptable:
  - subset of MILC ensembles broad enough to allow chiral and continuum extrapolation;
  - tree-level matching, with conservative estimate of one-loop correction ρ_-/ρ_+.
- Targets require $f_0(q^2)$ for all $q^2 \in [(M_B - M_D)^2, 0]$, *e.g.*, $q^2 = M_\pi^2, M_K^2$, so need a robust way to cover whole kinematic range:
  - map $w \mapsto z = \frac{\sqrt{1+w}-\sqrt{2}}{\sqrt{1+w}+\sqrt{2}}$, which maps form-factor kinematics onto interval $z \in [0, 0.0644]$;
  - appeal to unitarity to constrain coefficient of series expansion in z.
  - Vast literature on this, *e.g.*, Boyd, Grinstein, & Lebed [arXiv:hep-ph/9508211].

Chiral-continuum extrapolation for $h_+(w)$

- LO, NLO log (messy expression), NNLO analytic terms:

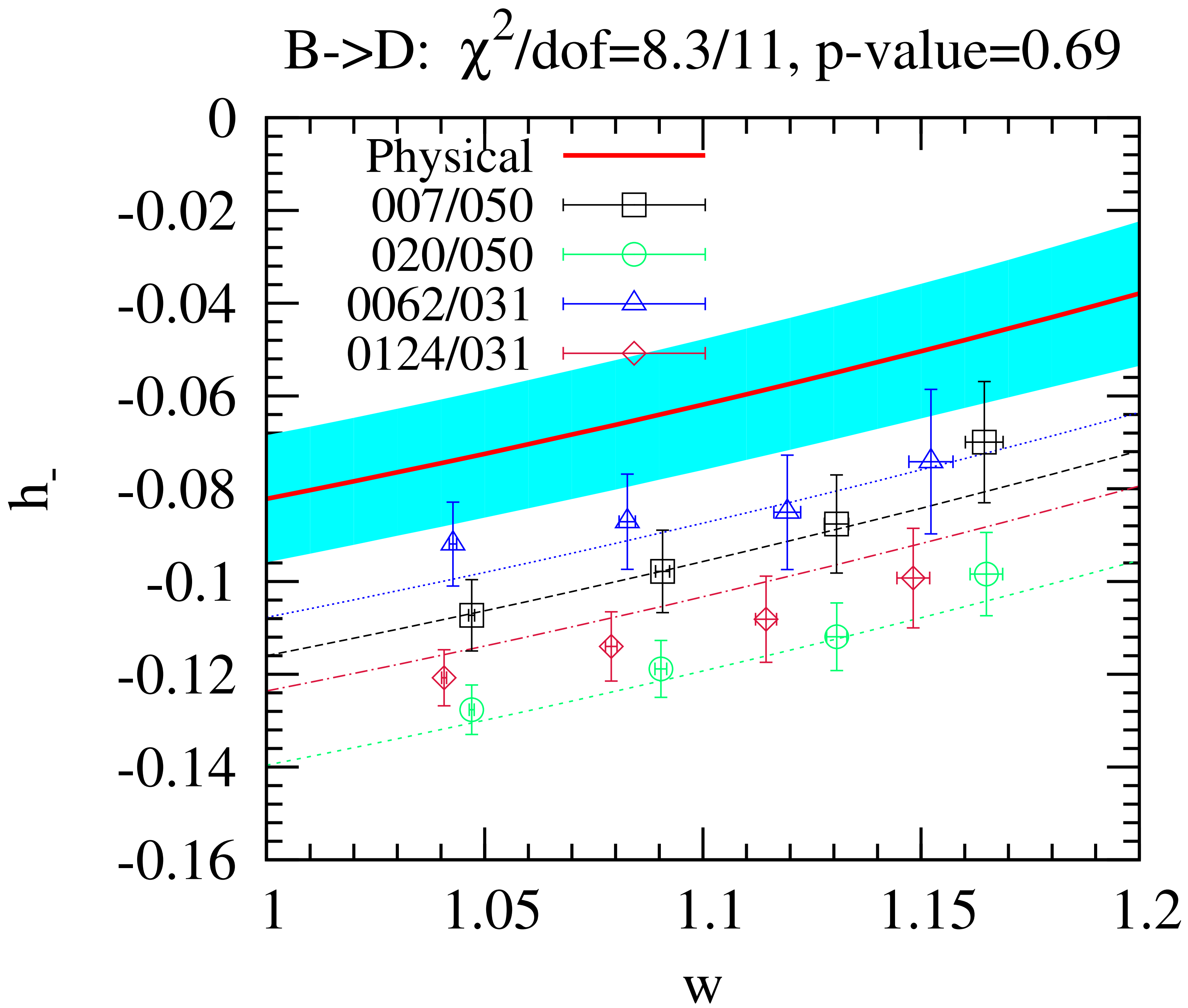


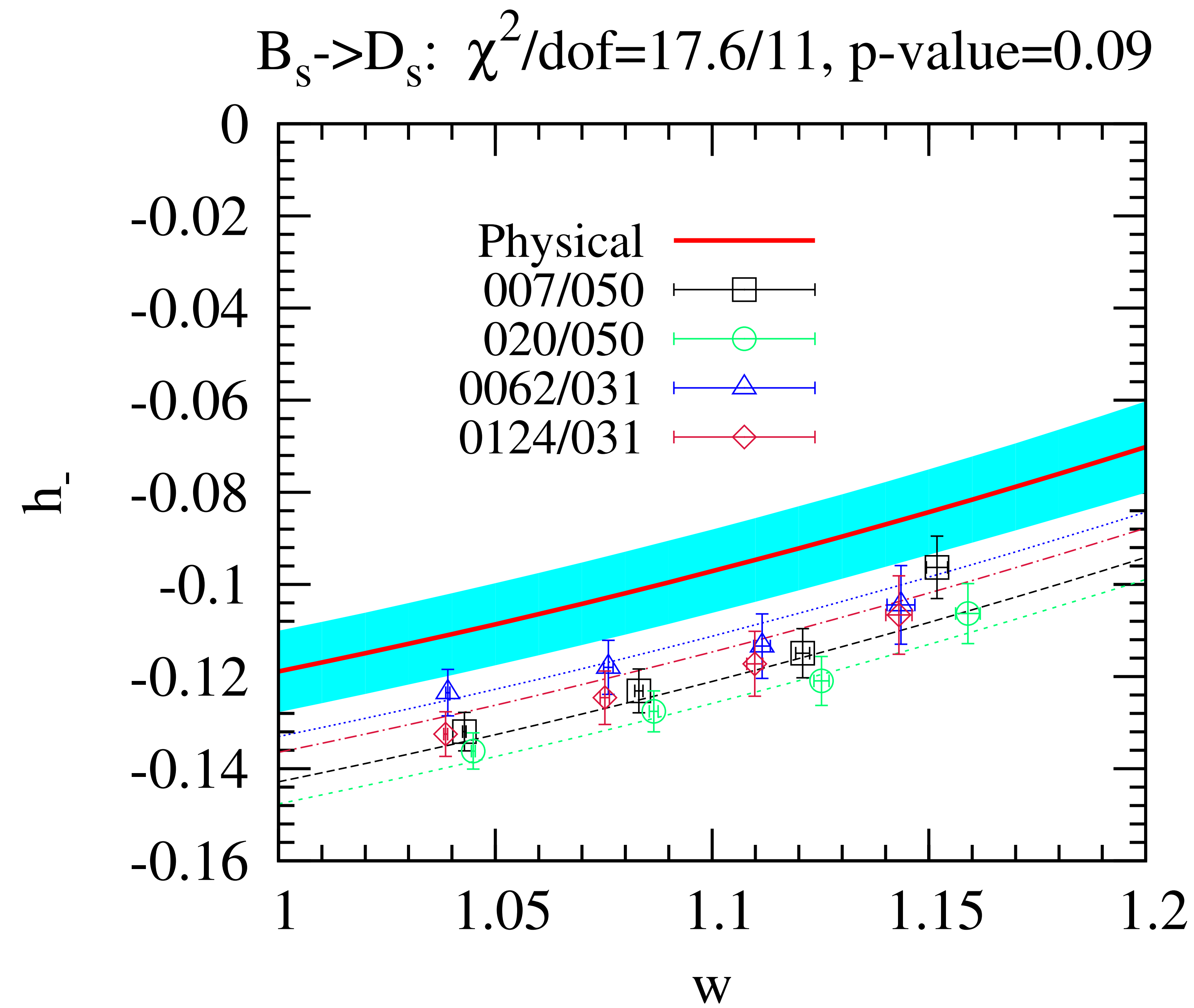


- Extrapolations are mild and straightforward.

Chiral-continuum extrapolation for $h_{-}(w)$

- LO, NLO, NNLO analytic (no one-loop logs):



