Chiral behavior of kaon semileptonic form factors in lattice QCD with exact chiral symmetry

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1. introduction

$K \to \pi$ decay form factors

$$\langle \pi^+(p')|V_\mu|K^0(p)\rangle = (p+p')_\mu\, f_+(q^2) + (p-p')_\mu\, f_-(q^2),$$

$$f_0(q^2) = f_+(q^2) + \frac{q^2}{M_K^2 - M_\pi^2} f_-(q^2), \qquad \xi(q^2) = \frac{f_-(q^2)}{f_+(q^2)}$$

- vector form factor at $q^2=0$: $f_+(0)$ $\Leftrightarrow$ determination of $|V_{us}|$
  - $M_{\pi,\mathrm{sim}} \lesssim M_K$: $N_f=3$: *RBC/UKQCD, 2008,2010*, $N_f=2$: *ETM, 2009*

    (see also MILC/FNAL/HPQCD, 2005; FNAL/MILC @ Lat'11)
  - Γ $\Rightarrow$ $|V_{us}|f_+(0) = 0.2163(5)$ (*FlaviaNet, 2010*) $\Leftrightarrow$ $f_+(0)$ w/ $\lesssim 1$ % accuracy
- other information of ME : $f_-(0)$, $\lambda_{+,0}^{\prime;\prime\prime}$ $\Leftrightarrow$ consistency with ChPT, exp't

1. introduction

this talk

report on JLQCD's calculation of $K \to \pi$ form factors in $N_f = 2+1$ QCD

- overlap quarks $\Rightarrow$ straightforward comparison w/ ChPT ($a=0$)
- all-to-all quark prop. $\Rightarrow$ precise calculation of relevant meson correlators

outline

- simulation method
- q^2 interpolation
- chiral extrapolation

2.1 simulations : setup

configurations

- $N_f = 2+1$ QCD
- Iwasaki gauge + overlap quarks + $\det[H_W^2]/\det[H_W^2+\mu^2]$ $\Rightarrow$ speed up, fix Q
- $a = 0.1120(5)(3)$ fm (M_Ω as input) $\Leftrightarrow$ $O((a\Lambda_{\rm QCD})^2) \approx 8\,\%$ error

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measurements of meson correlators

- 4 m_{ud}'s $\Rightarrow$ $M_\pi \simeq$ 290 – 540 MeV; $m_s =$ 0.080 $\Leftrightarrow$ $m_{s,\rm phys} =$ 0.081
- $16^3 \times 48$ or $24^3 \times 48$ (depending on m_{ud}) $\Rightarrow$ $M_\pi L \gtrsim$ 4
- in $Q=0$ sector $\Rightarrow$ fixed Q effects $\propto V^{-1}$; sub-% for $M_{\{\pi,K\}}$, $f_{\{\pi,K\}}$
- 50 conf $\times$ 50 HMC traj. for each (m_{ud},m_s)

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- w/ all-to-all propagator $\Rightarrow$ 160 - 240 low-modes + noise method
- twisted boundary conditions (TBCs) $\Rightarrow$ $q^2 \in [-0.1\,\mathrm{GeV}^2, q^2_{\rm max}]$
- reweighting $\Rightarrow$ additional $m_s =$ 0.060

2.2 simulations : ratio method

ratio method (*Hashimoto et al., 1999*)

$$C^{PQ}_{V_\mu}(\mathbf{p},\mathbf{p}') = \langle \mathcal{O}_Q(\mathbf{p}') V_{\mu,\mathrm{lat}} \mathcal{O}_P^\dagger(\mathbf{p})\rangle \sim \frac{\sqrt{Z_Q(\mathbf{p}')Z_P(\mathbf{p})}}{4E_P E_Q\, Z_V} \langle Q(\mathbf{p}')|V_\mu|P(\mathbf{p})\rangle e^{-E_Q\Delta t' - E_P \Delta t}$$

$$C^P(\mathbf{p}) = \langle \mathcal{O}_P(\mathbf{p}')\mathcal{O}_P^\dagger(\mathbf{p})\rangle \sim \frac{Z_P}{2E_P} e^{-E_Q\Delta t'} \quad (P, Q = K \text{ or } \pi)$$

