Determining V_{us} from semi leptonic Kaon decays

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CKM Matrix

Quarks may change their flavors through electro-weak interaction.

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$

Search deviations from unitarity → search for physics beyond the SM.

The most sensitive test of the unitarity of the CKM matrix is provided by the relation

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \delta$$

In the above unitarity test, $|V_{ud}|$ is precisely determined, and $|V_{ub}|$ is usually neglected as it is very small so that leaves us with $|V_{us}|$

Our previous result - $|V_{us}| = 0.9599(34)_{\text{stat}}\binom{+31}{-43}_{\chi}(14)_{\text{a}}$

dominated by uncertainty in chiral extrapolations

V_{us}

Semi-leptonic decays involve changes to flavors of quarks, or mixing of quarks, and from these processes we can determine CKM matrix elements

$K \rightarrow \pi l v (Kl3)$ semi-leptonic decay process leads to the determination of $|V_{us}|$

In a Kl3 decay, the decay rate can be written as

$$\Gamma = G_F{}^2 M_K{}^2 C^2 / 192\pi^2 \; S_{ew}(1 + \delta_{em}) \quad |V_{us}|^2 \; |f_+(0)|^2$$

Known constants Known corrections formfactor

The decay rate can be precisely estimated from experiments of semi-leptonic $K \rightarrow \pi$ decays and hence the value of the product $|f_+(0)|^2 |V_{us}|^2$

The determination of $f_+(0)$ using Lattice QCD is important in estimating V_{us}

$\boldsymbol{f}_+(0)$

$f_+(0)$ is defined from the $K \to \pi$ matrix element of the weak vector current (V_μ) at zero momentum transfer

$$\langle \pi(p') | V_\mu | K(p) \rangle = \left(p_\mu + p'_\mu\right) f_+(\boldsymbol{q^2}) + \left(p_\mu - p'_\mu\right) f_-(\boldsymbol{q^2})$$

where $\boldsymbol{q^2 = (p - p')^2}$

current conservation implies that $\boldsymbol{f}_+(0)$ =1 [SU(3) flavour limit $m_\pi{}^2 = m_K{}^2$]

Ademollo-Gatto Theorem : SU(3) breaking effects in

$$f_+(0) = 1 + f_2 + f_4 + O(p^6)\,, \quad f_n = O(m_\pi{}^n, K, \eta)$$

$$\Delta f = 1 + f_2 - f_+(0)$$

standard result from Leutwyler & Roos (1984) Δf=-0.016(8)

$\langle \pi(\boldsymbol{p_f}) | V_\mu | K(\boldsymbol{p_i}) \rangle$

Construct ratios of correlation functions , such that we can extract the matrix element $\langle \pi(\boldsymbol{p_f}) | V_\mu | K(\boldsymbol{p_i}) \rangle$

$$\langle \pi(\boldsymbol{p_f}) | V_\mu | K(\boldsymbol{p_i}) \rangle = \begin{cases} R_{1,p_i,p_f} = 4\sqrt{E_i E_f}\sqrt{\dfrac{C_{K\pi}(p_i,p_f)C_{\pi K}(p_f,p_i)}{C_K(p_i)C_\pi(p_f)}} \\ R_{2,p_i,p_f} = 2\sqrt{E_i E_f}\sqrt{\dfrac{C_{K\pi}(p_i,p_f)C_{\pi K}(p_f,p_i)}{C_{KK}(p_i,p_i)C_{\pi\pi}(p_f,p_f)}} \end{cases}$$

Where $C_K(p_i)$ is the Kaon two point functions

$C_{K\pi}(p_i, p_f)$ is the Kaon to Pion three point function and similarly …

So far ...

V_{us} CKM Matrix element

⬇

$\boldsymbol{f}_{+}(0)$ K→ π formfactor at zero momentum transfer

⬇

$\langle \pi(\boldsymbol{p}_f) | V_\mu | K(\boldsymbol{p}_i) \rangle$ $K \to \pi$ matrix element of the weak vector current (V_μ)

⬇

$R_{1,K,\pi}$, $R_{2,K,\pi}$ *Ratios of correlation functions*

So far …

And Happily lived ever after …

Existing Method

Measure 2-pt and 3-pt correlation functions from Lattice QCD simulations

Periodic boundary condition results in hadronic momenta (p) **quantized** as

$$p_i = \frac{2\pi}{L} n_i$$ where L is the cubic volume, n_i's are integers.