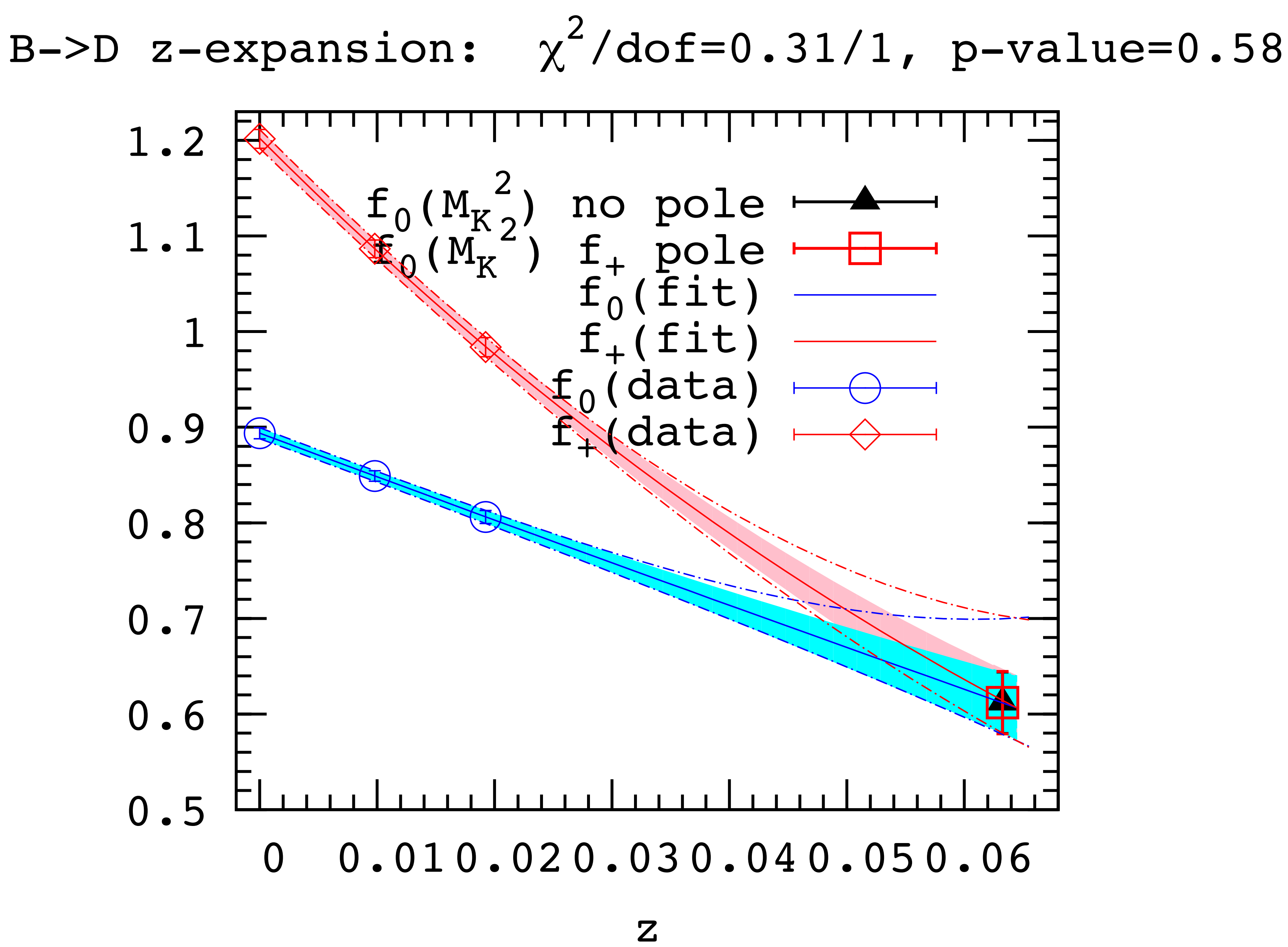


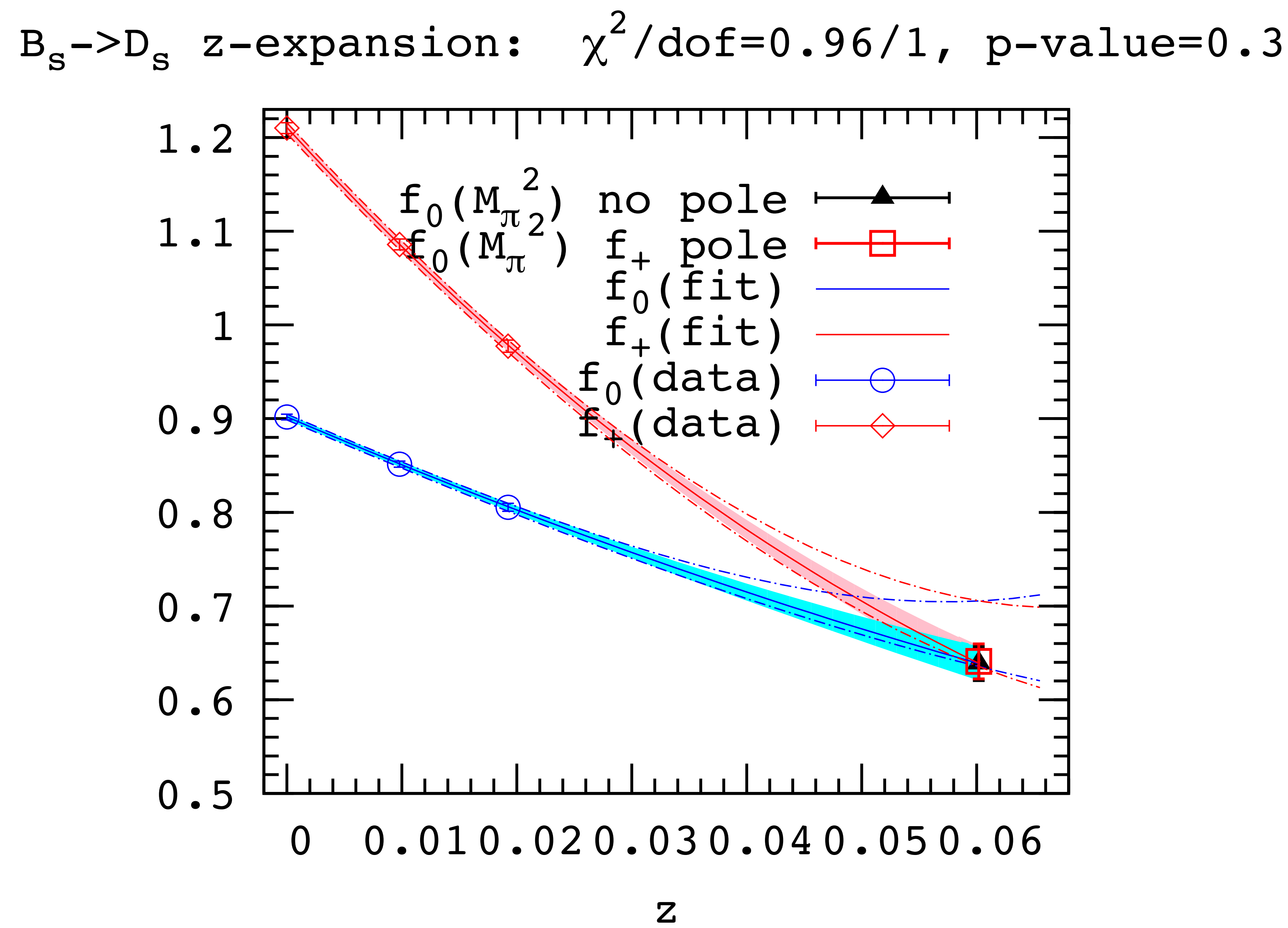
- Extrapolations are more significant but not large (note scale ÷ 2).

z Expansion

- Series in z: $f_i(z) = \frac{1}{P_i(z)\phi_i(z)} \sum_{n=0}^{\infty} a_n z^n$, with ϕ s.t. $\sum_n |a_n|^2 \leq 1$

