The effects of flavour symmetry breaking on hadron matrix elements II

Ashley N. Cooke
QCDSF – UKQCD

THE UNIVERSITY of EDINBURGH

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Outline

- Motivation
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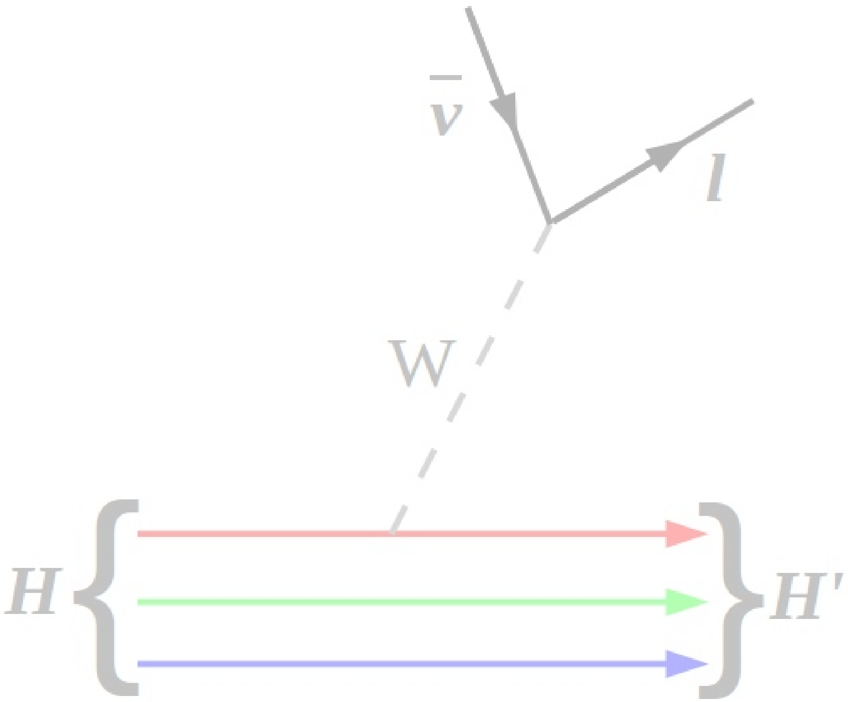
- Simulation Details
- Flavour Symmetry Breaking
- Recent Results
  - *Hyperon Semi-Leptonic Transitions*
  - *“Fan” plots*
  - *Hyperon Axial Charges*
- Summary & Future Prospects
- References & Acknowledgements



$$H \rightarrow H' l \nu_l$$

Hyperons are baryons containing at least one strange quark and we study them and their transitions primarily to:

• determine their form factors ⟶ One of several methods for calculating CKM matrix elements

• obtain axial charges from form factors at $q^2 = (p' - p)^2 = 0$ ⟶ Low-energy effective field theory description of octet baryons (F & D)

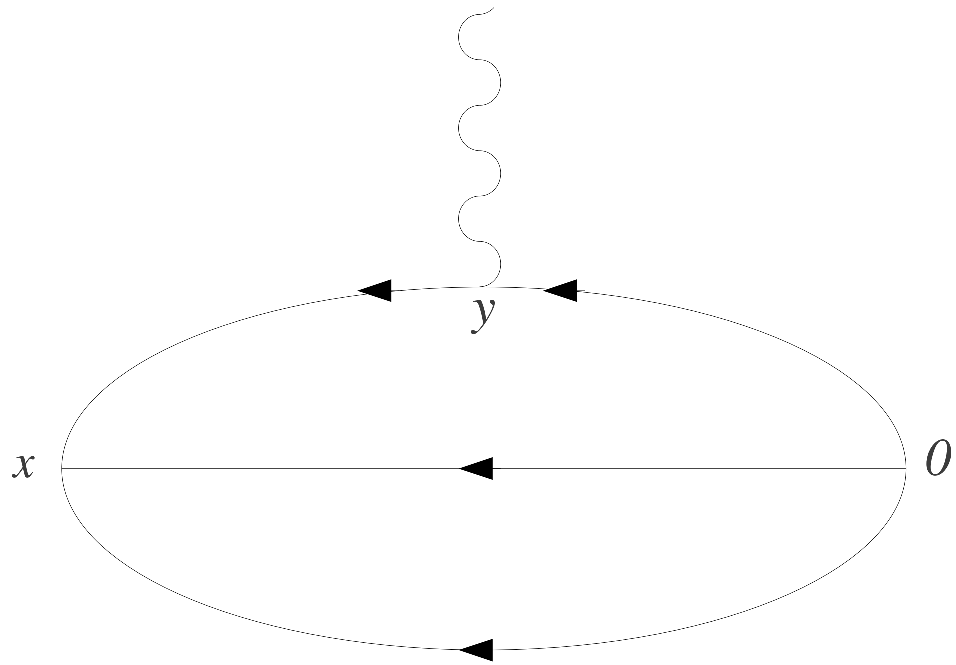
• examine the effects of *SU(3)* flavour symmetry breaking

• gain understanding of the internal hadron structure in general

Three Point Correlation Functions

$$\langle T(\chi(\vec{x},t_x)O(y,t_y)\bar{\chi}(0))\rangle$$

$$C_3(t_x,t_y)=\sum_{s,s'} e^{-E_{p'}(t_x-t_y)}e^{-E_p t_y}\langle\Omega|\chi|p's'\rangle\langle p's'|O|ps\rangle\langle ps|\bar{\chi}|\Omega\rangle$$



$$C_3(t_x,t_y)=\sum_{s,s'} e^{-E_{p'}(t_x-t_y)} e^{-E_p t_y} \langle \Omega|\chi|p's'\rangle \langle p's'|O|ps\rangle \langle ps|\bar{\chi}|\Omega\rangle$$

For transitions of the form $H \longrightarrow H'l\nu$ vector and axial-vector transitions are governed by three form factors each:

$$O_\mu^V(q)=f_1(q^2)\gamma_\mu+f_2(q^2)\sigma_{\mu\nu}\frac{q_\nu}{M_b+M_B}+f_3(q^2)i\frac{q_\mu}{M_b+M_B}$$

$$O_\mu^A(q)=g_1(q^2)\gamma_\mu\gamma_5+g_2(q^2)\sigma_{\mu\nu}\frac{q_\nu}{M_b+M_B}\gamma_5+g_3(q^2)i\frac{q_\mu}{M_b+M_B}\gamma_5$$

where

$$\langle H'(p',s')|V_\mu(x)+A_\mu(x)|H(p,s)\rangle=\bar{u}_b(p',s')(O_\mu^V(q)+O_\mu^A(q))u_B(p,s)$$

$$C_3(t_x, t_y) = \sum_{s,s'} e^{-E_{p'}(t_x - t_y)} e^{-E_p t_y} \langle \Omega | \chi | p' s' \rangle \langle p' s' | O | p s \rangle \langle p s | \bar{\chi} | \Omega \rangle$$

Wish to extract matrix elements. It is first worthwhile to define some lattice related form factors which can then be used to determine form factors of interest:

$$f_0(q^2) = f_1(q^2) + \frac{q^2}{M_H^2 + M_{H'}^2} f_3(q^2)$$

Note $f_0(0) = f_1(0)$, but generally $f_0(q^2_{max}) \sim f(0)$ as mass splitting is not too large for hyperons.

Also, can define

$$\tilde{g}_1(q^2) = g_1(q^2) - \frac{M_H - M_{H'}}{M_H + M_{H'}} g_2(q^2)$$

Ratios

$$C_3(t_x, t_y) = \sum_{s,s'} e^{-E_{p'}(t_x - t_y)} e^{-E_p t_y} \langle \Omega | \chi | p' s' \rangle \langle p' s' | O | p s \rangle \langle p s | \bar{\chi} | \Omega \rangle$$

Wish to extract matrix elements. Can take various ratios of 2-*pt* and 3-*pt* correlation functions to extract matrix elements of interest.

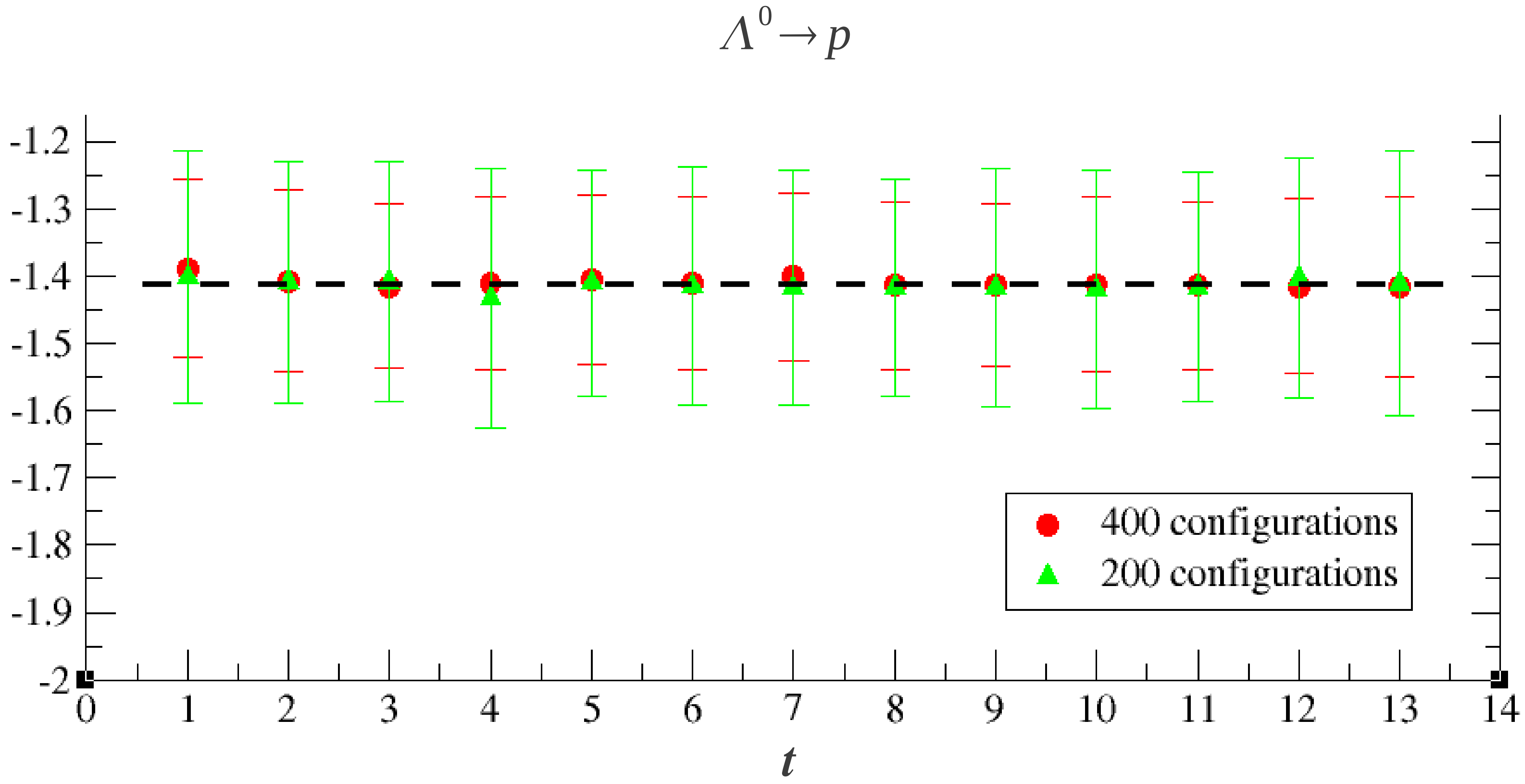
$$R(t; p, p') = \frac{C_3^{H \to H'}(t, t_{sink}; p, p')}{C_2^{H'}(t_{sink}, p')} \sqrt{\frac{C_2^H(t_{sink} - t, p) C_2^{H'}(t, p') C_2^{H'}(t_{sink}, p')}{C_2^{H'}(t_{sink} - t, p') C_2^H(t, p) C_2^H(t_{sink}, p)}}$$

$$\rightarrow \quad \tilde{g}_1(q^2_{max}) \qquad \text{OR} \qquad f_0(q^2_{max})$$

depending on choice of operator. Note renormalisation factors Z_A or Z_V also required here.

