Form factors for $\Lambda_b \to \Lambda$ transitions from lattice QCD

Stefan Meinel
The College of William and Mary

In collaboration with William Detmold, C.-J. David Lin, and Matthew Wingate

Lattice 2012, Cairns, Australia
Effective weak Hamiltonian for $b \rightarrow s$ transitions:

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} \left( C_i O_i + C'_i O'_i \right)$$

with

$$O_7 = \frac{e}{16\pi^2} m_b \bar{s} \sigma^{\mu\nu} P_R b \text{F}_{\mu\nu}^{(\text{e.m.)}}$$

$$O'_7 = \frac{e}{16\pi^2} m_b \bar{s} \sigma^{\mu\nu} P_L b \text{F}_{\mu\nu}^{(\text{e.m.)}}$$

$$O_9 = \frac{e^2}{16\pi^2} \bar{s} \gamma^\mu P_L b \bar{\ell} \gamma_\mu \ell,$$

$$O'_9 = \frac{e^2}{16\pi^2} \bar{s} \gamma^\mu P_L b \bar{\ell} \gamma_\mu \ell,$$

$$O_{10} = \frac{e^2}{16\pi^2} \bar{s} \gamma^\mu P_L b \bar{\ell} \gamma_\mu \gamma_5 \ell,$$

$$O'_{10} = \frac{e^2}{16\pi^2} \bar{s} \gamma^\mu P_R b \bar{\ell} \gamma_\mu \gamma_5 \ell.$$

In the Standard Model,

$$\frac{C'_7}{C_7} \approx \frac{m_s}{m_b}, \quad C'_9 = 0, \quad C'_{10} = 0.$$
Baryonic analogue of $B \rightarrow K^*\gamma$ and $B \rightarrow K^*\ell^+\ell^-$:

$$\Lambda_b \rightarrow \Lambda\gamma \quad \text{and} \quad \Lambda_b \rightarrow \Lambda\ell^+\ell^-$$  \hspace{1cm} (\Lambda_b \sim udb, \quad \Lambda \sim uds)

- $\Lambda_b \rightarrow \Lambda\mu^+\mu^-$ observed by CDF in 2011, new results expected from LHCb

- Unlike $K^* \rightarrow K\pi$, the secondary decay $\Lambda \rightarrow p\pi^-$ is a weak decay. Thanks to parity violation, the protons are preferentially emitted in the spin direction of the $\Lambda$ baryons: for completely polarized $\Lambda$,

$$\frac{dN}{d\Omega}[\Lambda \rightarrow p\pi^-] \propto (1 + a \, s_{\Lambda} \cdot \hat{p}_p), \quad \text{where} \quad a = 0.642(13). \quad [\text{PDG}]$$

- In $\Lambda_b \rightarrow \Lambda\gamma$ and $\Lambda_b \rightarrow \Lambda\ell^+\ell^-$ decays, different $\Lambda$ polarizations are sensitive to different combinations of $C_i$ and $C'_i$ [Mannel and Recksiegel 1997; Chen and Geng 2001]

Thus, $\Lambda_b \rightarrow \Lambda\gamma$ and $\Lambda_b \rightarrow \Lambda\ell^+\ell^-$ provide good sensitivity to the helicity structure of $\mathcal{H}_{\text{eff}}$. 
In general, 10 form factors

In the heavy-quark limit $m_b \to \infty$, only two independent form factors $F_1$ and $F_2$: [Mannel, Roberts, Ryzak, 1990]

$$\langle \Lambda(p', s') | \bar{s} \Gamma Q | \Lambda_Q(v, s) \rangle = \bar{u}(p', s') \left[ F_1 (p' \cdot v) + \gamma F_2 (p' \cdot v) \right] \Gamma u(v, s)$$

where $F_1$ and $F_2$ are functions of the $\Lambda$ energy in the $\Lambda_Q$ rest frame,

$$p' \cdot v = E_{\Lambda}$$

We also define

$$F_+ = F_1 + F_2,$$

$$F_- = F_1 - F_2$$
Lattice parameters

- Heavy quark: Eichten-Hill action with one level of HYP link smearing, \((\alpha_1, \alpha_2, \alpha_3) = (1.0, 1.0, 0.5)\) [Eichten and Hill 1990; Della Morte, Shindler, Sommer 2005]