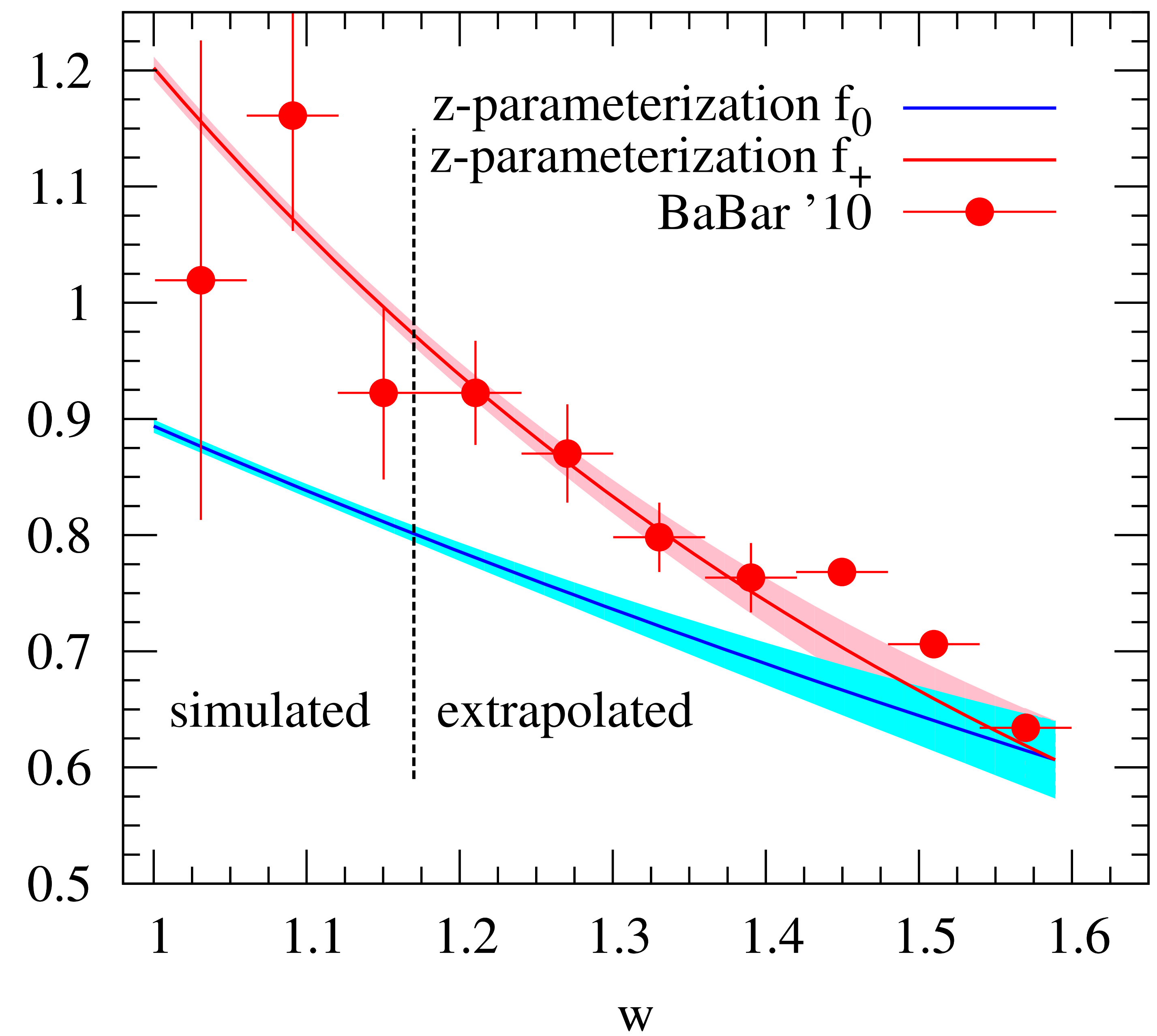
- Blaschke factor $P_i(z)$ chosen to factor out poles: here B_c^* for f_+; no change with or without.





The Shape of $f_+(q^2)$, whence $f_+(0)$

- With $|V_{cb}| = 41.4 \times 10^{-3}$, a reasonable value, compare our shape to BaBar:

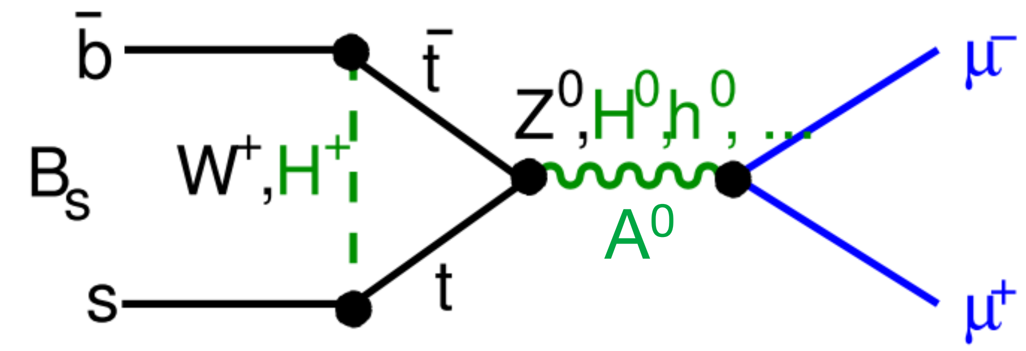


- Agreement with experiment bolsters confidence in $f_+(0) = f_0(0)$ ($q^2 = 0 \Leftrightarrow w = 1.589$); cutoff effects smallest for $w = 1$ ($q^2 = (M_B - M_D)^2$), so $f_0((M_B - M_D)^2)$ should be "easy".

New Physics in $B_s \to \mu^+\mu^-$ and f_s/f_d

arXiv:1202.6346 [hep-lat] (in PRD)

