Magnetic properties of the neutron in a uniform background field

Thomas Primer, Waseem Kamleh, Derek Leinweber and Matthias Burkardt

Lattice 2012

Introduction

The magnetic moment and magnetic polarisability are important fundamental properties which describe the response of a system to an applied static magnetic field.

These can be calculated on the lattice through the use of the background field method.

Background Field Method

Apply a uniform magnetic field over the whole lattice which produces a shift in the energy.

$$E(0) \to E'(B)$$

Background Field Method

Apply a uniform magnetic field over the whole lattice which produces a shift in the energy.

$$E(0) \to E'(B)$$

Small field expansion of the energy for a particle in a constant magnetic field:

$$E(B) = M + \frac{e|B|}{2M} + \vec{\mu} \cdot \vec{B} - \frac{4\pi}{2}\beta B^2 + \mathcal{O}(B^3)$$

Background Field Method

Apply a uniform magnetic field over the whole lattice which produces a shift in the energy.

$$E(0) \to E'(B)$$

Small field expansion of the energy for a particle in a constant magnetic field:

$$E(B) = M + \frac{e|B|}{2M} + \vec{\mu} \cdot \vec{B} - \frac{4\pi}{2}\beta B^2 + \mathcal{O}(B^3)$$

Magnetic moment μ and magnetic polarisability β.

Background Field Method

Consider the continuum case:

$$D_\mu = \partial_\mu + gG_\mu + qA_\mu$$

Lattice case:

$$U_\mu(x) \to U'_\mu(x) = U_\mu(x) U^{(B)}_\mu(x)$$

Background Field Method

Consider the continuum case:

$$D_\mu = \partial_\mu + gG_\mu + qA_\mu$$

Lattice case:

$$U_\mu(x) \to U'_\mu(x) = U_\mu(x) U_\mu^{(B)}(x)$$

The factor modifying the links has the form:

$$U_\mu^{(B)}(x) = e^{iaqA_\mu(x)}.$$

Background Field Method

Use Maxwell's equations to choose values for A_μ that give a constant magnetic field in the z-direction

$$
\begin{aligned}
A_y &= Bx \\
A_x &= \begin{cases} -N_x B y & \text{for } x = N_x - 1. \\ 0 & \text{elsewhere} \end{cases}
\end{aligned}
$$

Background Field Method

Use Maxwell's equations to choose values for A_μ that give a constant magnetic field in the z-direction

$$\begin{aligned} A_y &= Bx \\ A_x &= \begin{cases} -N_x B y & \text{for } x = N_x - 1. \\ 0 & \text{elsewhere} \end{cases} \end{aligned}$$

Requirement at the $x = N_x - 1$, $y = N_y - 1$ boundary:

$$qBa^2 = \frac{2\pi n}{N_x N_y}$$

Which acts as a quantisation condition on the field.

Simulation Details

$32^3 \times 64$ lattice, 2+1 flavour dynamical-QCD gauge field configurations, provided by the PACS-CS collaboration as part of the International Lattice Data Grid (ILDG) with $\beta = 1.9$, physical lattice spacing $a = 0.0907$ fm.

Simulation Details

$32^3 \times 64$ lattice, 2+1 flavour dynamical-QCD gauge field configurations, provided by the PACS-CS collaboration as part of the International Lattice Data Grid (ILDG) with $\beta = 1.9$, physical lattice spacing $a = 0.0907$ fm.

Correlation functions calculated using the standard χ_1 interpolating field

$$\chi_1 = (u^T C \gamma_5 d) u$$

with 100 sweeps of Gaussian smearing on the source.

Simulation Details

$32^3 \times 64$ lattice, 2+1 flavour dynamical-QCD gauge field configurations, provided by the PACS-CS collaboration as part of the International Lattice Data Grid (ILDG) with $\beta = 1.9$, physical lattice spacing $a = 0.0907$ fm.

Correlation functions calculated using the standard χ_1 interpolating field

$$\chi_1 = (u^T C\gamma_5 d)u$$

with 100 sweeps of Gaussian smearing on the source.

m_π	Configurations
622 MeV	360
512 MeV	360
388 MeV	360
282 MeV	360
151 MeV	not yet calculated

Simulation Details

Field strengths:

$$qBa^2 = +0.0061, -0.0123, +0.0184, +0.0245, -0.0368 - 0.0491.$$

Corresponding to $n =$ 1, -2, 3, 4, -6, -8 in the quantisation condition.