Calculate $f_+(q^2)$ for different q by constructing ratios

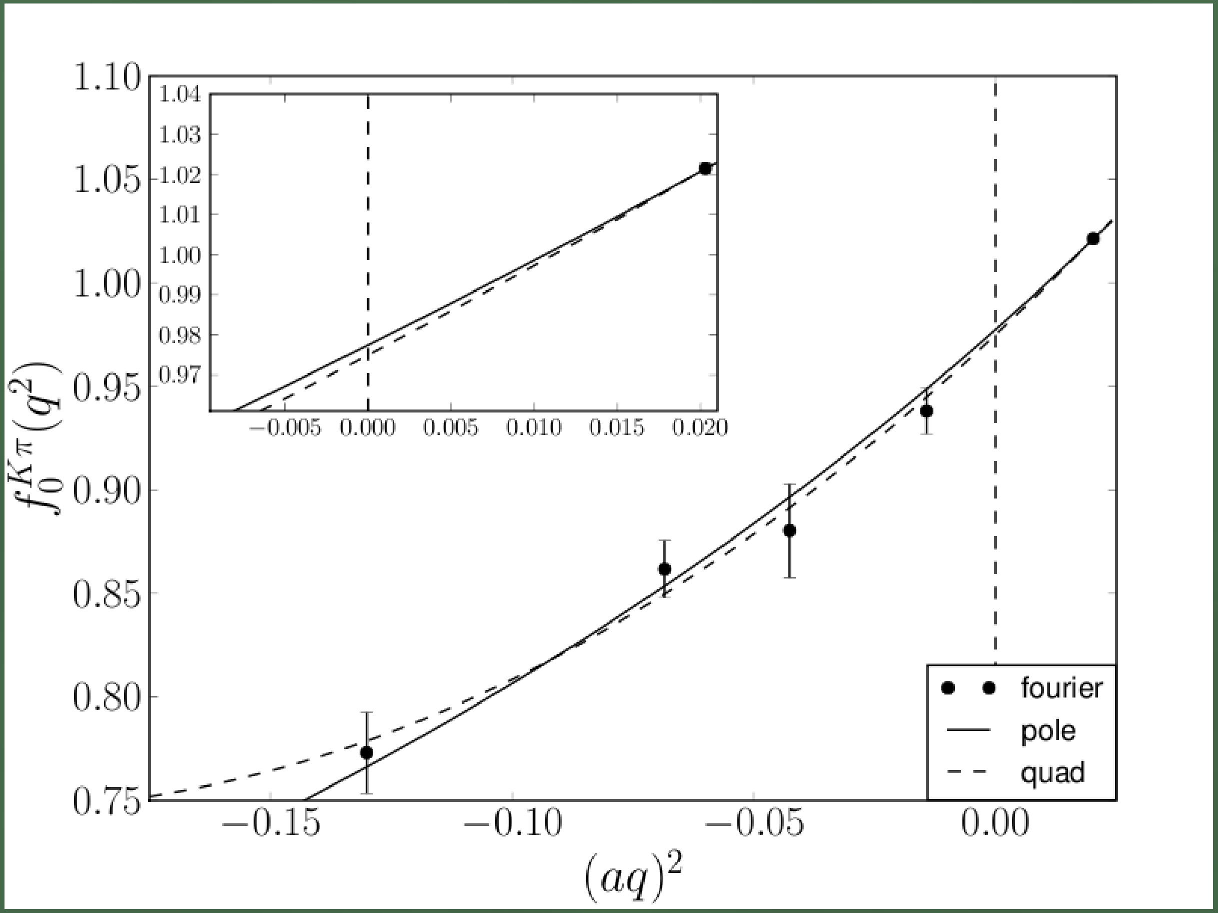
Find $f_+(0)$ by **interpolating** to $q^2 = 0$

Simulations performed at mass higher than the physical mass.

Extrapolate $f_+(0)$ result to that of actual physical mass of mesons. (chiral extrapolation)

Errors due to interpolation, extrapolation and finite volume effects.

Interpolation to $q^2 = 0$



$f_+(0)_{pole} = 0.9774(35)$

$f_+(0)_{quad} = 0.9749(59)$

~ 0.2% Systematic error from difference in pole and quadratic fits

Twisted boundary condition

Twisted boundary condition allows momenta smaller than $2\pi/L$ to be simulated

By partially twisting the boundary conditions of the quarks (θ_i,θ_f), the momentum transfer q^2 can be modified as follows