Standard & New Physics in $B \to \mu^+\mu^-$

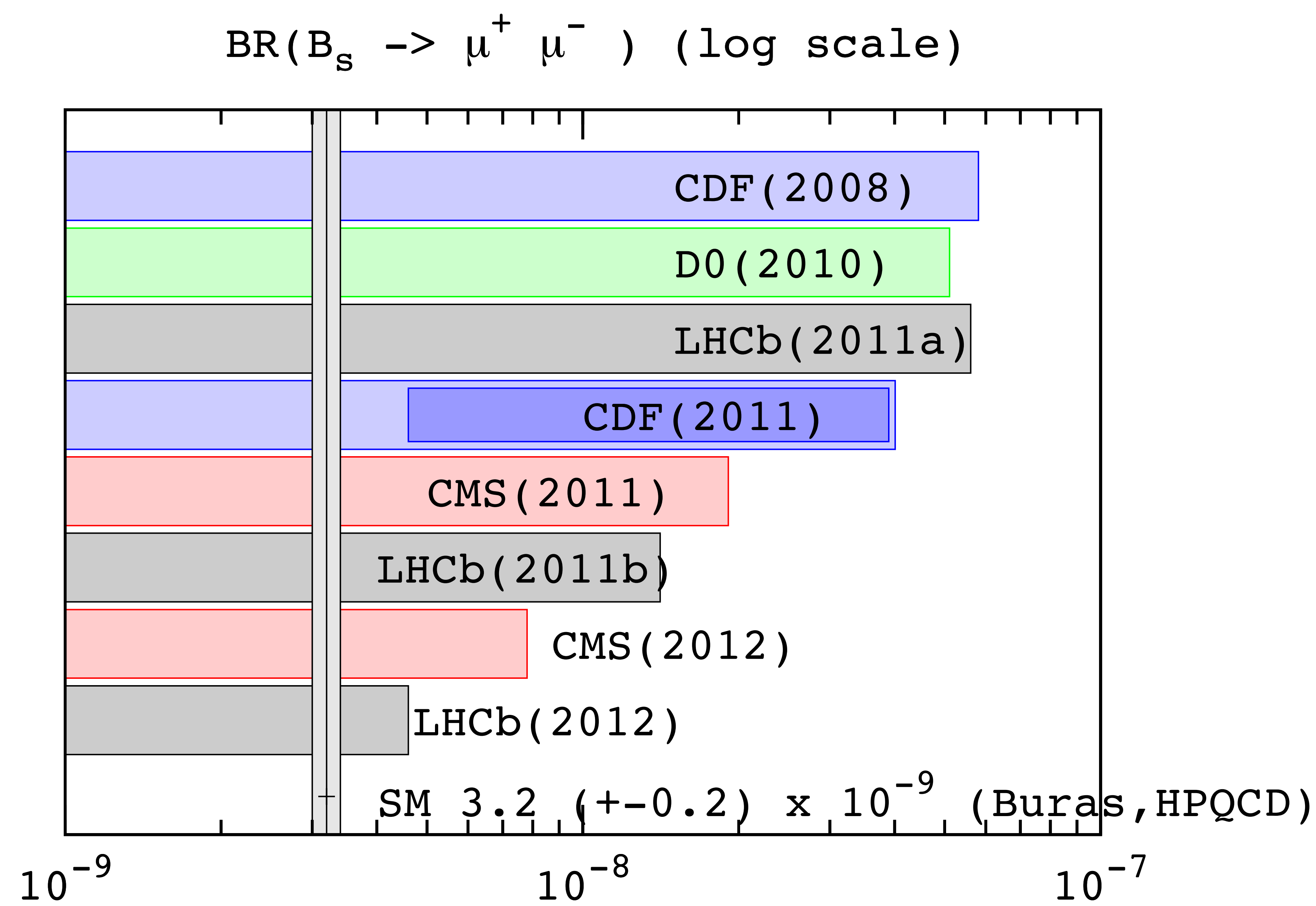


- The flavor-changing decay $B \to \mu^+\mu^-$ is interesting, because extensions of the SM can have a very different rate.

- In models with extended Higgs sector, *e.g.*, MSSM or Type II 2HDM,

$$\mathrm{BR}(B_q^0 \to \mu\mu) \propto \tau_{B_q} M_{B_q} f_{B_q}^2 G_F^4 m_\mu^2 m_t^4 \frac{M_W^4}{M_A^4} \tan^6 \beta$$

- Recent experimental progress has been dramatic:
  - no large enhancement;
  - destructive interference?!?



- This plot may be out of date next week.

Measurement at Hadron Colliders

- Measure branching ratio for $B_s \to \mu^+\mu^-$ relative to a well-known B^+ or B^0 mode:

$$\mathrm{BR}(B_s^0 \to \mu^+\mu^-) = \mathrm{BR}(B_q \to X)\frac{f_q}{f_s}\frac{\varepsilon_X}{\varepsilon_{\mu\mu}}\frac{N_{\mu\mu}}{N_X}.$$

- Fleischer, Serra, and Tuning [arXiv:1004.3982 [hep-ph]] propose using nonleptonic modes for fragmentation f_s/f_d, namely $\bar{B}_s \to D_s^+\pi^-$ and $\bar{B}^0 \to D^+K^-$ $(\bar{B}^0 \to D^+\pi^-)$:

$$\frac{f_s}{f_d} = 0.0743 \times \frac{\tau_{B^0}}{\tau_{B_s^0}}\left[\frac{\varepsilon_{DK}}{\varepsilon_{D_s\pi}}\frac{N_{D_s\pi}}{N_{DK}}\right]\frac{1}{\mathcal{N}_a}\left[\frac{f_0^{(s)}(M_\pi^2)}{f_0^{(d)}(M_K^2)}\right]^{-2}$$

- Here, the nonleptonic modes have been simplified with QCD factorization (BBNS):

$$\begin{aligned}\mathcal{A}(\bar{B}_s \to D_s^+\pi^-) &= G_F V_{cb} V_{ud}^* f_\pi f_0^{B_s\to D_s}(M_\pi^2)\, a_1^{(s)}(D_s\pi)\\ \mathcal{A}(\bar{B}^0 \to D_s^+K^-) &= G_F V_{cb} V_{us}^* f_K f_0^{B\to D}(M_K^2)\; a_1^{(d)}(DK)\end{aligned}$$

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$$\mathcal{A}(\bar{B}^0 \to D_s^+K^-) = G_F V_{cb} V_{us}^* f_K f_0^{B\to D}(M_K^2)\, a_1^{(d)}(DK)$$

$$\left[\frac{a_1^{(s)}(D_s\pi)}{a_1^{(d)}(DK)}\right]^{-2} \sim 1 + \text{few } \%$$

Results

arXiv:1202.6346 [hep-lat]

- Results: $$\frac{f_0^{(s)}(M_\pi^2)}{f_0^{(d)}(M_K^2)} = 1.046(44)(15)$$ $$\frac{f_0^{(s)}(M_\pi^2)}{f_0^{(d)}(M_\pi^2)} = 1.054(47)(17)$$

- With Fleischer, Serra, Tuning:

$$\frac{f_s}{f_d} = 0.283(27)_{\text{stat}}(19)_{\text{syst}}(24)_{\text{lat}}, \quad D_s\pi/DK$$
$$\frac{f_s}{f_d} = 0.286(16)_{\text{stat}}(21)_{\text{syst}}(26)_{\text{lat}}(22)_{\text{NE}}, \quad D_s\pi/D\pi$$

- LHCb semileptonic: $f_s/f_d = 0.268(8)_{\text{stat}}(^{+24}_{-22})_{\text{syst}}$
  PDG average of LEP/CDF: $0.288(24)$.

Source of error	$\delta(f_0^{(s)}/f_0^{(d)})$
Statistics $\oplus$ chiral-continuum	4.2%
z expansion	0.6%
Scale r_1	0.1%
Mistuned m_s	0.1%
Mistuned m_l	0.1%
Heavy-quark (κ) tuning	0.6%
Heavy-quark discretization	1.0%

- Error budget of $f_0^{(s)}(M_\pi^2)/f_0^{(d)}(M_K^2)$.
- Ratio consistent with lattice QCD calculations of other form factors
- Ratio different from QCD sum rules [arXiv:hep-ph/9307290], 1.30 ± 0.08.

