Hyperon Nucleon Interactions and Dense Matter

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Cairns, Australia
... to make predictions for the structure and interactions of nuclei using lattice QCD.
Core-Collapse Supernova and the Heavy Elements

SN1987a

Black-Hole or Neutron Star?

(Nuclear EoS)

(Mezzacappa et al)

Wednesday, June 27, 2012
Core-Collapse Supernova and the Heavy Elements

(Mezzacappa et al)

\[ e^- \rightarrow K^- + \text{neutrino} \]
\[ n + e^- \rightarrow \Sigma^- + \text{neutrino} \]

Role depends upon Interactions

Nuclear EoS
Hyperons in Matter: Present Predictive Capability

Mean-Field Models

( Schaffner-Bielich (2010) )

(Whittenbury (2012))
Multi-Volume Study by NPLQCD
2009 - 2011

lattice spacing : $b \sim 0.123$ fm
pion mass : $m_\pi \sim 390$ MeV
fermion action : Clover
anisotropy : $\xi_t \sim 3.5$

$L \sim 2$ fm
$L \sim 2.5$ fm
$L \sim 3$ fm
$L \sim 4$ fm

resources : $\sim 80 \times 10^6$ core hrs
$m_\pi L \sim 4, 5, 6, 8 \quad m_\pi T \sim 9, 9, 9, 18$
Multi-Volume Study by NPLQCD  
2009 - 2011

<table>
<thead>
<tr>
<th>L</th>
<th>cfgs</th>
<th>srcs</th>
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<tr>
<td>24</td>
<td>2215</td>
<td>390,000</td>
</tr>
<tr>
<td>32</td>
<td>739</td>
<td>135,000</td>
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</table>

<table>
<thead>
<tr>
<th>$L^3 \times T$</th>
<th>$16^3 \times 128$</th>
<th>$20^3 \times 128$</th>
<th>$24^3 \times 128$</th>
<th>$32^3 \times 256$</th>
<th>Extrapolation</th>
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<tbody>
<tr>
<td>$L$ (fm)</td>
<td>$\sim 2.0$</td>
<td>$\sim 2.5$</td>
<td>$\sim 3.0$</td>
<td>$\sim 4.0$</td>
<td>$\infty$</td>
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<tr>
<td>$m_\pi L$</td>
<td>3.86</td>
<td>4.82</td>
<td>5.79</td>
<td>7.71</td>
<td>$\infty$</td>
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<tr>
<td>$m_\pi T$</td>
<td>8.82</td>
<td>8.82</td>
<td>8.82</td>
<td>17.64</td>
<td>$\infty$</td>
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<tr>
<td>$M_N$ (t.l.u.)</td>
<td>0.21004(44)(85)</td>
<td>0.20682(34)(45)</td>
<td>0.20463(27)(36)</td>
<td>0.20457(25)(38)</td>
<td>$0.20455(19)(17)$</td>
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<tr>
<td>$M_A$ (t.l.u.)</td>
<td>0.22446(45)(78)</td>
<td>0.22246(27)(38)</td>
<td>0.22074(20)(42)</td>
<td>0.22054(23)(31)</td>
<td>$0.22064(15)(19)$</td>
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<tr>
<td>$M_\Sigma$ (t.l.u.)</td>
<td>0.2286(38)(67)</td>
<td>0.22752(32)(43)</td>
<td>0.22791(24)(31)</td>
<td>0.22726(24)(43)</td>
<td>$0.22747(17)(19)$</td>
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<tr>
<td>$M_\Xi$ (t.l.u.)</td>
<td>0.24192(38)(63)</td>
<td>0.24101(27)(38)</td>
<td>0.23975(20)(32)</td>
<td>0.23974(17)(31)</td>
<td>$0.23978(12)(18)$</td>
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</tbody>
</table>
1) Maiani-Testa Theorem

2) Luscher : Measure energy-eigenvalues of the two-hadron system
Explicitly, the stationary effective Schrödinger equation in the centre-of-mass frame reads

\[
-\frac{1}{2\mu} \Delta \psi(r) + \frac{1}{2} \int d^3 r' \, U_E(r, r') \psi(r') = E \psi(r),
\]

where the parameter \( E \) is related to the true energy \( W \) of the system through

\[
W = 2\sqrt{m^2 + mE}.
\]

The "potential" \( U_E(r, r') \) is the Fourier transform of the modified Bethe-Salpeter kernel \( \tilde{U}_E(k, k') \) introduced in ref.[3]. It depends analytically on \( E \) in the range \(-m < E < 3m\) and is a smooth function of the coordinates \( r \) and \( r' \), decaying exponentially in all directions. Furthermore, the potential

