# Neutron-Antineutron Oscillations on the Lattice

$N$

$\overline{N}$

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# Why should we care?

- Other particle/antiparticle mixings occur

$$K^0 \leftrightarrow \overline{K^0}$$
$$B^0 \leftrightarrow \overline{B^0}$$

- Expect baryon number number to be broken

  - Baryon-antibaryon asymmetry

$$\Delta B = 1 \text{ (Proton Decay)}$$
$$\Delta B = 2 \text{ } (N\overline{N} \text{ Oscillations})$$

- Natural in GUT theories with Majorana neutrinos

  - Usual sphaleron process: $B - L = 0$

Mohapatra, Marshak 1980

$\nu = \overline{\nu} \;\Rightarrow \Delta L = 2$ $\qquad B - L = 0 \;\Rightarrow \Delta B = 2$

# Basic Idea

- BSM physics leads to off-diagonal mixing

$$H = \begin{pmatrix} E_n & \delta m \\ \delta m & E_{\bar{n}} \end{pmatrix} = \begin{pmatrix} E + V & \delta m \\ \delta m & E - V \end{pmatrix}$$

$$V = 0 \;\; \Rightarrow \text{Free System}$$

- Transition Probability

$$P_{n \to \bar{n}}(t) = \frac{\delta m^2}{\delta m^2 + V^2} \sin^2 \left[ \sqrt{\delta m^2 + V^2}\; t \right] \qquad \tau_{n\bar{n}} = \frac{1}{\delta m}$$

- Estimates for ruling out large classes of GUTs

$$\tau_{n\overline{n}} > 10^{10} - 10^{11} \text{ sec}$$

# Experimental progress

- Neutron-antineutron annihilation signals

Primary channel $n\overline{n} \to 5\pi$ ("Zero background" signal)

- Two Types of experimental searches

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1. Neutron-antineutron annihilation in nuclei

Friedman, Gal 2008

-Nuclear suppression: $\tau_{\mathrm{Nucl}} = (3 \times 10^{22}) \frac{\tau_{n\overline{n}}^2}{\mathrm{sec}}$

Super-K bounds (2011) $\tau_{n\overline{n}} > 3.5 \times 10^8$ sec

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2. Free, Cold neutron annihilation with target

ILL bound (1993) $\tau_{n\bar{n}} > 0.86 \times 10^8$ sec

# Experimental prospects

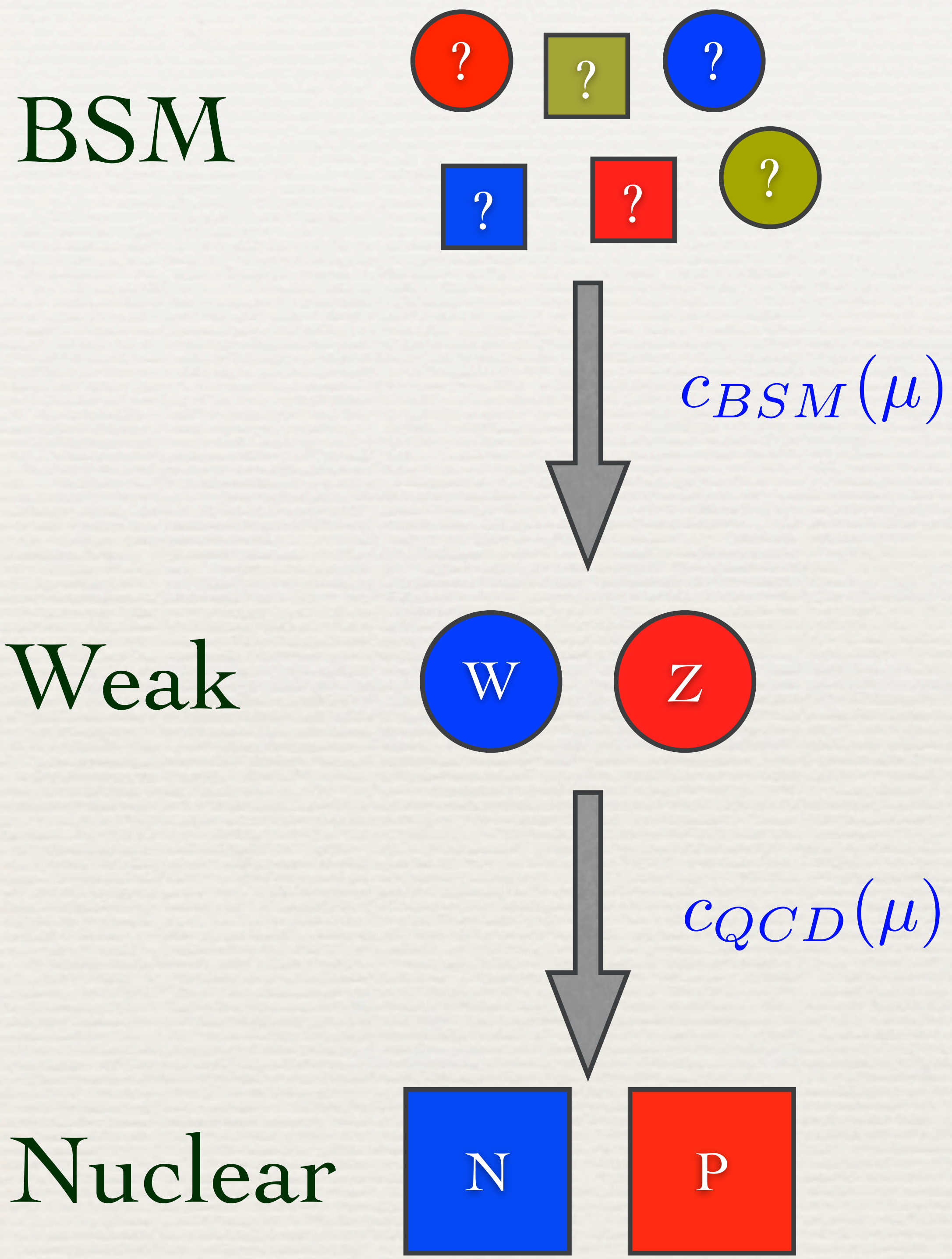
- Cost Estimates (Project X meeting last week)

$$\tau_{n\bar{n}} \gtrsim 3 \times 10^9 \text{ sec:} \qquad \sim \$10 \text{ million}$$

$$\tau_{n\bar{n}} \gtrsim 1 \times 10^{11} \text{ sec:} \qquad \gtrsim \$200 \text{ million}$$

Bottom line: Lattice allows for rigorous, first-principle understanding of QCD input

# Origin of Oscillations




- Running of BSM interaction to nuclear scale

$$\frac{1}{\tau_{n\overline{n}}} = \delta m = c_{BSM}(\mu) c_{QCD}(\mu) \langle \overline{n} | \mathcal{O} | n \rangle$$

# Where Lattice Can Help

- Is BSM running non-perturbative?
  - Model-dependent (assume pert. models for now)
- Is QCD running non-perturbative?
  - Should be calculated (pert. running reasonable)
- What is neutron-antineutron matrix element?
  - Inherently non-perturbative question
- What is effect in nuclei?
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# Six-quark Operators