New Physics in $\mathrm{BR}(B \to D\tau\nu)/\mathrm{BR}(B \to Dl\nu)$

arXiv:1206.4992 [hep-lat]

Semitauonic vs. semimuonic $B \to D$ decays

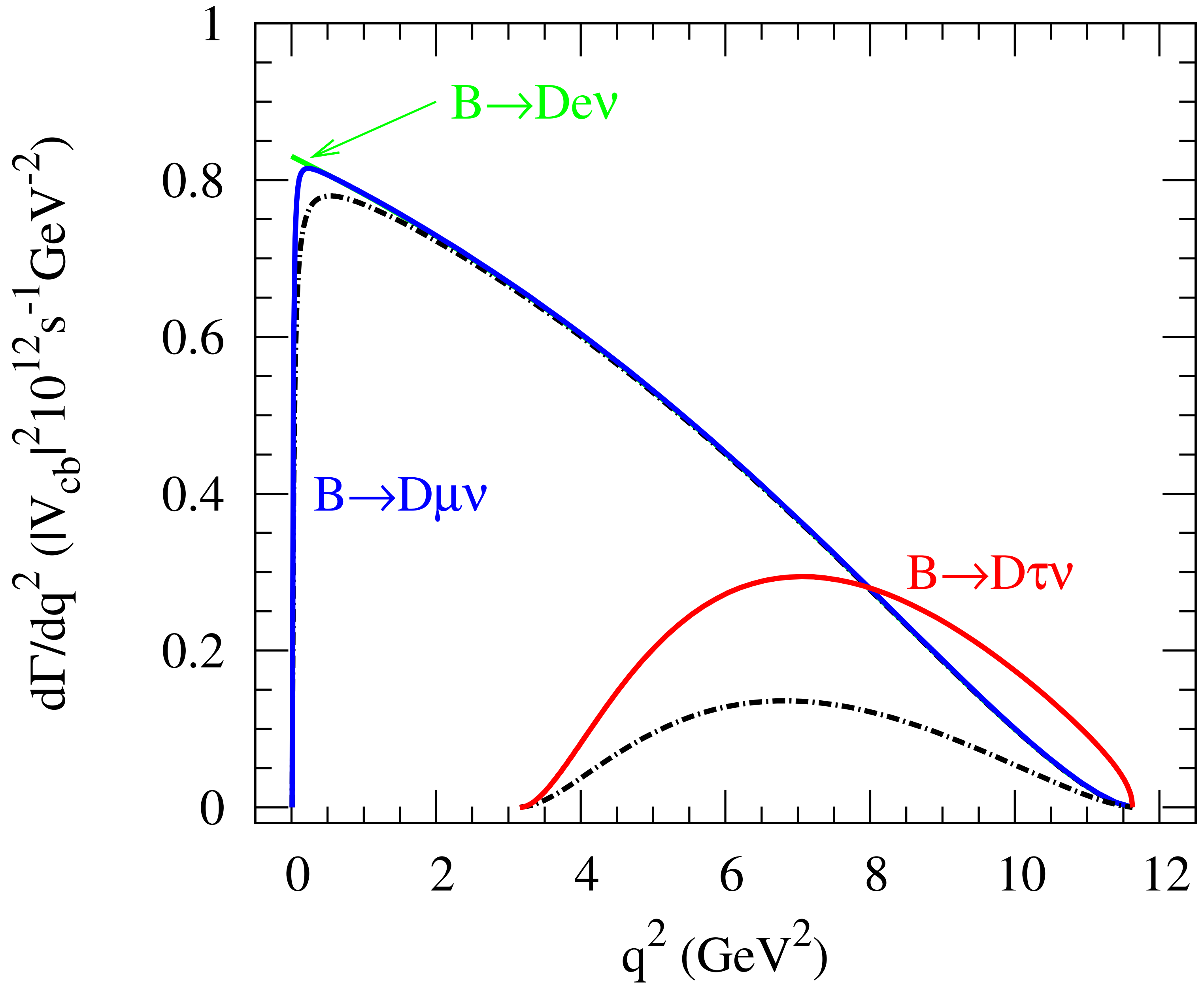
- B-factory experiments BaBar and Belle have been trying to measure

$$R(D) = \frac{\mathrm{BR}(B \to D\tau\nu)}{\mathrm{BR}(B \to D\mu\nu)}, \qquad R(D^*) = \frac{\mathrm{BR}(B \to D^*\tau\nu)}{\mathrm{BR}(B \to D^*\mu\nu)}.$$

- Large m_τ makes the numerator sensitive to $f_0(q^2)$ in the SM ($f_0(q^2)$ for D^*).

- Theoretical uncertainties from f_+ but not "f_0/f_+" partly cancel in ratio.