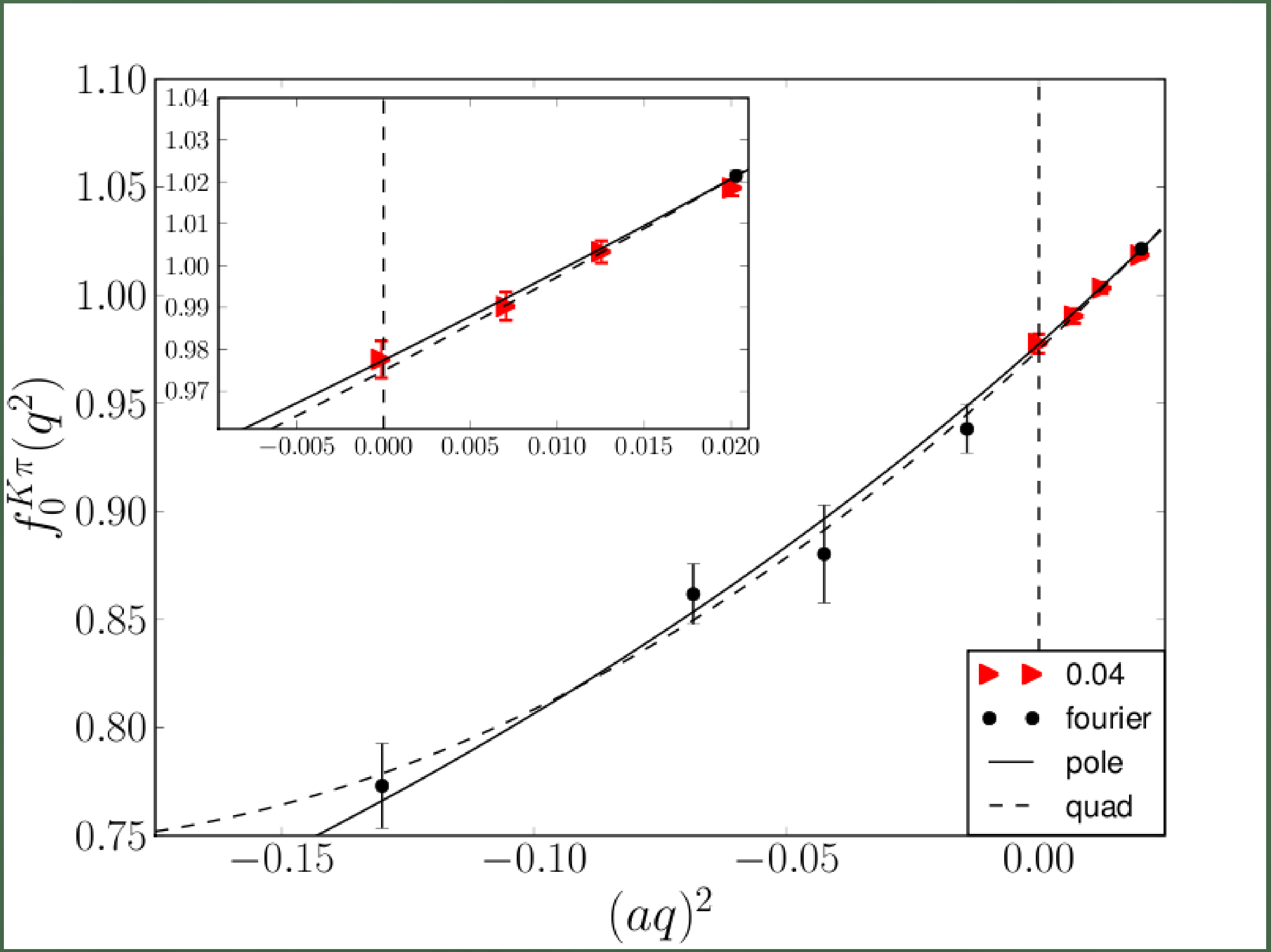
$$q^2 = \left\{ \left[E_i(\vec{p}_i) - E_f(\vec{p}_f) \right]^2 - \left[\left(p_{\overrightarrow{FT},i} + \frac{\vec{\theta}_i}{L} \right) - \left(p_{\overrightarrow{FT},f} + \frac{\vec{\theta}_f}{L} \right) \right] \right\}$$

We can evaluate $K \to \pi$ formfactor by applying twist to Kaon or pion such that $q^2 = 0$. The twist angle can be evaluated as

$$|\vec{\theta}_K| = L\sqrt{\frac{({m_K}^2 + {m_\pi}^2)}{2m_\pi} - {m_K}^2} \qquad |\vec{\theta}_\pi| = 0 \quad \text{or}$$

$$|\vec{\theta}_\pi| = L\sqrt{\frac{({m_K}^2 + {m_\pi}^2)}{2m_K} - {m_\pi}^2} \qquad |\vec{\theta}_K| = 0$$

Twisted boundary results



$f_+(0) = 0.9757(44)$

[PA Boyle et al arXiv:1004.0886v1]