Simulation Details

Field strengths:

$$qBa^2 = +0.0061, -0.0123, +0.0184, +0.0245, -0.0368 - 0.0491.$$

Corresponding to $n = 1$, -2, 3, 4, -6, -8 in the quantisation condition.

Calculate correlation functions with $q_d(B_n)$ and $q_u(B_{-2n})$ for a baryon in a field of strength $-3B_n$.

Magnetic Moment

$$E = M + \frac{e|B|}{2M} + \vec{\mu} \cdot \vec{B} - \frac{4\pi}{2}\beta B^2 + \mathcal{O}(B^3)$$

Magnetic Moment

$$E = M + \frac{e|B|}{2M} + \vec{\mu} \cdot \vec{B} - \frac{4\pi}{2}\beta B^2 + \mathcal{O}(B^3)$$

Taking the difference of spin-up and spin-down:

$$\frac{1}{2}(E_{\uparrow} - E_{\downarrow}) = \vec{\mu} \cdot \vec{B}$$

Magnetic Moment

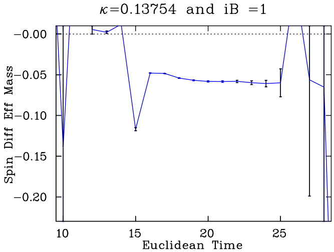
$$E = M + \frac{e|B|}{2M} + \vec{\mu} \cdot \vec{B} - \frac{4\pi}{2} \beta B^2 + \mathcal{O}(B^3)$$

Taking the difference of spin-up and spin-down:

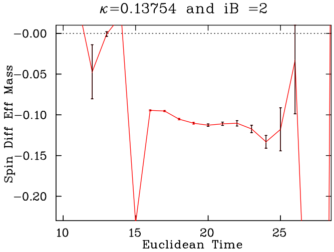
$$\frac{1}{2}(E_\uparrow - E_\downarrow) = \vec{\mu} \cdot \vec{B}$$

In terms of correlation functions:

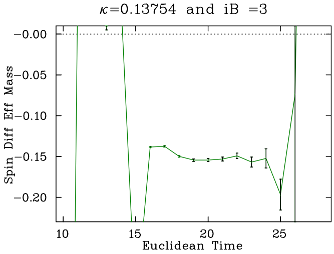
$$\Delta E(B) = \frac{1}{2}\left(ln \left(\frac{G_\uparrow(B,t)}{G_\uparrow(0,t)} \frac{G_\downarrow(0,t)}{G_\downarrow(B,t)} \right) \right)_{fit}$$



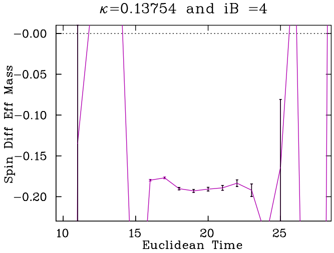
κ=0.13754 and iB =1
−0.00
−0.05
−0.10
−0.15
−0.20
Spin Diff Eff Mass
10
15
20
25
Euclidean Time



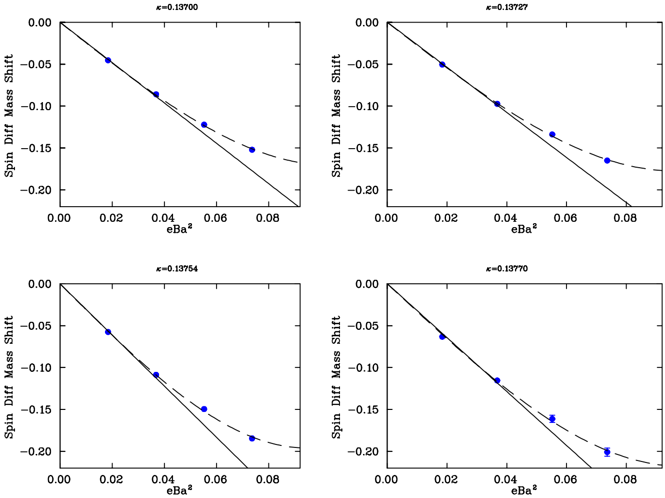
κ=0.13754 and iB =2
−0.00
−0.05
−0.10
−0.15
−0.20
Spin Diff Eff Mass
10
15
20
25
Euclidean Time