- Contribution from f_0 is about half the total rate for



Experimental Measurements

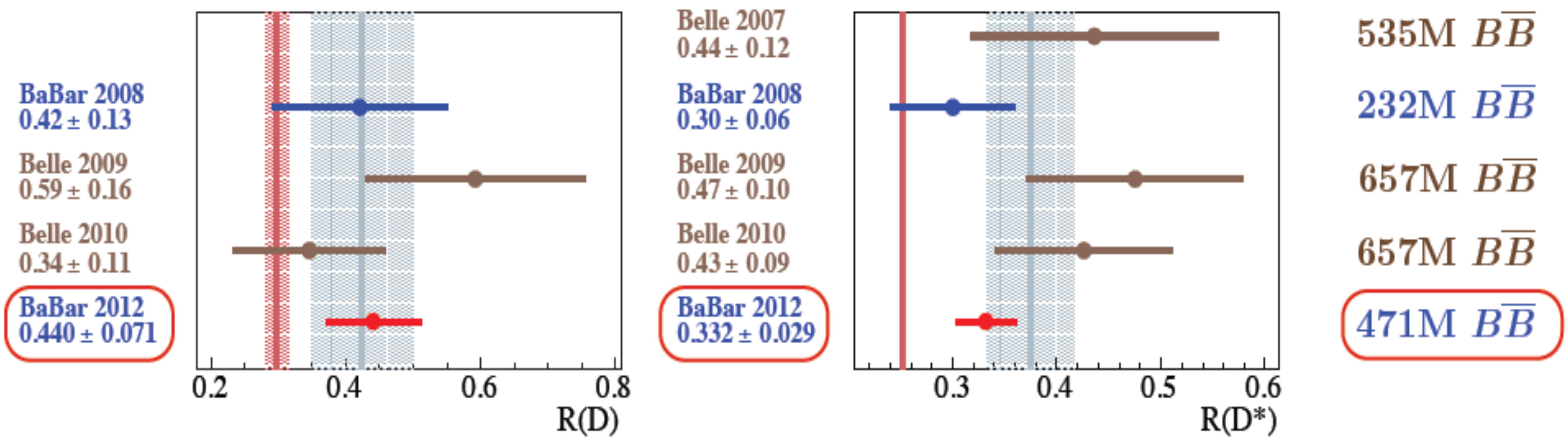
plot from Vera Lüth @ FPCP

- BaBar has now observed $B \to D^{(*)}\tau\nu$ at 6.8σ [arXiv:1205.5442 [hep-ex]]:

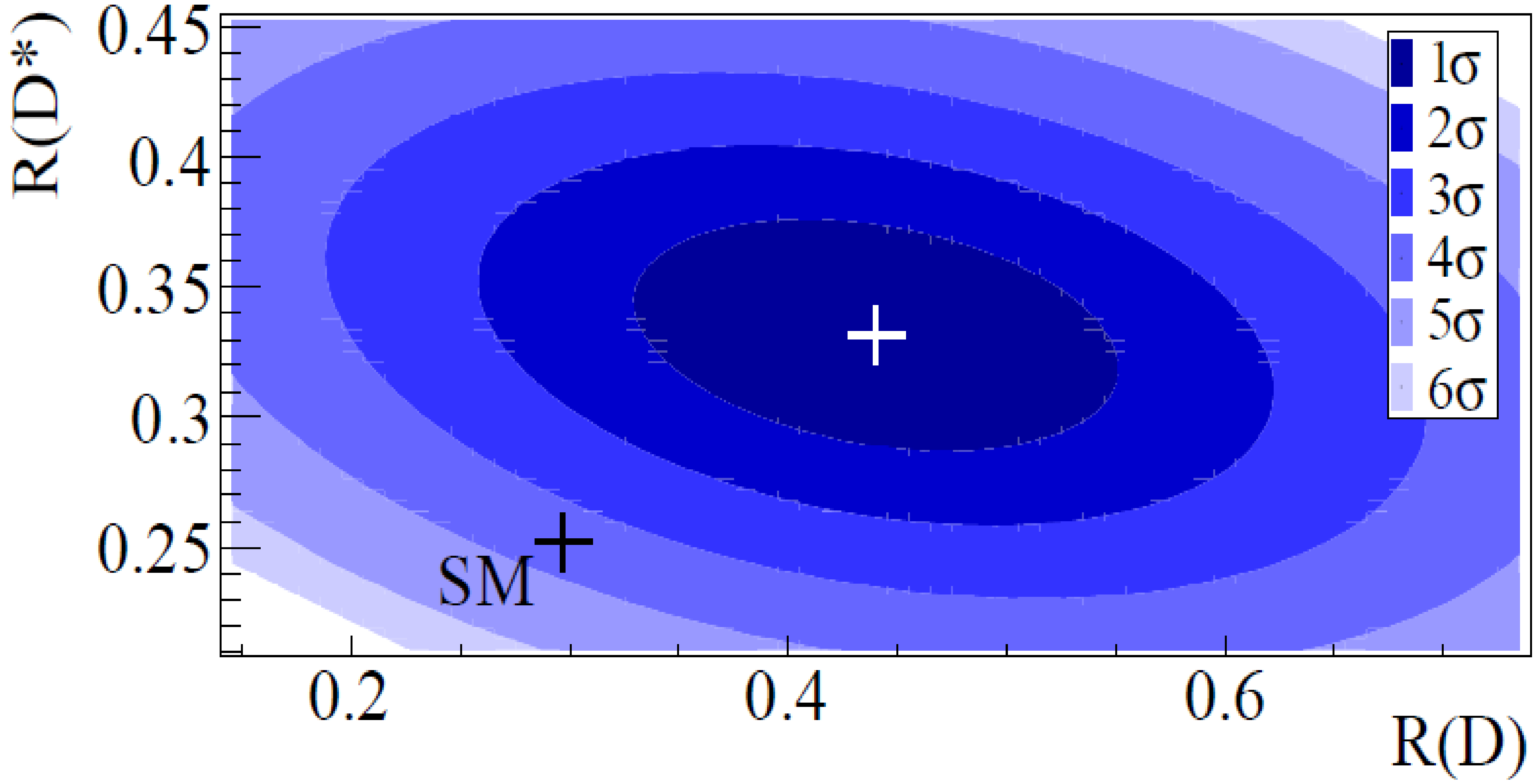
$$R(D) = 0.440 \pm 0.058 \pm 0.042, \qquad R(D^*) = 0.332 \pm 0.024 \pm 0.018.$$

- With info from expt (f_+; V, A_1, A_2) and from HQ scaling & quenched QCD (f_0; A_0) [Fajfer, Kamenik, Nišandžić, arXiv:1203.2654 [hep-ph] ← Tantalo *et al.*, arXiv:0707.0587 [hep-lat]]:

$$R_{\text{SM}}(D) = 0.297 \pm 0.017, \qquad R_{\text{SM}}(D^*) = 0.252 \pm 0.003.$$



- The measurements deviate from the SM predictions by $2.0\sigma \oplus 2.7\sigma = 3.4\sigma$.