Results

We simulate at

N_f = 2+1 dynamical flavours

Iwasaki gauge action + Domain Wall Fermion (DWF) action

1.73^{-1} , $24^3 \times 64 \times 16$

2.28^{-1} , $32^3 \times 64 \times 16$ (fine) ⟵ **New data**

Iwasaki + Dislocation-Suppressing-Determinant-Ratio DSDR + DWF

1.35^{-1} , $32^3 \times 64 \times 32$ (coarse) ⟵ **New data**

With different strange quark masses

Using HPC facilities

QCDOC, BlueGene-P and BlueGene-Q

Global fit

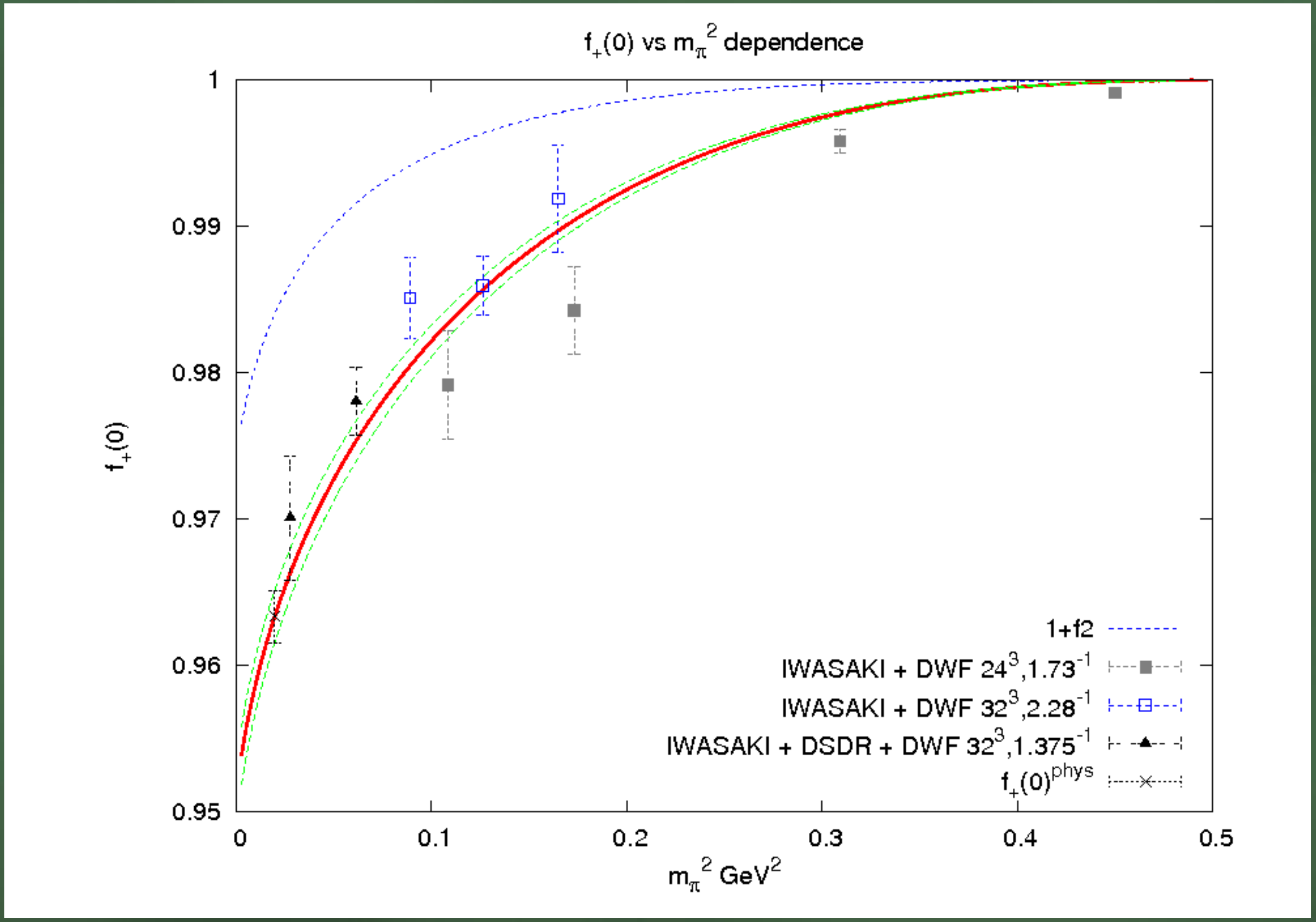
Simultaneous fit all data to q^2 , ${m_\pi}^2$, ${m_K}^2$ using the ansatz

$$f_0(q^2,{m_\pi}^2,{m_K}^2) = \frac{1+f_2+({m_K}^2-{m_\pi}^2)^2\{A_0+A_1({m_K}^2+{m_\pi}^2)\}}{1-q^2/(M_0+M_1({m_K}^2+{m_\pi}^2))^2}$$