(partially) cancel Z_V, $\exp[-E_{\pi(K)}\Delta t]$, fluctuation in $C^{PQ}_{V_\mu}(\mathbf{p},\mathbf{p}')$ and $C^P(\mathbf{p})$

$$R = \frac{C^{K\pi}_{V_4}(\mathbf{0},\mathbf{0})\, C^{\pi K}_{V_4}(\mathbf{0},\mathbf{0})}{C^{KK}_{V_4}(\mathbf{0},\mathbf{0})\, C^{\pi\pi}_{V_4}(\mathbf{0},\mathbf{0})} \rightarrow \frac{(M_K+M_\pi)^2}{4M_K M_\pi} f_0(q^2_{\mathrm{max}})^2 \quad (q^2_{\mathrm{max}} = (M_K - M_\pi)^2)$$

$$\tilde{R} = \frac{C^{K\pi}_{V_4}(\mathbf{p},\mathbf{p}')\, C^K(\mathbf{0})\, C^\pi(\mathbf{0})}{C^{K\pi}_{V_4}(\mathbf{0},\mathbf{0})\, C^K(\mathbf{p})\, C^\pi(\mathbf{p}')} \rightarrow \left\{1 + \frac{E_K(\mathbf{p}) - E_\pi(\mathbf{p}')}{E_K(\mathbf{p}) + E_\pi(\mathbf{p}')}\xi(q^2)\right\} \frac{f_+(q^2)}{f_0(q^2_{\mathrm{max}})}$$

$$R_k = \frac{C^{K\pi}_{V_k}(\mathbf{p},\mathbf{p}')\, C^{KK}_{V_4}(\mathbf{p},\mathbf{p}')}{C^{K\pi}_{V_4}(\mathbf{p},\mathbf{p}')\, C^{KK}_{V_k}(\mathbf{p},\mathbf{p}')} \rightarrow \text{a function of } \xi(q^2)$$

all-to-all prop. + ratio method $\Rightarrow$ $\Delta[f_{+,0}(q^2)] \lesssim 1\,\%$, $\Delta[\xi(q^2)] \sim 10 - 30\,\%$

2.3 simulations : reweighting

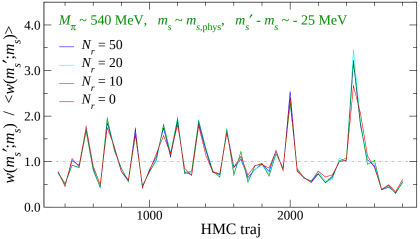
reweighting $\Rightarrow$ can impair stat. accuracy

$$\langle C_{V_\mu}^{PQ} \rangle_{m_s'} = \langle C_{V_\mu}^{PQ} \, \tilde{w}(m_s', m_s) \rangle_{m_s}$$

$$\tilde{w}(m_s', m_s) = \frac{w(m_s', m_s)}{\langle w(m_s', m_s) \rangle_{m_s}}$$

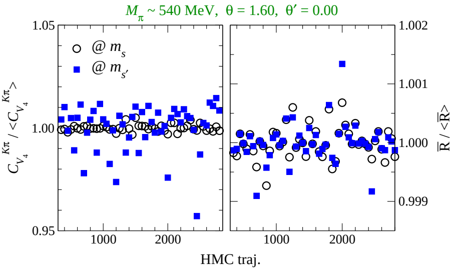
$$w(m_s', m_s) = \det[D(m_s')] / \det[D(m_s)]$$

$$= \frac{\prod_{k=1}^{N_e} \lambda_k(m_s')}{\prod_{k=1}^{N_e} \lambda_k(m_s)} \times \frac{1}{N_r} \sum_{r=1}^{N_r} e^{-\frac{1}{2} \xi_r^\dagger \frac{D(m_s')}{D(m_s)} \xi_r}$$



- $24^3(16^3) \times 48$, $\approx$ 200 low-modes

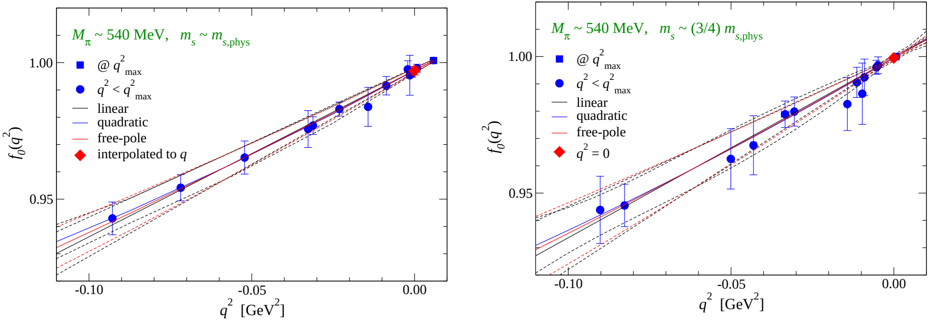
  $\Rightarrow$ do not need large N_r
- largely increase $\Delta[C_{V_\mu}^{PQ}]$
- at most $\times$ 2 for ratios



all-to-all + ratio + reweight $\Rightarrow$ $\Delta[f_{+,0}(q^2)] \lesssim 1.5\,\%$, $\Delta[\xi(q^2)] \sim 20 - 40\,\%$