$$q^2_{max} = (M_H - M_{H'})^2$$

$f_0(q^2_{max})$ Ratio Example



Tuning the Quark Masses

Lattice $N_f = 2 + 1$ calculations typically involve extrapolating first $m_s \rightarrow m_s^*$ and then $m_l \rightarrow m_l^*$

1102.5300 [hep-lat]

Alternative approach is to keep singlet quark mass constant, *i.e.*

$$\bar{m} = \frac{1}{3}(m_u + m_d + m_s) = \frac{1}{3}(2\mathrm{m}_l + m_s) = constant$$

Advantages

- Physical masses (e.g. Kaon) are approached from below
- A wide range of masses for both flavours can be explored
- Flavour singlet quantities are flat at the symmetric point which is useful for scaling
- Can work in "inverse world" ($m_l > m_s$)

To explore the full effects of *SU(3)* flavour symmetry breaking it makes sense to start at the flavour symmetric point ($m_l = m_s$) and change values such that plots "fan-out"

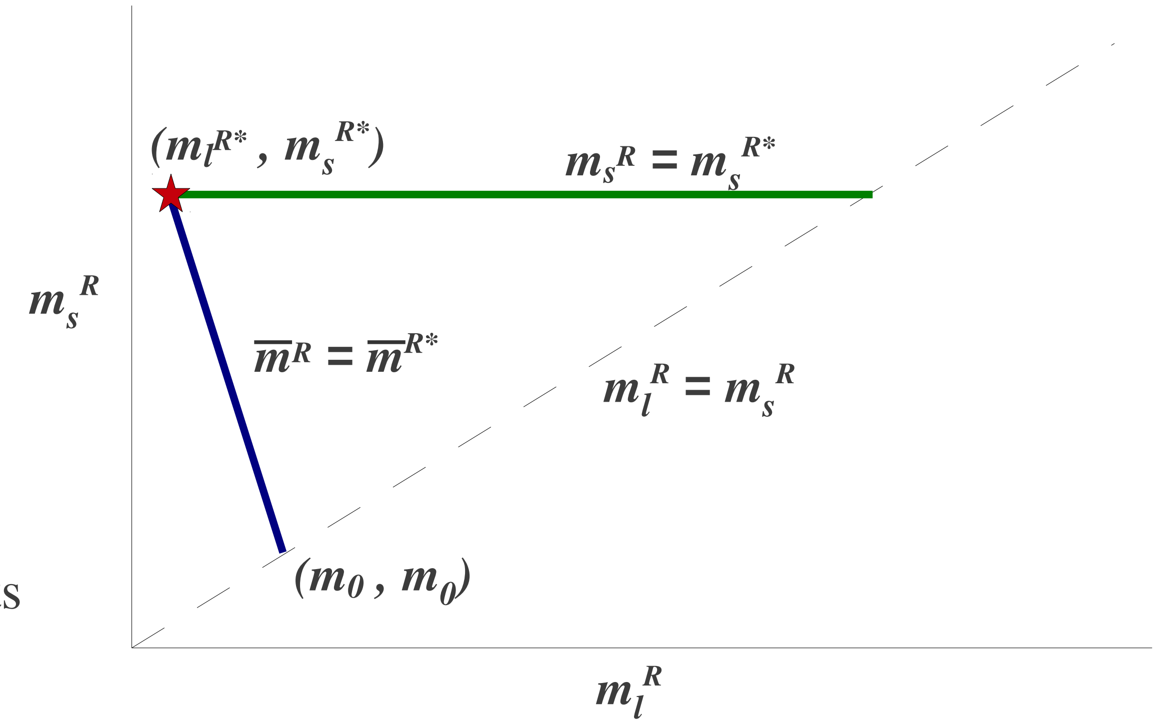
Flavour Symmetric Line

Start from some point on the flavour symmetric line; $(m_l\ ,\ m_s) = (m_0\ ,\ m_0)$

Decide point on symmetric line by fixing to some dimensionless flavour singlet quantity

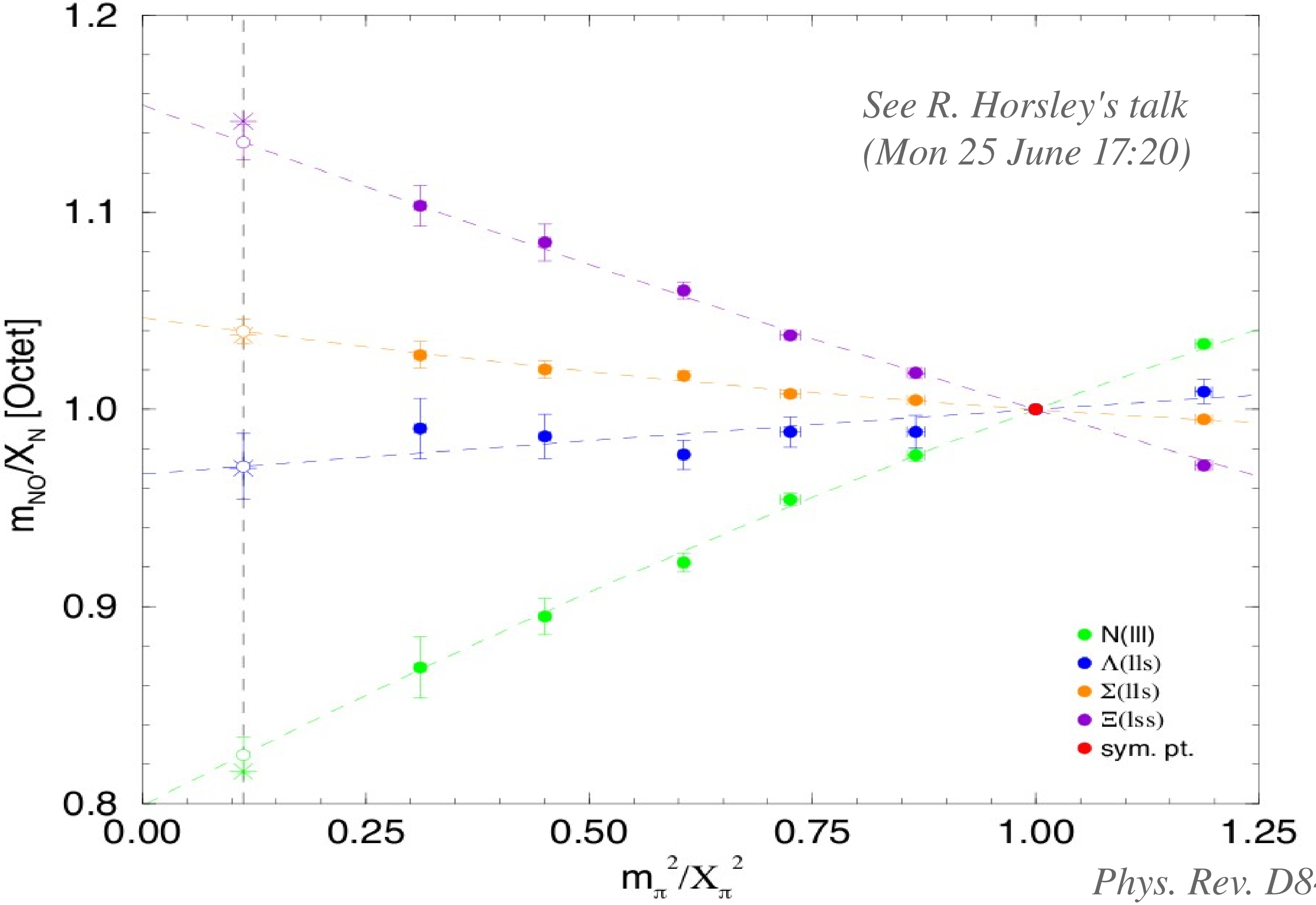
Keep sum of quark masses fixed as physical point is approached

$$\bar{m} = \frac{1}{3}(m_u + m_d + m_s)$$
$$= \frac{1}{3}(2m_l + m_s)$$
$$= constant$$



Using group theoretic arguments fits can be highly constrained...

Mass Fan Plot



Phys. Rev. D84 054509

Simulation Details

$N_f = 2+1$ flavours of dynamical non-perturbatively $O(a)$ improved clover fermions and tree-level Symanzik improved gluon action

Phys.Rev.D79 094507

At present 3-*pt* functions only calculated on $24^3 \times 48$ ensembles

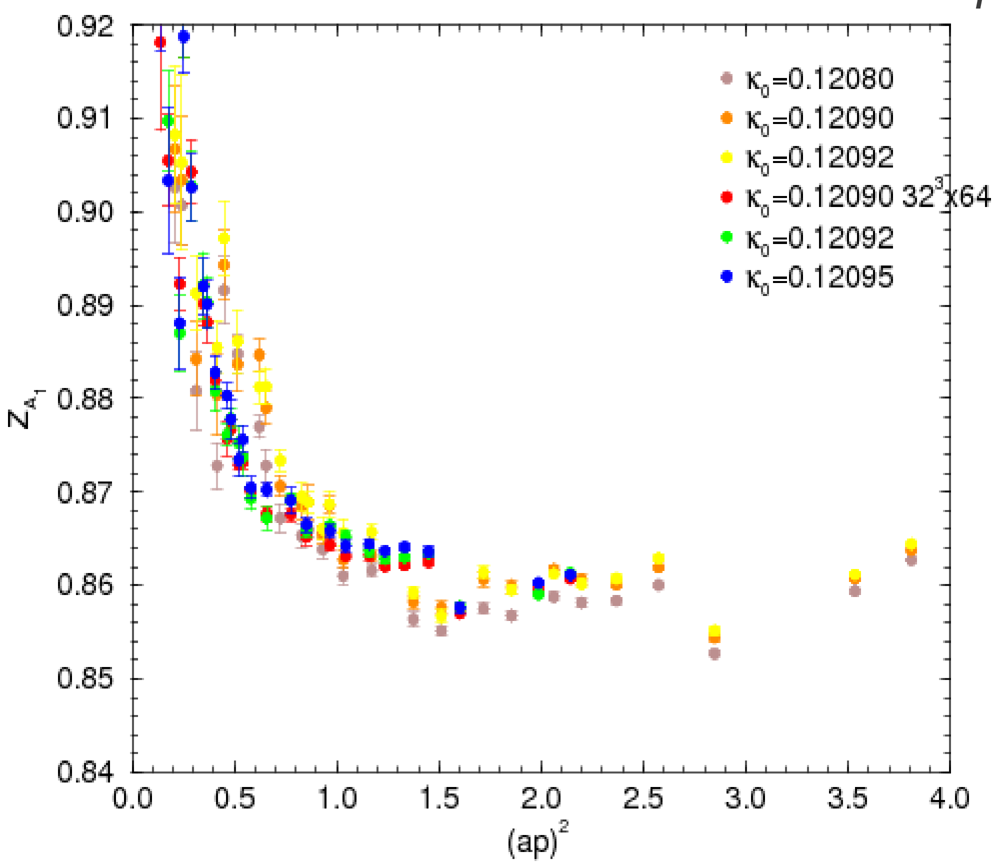
κ_l	κ_s	Lattice Volume	β	M_π[MeV]	M_K[MeV]	Number of confs
0.12083	0.12104	$24^3 \times 48$	5.50	462	402	~ 400-500
0.12090	0.12090	$24^3 \times 48$	5.50	425	425	~ 400-500
0.12095	0.12080	$24^3 \times 48$	5.50	394	437	~ 400-500
0.12100	0.12070	$24^3 \times 48$	5.50	358	453	~ 400-500
0.12104	0.12062	$24^3 \times 48$	5.50	337	460	~ 400-500