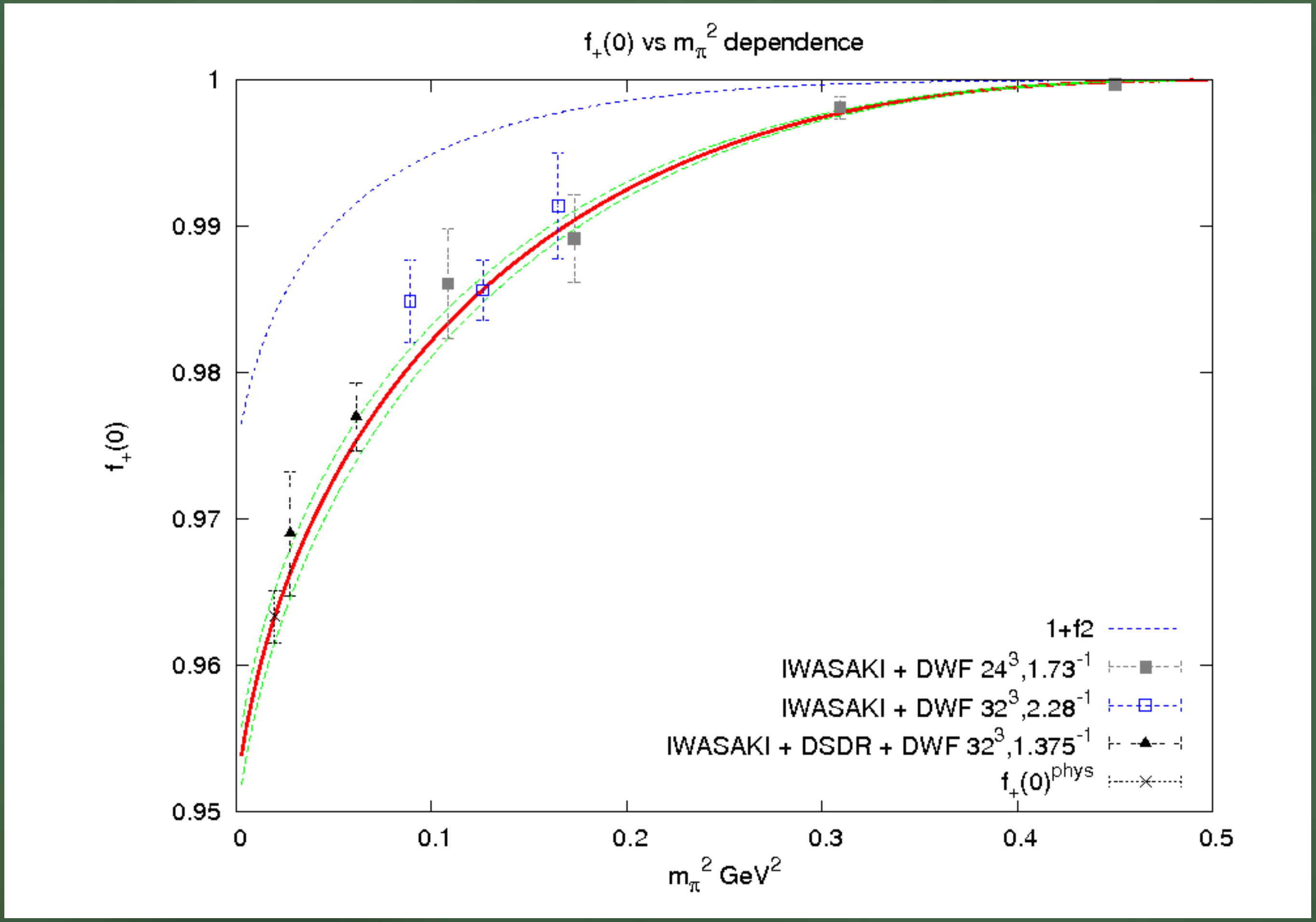
With four fit parameters A_0, A_1 , M_0, M_1

Expression motivated from the Ademollo-Gatto Theorem

$f_+(0)$ vs $m_\pi{}^2$ dependence
-no strange quark mass correction



$f_+(0)$ vs $m_\pi{}^2$ dependence with correction for strange quark mass $f_0(q^2, m_\pi{}^{latt}, m_K{}^{latt})$ - $f_0(q^2, m_\pi{}^{phys}, m_K{}^{phys})$