Rao, Shrock (1982)

Three pairs of quarks:

1. $u^T C u$ or $u^T C d$ or $d^T C d$

2. $u_L^T C d_L$ or $u_R^T C d_R$

3. $$\Gamma^s_{ijklmn} = \epsilon_{mik}\epsilon_{njl} + \epsilon_{nik}\epsilon_{mjl} + \epsilon_{mjk}\epsilon_{nil} + \epsilon_{njk}\epsilon_{mil}$$

$$\Gamma^a_{ijklmn} = \epsilon_{mij}\epsilon_{nkl} + \epsilon_{nij}\epsilon_{mkl}$$

# Six-quark Operators

Rao, Shrock (1982)

$\chi_i = L, R$

1. $\mathcal{O}^1_{\chi_1\chi_2\chi_3} = (u^T_{i\chi_1} C u_{j\chi_1})(d^T_{k\chi_2} C d_{l\chi_2})(d^T_{m\chi_3} C d_{n\chi_3})\Gamma^s_{ijklmn}$

2. $\mathcal{O}^2_{\chi_1\chi_2\chi_3} = (u^T_{i\chi_1} C d_{j\chi_1})(u^T_{k\chi_2} C d_{l\chi_2})(d^T_{m\chi_3} C d_{n\chi_3})\Gamma^s_{ijklmn}$

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$$\chi_i = L, R \qquad \mathcal{O}^1_{\chi_1 LR} = \mathcal{O}^1_{\chi_1 RL} \qquad \mathcal{O}^{2,3}_{LR\chi_3} = \mathcal{O}^{2,3}_{RL\chi_3}$$

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18 Independent Operators

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Caswell, Milutinovic, Sejanovic (1983)

18 Independent Operators → 14 Indep. Operators

# Six-quark Operators

Rao, Shrock (1982)

If invariant under $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ :

(Six Operators)

$$
\begin{aligned}
\mathcal{P}_1 &= \mathcal{O}^1_{RRR} \\
\mathcal{P}_2 &= \mathcal{O}^2_{RRR} \\
\mathcal{P}_3 &= \mathcal{O}^3_{RRR} \\
\mathcal{P}_4 &= 2\mathcal{O}^3_{LRR} \\
\mathcal{P}_5 &= 4\mathcal{O}^3_{LLR} \\
\mathcal{P}_6 &= 4(\mathcal{O}^1_{LLR} - \mathcal{O}^2_{LLR})
\end{aligned}
$$

- Matrix elements cannot be calculated perturbatively

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(Six Operators)

$$\mathcal{P}_1 = \mathcal{O}^1_{RRR}$$

(Four Operators)

$$\mathcal{P}_2 = \mathcal{O}^2_{RRR}$$

Caswell, Milutinovic, Sejanovic (1983)

$$\mathcal{P}_3 = \mathcal{O}^3_{RRR}$$

$$\mathcal{P}_4 = 2\mathcal{O}^3_{LRR}$$

$$\mathcal{P}_6 = -3\mathcal{P}_5$$

$$\mathcal{P}_5 = 4\mathcal{O}^3_{LLR}$$

$$\mathcal{P}_2 - \mathcal{P}_1 = 3\mathcal{P}_3$$

$$\mathcal{P}_6 = 4(\mathcal{O}^1_{LLR} - \mathcal{O}^2_{LLR})$$

- Matrix elements cannot be calculated perturbatively

# Current Understanding of Matrix Elements

MIT bag Model: (Rao, Shrock 1982)

- -Model dependent estimation
- -Results roughly consistent with DA
- -No QCD input

Lattice Motivation:

- -Numerical QCD calculation
- -Quantification of uncertainties
- -Pinpoint target sensitivity for experiment
- -Large enhancements/suppressions?

# Lattice Calculation

## Correlation Functions via path integral:

$$C_{\mathcal{O}} = \langle \mathcal{O} \rangle = \int d[U]\ \mathcal{O}\ \det(D_{lat}(U)) e^{-S_G(U)}$$

$$C_{NN}(t) = \langle \overline{N}(t) N(0) \rangle \to |\langle N | n \rangle|^2 e^{-m_n t}$$

$$C_{\overline{NN}}(t) = \langle N(t) \overline{N}(0) \rangle \to |\langle \overline{N} | \overline{n} \rangle|^2 e^{-m_n t}$$

$$C_{\overline{N}\mathcal{O}N}(t_1, t_2) = \langle N(t_1) \mathcal{O}(0) \overline{N}(t_2) \rangle \to \langle \overline{N} | \overline{n} \rangle \langle N | n \rangle e^{-m_n (t_1 + t_2)} \langle n | \mathcal{O} | \overline{n} \rangle$$

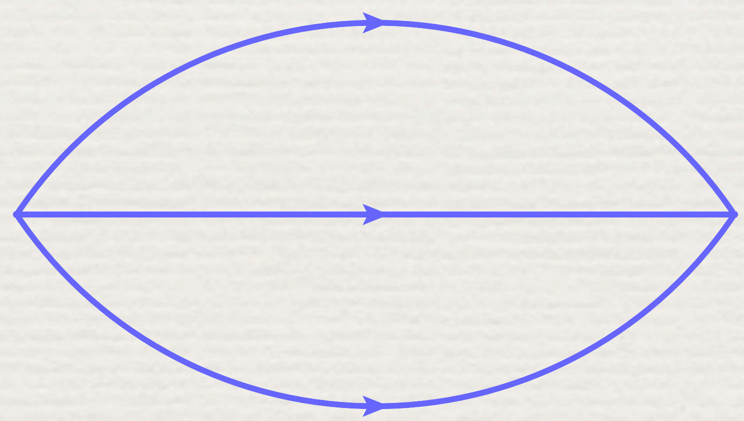
$$\mathcal{R} = \frac{C_{\overline{N}\mathcal{O}N}(t_1, t_2)}{C_{\overline{NN}}(t_1 + t_2)} \left[ \frac{C_{NN}(t_1) C_{\overline{NN}}(t_2) C_{\overline{NN}}(t_1 + t_2)}{C_{\overline{NN}}(t_1) C_{NN}(t_2) C_{NN}(t_1 + t_2)} \right]^{\frac{1}{2}} \to \langle \overline{n} | \mathcal{O} | n \rangle$$

# Lattice Contractions

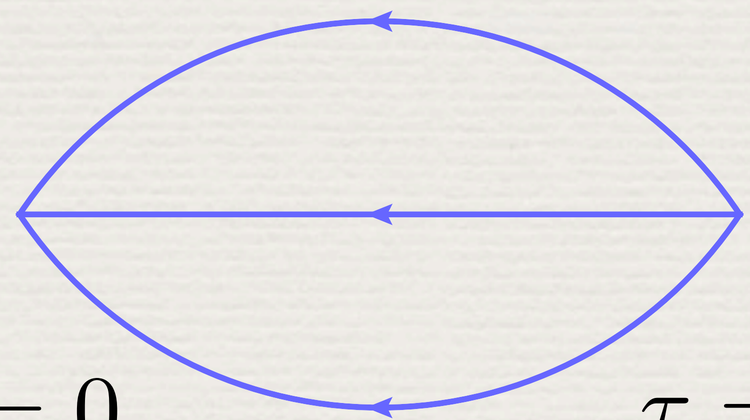
## Propagator Contractions:

$$\overline{q}_{i'}^{\alpha'}(y)\; q_i^{\alpha}(x) = S_{i'i}^{\alpha'\alpha}(y,x) \qquad\qquad S^{\dagger} = \gamma_5 S \gamma_5$$