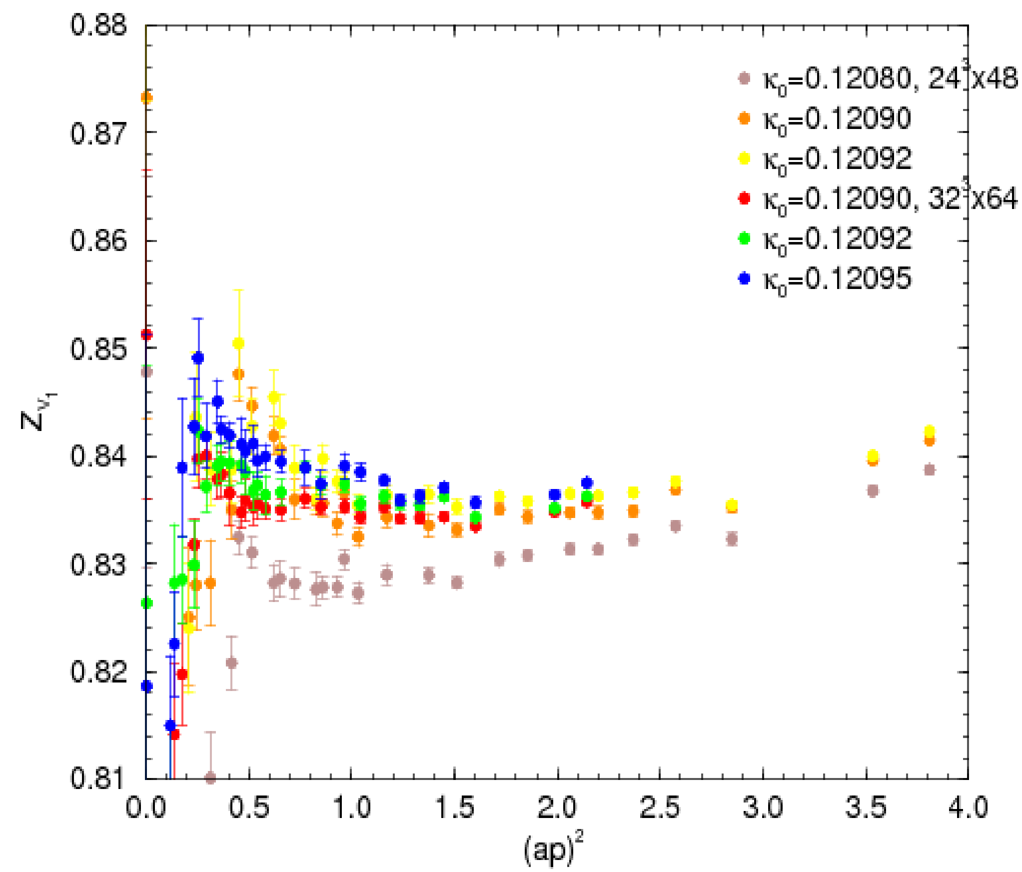
a = 0.078 fm

Z_A & Z_V

Using renormalisation constants determined by RI-MOM. Still some work to do on this but preliminary results are Z_A ~ 0.865 and Z_V ~ 0.842. Can also consider ratios where such constants cancel...

β=5.50





Flavour Symmetry Breaking (Recap I) (13/22)

Quick review of key points from previous talk...

See P. Rakow's talk (Previous to this talk)

Can express amplitudes of octet transition matrix elements in terms of 10 independent parameters (f and d in the $SU(3)$ symmetric limit and 8 to first order in δm_l)

Consider the following diagonal matrix element

$$\langle \Xi^- | \eta | \Xi^- \rangle = \langle \Lambda^0 | \frac{1}{\sqrt{6}} (\bar{u} \Gamma u + \bar{d} \Gamma d - 2 \bar{s} \Gamma s) | \Xi^- \rangle$$

This can be expanded in terms of these parameters

$$\langle \Xi^- | \eta | \Xi^- \rangle = -\sqrt{3} f - d + (g_1 + h_2) \delta m_l$$

Can construct 2 sets of amplitudes with the same values at the symmetric point but "fan" out as the mass splitting increases

Flavour Symmetry Breaking (Recap II) (14/22)

"D-fan" $(\delta m_q = m_q - \bar{m})$

$$D_1 = -(\langle \bar{N}|\eta|N\rangle + \langle \bar{\Xi}|\eta|\Xi\rangle) = 2\mathrm{d} - 2\mathrm{g}_1 \delta m_l$$

$$D_2 = \langle \bar{\Sigma}|\eta|\Sigma\rangle = 2\mathrm{d} + (g_1 + 2\sqrt{3} g_3)\delta m_l$$

$$D_3 = -\langle \bar{\Lambda}|\eta|\Lambda\rangle = 2\mathrm{d} - (g_1 + 2 g_2)\delta m_l$$

$$D_4 = \frac{1}{\sqrt{3}}(\langle \bar{N}|\pi|N\rangle + \langle \bar{\Xi}|\pi|\Xi\rangle) = 2\mathrm{d} - \frac{4}{\sqrt{3}} g_3 \delta m_l$$

$$D_5 = \Re\langle \bar{\Sigma}|\pi|\Lambda\rangle = 2\mathrm{d} + (g_2 - \sqrt{3} g_3)\delta m_l$$

$$D_6 = \frac{1}{\sqrt{6}}\Re(\langle \bar{N}|K|\Sigma\rangle + \langle \bar{\Sigma}|K|\Xi\rangle) = 2\mathrm{d} + \frac{2}{\sqrt{3}} g_3 \delta m_l$$

$$D_7 = -\Re(\langle \bar{N}|K|\Lambda\rangle + \langle \bar{\Lambda}|K|\Xi\rangle) = 2\mathrm{d} - 2 g_2 \delta m_l$$

Note: values are the same at symmetric point. Highly constrained; only 3 slope parameters.

Flavour Symmetry Breaking (Recap III) (15/22)

"F-fan" $(\delta m_q = m_q - \bar{m})$

$$F_1 = \frac{1}{\sqrt{3}}(\langle \bar{N}|\eta|N\rangle - \langle \bar{\Xi}|\eta|\Xi\rangle) = 2\mathrm{f} - \frac{2}{\sqrt{3}} h_2 \delta m_l$$

$$F_2 = \langle \bar{N}|\pi|N\rangle + \langle \bar{\Xi}|\pi|\Xi\rangle = 2\mathrm{f} + 4 h_1 \delta m_l$$

$$F_3 = \langle \bar{\Sigma}|\pi|\Sigma\rangle = 2\mathrm{f} + (-2\mathrm{h}_1 + \sqrt{3} h_2) \delta m_l$$

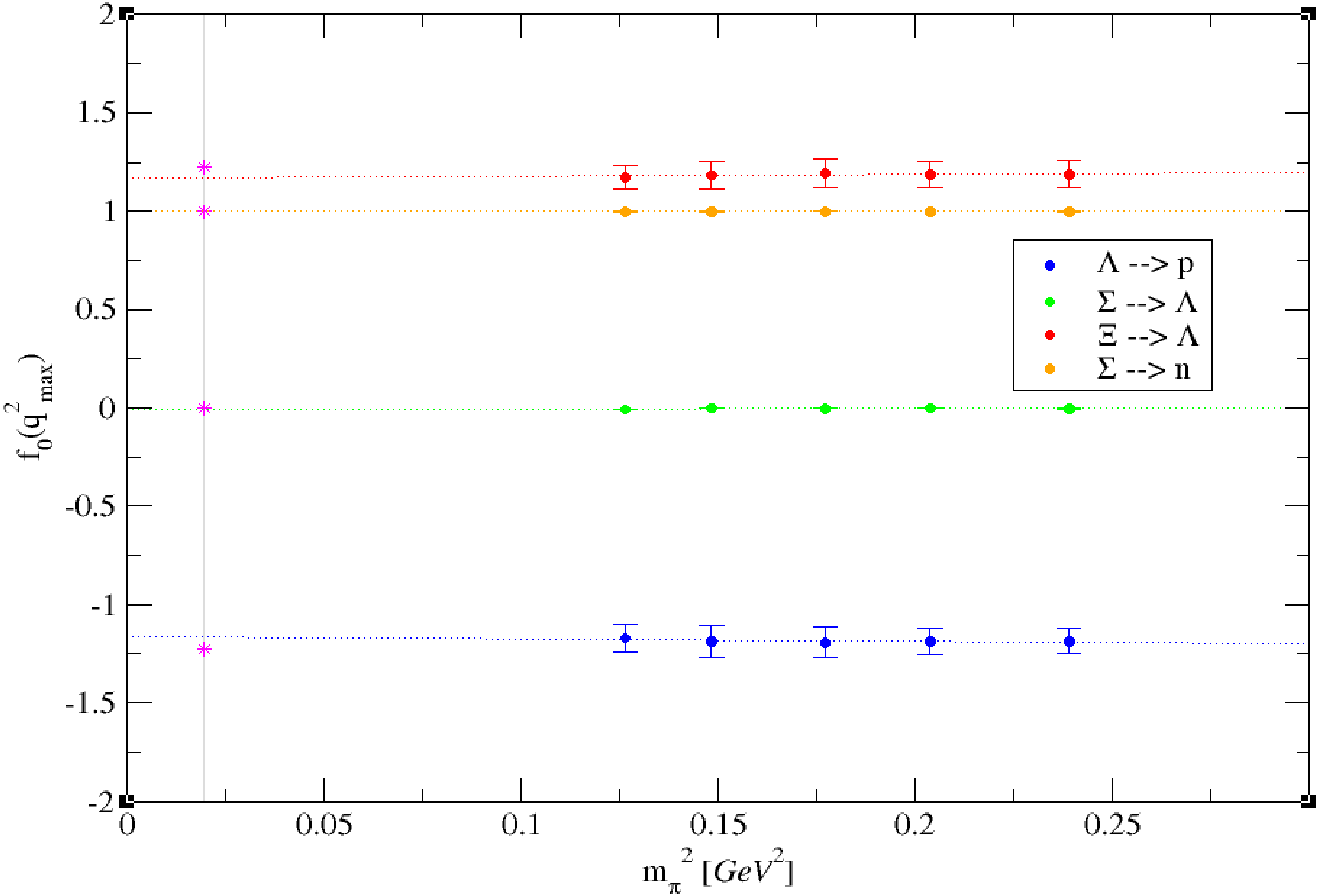
$$F_4 = \frac{1}{\sqrt{2}} \Re(\langle \bar{\Sigma}|K|\Xi\rangle - \langle \bar{N}|K|\Sigma\rangle) = 2\mathrm{f} - 2 h_1 \delta m_l$$

$$F_5 = \frac{1}{\sqrt{3}} \Re(\langle \bar{\Lambda}|K|\Xi\rangle - \langle \bar{N}|K|\Lambda\rangle) = 2\mathrm{f} + \frac{2}{\sqrt{3}}(\sqrt{3} h_1 - h_2) \delta m_l$$

Note: values are the same at symmetric point. Highly constrained; only 2 slope parameters.