3. q^2-dependence

$f_0(q^2)$ vs q^2



- reweighting $\Rightarrow$ slightly larger error
- small curvature in simulated region of q^2

  $\Leftrightarrow$ $M_{\rm pole}$=1.2–1.3 GeV @ $m_{q,{\rm phys}}$ *(PDG, 2010)* $\Rightarrow$ $q^4/M_{\rm pole}^4 \lesssim$ 0.6%
- well described by any of

$$f_0(q^2) = f_0(0)(1 + a_1 q^2), \quad f_0(0)(1 + a_1 q^2 + a_2 q^4), \quad \frac{f_0(0)}{1 - q^2/M_{\rm pole}^2}$$

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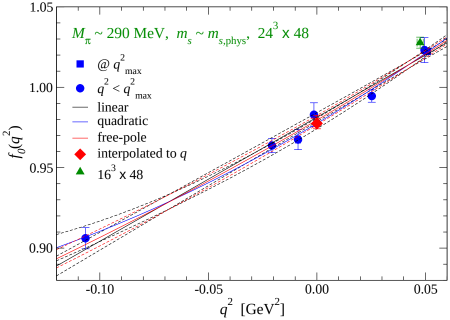
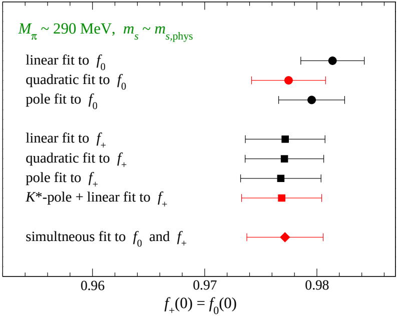


fit results for $f_{\{+,0\}}(0)$



- small finite volume effects $\Leftrightarrow$ lattice boundaries + fixed Q

  $16^3 \times 48 \Leftrightarrow 24^3 \times 48$: $\lesssim$ 0.8% ($\lesssim$ 2.1σ) $\Rightarrow$ $24^3 \times 48 \Leftrightarrow V = \infty$: even smaller

- $f_+(0)$ $(= f_0(0))$, $f'_{\{+,0\}}(0)$: stable against choice of fit form

- employ simultaneous fit to $f_0(q^2)$ and $f_+(q^2)$ $\Rightarrow$ $f_+(0)$, $f'_{\{+,0\}}(0)$

4.1 chiral fit : $f_+(0)$

chiral expansions *(cf. JLQCD, 2008)*

$$f_+(0) = 1 + f_2 + f_4 + \cdots = 1 + f_2 + \Delta f$$

- $x_{\pi(K)} \equiv M^2_{\pi(K)}/(4\pi F_0)^2$ ("x-expansion")

  small $F_0 = 52.5(5.1)$ MeV $\ll F_\pi$ $\Rightarrow$ enhanced chiral corrections $\mathcal{O}_{2n} = (M_\pi/F_0)^{2n}$

- $\xi_{\pi(K)} \equiv M^2_{\pi(K)}/(4\pi F_\pi)^2$ ("ξ-expansion")

  better convergence for $M_{\{\pi,K\}}$, $F_{\{\pi,K\}}$ *(JLQCD, 2008 ($N_f = 2$) ; 2009 ($N_f = 2+1$))*

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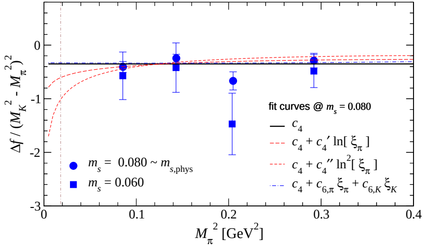
fitting w/ ξ-expansion