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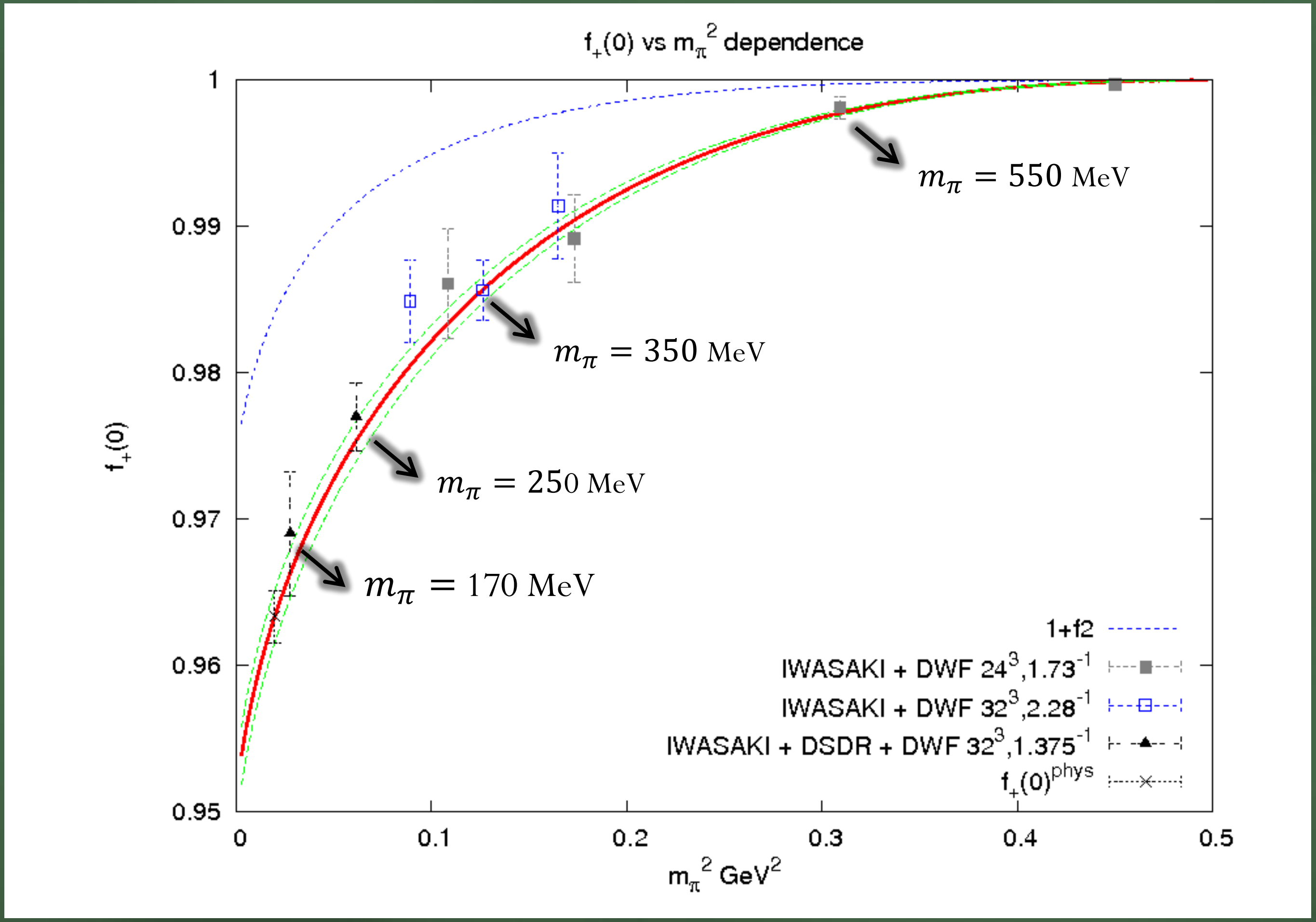