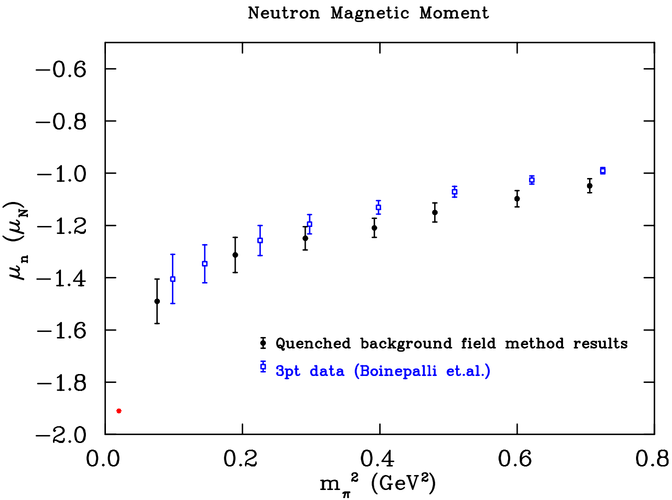


κ=0.13754 and iB =3
Spin Diff Eff Mass
−0.00
−0.05
−0.10
−0.15
−0.20
10
15
20
25
Euclidean Time

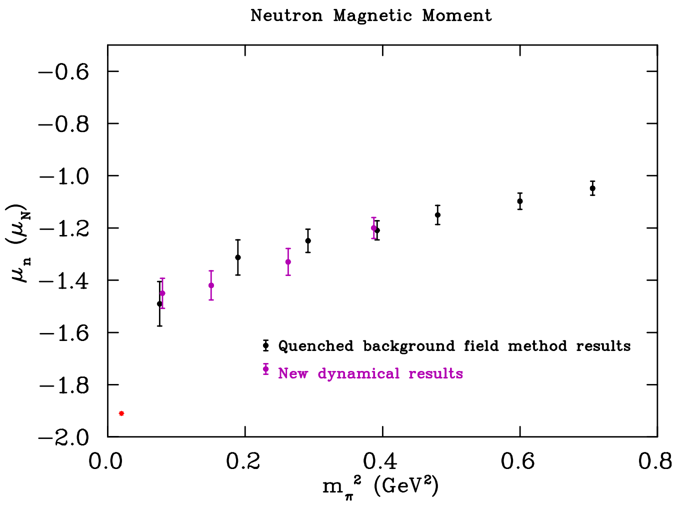


κ=0.13754 and iB =4
Spin Diff Eff Mass
−0.00
−0.05
−0.10
−0.15
−0.20
10
15
20
25
Euclidean Time


κ=0.13700
κ=0.13727
κ=0.13754
κ=0.13770
Spin Diff Mass Shift
eBa²
0.00
−0.05
−0.10
−0.15
−0.20
0.00
0.02
0.04
0.06
0.08



Neutron Magnetic Moment
−0.6
−0.8
−1.0
−1.2
−1.4
−1.6
−1.8
−2.0
0.0
0.2
0.4
0.6
0.8
μ_n (μ_N)
m_π^2 (GeV²)
Quenched background field method results
3pt data (Boinepalli et.al.)


Neutron Magnetic Moment
−0.6
−0.8
−1.0
−1.2
−1.4
−1.6
−1.8
−2.0
0.0
0.2
0.4
0.6
0.8
μ_n (μ_N)
m_π^2 (GeV^2)
Quenched background field method results
New dynamical results

Negative Parity Neutron

Lowest lying negative parity nucleon state $n^*(1/2-)$

Correlation function can be written as

$$
\begin{aligned}
G(t,\vec{p}) &= \sum_{B^+} \lambda_{B^+} \bar{\lambda}_{B^+} e^{-E_{B^+} t} \frac{\gamma \cdot p_{B^+} + M_{B^+}}{2E_{B^+}} \\
&- \sum_{B^-} \lambda_{B^-} \bar{\lambda}_{B^-} e^{-E_{B^-} t} \frac{-\gamma \cdot p_{B^-} + M_{B^-}}{2E_{B^-}}
\end{aligned}
$$