- 2+1 flavor RBC/UKQCD ensembles [Aoki et al., 2011]

<table>
<thead>
<tr>
<th>(L^3 \times T)</th>
<th>(a_{s}^{\text{sea}})</th>
<th>(a_{u,d}^{\text{sea}})</th>
<th>(a) (fm)</th>
<th>(a_{s}^{\text{val}})</th>
<th>(a_{u,d}^{\text{val}})</th>
<th>(m_{\pi}^{(\text{vv})}) (MeV)</th>
<th>(N_{\text{meas}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>24(^3) \times 64</td>
<td>0.04</td>
<td>0.005</td>
<td>0.1119(17)</td>
<td>0.04</td>
<td>0.001</td>
<td>245(4)</td>
<td>2705</td>
</tr>
<tr>
<td>24(^3) \times 64</td>
<td>0.04</td>
<td>0.005</td>
<td>0.1119(17)</td>
<td>0.04</td>
<td>0.002</td>
<td>270(4)</td>
<td>2683</td>
</tr>
<tr>
<td>24(^3) \times 64</td>
<td>0.04</td>
<td>0.005</td>
<td>0.1119(17)</td>
<td>0.04</td>
<td>0.005</td>
<td>336(5)</td>
<td>2780</td>
</tr>
<tr>
<td>24(^3) \times 64</td>
<td>0.04</td>
<td>0.005</td>
<td>0.1119(17)</td>
<td>0.03</td>
<td>0.005</td>
<td>336(5)</td>
<td>1192</td>
</tr>
<tr>
<td>32(^3) \times 64</td>
<td>0.03</td>
<td>0.004</td>
<td>0.0849(12)</td>
<td>0.03</td>
<td>0.002</td>
<td>227(3)</td>
<td>1918</td>
</tr>
<tr>
<td>32(^3) \times 64</td>
<td>0.03</td>
<td>0.004</td>
<td>0.0849(12)</td>
<td>0.03</td>
<td>0.004</td>
<td>295(4)</td>
<td>1919</td>
</tr>
<tr>
<td>32(^3) \times 64</td>
<td>0.03</td>
<td>0.006</td>
<td>0.0848(17)</td>
<td>0.03</td>
<td>0.006</td>
<td>352(7)</td>
<td>2785</td>
</tr>
</tbody>
</table>
Form factors are defined in HQET. Thus, need continuum HQET current (not yet QCD current!)

\[
J^{(HQET)}_\Gamma(m_b) = U(m_b, a^{-1}) \cdot \mathcal{Z} \left[ \left( 1 + c^{(ma)} \frac{m_s a}{1 - (w_0^{MF})^2} \right) J^{(LHQET)}_\Gamma + c^{(pa)} a J^{(LHQET)}_{\Gamma_D} \right]
\]

with

\[
J^{(LHQET)}_\Gamma = \overline{Q} \Gamma s, \quad J^{(LHQET)}_{\Gamma_D} = \overline{Q} \Gamma \gamma \cdot \nabla s.
\]

LHQET → HQET matching coefficients are independent of \( \Gamma \), as required by heavy-quark symmetry.

We use one-loop results for \( \mathcal{Z} \), \( c^{(ma)} \), \( c^{(pa)} \) from [Ishikawa et al. 2011]
Three-point function

\[ C^{(3)}_{\delta \alpha} (\Gamma, \mathbf{p}', t, t') \]

\[ = \sum_y e^{-i\mathbf{p}' \cdot (\mathbf{x} - \mathbf{y})} \left\langle \Lambda_\delta (x_0, \mathbf{x}) \ J^{(\text{HQET})\dagger}_\Gamma (x_0 - t + t', \mathbf{y}) \ \bar{\Lambda}_{Q\alpha} (x_0 - t, \mathbf{y}) \right\rangle \]