Don't expect all such transitions to "fan" however. Ademolo-Gatto theorem protects f_0 from leading order SU(3) corrections

$f_0(q^2_{max})$



Λ --> p
Σ --> Λ
Ξ --> Λ
Σ --> n
$f_0(q^2_{max})$
m_π^2 [GeV^2]
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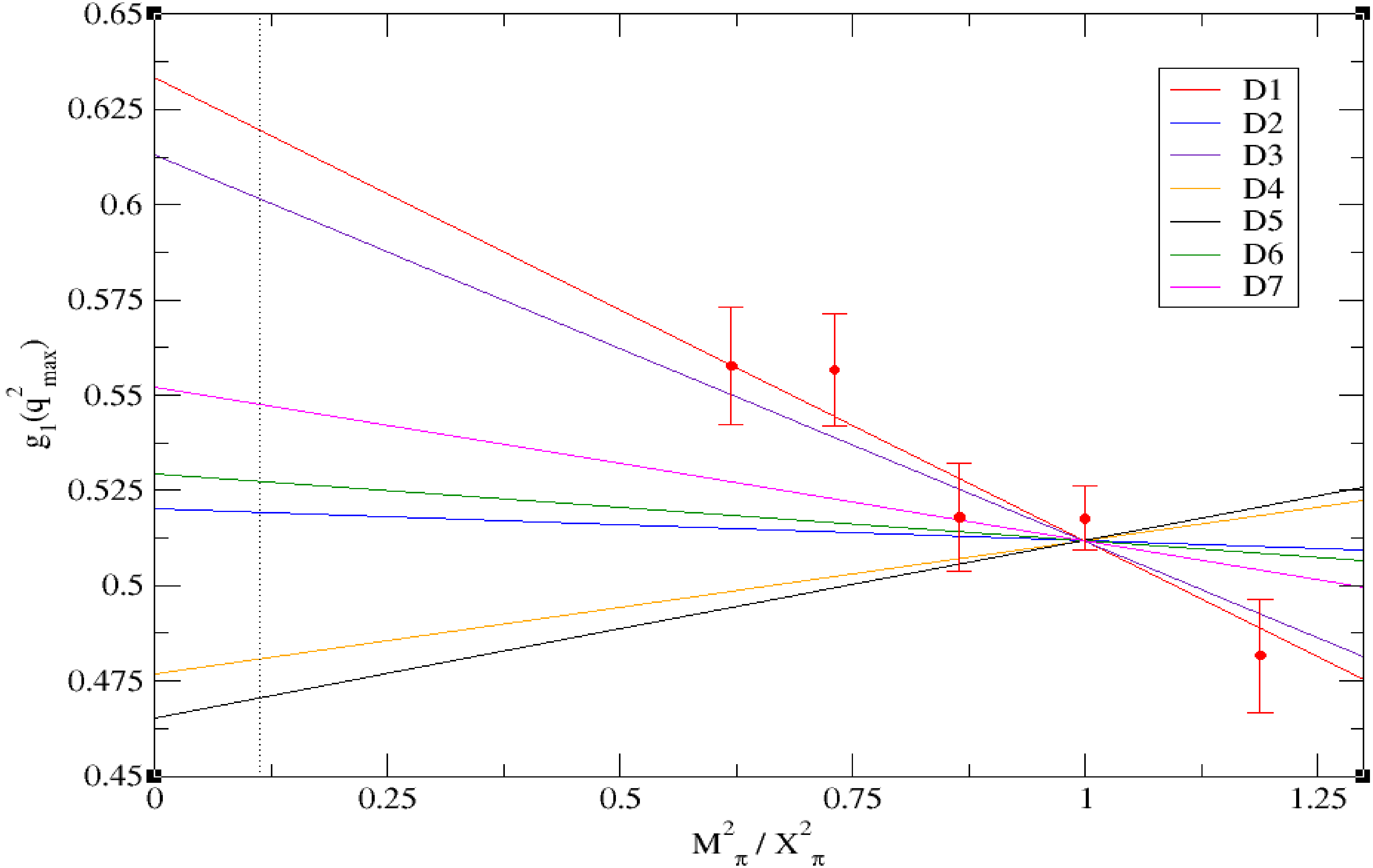
D-fan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$g_1(q^2_{max})$
M^2_π / X^2_π
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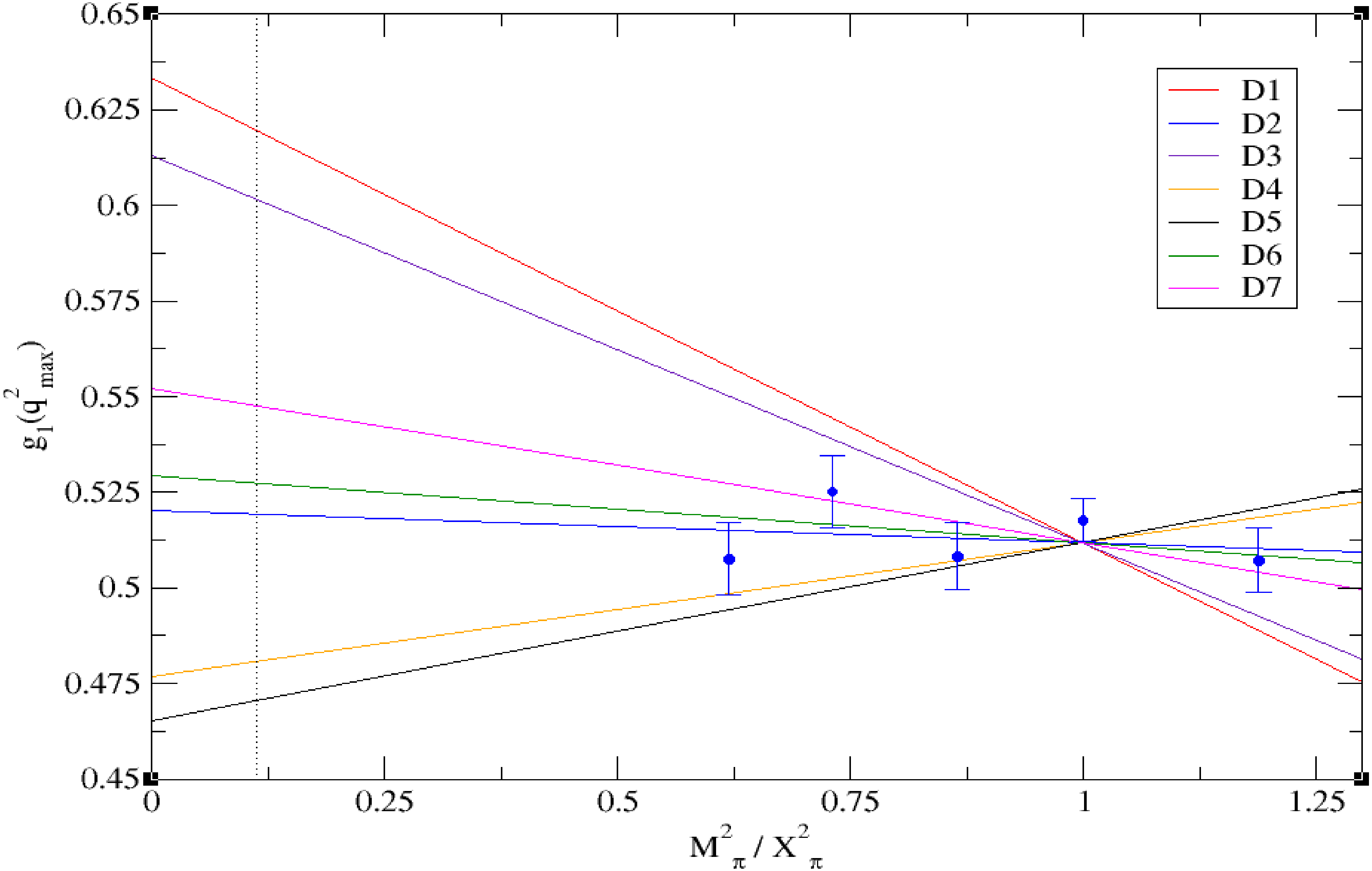
D-fan