Negative Parity Neutron

Lowest lying negative parity nucleon state $n^*(1/2-)$

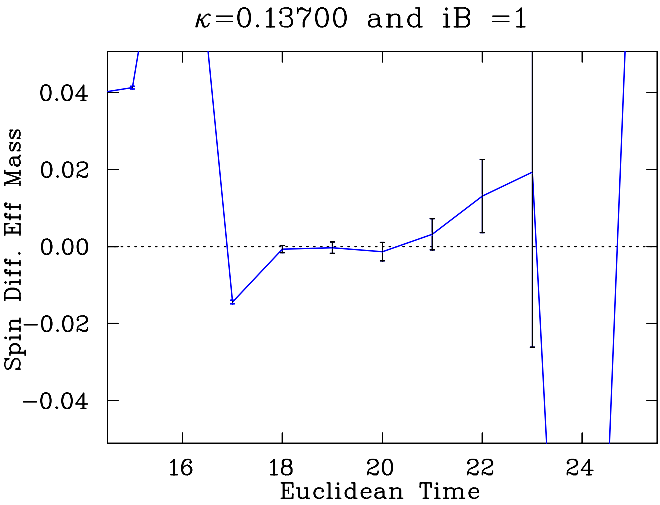
Correlation function can be written as

$$G(t,\vec{p}) = \sum_{B^+} \lambda_{B^+} \bar{\lambda}_{B^+} e^{-E_{B^+}t} \frac{\gamma \cdot p_{B^+} + M_{B^+}}{2E_{B^+}} - \sum_{B^-} \lambda_{B^-} \bar{\lambda}_{B^-} e^{-E_{B^-}t} \frac{-\gamma \cdot p_{B^-} + M_{B^-}}{2E_{B^-}}$$

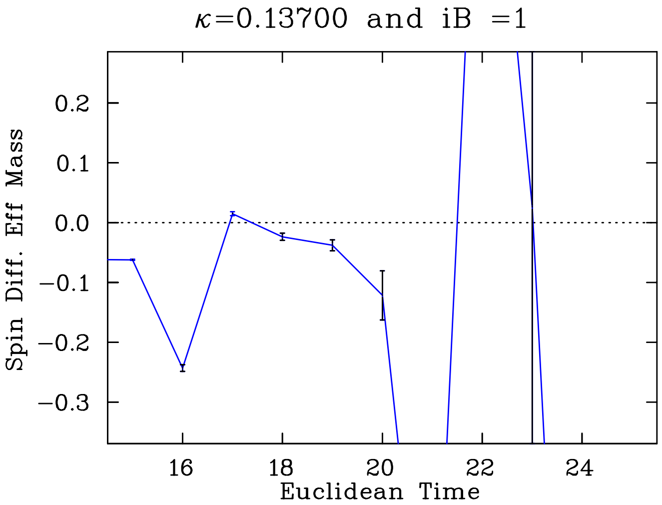
After setting $\vec{p} = 0$ to give $E_{B^\pm} = M_{B^\pm}$ the negative parity states are projected into the 3,3 and 4,4 components of the Dirac matrix via

$$\Gamma_\pm = \frac{1}{2}(1 \pm \gamma_0)$$

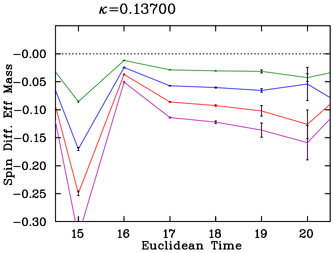
Negative Parity Neutron

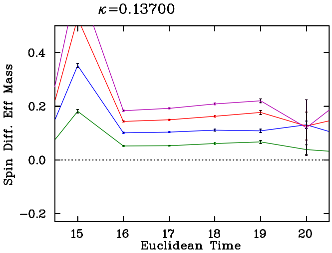


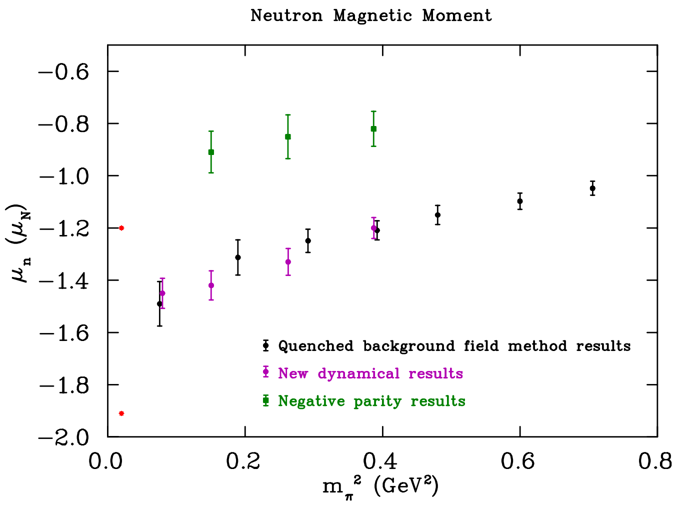
Negative Parity Proton



Using interpolating field $\chi_2 = (u^T C d)\gamma_5 u$:






Neutron Magnetic Moment
−0.6
−0.8
−1.0
−1.2
−1.4
−1.6
−1.8
−2.0
0.0
0.2
0.4
0.6
0.8
μ_n (μ_N)
m_π^2 (GeV^2)
Quenched background field method results
New dynamical results
Negative parity results

Magnetic Polarisability

$$E = m + \frac{e|B|}{2M} + \vec{\mu} \cdot \vec{B} - \frac{4\pi}{2}\beta B^2 + \mathcal{O}(B^3)$$

Magnetic Polarisability

$$E = m + \frac{e|B|}{2M} + \vec{\mu} \cdot \vec{B} - \frac{4\pi}{2} \beta B^2 + \mathcal{O}(B^3)$$

Taking the average of spin-up and spin-down:

$$\frac{1}{2}\left((E_\uparrow - m_\uparrow) + (E_\downarrow - m_\downarrow)\right) = \frac{0 \cdot |B|}{2M} - \frac{4\pi}{2} \beta B^2$$

Magnetic Polarisability

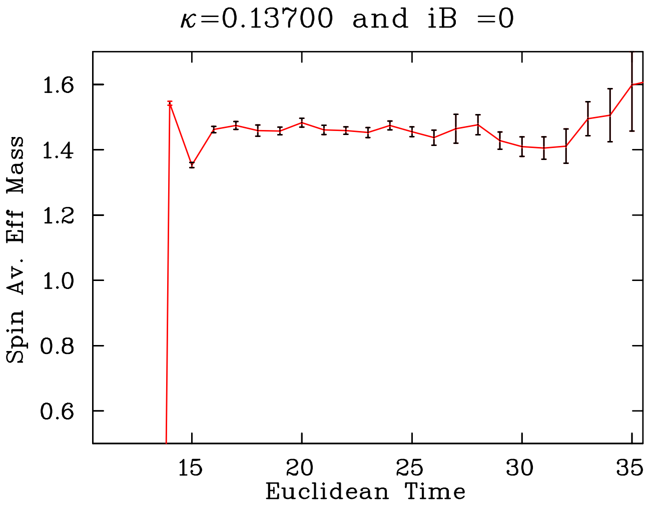
$$E = m + \frac{e|B|}{2M} + \vec{\mu} \cdot \vec{B} - \frac{4\pi}{2}\beta B^2 + \mathcal{O}(B^3)$$

Taking the average of spin-up and spin-down:

$$\frac{1}{2}\left((E_\uparrow - m_\uparrow) + (E_\downarrow - m_\downarrow)\right) = \frac{0 \cdot |B|}{2M} - \frac{4\pi}{2}\beta B^2$$

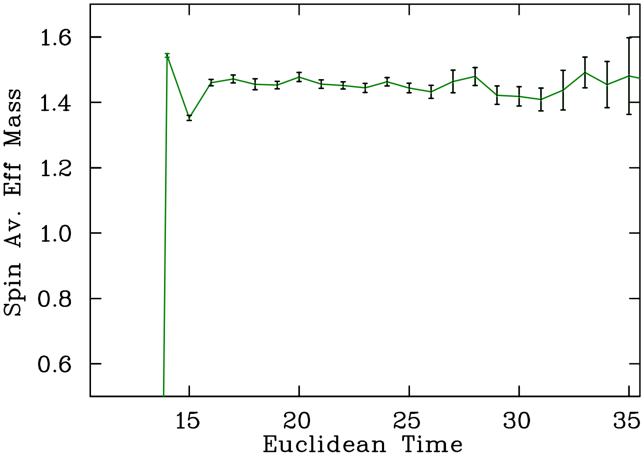
In terms of correlation functions:

$$\Delta E(B) = \frac{1}{2}\left(ln \left(\frac{G_\uparrow(B,t)}{G_\uparrow(0,t)} \frac{G_\downarrow(B,t)}{G_\downarrow(0,t)} \right)\right)_{fit}$$