$C_{NN}(t)$

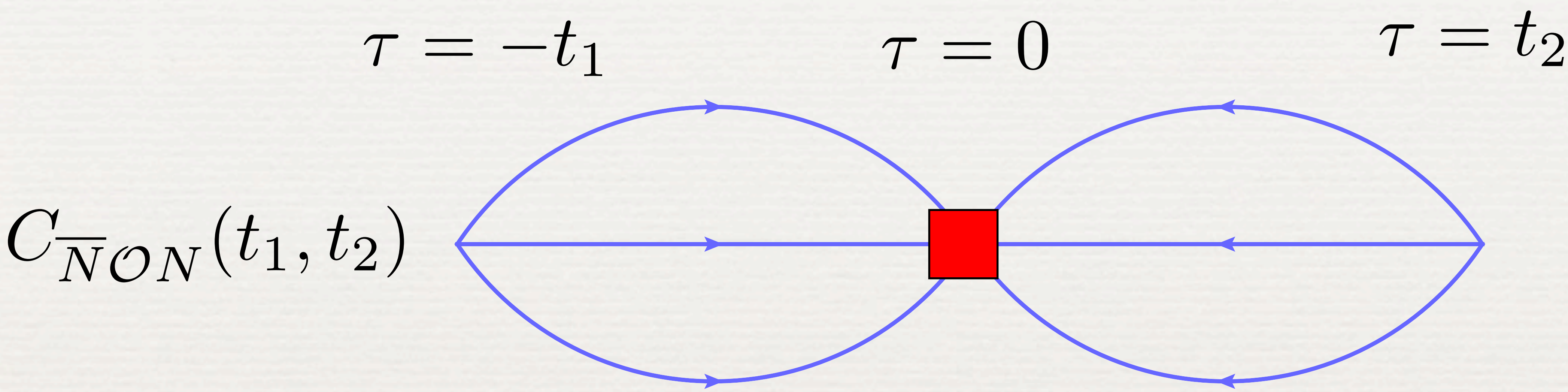


$C_{\overline{NN}}(t)$



$\tau = 0$ $\qquad\qquad$ $\tau = t$

# Lattice Contractions

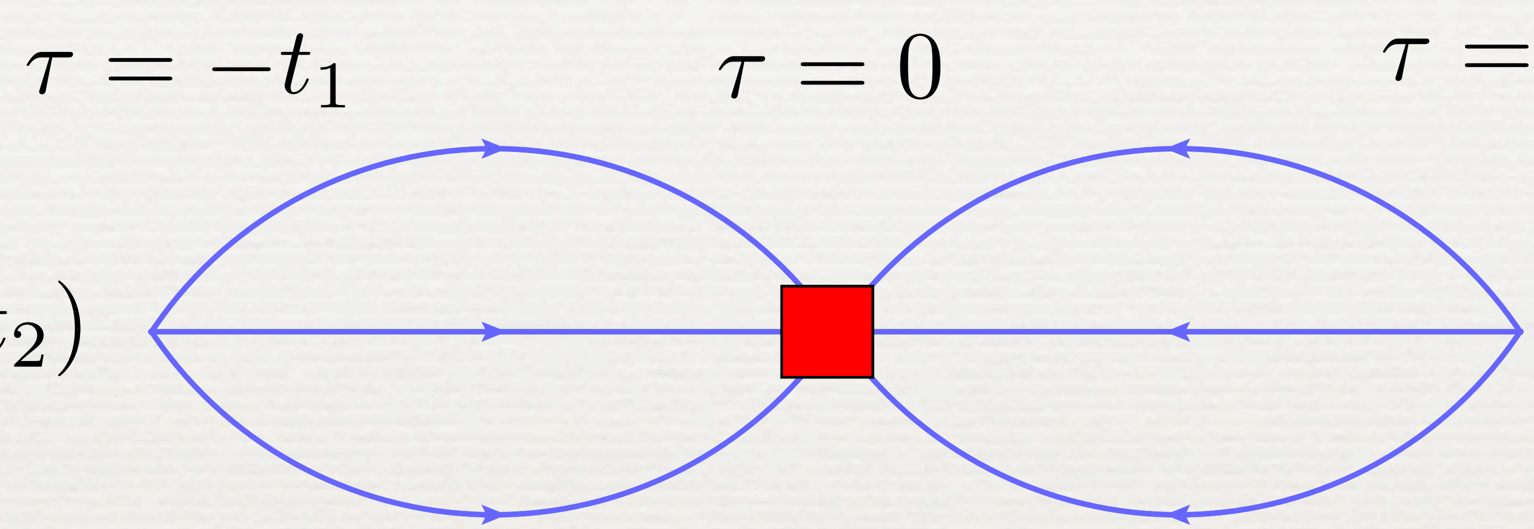



No Dis. Diagrams

1 Propagator → One measurement ALL time insertion

# Lattice Contractions

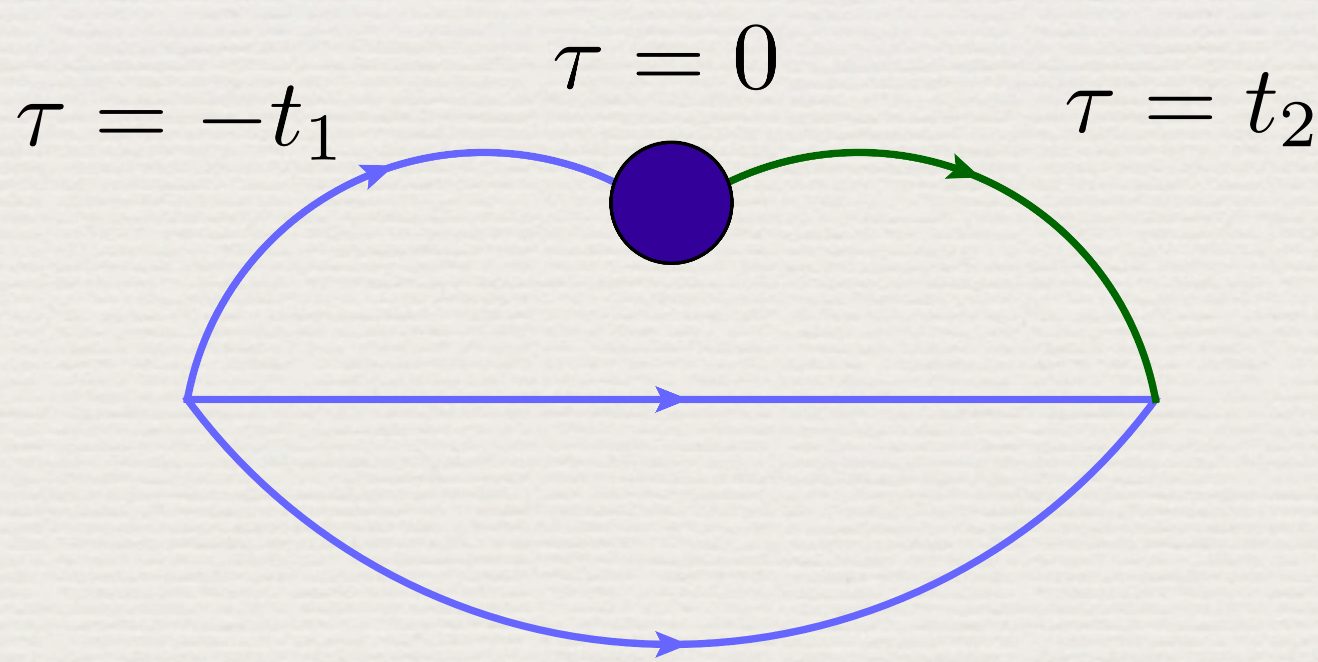
$C_{\overline{N}\mathcal{O}N}(t_1, t_2)$