Only need light-quark propagators with source at \((x_0, \mathbf{x})\).
Backward three-point function

\[ C_{\alpha \delta}^{(3,bw)}(\Gamma, \mathbf{p}', t, t-t') = \sum_y e^{-i \mathbf{p}' \cdot (\mathbf{y} - \mathbf{x})} \left\langle \Lambda_{Q \alpha}(x_0 + t, \mathbf{y}) \ J_{\Gamma}^{(\text{HQET})}(x_0 + t', \mathbf{y}) \ \bar{\Lambda}_\delta(x_0, \mathbf{x}) \right\rangle \]

Only need light-quark propagators with source at \((x_0, \mathbf{x})\).
Double ratios

For arbitrary $\Gamma$, define

$$R(\Gamma, p', t, t') = \frac{4 \text{Tr} \left[ C(3)(\Gamma, p', t, t') \ C(3,bw)(\Gamma, p', t, t-t') \right]}{\text{Tr}[C(2,\Lambda)(p', t)] \text{Tr}[C(2,\Lambda_b)(t)]}$$

Then form the combinations

$$R_+(p', t, t') = \frac{1}{4} \left[ R(1, p', t, t') + R(\gamma^2 \gamma^3, p', t, t') + R(-\gamma^3 \gamma^1, p', t, t') + R(\gamma^1 \gamma^2, p', t, t') \right],$$

$$R_-(p', t, t') = \frac{1}{4} \left[ R(\gamma^1, p', t, t') + R(\gamma^2, p', t, t') + R(\gamma^3, p', t, t') + R(-i \gamma_5, p', t, t') \right].$$

The spectral decomposition gives

$$R_+(p', t, t') = \frac{E_\Lambda + M_\Lambda}{E_\Lambda} [F_+]^2 + \text{(excited state contribs.)}$$

$$R_-(p', t, t') = \frac{E_\Lambda - M_\Lambda}{E_\Lambda} [F_-]^2 + \text{(excited state contribs.)}$$
Double ratios: example results at coarse lattice spacing

\[ p'^2 = 4 \times (2\pi/L)^2, \quad a = 0.112 \text{ fm}, \quad am^{(\text{val})}_s = 0.04, \quad am^{(\text{val})}_{u,d} = 0.005 \]
Double ratios: example results at fine lattice spacing

\[ p'^2 = 4 \times (2\pi/L)^2, \quad a = 0.085 \text{ fm}, \quad am_s^{(\text{val})} = 0.03, \quad am_{u,d}^{(\text{val})} = 0.004 \]

\[ t/a = 8 \quad t/a = 11 \quad t/a = 14 \]
Now compute

\[ R_+(p', t) = \sqrt{\frac{E_\Lambda}{E_\Lambda + M_\Lambda}} R_+(p', t, t/2) \]

\[ R_-(p', t) = \sqrt{\frac{E_\Lambda}{E_\Lambda - M_\Lambda}} R_-(p', t, t/2) \]

where \( E_\Lambda \) and \( M_\Lambda \) are from fits of the \( \Lambda \) two-point functions.

We know that

\[ F_\pm(p') = \lim_{t \to \infty} R_\pm(p', t). \]
$p' = 0$

$a = 0.112 \text{ fm}$

$a = 0.085 \text{ fm}$

$R_+ (p' = 0, t)$ has practically no $t$ dependence.
Extrapolation to infinite source-sink separation

\[ p' \, p' = 1 \times (2\pi/L)^2 \]

\[ a = 0.112 \text{ fm} \]

\[ a = 0.085 \text{ fm} \]

Fit using \( R_\pm(t) = F_\pm + a_\pm \exp[-\delta t] \)
Extrapolation to infinite source-sink separation

\[ p'^2 = 4 \times (2\pi/L)^2 \]

\[ a = 0.112 \text{ fm} \]

Fit using \( R_\pm(t) = F_\pm + a_\pm \exp[-\delta t] \)
Form factor results

$q^2$ computed using physical $\Lambda_b$ and $\Lambda$ masses.
Simple dipole fit, no chiral extrapolation.
Calculation of differential branching fraction