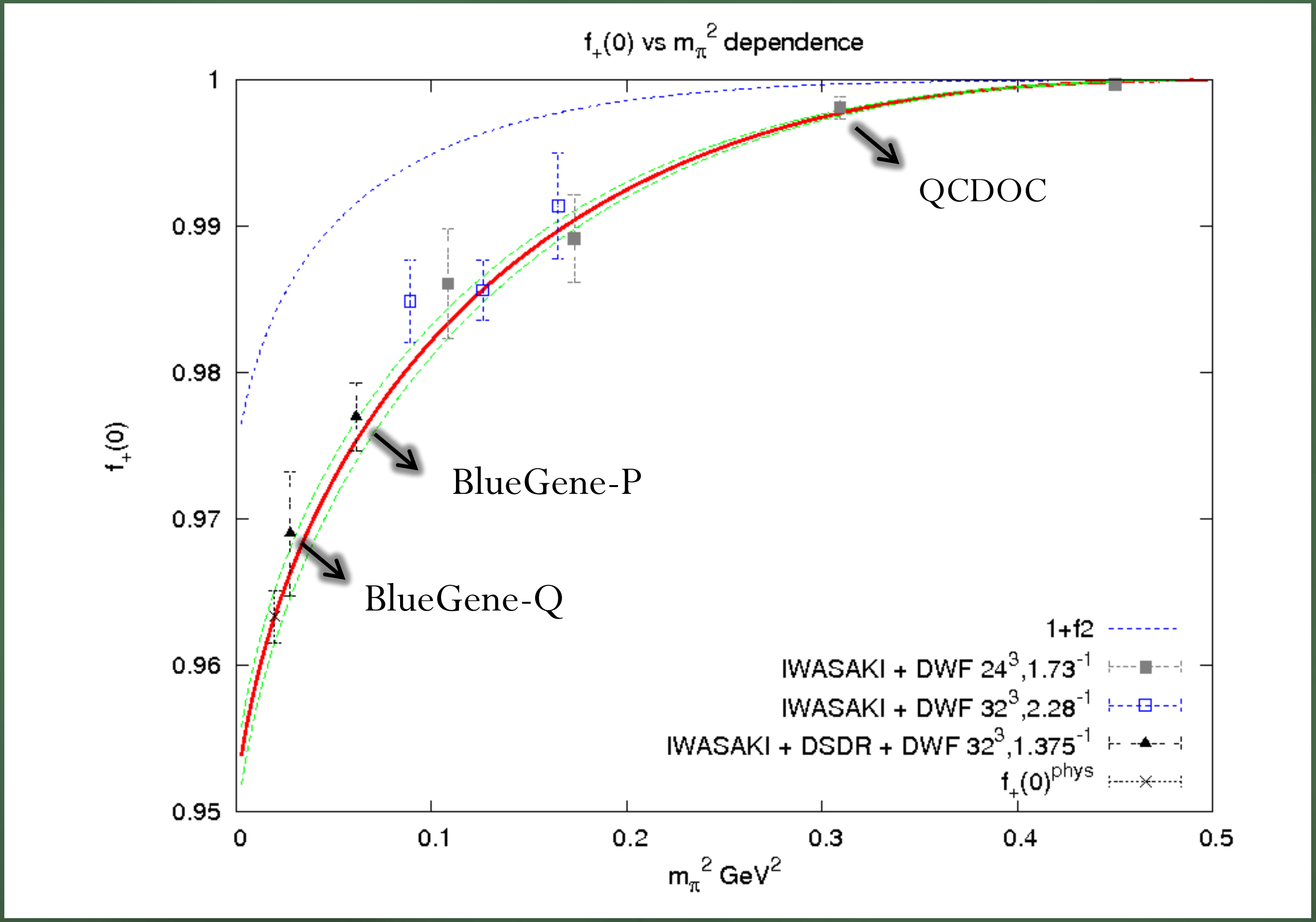
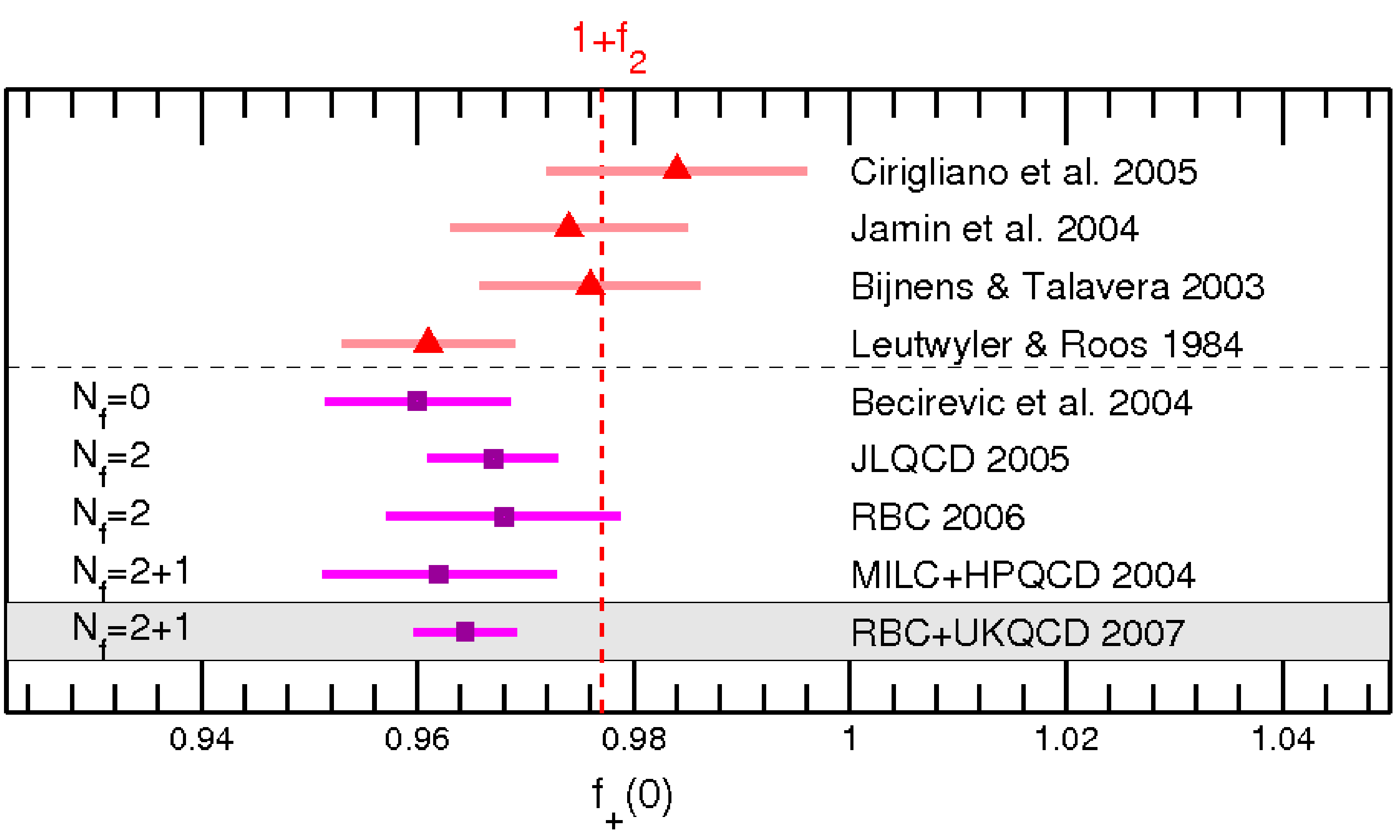