No Dis. Diagrams

1 Propagator → One measurement ALL time insertion

Typical 3-point




Often Dis. Diagrams

2 Propagators → One measurements One time insertion

# Executive Summary

- Advantages of Neutron-Antineutron calculations

  For same cost:

  More Statistics

  All Operator Insertions

  No Quark Loop or Disconnected Diagrams

- Disadvantages of Neutron-Antineutron calculations

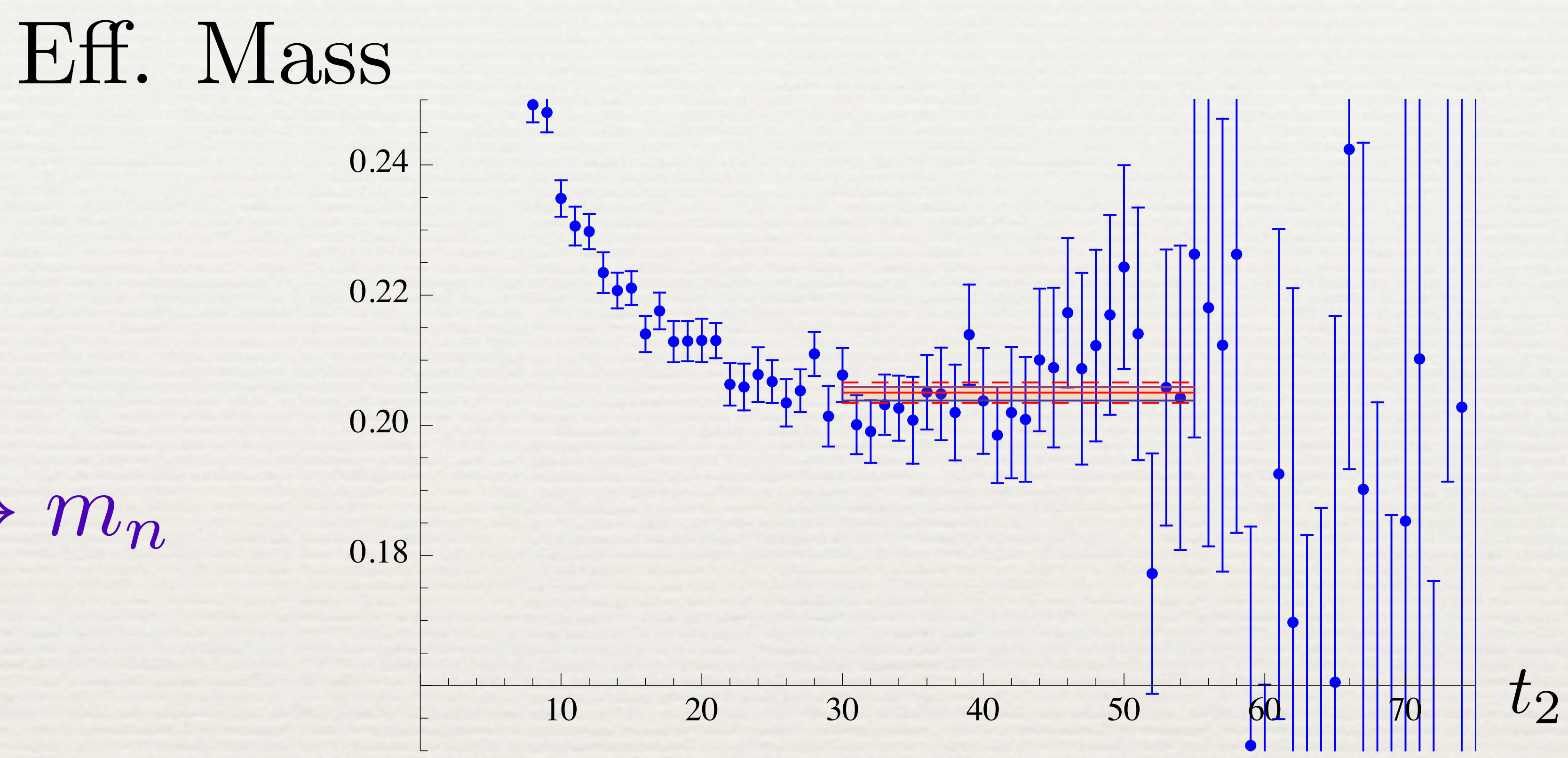
  More Propagator Multiplications → Potentially Worse Signal

# Lattice Details

- $32^3 \times 256$ anisotrpoic clover-Wilson lattices
- $m_\pi \sim 390$ MeV
- $a_t \sim 0.04$ fm, $a_s \sim 0.125$ fm
- $L \sim 4$ fm
- 159 configurations (every 4th trajectory)
- 7268 propagators (Gaussian smeared sources)

# Nucleon Effective Mass

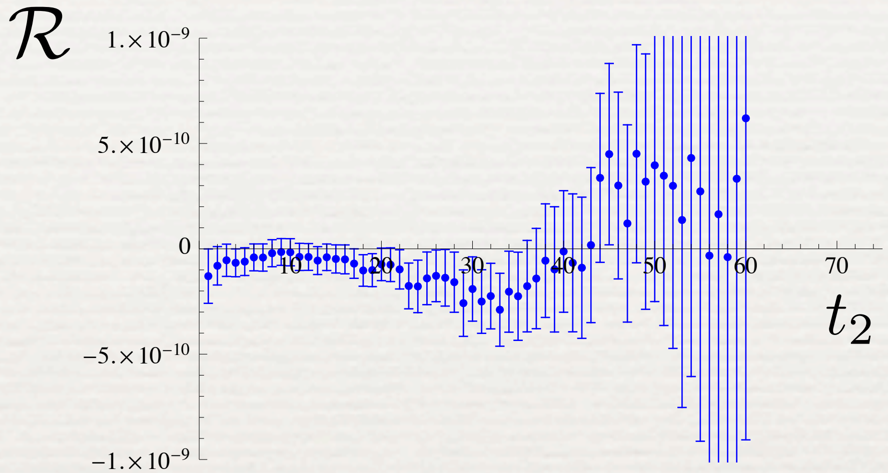
$$\frac{C_{NN}(t+1)}{C_{NN}(t)} \to m_n$$