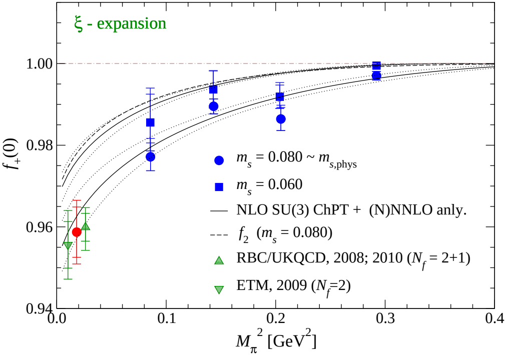
- NLO *(Gasser-Leutwyler, 1985)*
  Ademollo-Gatto $\Rightarrow$ $f_2(F_\pi, M_{\{\pi,K,\eta\}})$
- NNLO and higher orders
  (Post-Schilcher, 2001; Bijnens-Talavera, 2003)
  $\Delta f(F_\pi, \{L_i\}, \{C_i\}, M_{\{\pi,K,\eta\}})$

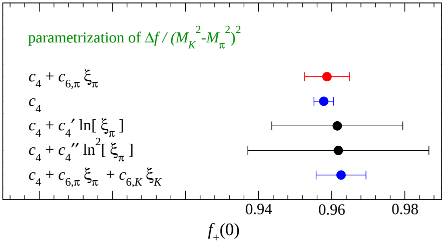


- modeling Δf (as in previous studies)

$$\Delta f = (M_K^2 - M_\pi^2)^2 (\, c_4 \quad + c_4' \ln[\xi_\pi], \quad c_4'' \ln^2[\xi_\pi], \quad c_{6,\pi}\xi_\pi, \quad c_{6,K}\xi_K)$$

4.1 chiral fit : $f_+(0)$





fit results

- parameterizations of $\Delta f/(M_K^2 - M_\pi^2)^2$
  - $c_4 + c_{6,\pi}\,\xi_\pi \;\Rightarrow$ central value
  - c_4
  - $c_4 + c'_{4,\pi}\ln[\xi_\pi]$ (ill-determined c_X)
  - $c_4 + c''_{4,\pi}\ln^2[\xi_\pi]$ (ill-determined c_X)
  - $c_4 + c_{6,\pi}\,\xi_\pi + c_{6,K}\,\xi_K$
- assume $O((a\Lambda)^2)$ error in $f_2 + \Delta f$

$$f_+(0) = 0.959(6)_{\rm stat}(4)_{\rm chiral}(3)_{a\neq 0}$$
$$= 1 - 0.024\,[f_2] - 0.018(8)\,[\Delta f]$$

$$|V_{us}| = 0.2256(19)$$

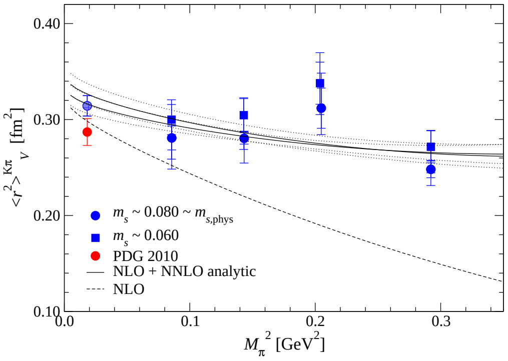
$$\Delta_{\rm CKM} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1$$
$$= -1(9)\times 10^{-4}$$

4.2 chiral fit : $\langle r^2\rangle_V^{K\pi}$

slope of $f_+(q^2)$

- $f_+(q^2) \;=\; f_+(0)\left\{1+(1/6)\langle r^2\rangle_V^{K\pi} q^2+\cdots\right\}$
- SU(3) NLO ChPT *(Gasser-Leutwyler, 1985)* + NNLO analy., ξ-expansion,

$$\langle r^2\rangle_V^{K\pi} \;=\; 12L_9^r/F_\pi^2 - (1/F_\pi^2)\,\text{"logs"} + c_\pi\xi_\pi + c_K\xi_K$$



- significant NNLO @ $m_{ud(s),\mathrm{sim}}$

  $\Rightarrow$ milder m_{ud}-dependence

  $\Leftrightarrow$ $\langle r^2\rangle_V^{\pi}$, $\langle r^2\rangle_V^{K}$ *(JLQCD, $N_f = 2, 3$)*

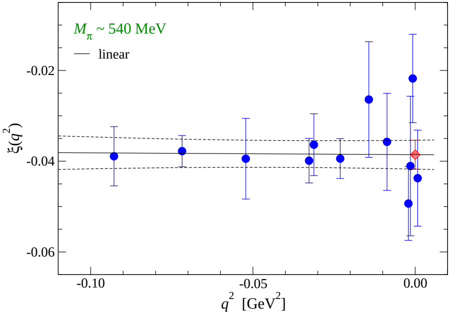
- $L_9^r(M_\rho)$: consistent w/ pheno.
  - $L_9^r = 4.3(0.6)(0.3)\times 10^{-3}$
  - pheno. : $5.91(4)\times 10^{-3}$