Taking \( U \) to be energy-independent is a model-dependent assertion and not a QCD prediction.
Two-Particle Energy Levels (Luscher)

Below Inelastic Thresholds:
Measure on lattice

\[ \delta E = 2\sqrt{p^2 + m^2} - 2m \]

\[ p \cot \delta(p) = \frac{1}{\pi L} S \left( \left( \frac{Lp}{2\pi} \right)^2 \right) \]

\[ S(\eta) \equiv \sum_j^{\Lambda_j} \frac{1}{|j|^2 - \eta} - 4\pi \Lambda_j \]

Gives the scattering amplitude at \( \delta E \)
Luscher Eigenvalue Relation

Bound-state or Scattering state?

\[ p \cot \delta(p) = \frac{1}{\pi L} S \left( \left( \frac{Lp}{2\pi} \right)^2 \right) \]

Non-interacting particles

\[ V = 0 \quad \rightarrow \quad a = r = 0 \]
\[ S = \infty \]

\[ k = \frac{2\pi}{L} n \]
\[ n = (nx, ny, nz) \]
\( n \Sigma^- \) at a pion mass of \(~390\) MeV
Effective Field Theory introduced by Weinberg in the early 1990's to systematize nuclear forces

- Low-energy EFT of QCD
- Chiral symmetries of QCD
- Quark mass dependence
- Renormalization Group
- Softer Interactions
  - $V_{\text{lowk}}$, SRG

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<thead>
<tr>
<th></th>
<th>2N force</th>
<th>3N force</th>
<th>4N force</th>
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<td><strong>LO</strong></td>
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</table>
LO Weinberg in $n\Sigma^-$

1: local 4-baryon contact interaction (1 parameter)
2: one meson exchange (0 new parameters)
Convergence of (Systematic) Expansions of Potentials

Figure 3: Chiral expansion of the isovector-tensor (upper row) and isoscalar central (lower row) long-range potentials $\tilde{W}_T(r)$ and $\tilde{V}_C(r)$, respectively. The left (right) panel shows the results for the EFT without (with) explicit $\Delta(1232)$ degrees of freedom. The light-shaded band shows the estimation of the intrinsic model dependence associated with the short-range components as explained in the text (only shown for the theory without deltas).
Require regulation - use simple compact shape
- square-well, Gaussian, exponential
- one length scale, one coupling
$^3S_1$: 3-d Schrodinger, Modifications to Luscher’s Relation?

- diagonalize $H$ in full 3-d mom. space (constrained by $L$ and $a$)
- choose $R << 1/m_{\text{mes}}$ and fix $C_0$ to energy-eigenvalue
- extended repulsive core - Luscher?
- use both Luscher and Weinberg EFT
  - phase-shifts are same within uncertainties!
$^1S_0$: 3-d Schrödinger, Modifications to Luscher’s Relation?

Attractive core is compact - Luscher’s relation is valid
Hyperon Nucleon Interactions

Haidenbauer and Meissner (2007)
Hyperon Nucleon Interactions

Meissner+Haidenbauer - Experiment + YN-EFT (LO)
Hyperon Nucleon Interactions

NPLQCD - Lattice QCD + YN-EFT (LO)
Binding in the Singlet Channel

\[ B(\text{MeV}) \]

\[ m_\pi(\text{MeV}) \]

\[ \Sigma^1 S_0 \text{ LQCD stat} \]

\[ \Sigma^1 S_0 \text{ LQCD stat+sys} \]
\[ \Delta E = -\frac{1}{\pi \mu} \int_0^{k_f} dk \ k \left[ \frac{3}{2} \delta_{3S_1}(k) + \frac{1}{2} \delta_{1S_0}(k) \right] \]

Cancellation between channels in dense matter energy-shift of hyperon
SU(3) Limit - Isotropic Clover

\[ \Delta E (\text{MeV}) \]

- deuteron
- nn
- H–dib
- nΛ (1s0)
- nΛ (3s1)
- nΣ (1s0)
- nΣ (3s1)
- nΞ (3s1)
- pΞ (3s1)

NPLQCD: e-Print: arXiv:1206.5219 [hep-lat]
• LQCD calculations of $n\Sigma^-$
• Luscher and 3-d SE consistent
• Extrapolation with (W)EFT
• Consistent with phase-shift analysis of (limited) expt data
• Future LQCD calculations will supersede experiment
• Refine understanding of dense matter
  • Further theoretical developments required
Hyperon Nucleon Interactions

- LQCD + (W)EFT consistent with limited data sets
- Next generation of LQCD will be more precise than expt