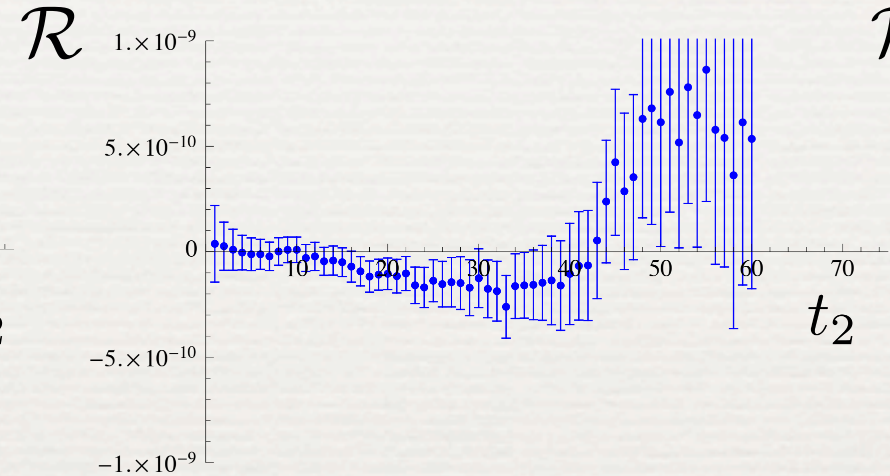

$$M_N = 1.148(\pm 0.0088)(+0.0048)(-0.0068)\ \mathrm{GeV}$$

# N-NBar Matrix Element

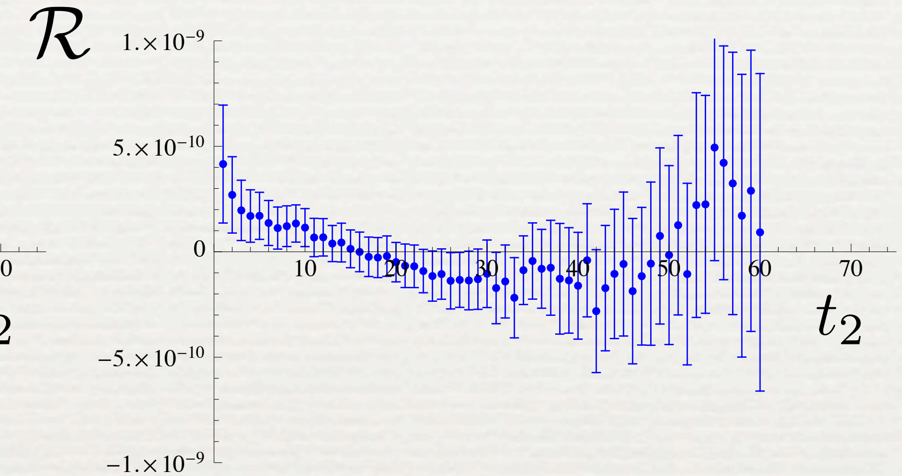
$t_1 = 5$



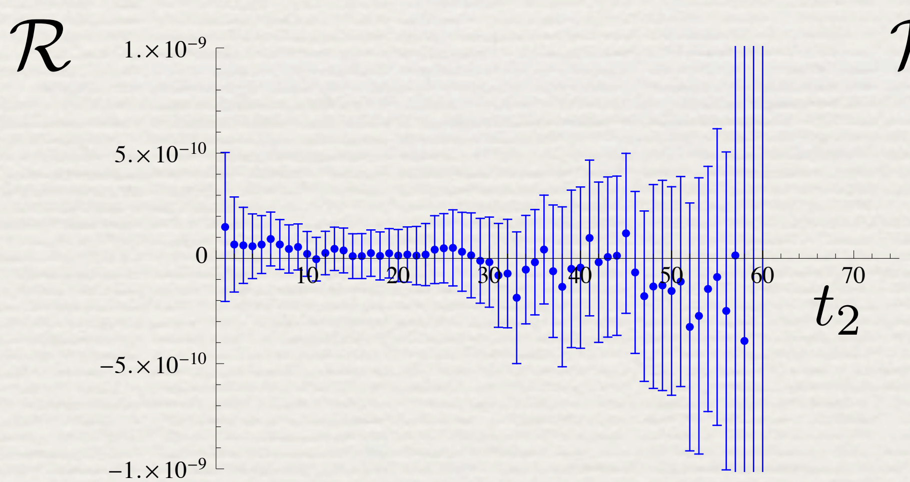

$t_1 = 10$



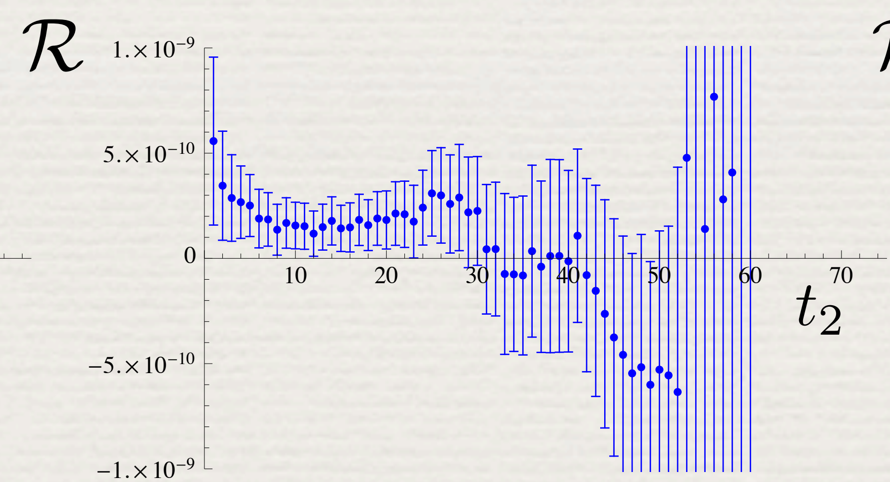