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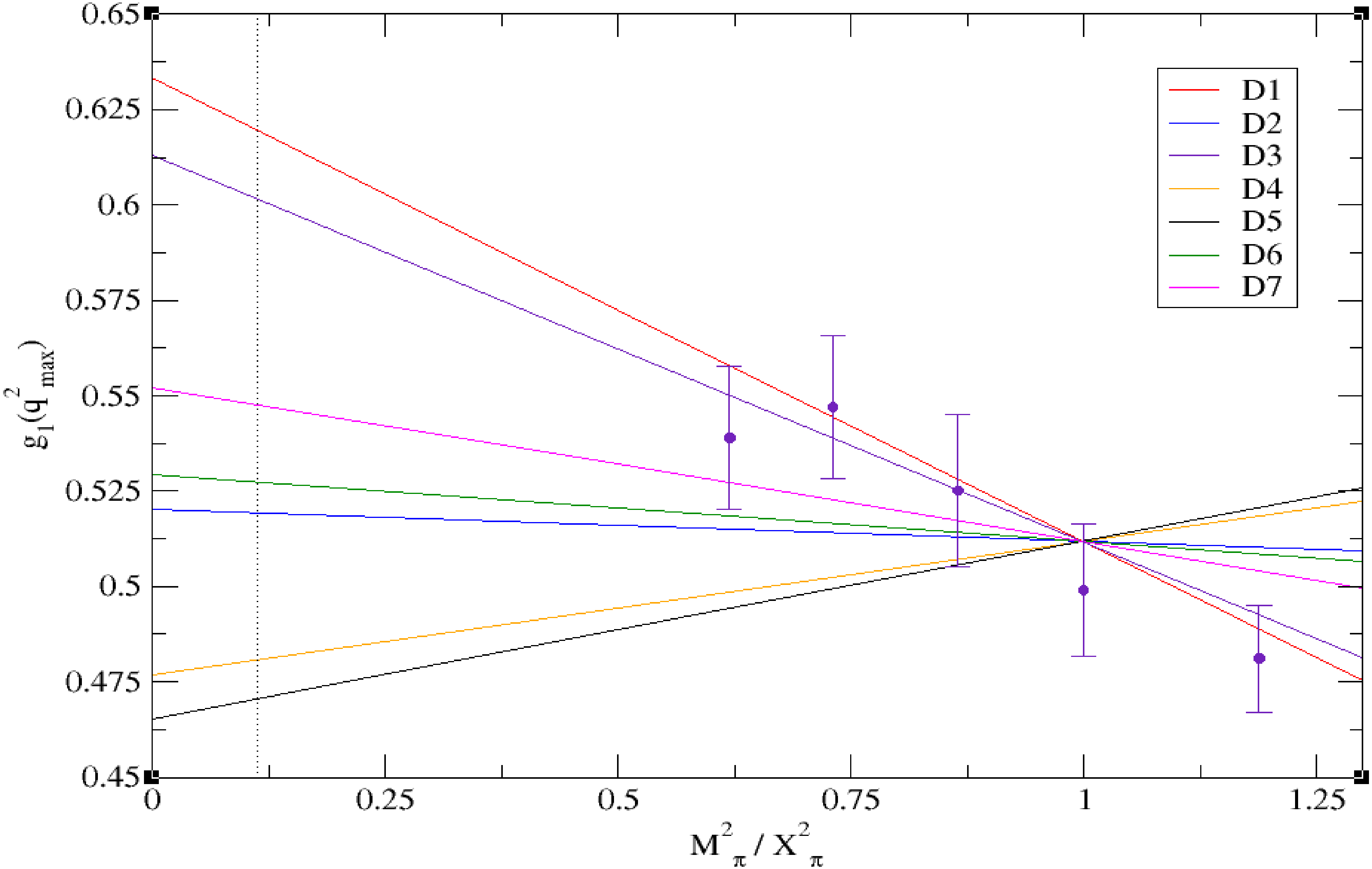
D-fan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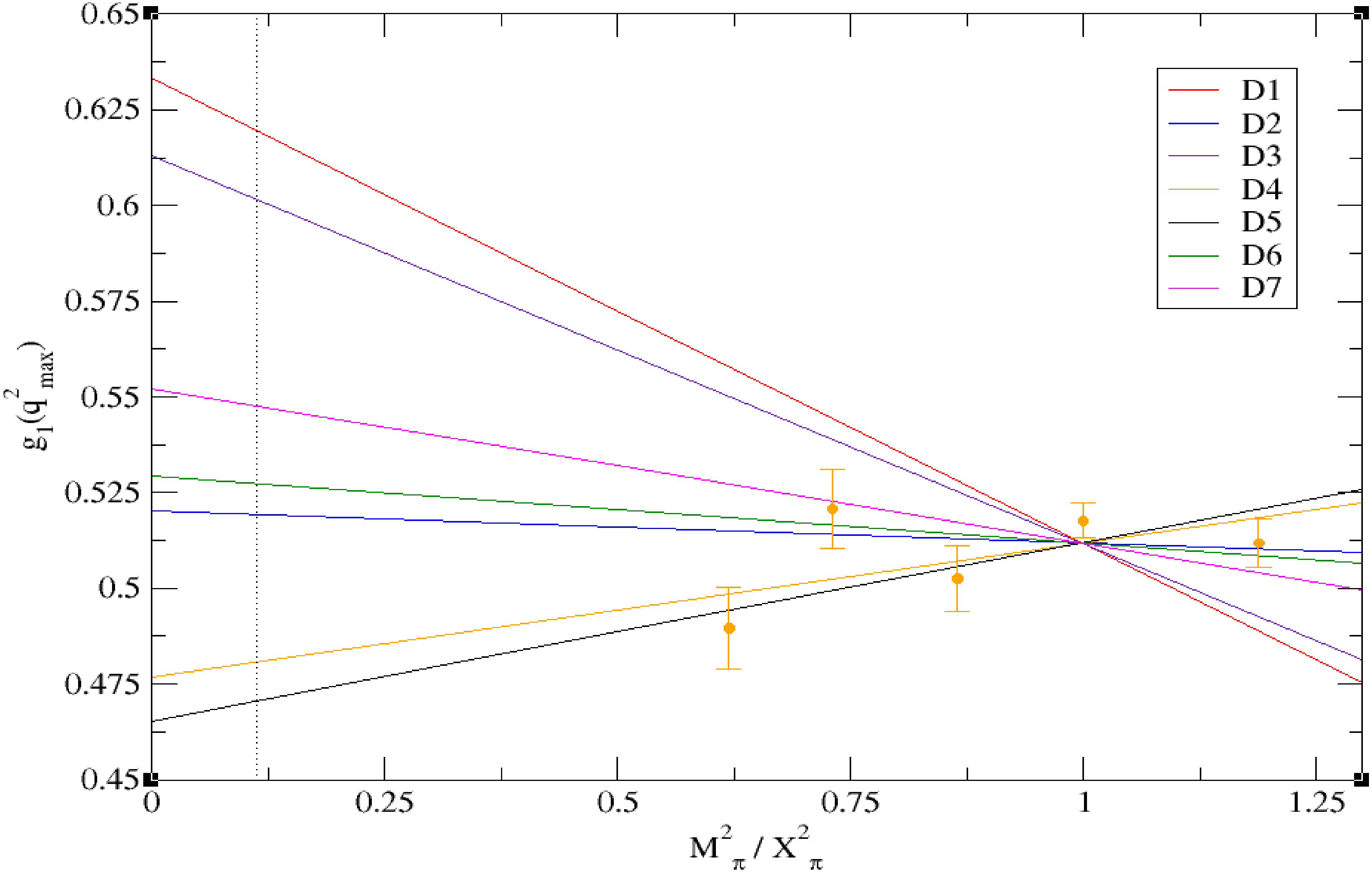

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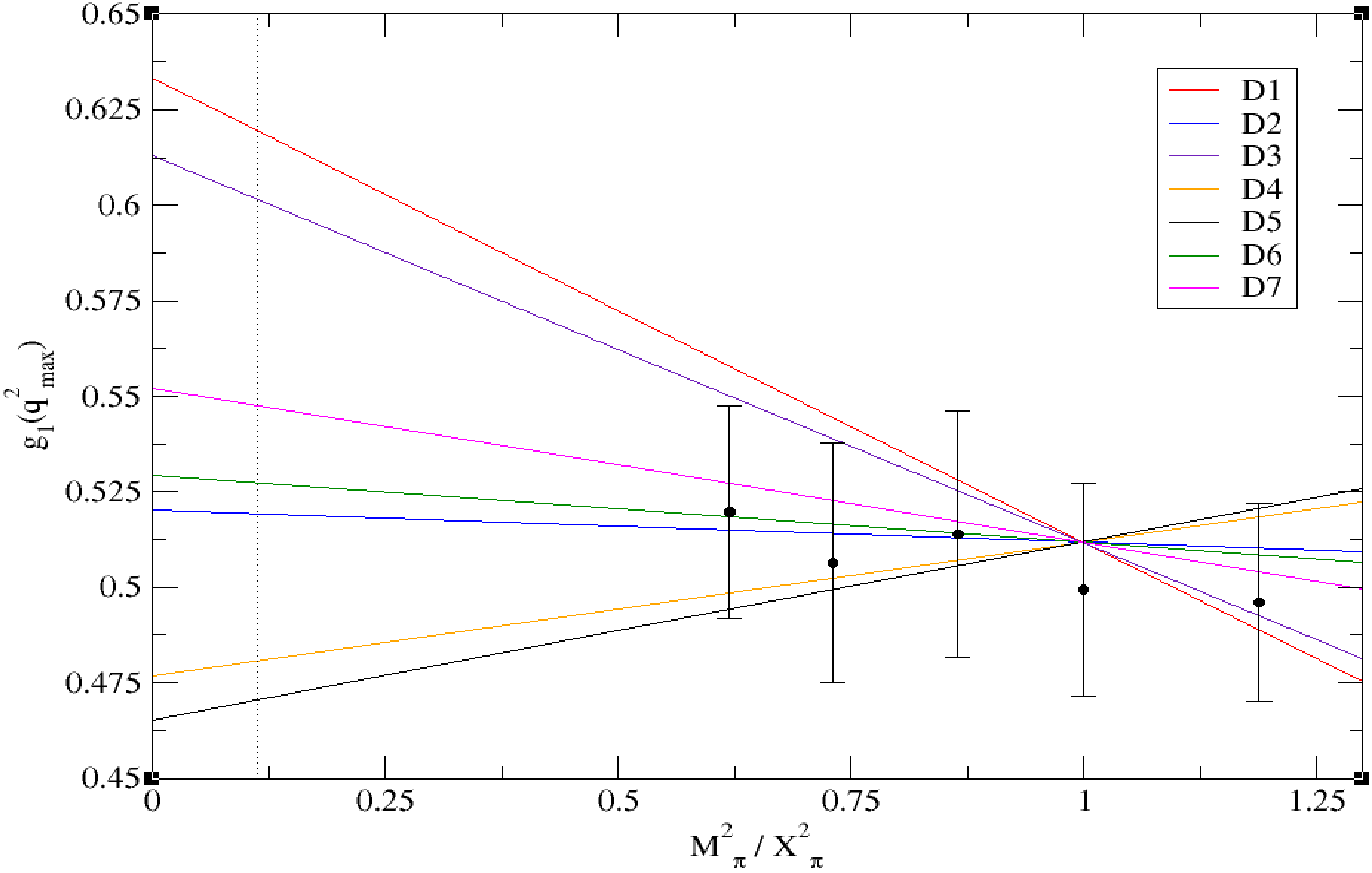
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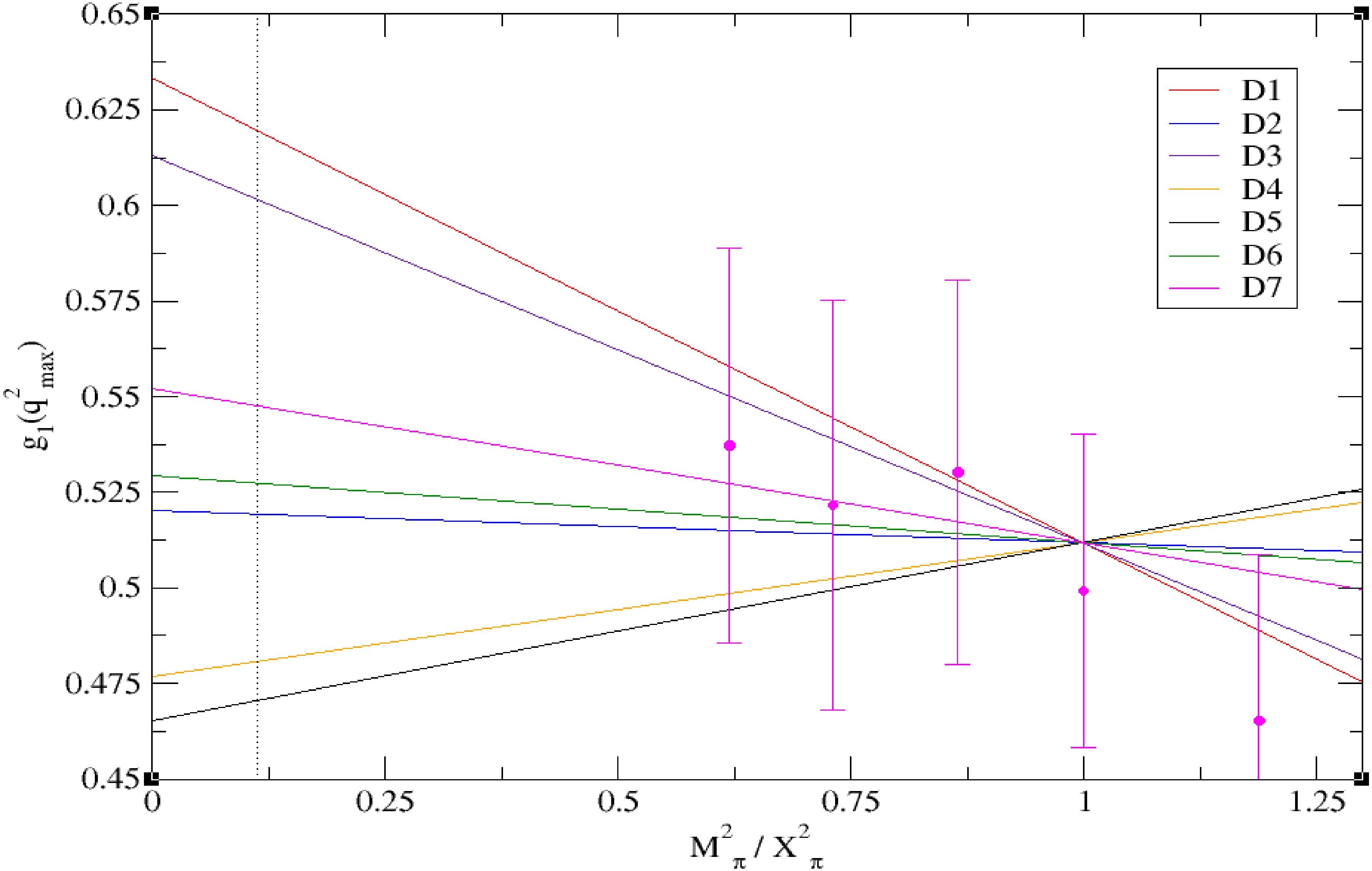
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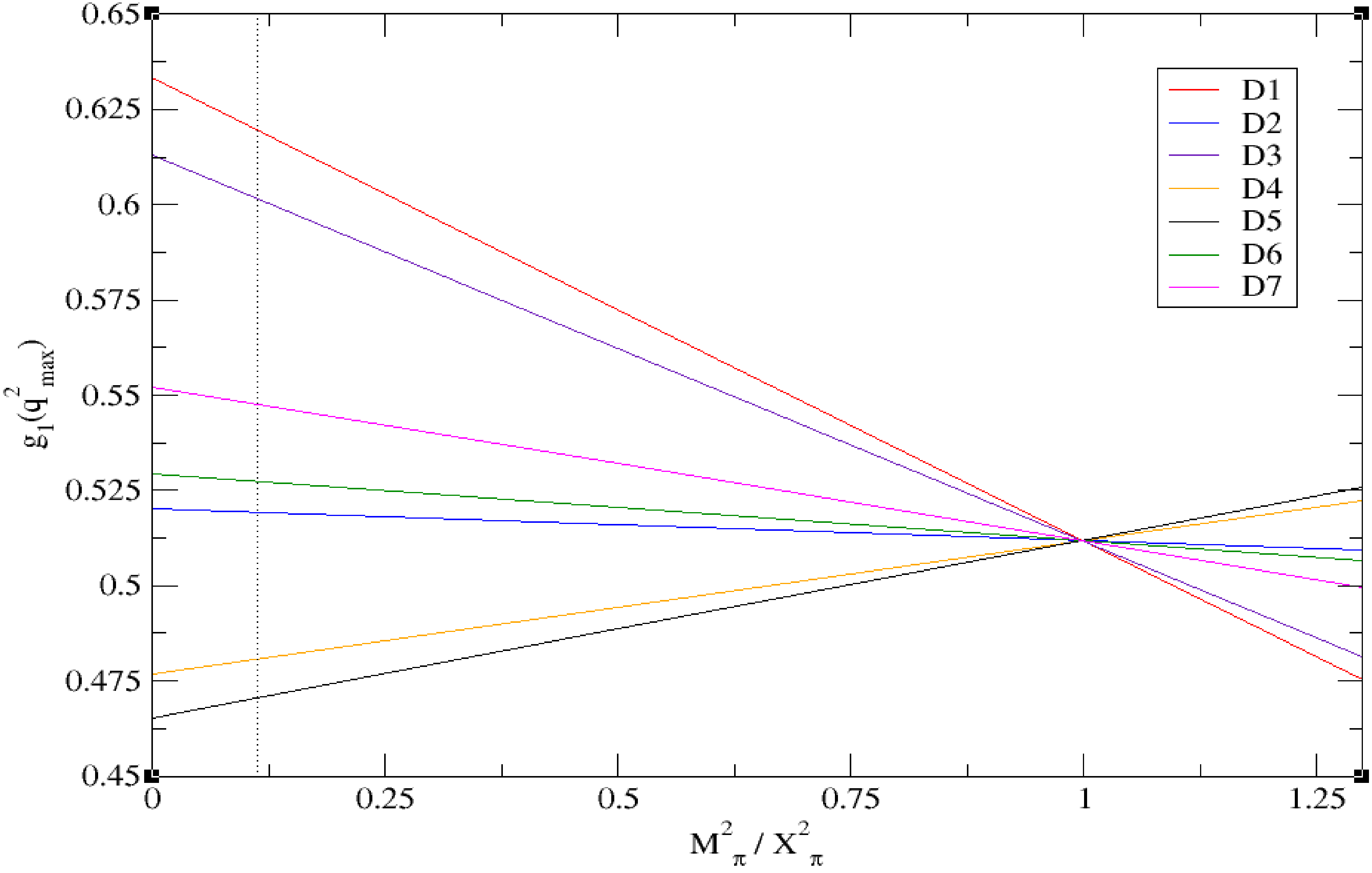