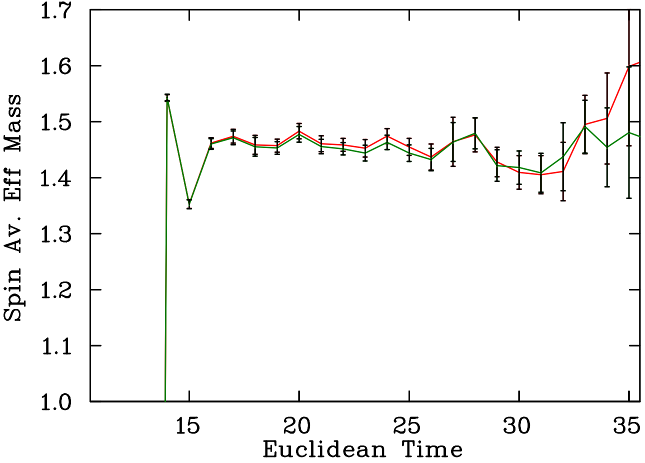


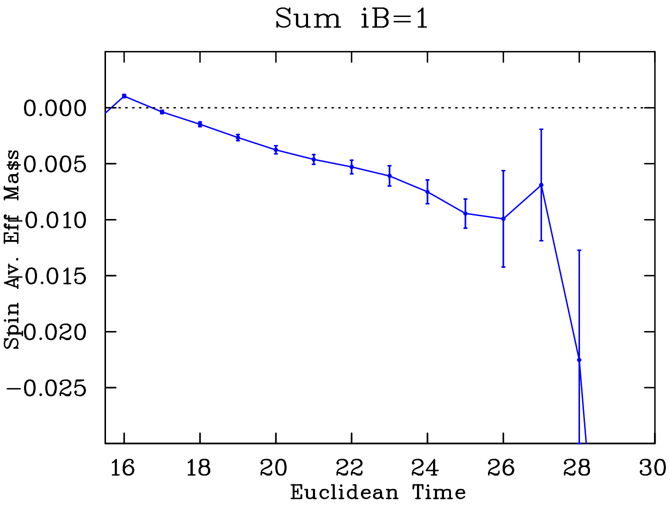
κ=0.13700 and iB =0
1.6
1.4
1.2
1.0
0.8
0.6
Spin Av. Eff Mass
15
20
25
30
35
Euclidean Time

κ=0.13700 and iB =1

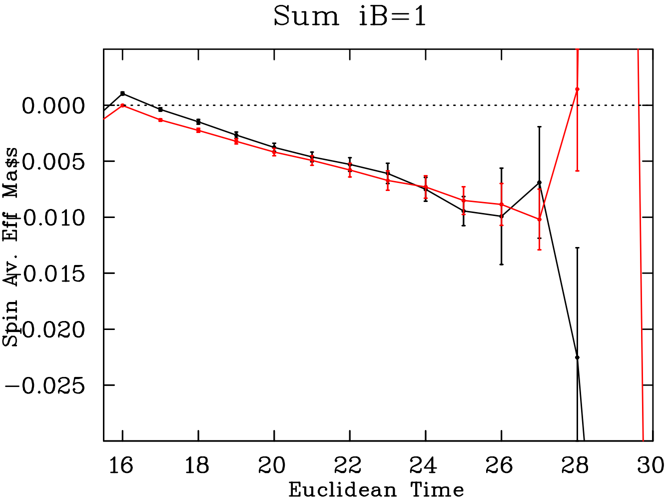


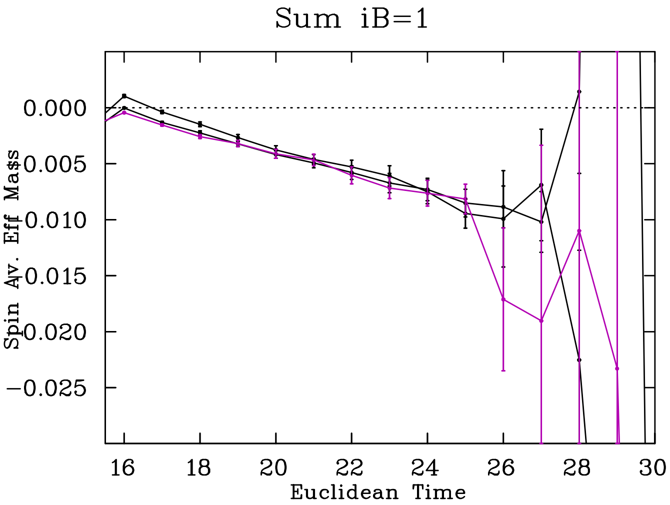
κ=0.13700





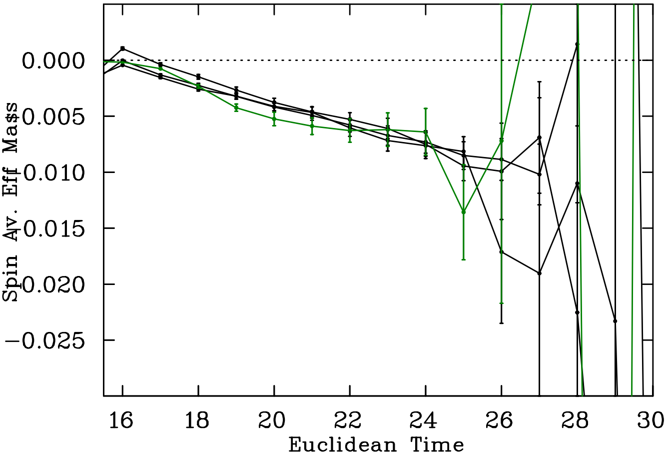
Sum iB=1
0.000
−0.005
−0.010
−0.015
−0.020
−0.025
Spin Av. Eff Mass
16
18
20
22
24
26
28
30
Euclidean Time