$t_1 = 15$



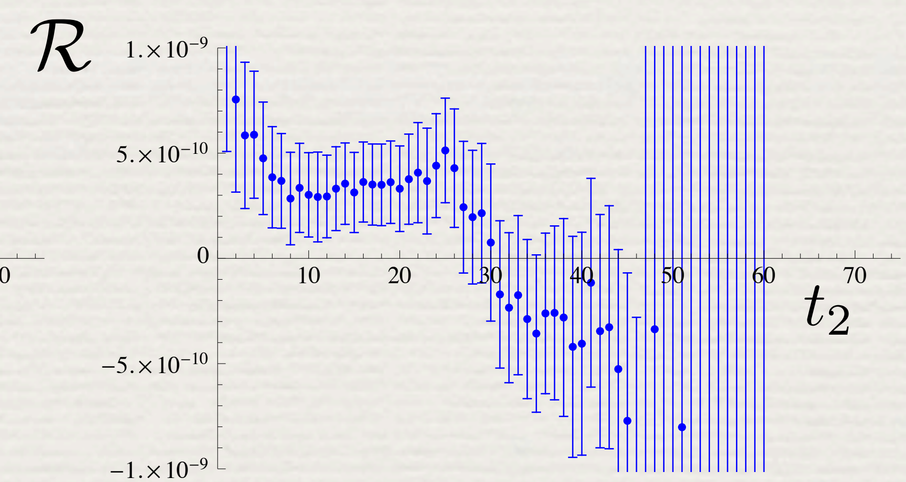

$t_1 = 20$




$t_1 = 25$




$t_1 = 30$

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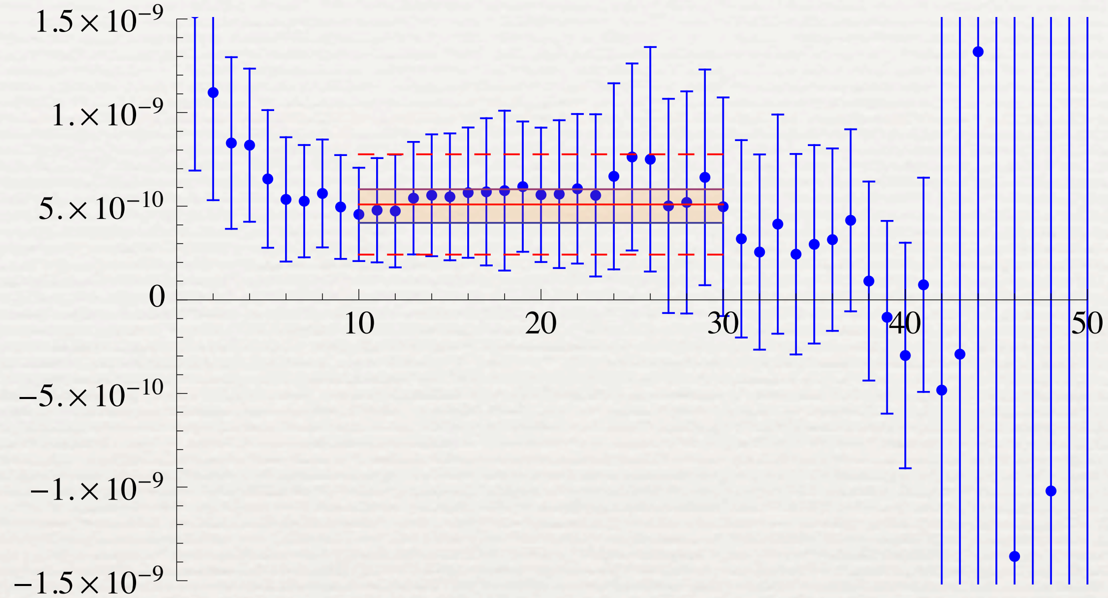
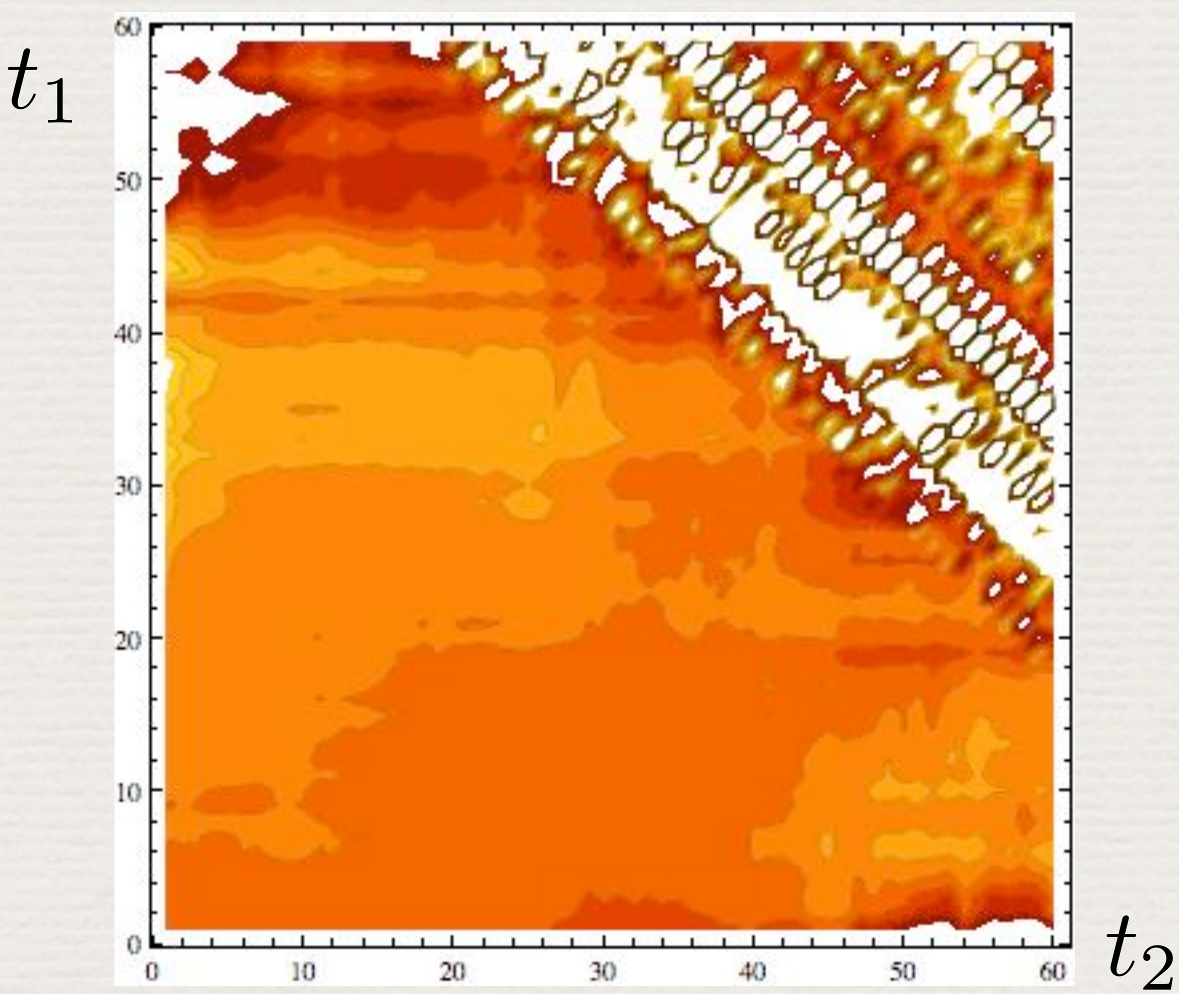
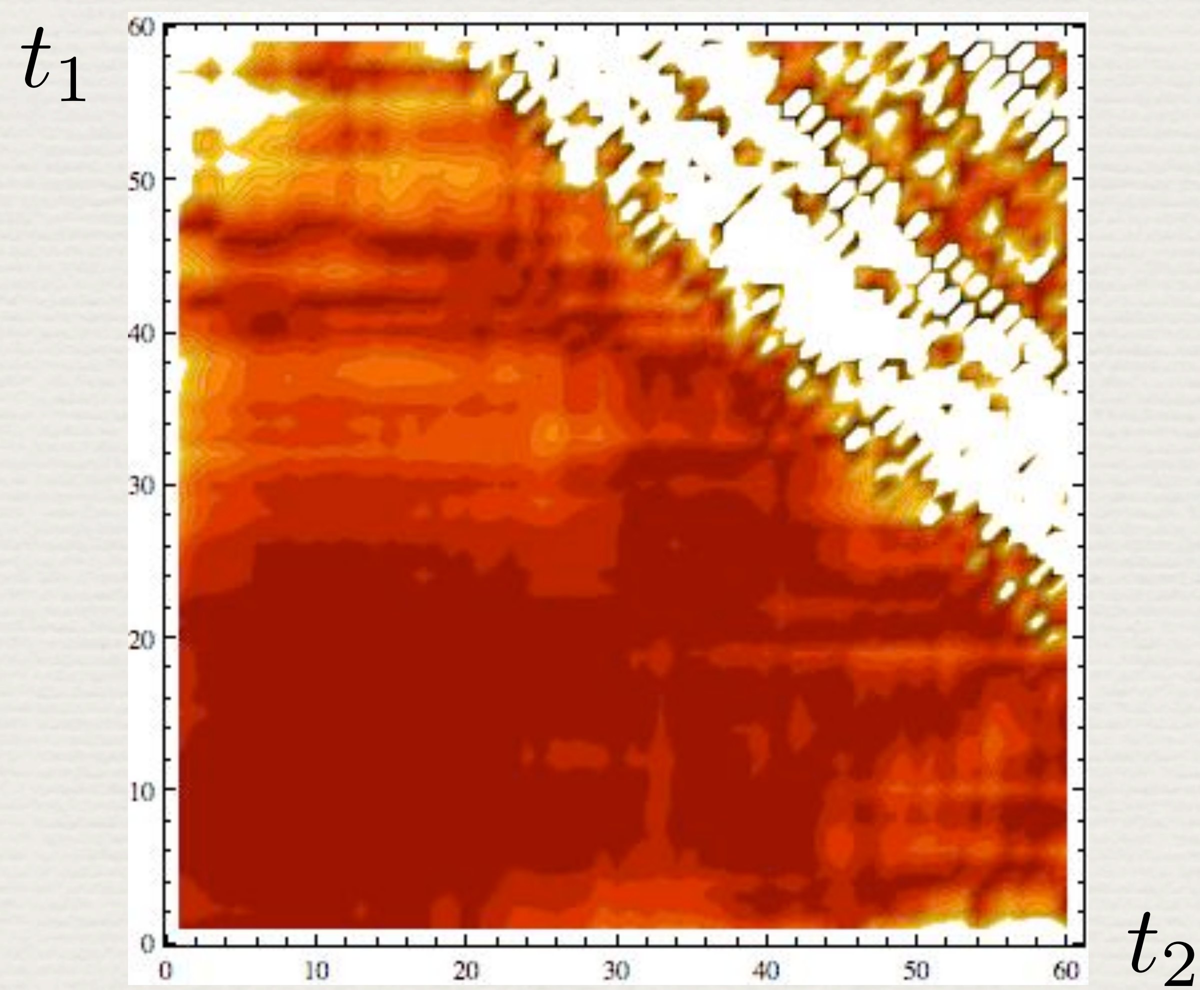