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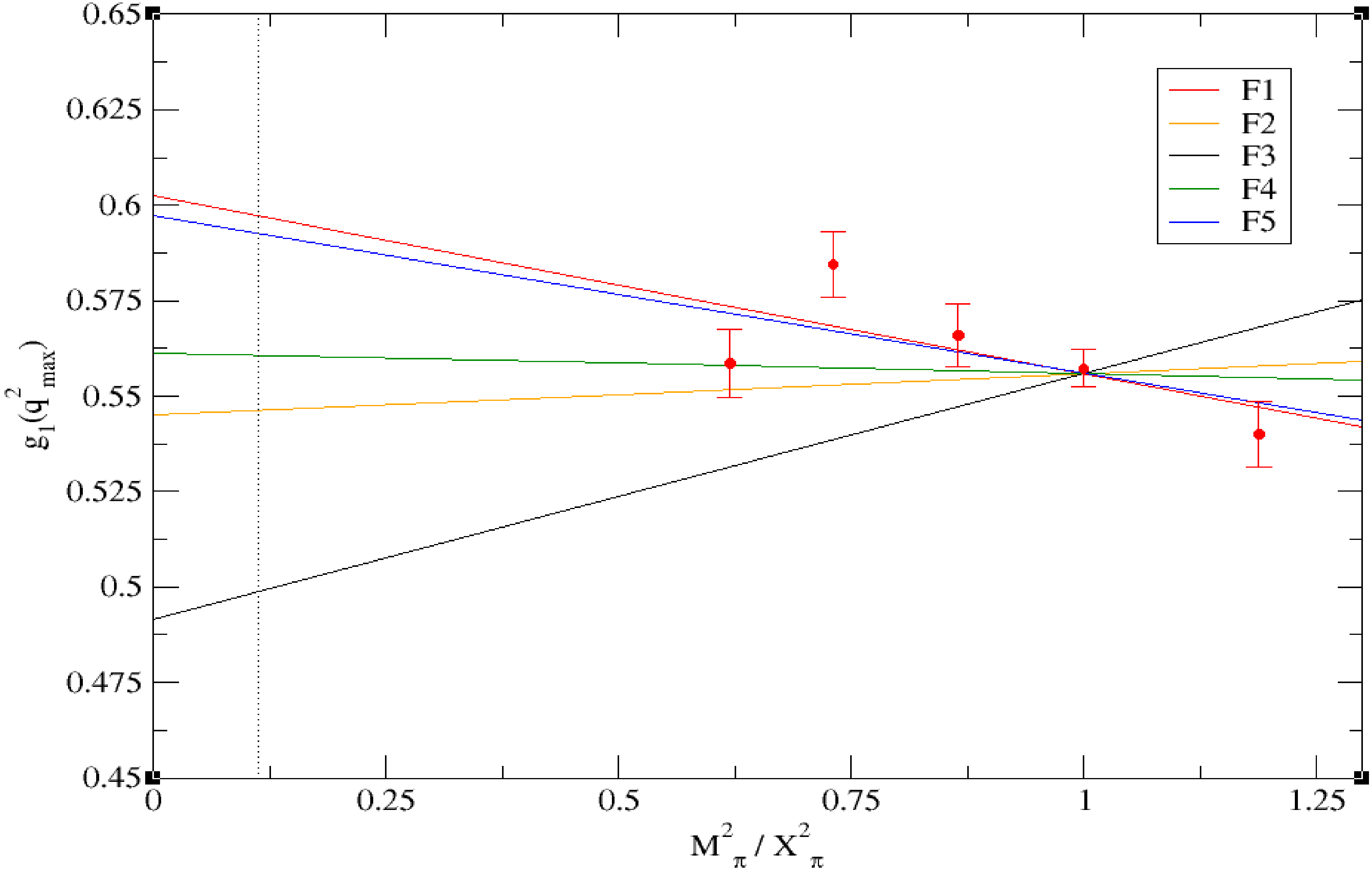
F-fan



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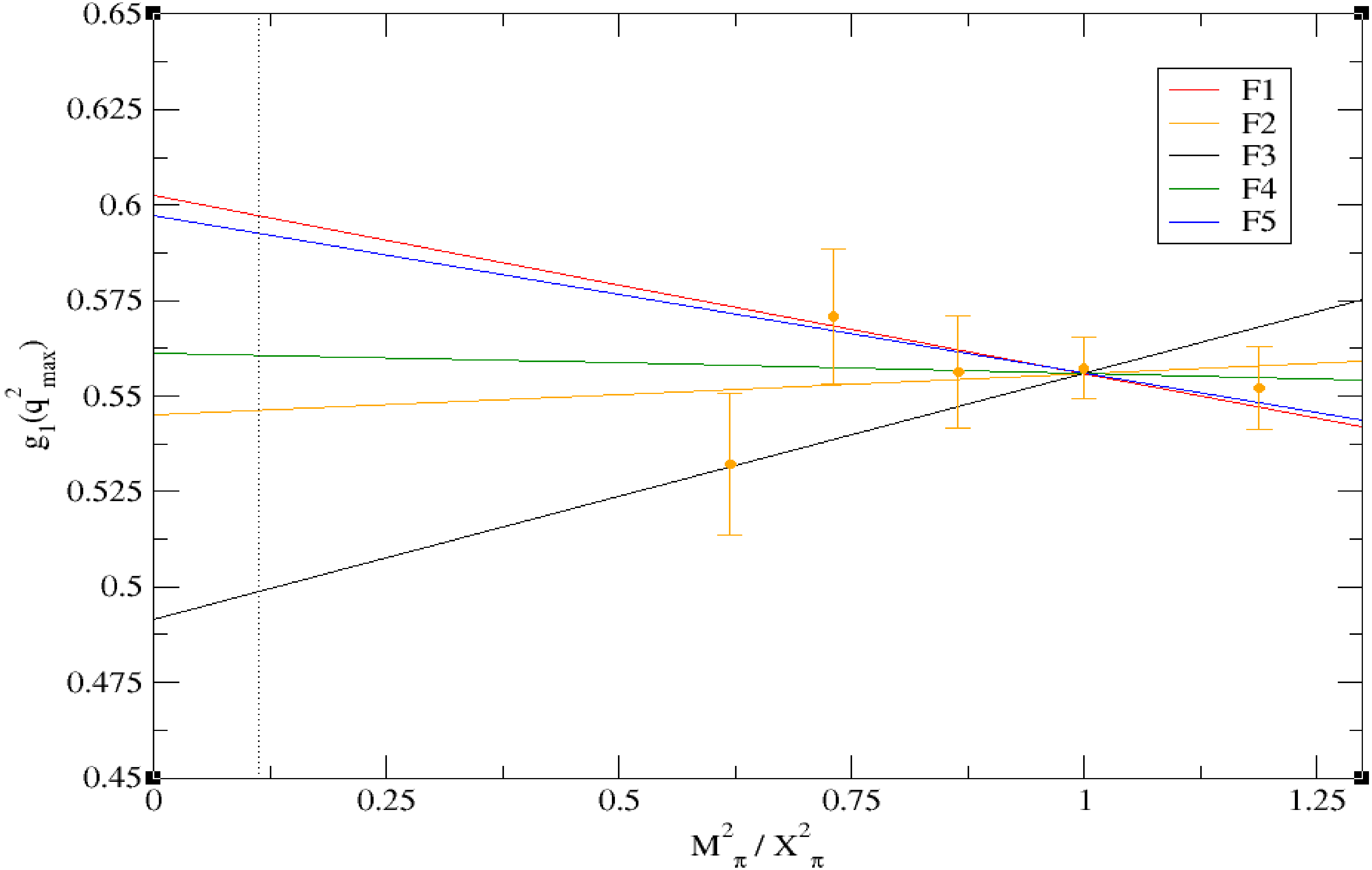