Sum iB=1
0.000
−0.005
−0.010
−0.015
−0.020
−0.025
Spin Av. Eff Mass
16
18
20
22
24
26
28
30
Euclidean Time

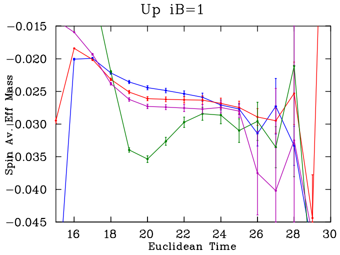


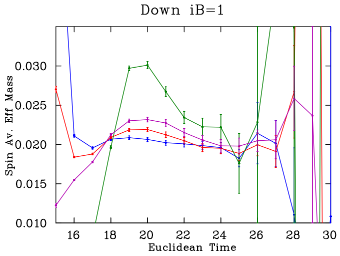
Sum iB=1
0.000
−0.005
−0.010
−0.015
−0.020
−0.025
Spin Av. Eff Mass
16
18
20
22
24
26
28
30
Euclidean Time

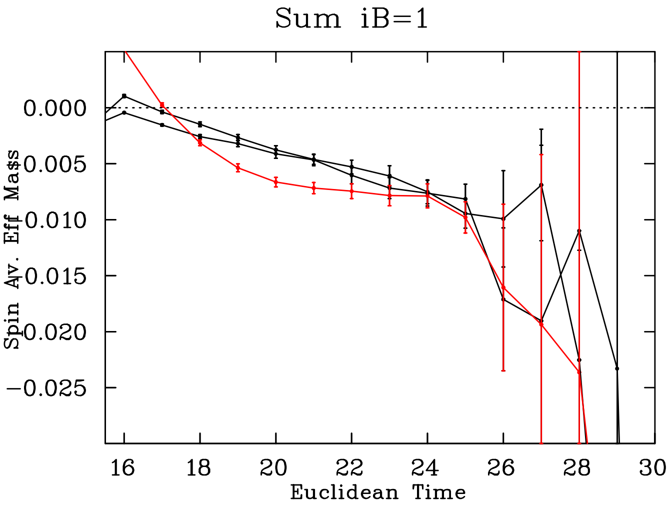
Sum iB=1



0.000
−0.005
−0.010
−0.015
−0.020
−0.025
Spin Av. Eff Mass
16
18
20
22
24
26
28
30
Euclidean Time


Up iB=1
Spin Av. Eff Mass
Euclidean Time
−0.015
−0.020
−0.025
−0.030
−0.035
−0.040
−0.045
16
18
20
22
24
26
28
30


Down iB=1
Spin Av. Eff Mass
Euclidean Time
0.030
0.025
0.020
0.015
0.010
16
18
20
22
24
26
28
30


Sum iB=1
0.000
−0.005
−0.010
−0.015
−0.020
−0.025
Spin Av. Eff Mass
16
18
20
22
24
26
28
30
Euclidean Time

The Origin of the Background Field

Does the location of the origin of the background field matter?

The Origin of the Background Field

Does the location of the origin of the background field matter?

$$U_\mu^{(B)}(x) \to U_\mu^{'(B)}(x) = U_\mu^{(B)}(x - x_0)$$

The physical value of the field B is not affected.