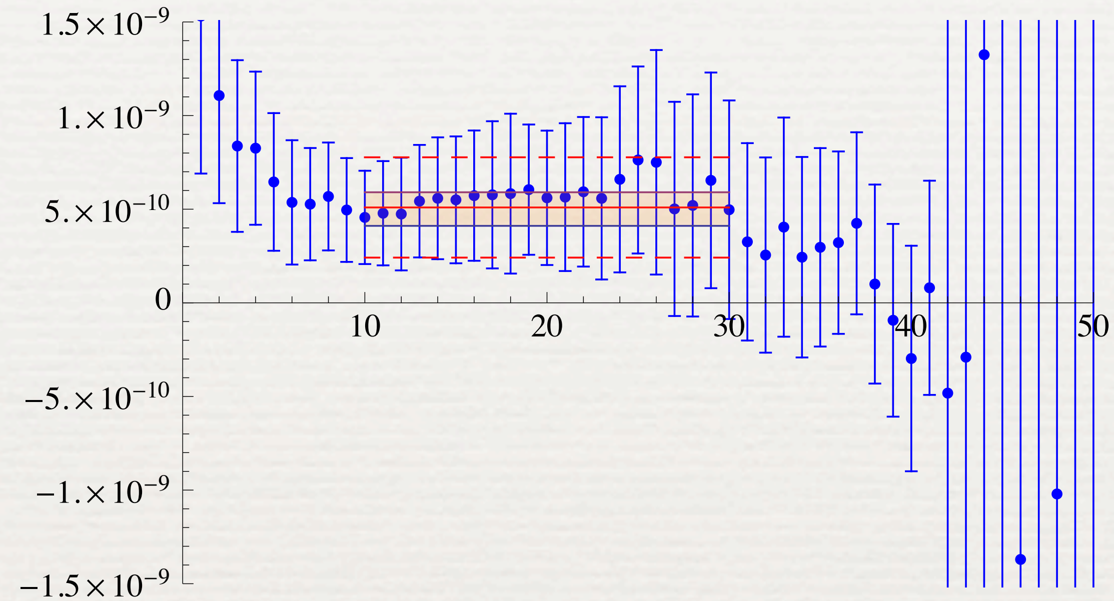

$$\langle \overline{n} | \mathcal{O}^1_{RRR} | n \rangle = 1.57(\pm 0.85)(+0.25)(-0.30) \ \times 10^{-5} \ \text{GeV}^6$$