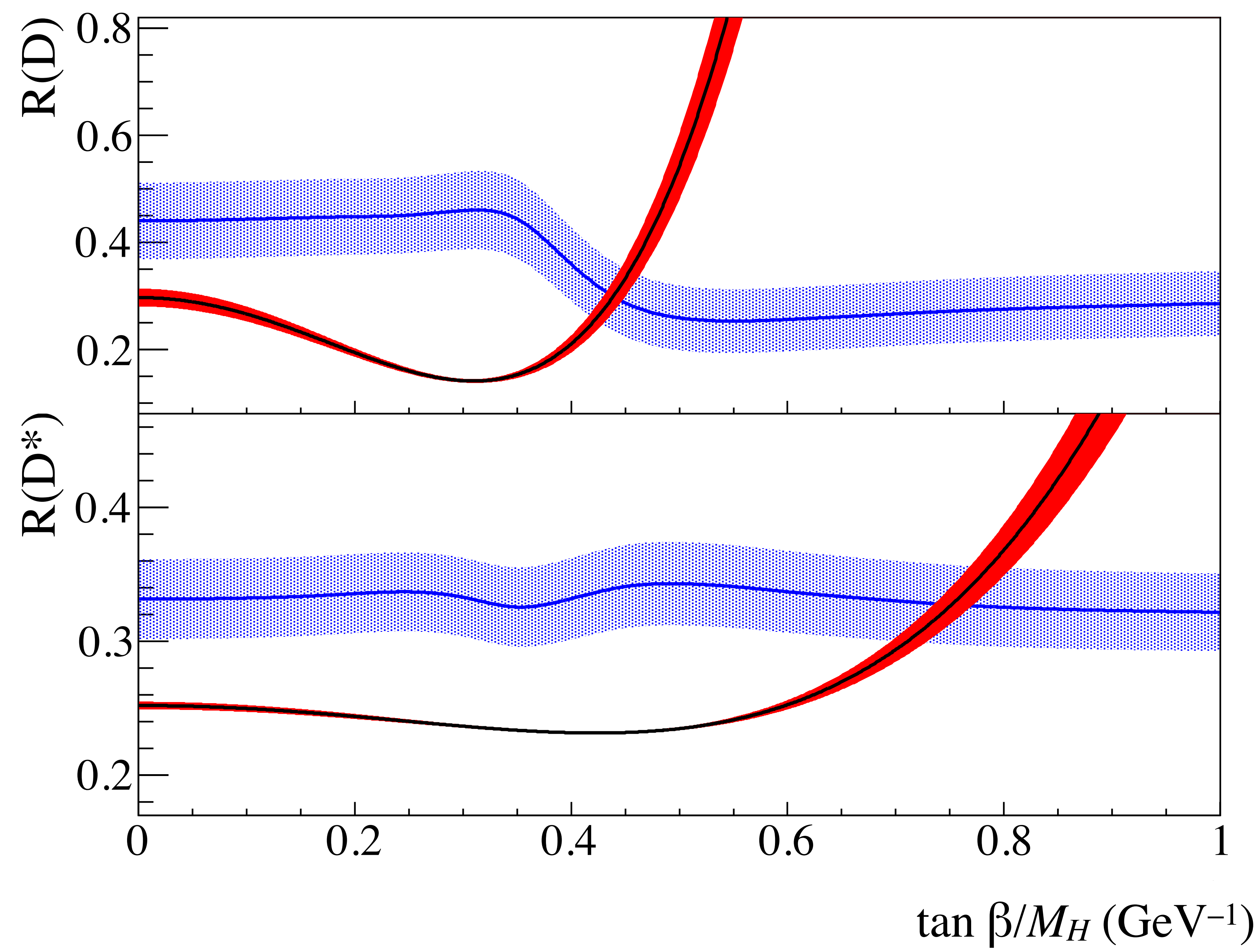


Vera Lüth @ FPCP;
Richard Kass @ CIPANP

- We aim to check $R_{\mathrm{SM}}(D)$ and $R_{\mathrm{SM}}(D^*)$ by using lattice-QCD calculations of the form factors.
  - Similar work by Bečirević, Košnik, and Tayduganov [arXiv:1206.4977 [hep-lat]].
- For $R(D)$, we have enough information from the previous analysis.
- For $R(D^*)$, we will have to calculate all four pseudoscalar-to-vector form factors.

Charged Higgs Test

- The charged Higgs interferes destructively in both channels and then overwhelms the SM.



- BaBar finds no value of $\tan\beta/M_H$ [in 2HDM II] that can describe both $R(D)$ and $R(D^*)$. This model is (for large $\tan\beta$) special: $G_S \approx (\tan\beta/M_H)^2 \approx G_P$.

General BSM Expression

arXiv:0812.2030 [hep-lat]

- In general, BSM physics could alter the rate in many ways:

$$\frac{d\Gamma}{dq^2} = \frac{1}{16\pi^3}|p_D|\left(1-\frac{m_\ell^2}{q^2}\right)^2\left\{|p_D|^2\frac{2q^2+m_\ell^2}{3q^2}\left|G_F V_{cb}^* + \bar{G}_V^{\ell cb}\right|^2 |f_+(q^2)|^2\right.$$
$$- \frac{2m_\ell}{M_B}|p_D|^2\Re\left[\left(G_F V_{cb}^* + \bar{G}_V^{\ell cb}\right) G_T^{\ell cb*} f_+(q^2) f_2^*(q^2)\right] + |p_D|^2\frac{q^2+2m_\ell^2}{3M_B^2}\left|G_T^{\ell cb}\right|^2 |f_2(q^2)|^2$$
$$\left. + \frac{q^2}{4M_B^2}\left|m_\ell\left(G_F V_{cb}^* + \bar{G}_V^{\ell cb}\right)\frac{M_B^2-M_D^2}{q^2} + G_S^{\ell cb}\frac{M_B^2-M_D^2}{m_c-m_s}\right|^2 |f_0(q^2)|^2\right\}$$

- BaBar interprets the discrepancy with the Type II two-Higgs-doublet model (2HDM II), for which $G_S^{\ell cb} = G_F V_{cb} m_\ell (m_c + m_b \tan^2\beta)/M_{H^\pm}^2$.

Our Results

- With our form factors, discussed above, we find $R_{\mathrm{SM}}(D) = 0.316(12)_{\mathrm{stat}}(7)_{\mathrm{syst}}$, with leading systematic being the chiral-continuum extrapolation and the z expansion.

- The agrees with earlier estimates but reduces the tension between BaBar and the SM.

- We also trace out the curve for 2HDM II and find a significantly different crossing point.

- Outlook is to look at all four form factors to examine $R(D^*)$, where BaBar has an even larger discrepancy.