    (Bijnens, 2007, ξ-expansion)

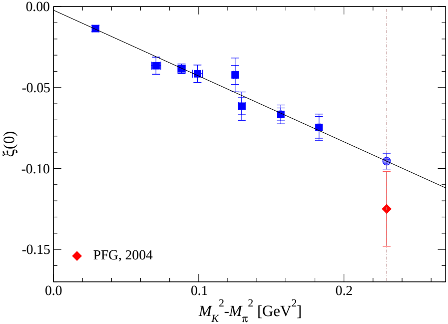
- $\langle r^2\rangle_V^{K\pi}$: consistent w/ exp't

4.3 chiral fit : $\xi(0) = f_-(0)/f_+(0)$

$\xi(q^2)$ vs q^2



$\xi(0)$ vs $M_K^2 - M_\pi^2$



- mild q^2 dependence w/ our statistical accuracy

$$cf. \quad f_{-,\text{analy}}(q^2) = (M_K^2 - M_\pi^2)\,(4L_5^r/F_\pi^2 - 2L_9^r/F_\pi^2)$$ *(Bijnens-Talavera 2003)*

- (for simplicity) fit form linear in $(M_K^2 - M_\pi^2)$: $\xi(0) = c_0 + c_1(M_K^2 - M_\pi^2)$
  - vanish in SU(3) limit as expected : $c_0 = -0.0022(25)$
  - consistent w/ experiment : $\xi(0) = -0.125(23)$ *(PDG 2004, K_{l3}^+)*
  - one-loop chiral log.s (and two-loops?) should be included *(Bijnens-Talavera 2003)*

5. summary

kaon semileptonic form factors in $N_f = 2+1$ QCD with overlap quarks

- techniques
  - all-to-all propagators $\Rightarrow$ precise determination of $f_{+,0}(q^2)$
  - TBCs $\Rightarrow$ precise determination of $f_+(0)$, $\xi(0)$, $\langle r^2\rangle_{V,S}^{K\pi}$
  - reweighting $\Rightarrow$ additional m_s

- chiral fits
  - $f_+(0) = 0.959(6)_{\rm stat}\,(4)_{\rm chiral}\,(3)_{\rm a\neq 0}$

    $\Rightarrow\ |V_{us}| = 0.2256(19), \quad |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -1\,(9)\times 10^{-4}$
  - $\langle r^2\rangle_V^{K\pi}$, $\xi(0)$: reasonably consistent w/ exp't

- future directions
  - further refinements $\Leftrightarrow$ more rigorous treatment of NNLO $\Leftrightarrow$ overlap quarks
  - π, K EM form factors $\Leftrightarrow$ test of ChPT
  - D, B meson decays $\Leftrightarrow$ flavor physics

6.1 appendix : F_0

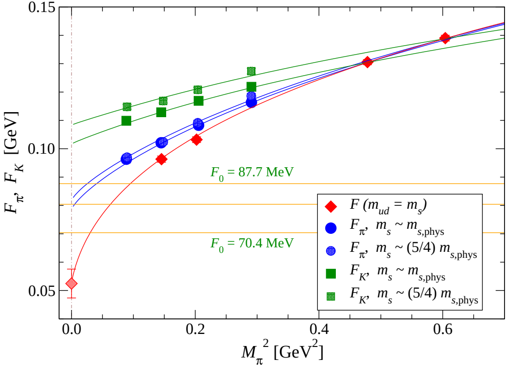
F_0

- phenomenological estimates

  (Bijnens @ Lattice'07; $\mu = 0.77$ GeV)

$10^3 L_4^r$	$\equiv 0$	$\equiv 0.5$	$\equiv 0.2$
$10^3 L_6^r$	$\equiv 0$	$\equiv 0.1$	$\equiv 0$
F_0[MeV]	87.7	70.4	80.4

- JLQCD's analysis *(Noaki @ Lattice'10)*
  - NNLO fit to $M_{\{\pi,K\}}$, $F_{\{\pi,K\}}$
  - chiral expansion w/ M_π^2/F_π^2 instead of M_π^2/F_0
    $\Rightarrow$ better convergence
  - single lattice spacing $\sim$ 0.11 fm
  - $F_0 = 52.5(5.1)$ MeV



small F_0 $\Rightarrow$ enhance chiral corrections : NLO $\propto 1/F_0^2$; NNLO $\propto 1/F_0^4$