# N-NBar Matrix Element

$\mathcal{R} \rightarrow \langle \overline{n} | \mathcal{O} | n \rangle$

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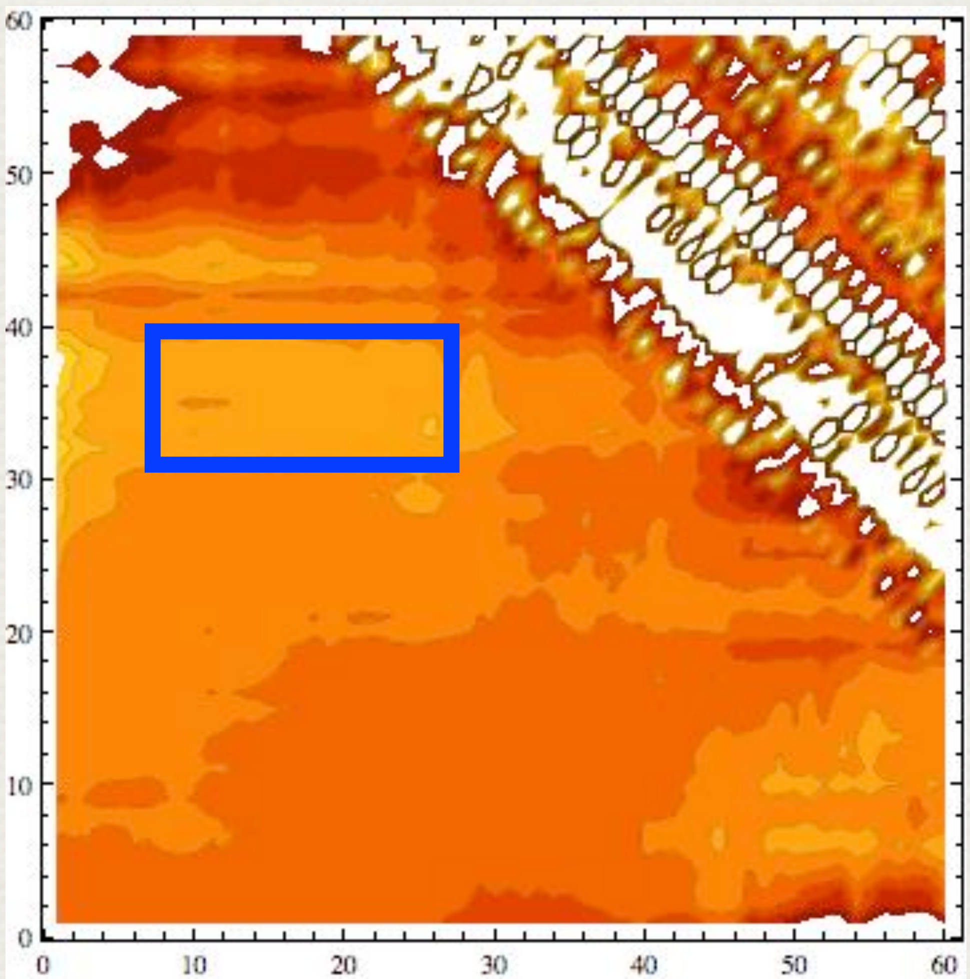


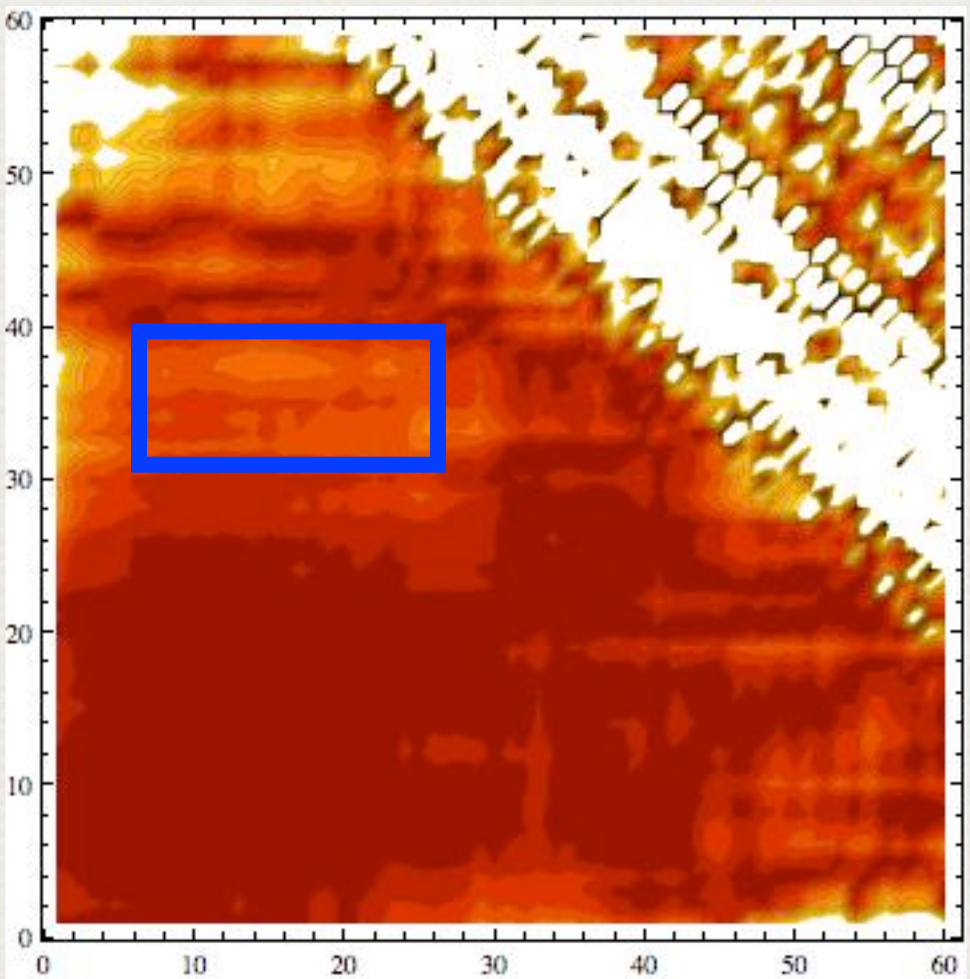

# VERY Preliminary Results

| | Lattice | MIT Bag Model |
|---|---|---|
| $\langle \overline{n} \vert \mathcal{P}_1 \vert n \rangle$ | $1.57(\pm 0.85)(+0.25)(-0.30)$ | $-6.56$ |
| $\langle \overline{n} \vert \mathcal{P}_2 \vert n \rangle$ | $-0.20(\pm 0.14)(+0.14)(-0.12)$ | $1.64$ |
| $\langle \overline{n} \vert \mathcal{P}_3 \vert n \rangle$ | $-0.24(\pm 0.26)(+0.10)(-0.07)$ | $2.73$ |
| $\langle \overline{n} \vert \mathcal{P}_4 \vert n \rangle$ | $-0.02(\pm 0.39)(+0.07)(-0.18)$ | $-6.36$ |
| $\langle \overline{n} \vert \mathcal{P}_5 \vert n \rangle$ | $0.34(\pm 0.82)(+0.27)(-0.57)$ | $9.64$ |
| $\langle \overline{n} \vert \mathcal{P}_6 \vert n \rangle$ | $-2.07(\pm 1.10)(+1.28)(-0.77)$ | $-28.92$ |
| | $\times 10^{-5}\ \mathrm{GeV}^6$ | $\times 10^{-5}\ \mathrm{GeV}^6$ |

# Systematic Effects

- Unphysical Pion Mass
  - No chiral extrapolation (yet...)
  - Real-world dynamics could differ
- Unphysical discretization effects
  - Most violent case should not occur
  - Beneficial to quantify
- Excited State Contamination
  - Range of operator insertions help some
- Volume Effects

# Future Outlook

## Currently in progress:

- Independent analysis checks
- L = 20, 390 MeV pions
- L = 32, 240 MeV pions
- Lattice Renormalization

## Feasible in the next year or two:

- Physical Point Calculation
- Chiral Fermion Calculation

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