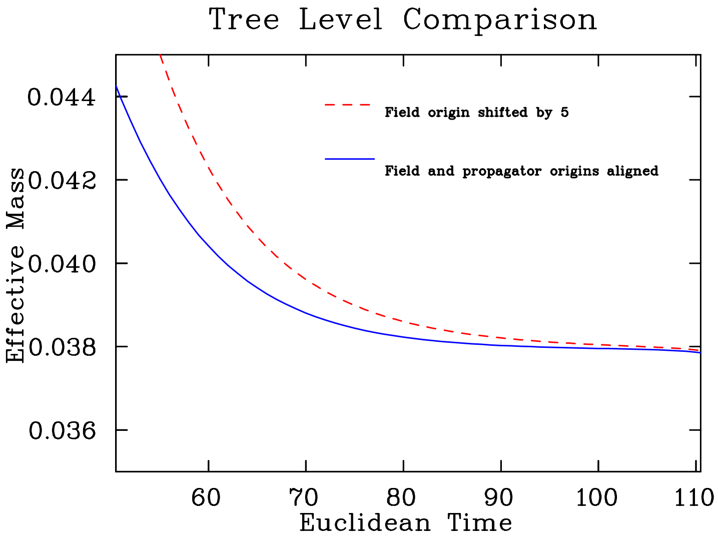
The Origin of the Background Field

Does the location of the origin of the background field matter?

$$U_\mu^{(B)}(x) \to U_\mu^{'(B)}(x) = U_\mu^{(B)}(x - x_0)$$

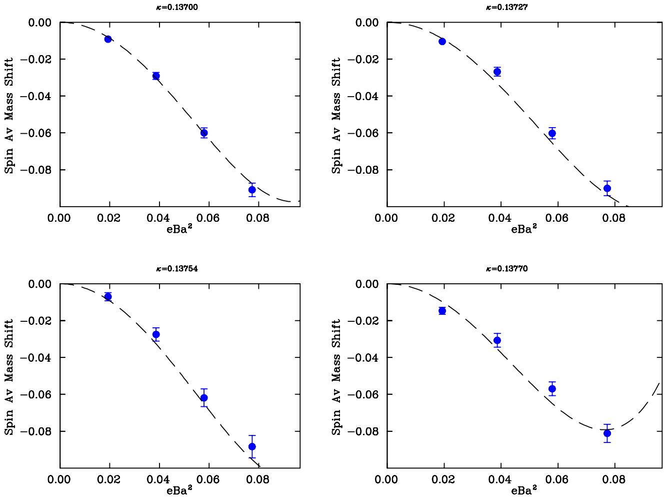
The physical value of the field B is not affected.

- Errors and noise make it hard to tell if there is a meaningful change in the overlaps.
- Use tree level calculations as a check.
- Done by setting gauge field to 1 everywhere which turns off QCD interactions.

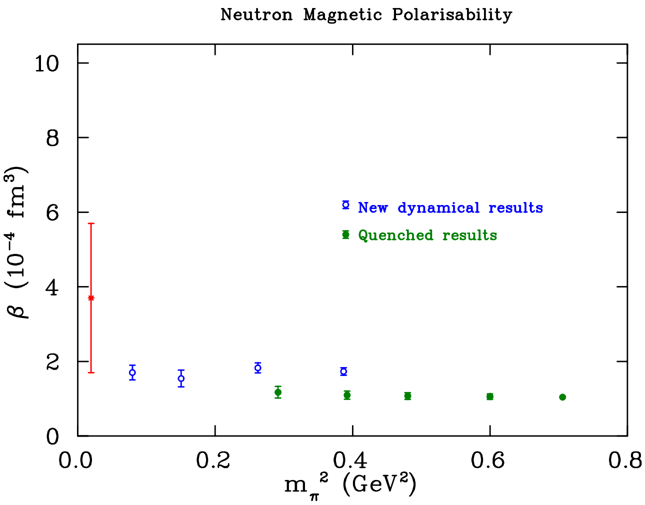

Tree Level Comparison
Field origin shifted by 5
Field and propagator origins aligned
Effective Mass
0.044
0.042
0.040
0.038
0.036
60
70
80
90
100
110
Euclidean Time

The Origin of the Background Field

- Only the overlap is affected, not the ground state asymptote value.
- No effect is seen at all for a point source.
- Ideal ground state approach is seen when the origin of the quark props is aligned with the origin of the background field.



κ=0.13700
κ=0.13727
κ=0.13754
κ=0.13770
Spin Av Mass Shift
eBa²
0.00
−0.02
−0.04
−0.06
−0.08
0.00
0.02
0.04
0.06
0.08



Neutron Magnetic Polarisability
New dynamical results
Quenched results
$\beta\ (10^{-4}\ \mathrm{fm}^3)$
$m_\pi^2\ (\mathrm{GeV}^2)$
0
2
4
6
8
10
0.0
0.2
0.4
0.6
0.8

Work to be completed

- Choosing a smearing prescription for extracting good polarisability results.
- Calculation of propagators for the lightest quark mass and subsequent results at that mass.
- Correlation matrix analysis to better isolate states, in particular the negative parity states.