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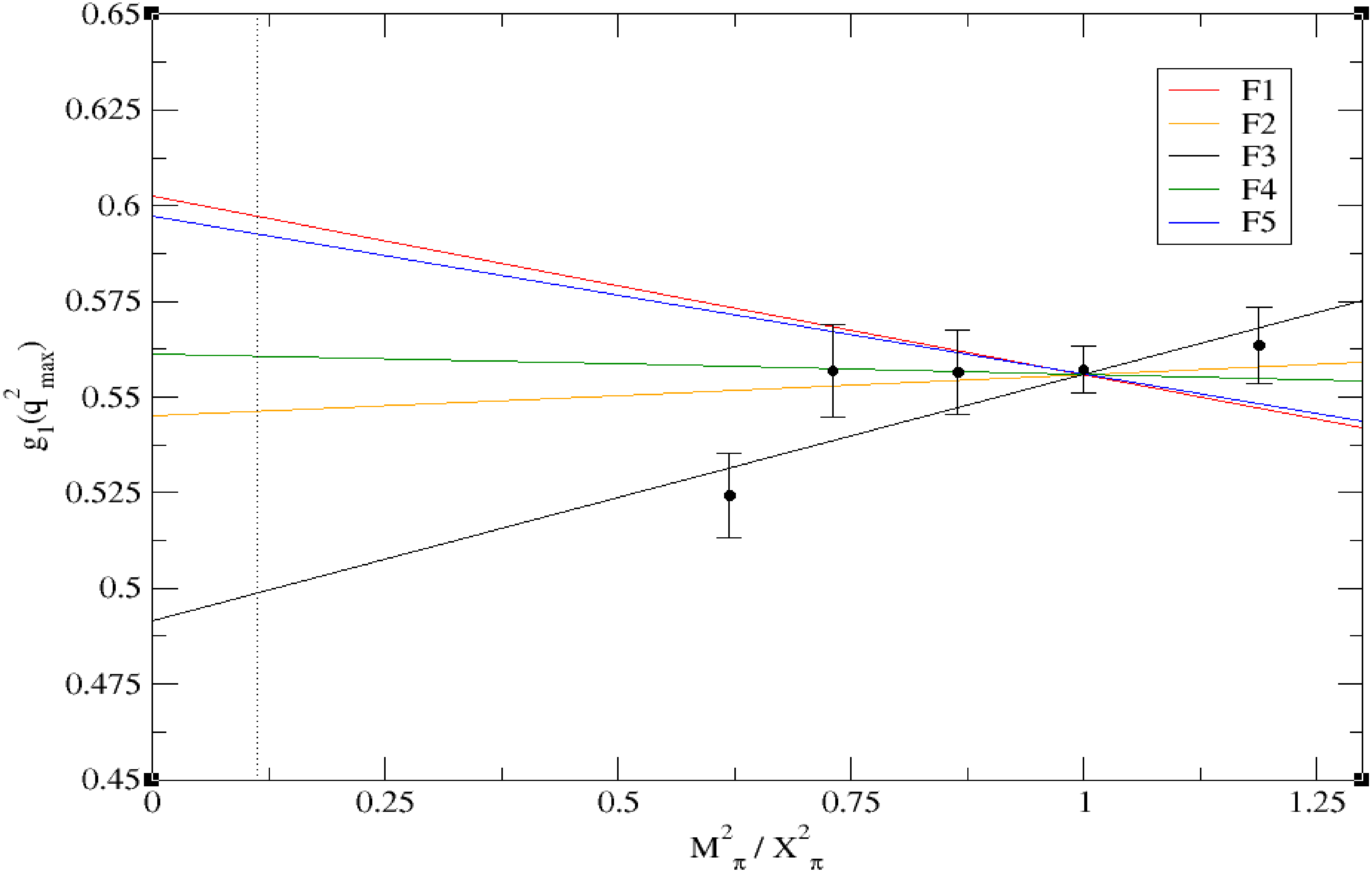

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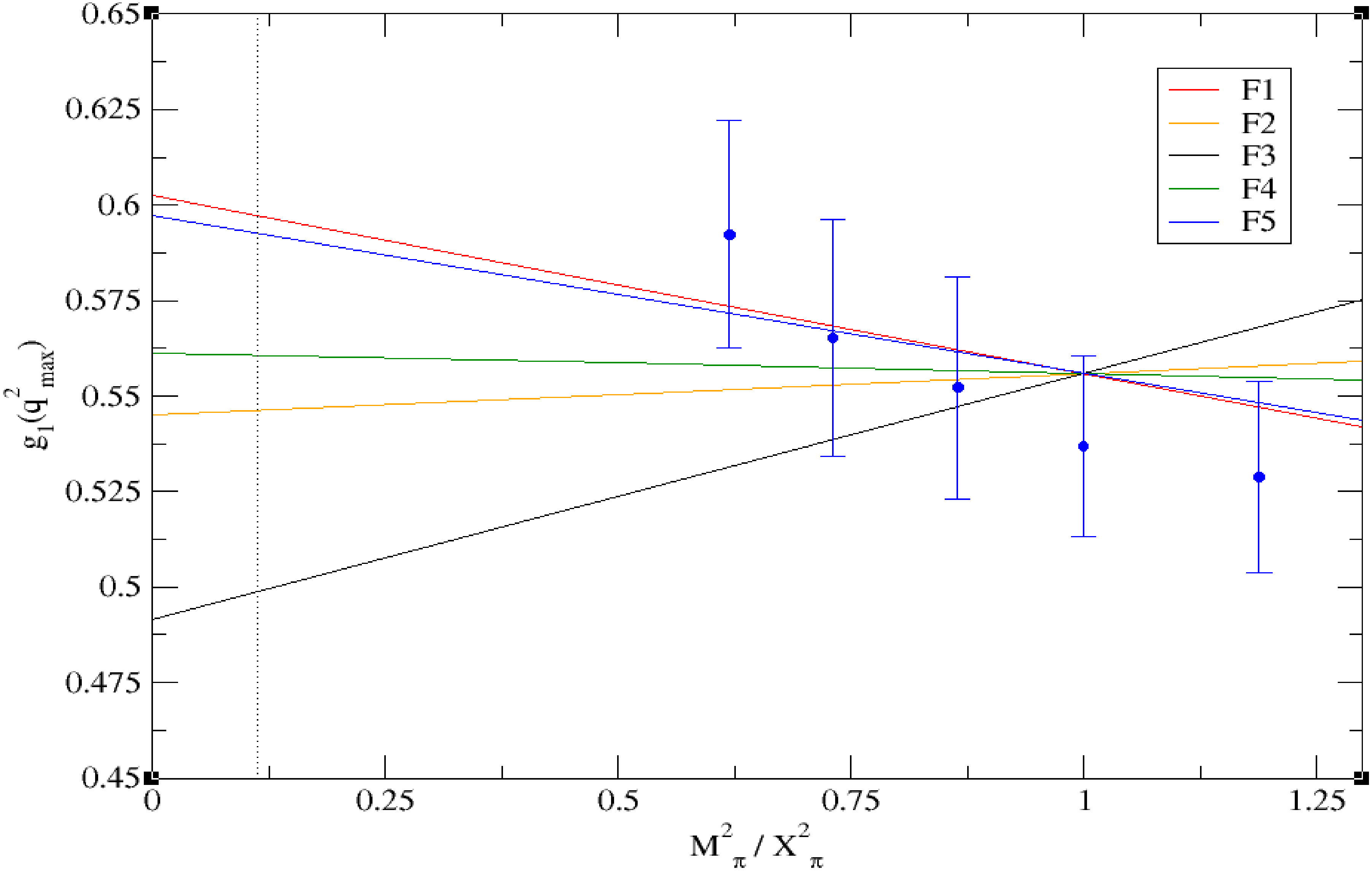