$f_+(0)$ vs $m_\pi{}^2$ dependence with correction
$f_0(q^2, m_\pi{}^{latt}, m_K{}^{latt})$ - $f_0(q^2, m_\pi{}^{phys}, m_K{}^{phys})$



Comparison of Lattice results (blue squares) with various model estimates based on $_{\chi}$PT (red triangles)



PA Boyle et al PRL 100,141601(2008)

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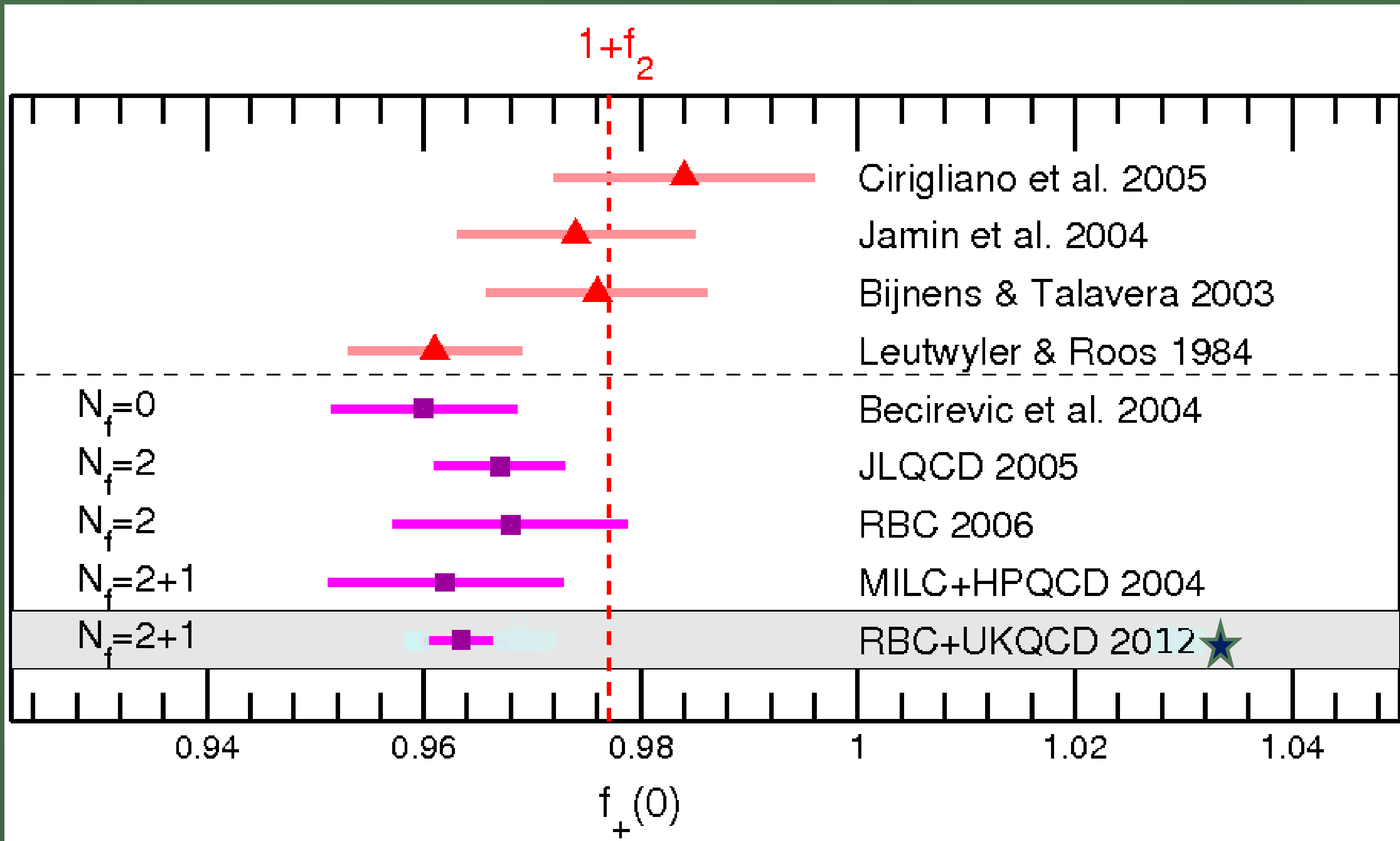

★ RBC+UKQCD 2012 - Expected Preliminary result

Conclusion

We can determine CKM Matrix element V_{us} precisely using kl3 semi-leptonic decays

By appropriate choice of twisted boundary conditions systematic errors due to q^2 interpolation can be removed

Uncertainty dominated by Chiral extrapolation

Measurement made on different lattice spacing and volume with m_π as low as 170 MeV

Chiral extrapolation error for $\boldsymbol{f}_+^{K\pi}(0)$ can be almost halved

Outlook :

Larger volume, smaller spacing, Exascale Computing


धन्यवाद
Obrigado!
Gracias
תודה
THANK YOU
Ευχαριστώ
Merci
Vielen Dank
شكراً
Lattice 2012