In the Standard Model, after RG running to $\mu = m_b$, and after integrating out the photon field,

$$
H_{\text{eff}}(q) = \frac{G_F \alpha_{\text{em}}}{\sqrt{2\pi}} V_{tb} V_{ts}^* \left[ \left( C_{9}^{\text{eff}} \bar{s} \gamma_\mu P_L b - C_{7}^{\text{eff}} \frac{2m_b}{q^2} i q^\nu \bar{s} \sigma_{\mu\nu} P_R b \right) \bar{\ell} \gamma^\mu \ell 
+ \left( C_{10}^{\text{eff}} \bar{s} \gamma_\mu P_L b \right) \bar{\ell} \gamma^\mu \gamma_5 \ell \right]
$$

where

$$
C_{7}^{\text{eff}} = -0.304, \\
C_{9}^{\text{eff}} = 4.211 + Y(q^2), \\
C_{10}^{\text{eff}} = -4.103
$$

Wilson coefficients from [Altmannshofer et al. 2011], based on [Buras and Münz 1995]
Calculation of differential branching fraction

$\mathcal{H}_{\text{eff}}$ is defined in QCD (in NDR, $\mu = m_b$). When computing matrix elements, insert HQET-to-QCD matching factors, which do depend on $\Gamma$.

$$J^{(\text{QCD})}_\Gamma(m_b) = C_\Gamma(m_b, m_b) J^{(\text{HQET})}_\Gamma(m_b)$$

<table>
<thead>
<tr>
<th>$\Gamma$</th>
<th>$C_\Gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1, \gamma_5$</td>
<td>$1 + 2\alpha_s/(3\pi)$</td>
</tr>
<tr>
<td>$\gamma_0, \gamma_0\gamma_5$</td>
<td>$1 - 2\alpha_s/(3\pi)$</td>
</tr>
<tr>
<td>$\gamma_j, \gamma_j\gamma_5$</td>
<td>$1 - 4\alpha_s/(3\pi)$</td>
</tr>
<tr>
<td>$\sigma_{\mu\nu}, \sigma_{\mu\nu}\gamma_5$</td>
<td>$1 - 4\alpha_s/(3\pi)$</td>
</tr>
</tbody>
</table>

One-loop results in NDR, for $v = 0$ [Eichten, Hill, 1989]

Thus, for example

$$\left\langle \Lambda \mid \bar{s}\gamma_0 b \mid \Lambda_Q \right\rangle_{\text{QCD}} = \left[ 1 - 2\alpha_s/(3\pi) \right] \underbrace{\bar{u}_\Lambda \left[ F_1 + \gamma_0 F_2 \right] \gamma_0 u_{\Lambda_Q}}_{= F_+ \quad \bar{u}_\Lambda u_{\Lambda_Q}}$$

$$\left\langle \Lambda \mid \bar{s}\gamma_j b \mid \Lambda_Q \right\rangle_{\text{QCD}} = \left[ 1 - 4\alpha_s/(3\pi) \right] \underbrace{\bar{u}_\Lambda \left[ F_1 + \gamma_0 F_2 \right] \gamma_j u_{\Lambda_Q}}_{= F_- \quad \bar{u}_\Lambda \gamma_j u_{\Lambda_Q}}.$$
Differential branching fraction

Without long-distance contributions, in Standard Model:

[Graph showing differential branching fraction as a function of $q^2$ (GeV$^2$) with data points and error bars.]

Only statistical uncertainty shown. There are $\mathcal{O}(\Lambda_{QCD}/m_b, |p_\Lambda|/m_b)$ errors from static approximation.
Conclusions

Summary:

- first lattice calculation of $\Lambda_b \rightarrow \Lambda$ form factors
- ratio method for extracting form factors works well
- results are surprisingly precise, future calculation with non-static $b$-quarks seems worthwhile

To Do:

- chiral/continuum extrapolation
- analysis of systematic uncertainties
- more phenomenology: $\Lambda$ polarization asymmetries ($\Lambda_b \rightarrow p\pi\ell^+\ell^-$ angular distribution)