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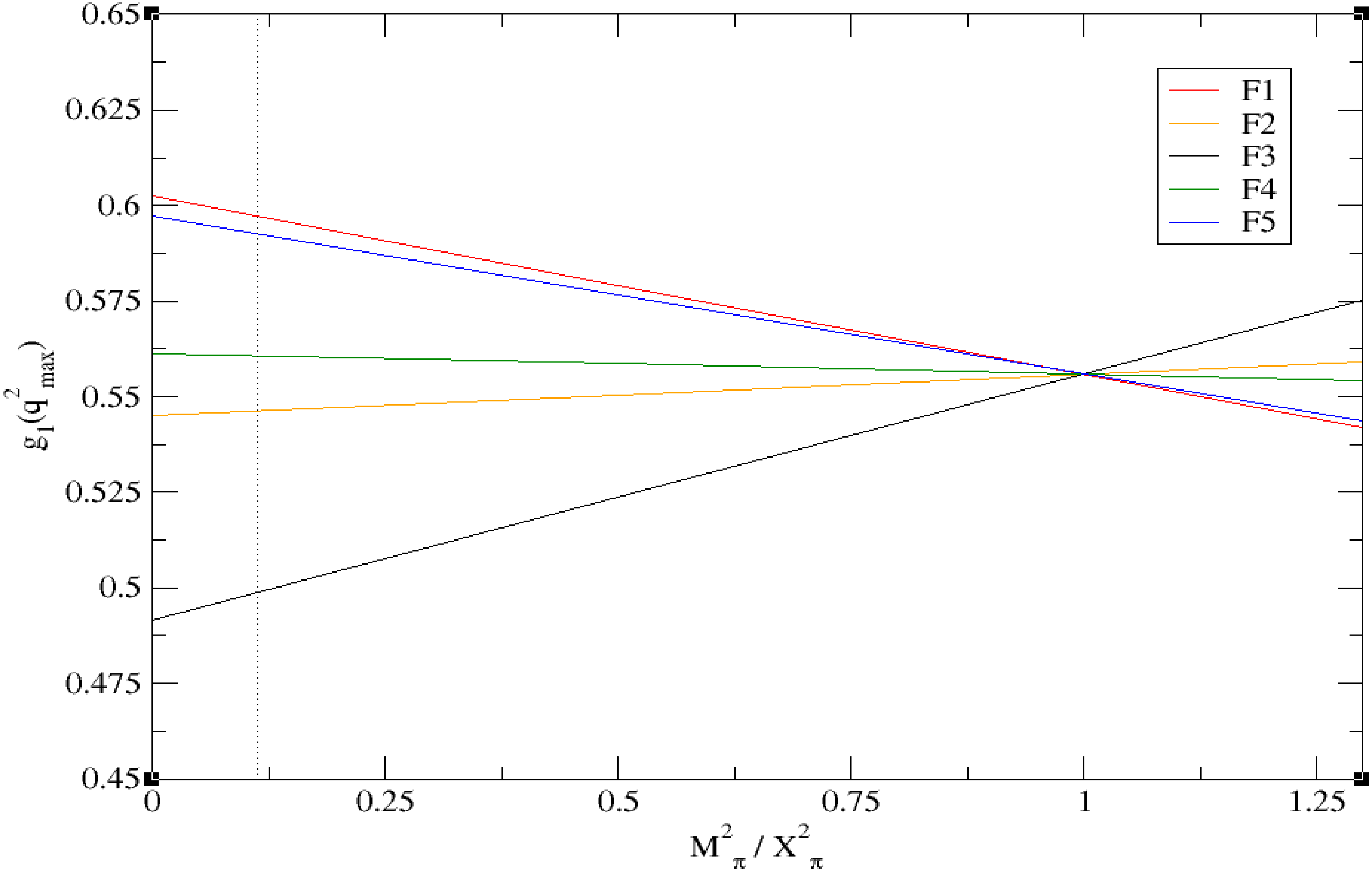
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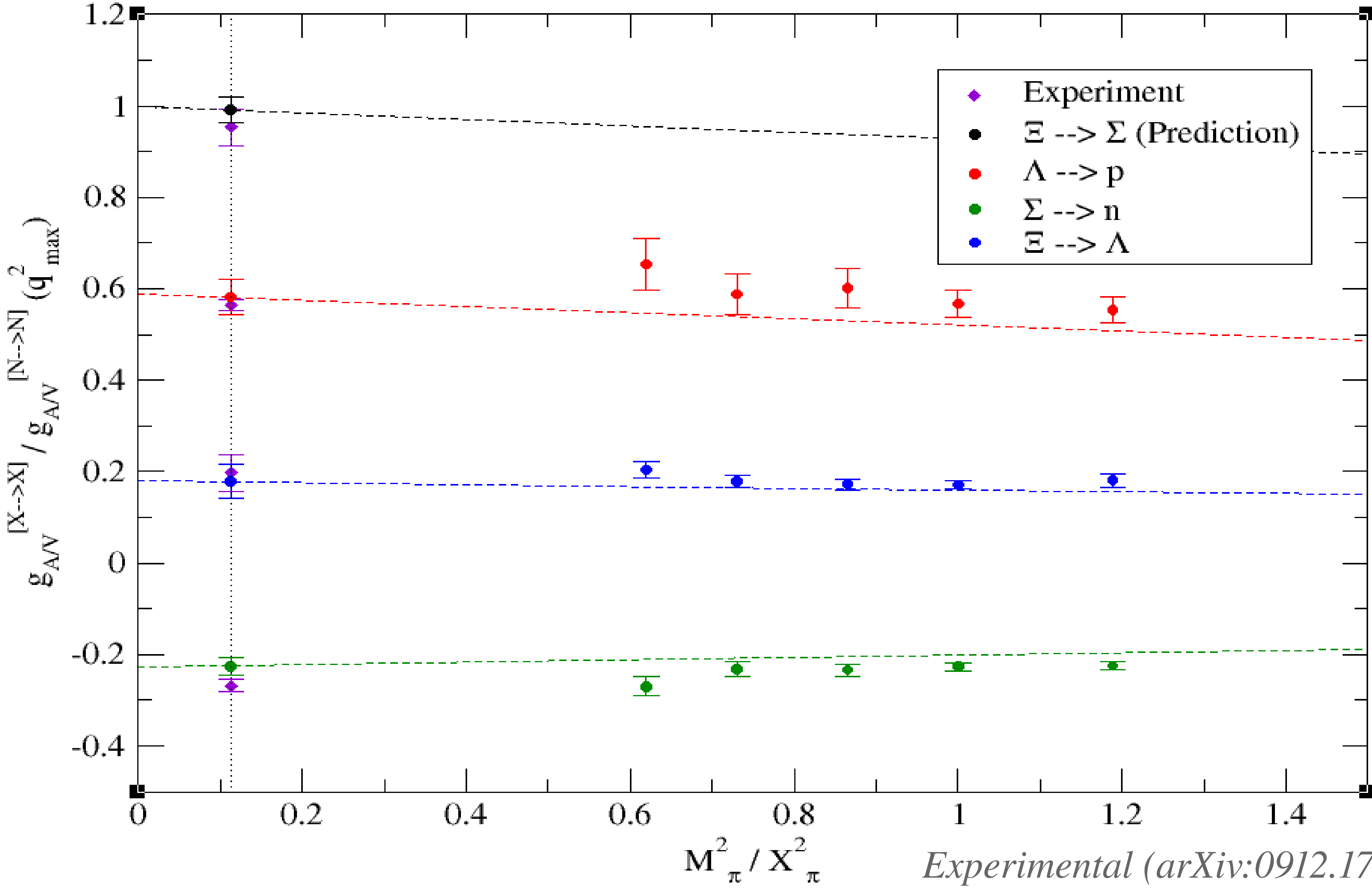
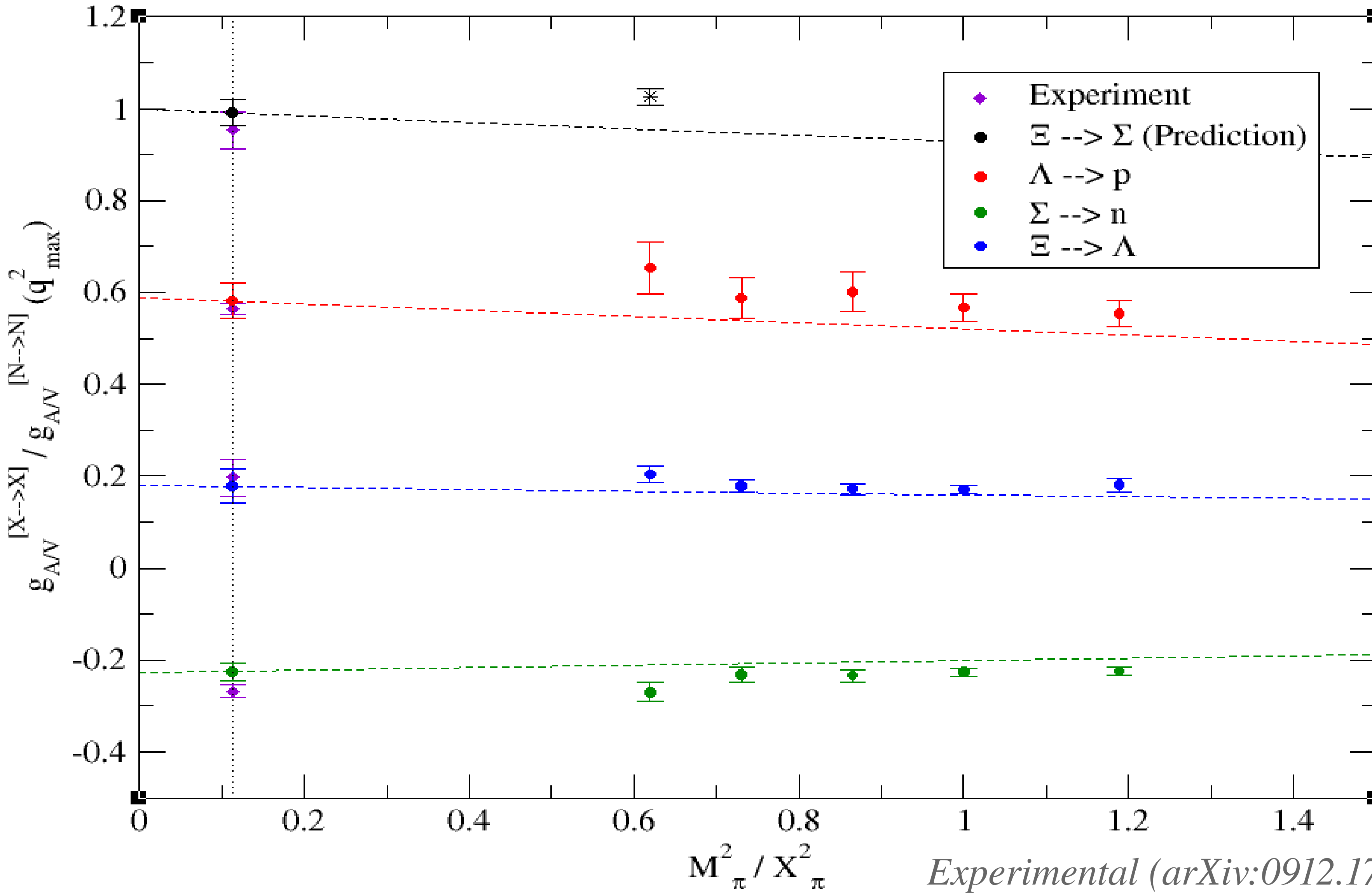


Experimental (arXiv:0912.1765 hep-ph)

Axial Charges



Experimental (arXiv:0912.1765 hep-ph)

Long term aim is for precise theoretical estimate of the CKM matrix element $|V_{us}|$ via hyperon semi-leptonic decays. Relate form factors and experimental decay rates

$$\Gamma = \frac{G_F^2}{60\pi^3}(M_B - M_{B'})^2(1-3\delta)\left|V_{us}\right|^2\left|f_1(0)\right|^2\left(1+3\left|\frac{g_1(0)}{f_1(0)}\right|^2+\ldots\right)$$

arXiv:hep-ph/0307298

PDG reports hyperon decay theoretical value of

$$\left|V_{us}\right| = 0.2250 \pm 0.0027$$

Focus on $|V_{us}|$ as this term dominates uncertainty in the CKM unitarity relation

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

which is a test of the Standard Model. Deviation from unity would indicate existence of new physics....

See also work by S. Sasaki & T. Yamazaki (arXiv:0811.1406)

Summary & Further Work

Summary

- Hyperon semileptonic decays useful as alternative determination of $|V_{us}|$
- Consider various ratios of correlation functions to determine form factors
- Incorporate methods which highly restrict lattice extrapolation fits

Future Work

- Run simulations with higher number of configurations, larger lattices and lighter masses, etc.
- Improve renormalisation
- Evaluate form factors at zero momentum transfer in order to extract all necessary values to determine $|V_{us}|$
- Include twisted Boundary Conditions to remove interpolating error *arXiv:1004.0886 hep-lat*

Acknowledgements

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