# Continuum Light Hadronic Observables from 2+1 flavor DWF QCD

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# Recent RBC/UKQCD 2+1 flavor DWF ensembles



Current analysis uses all 3 ensembles

# Improving Domain Wall Fermions via DSDR

- When underlying gauge field changes topology, the DWF modes can extend farther in the fifth dimension
- This gives a non-perturbative contribution to residual chiral symmetry breaking
- Becomes problematic at strong coupling
- Add ratio of determinants of twisted Wilson fermions to suppress these gauge field dislocations
- Tune to minimize residual mass while still preserving toplogical ergodicity



$$\frac{\det \left[ D_W (-M + i\varepsilon_f \gamma^5)^{\dagger} D_W (-M + i\varepsilon_f \gamma^5) \right]}{\det \left[ D_W (-M + i\varepsilon_b \gamma^5)^{\dagger} D_W (-M + i\varepsilon_b \gamma^5) \right]} = \prod_i \frac{\lambda_i^2 + \varepsilon_f^2}{\lambda_i^2 + \varepsilon_b^2}$$

 $\lambda_i$  are eigenvalues of the Hermitian Wilson operator

• DSDR = Dislocation Suppressing Determinant Ratio

#### Force Gradient Integrator (FGI)

- Proposed by Clark and Kennedy. Implemented (and simplified) in CPS by Hantao Yin
- For  $16^3 \times 32 \times 16$  volumes, no speed-up compared to  $O(\delta \tau^2)$  Omelyan



- For larger volumes, where  $\delta H$  grows with volume, force gradient may be helpful
- Tests on  $48^3 \times 64 \times 16$  with 220 MeV pions using FGI and retuning Hasenbush masses, 184 minutes/accepted configuration went down to 108 minutes/accepted configuration.
- For DWF+ID ensembles analyzed here, lattice is  $32^3 \times 64 \times 32$ . For m = 0.001, FGI used with 5 intermediate Hasenbusch preconditioning masses, all at top integration level.

# m<sub>res</sub>, Input Masses, Reweighting Range for Strange Quark



 $m_{\pi}$  (unitary, degenerate quarks) and a  $^2$  for DWF ensembles

am <sub>res</sub>	0.000666(8)	0.00308(6)	0.001842(7)
L <sub>s</sub>	16	16	32
lightest input dynamical quark mass (am <sub>l</sub> )	0.004	0.005	0.001
input dynamical heavy quark mass	0.03	0.04	0.045
am <sub>s</sub> - am <sub>res</sub> (from fits)	0.0263(9)	0.0336(13)	0.0467(6)

#### Checking Scaling at Unphysical Masses



- Total light and strange quark masses, using bare quark normalization for 32 DWF+I
- Can check scaling at unphysical quark masses by interpolating/extrapolating data to masses where  $m_{ll}/m_{hhh}$  and  $m_{lh}/m_{hhh}$  are identical on different ensembles.

# Scaling at unphysical light quark mass



m<sub>lh</sub>/m<sub>hhh</sub>

0.98

• DWF+ID: 1/a = 1.37 GeV (RBC/UKQCD to appear)

(RDC/ORQCD to appear)  $\int_{0.94}^{0.96} \frac{1}{f_{lh}/m_{hhh}} \frac{1}{r_0 f_{ll}} \frac{1}{f_{lh}/m_{hh}}$ See few percent scaling errors from  $1/a = 1.73 \text{ GeV} \rightarrow \infty$ , with larger O(5%) errors from 1/a = 1.37 GeV

 $r_0 m_{11}$ 

 $m_{\parallel}/m_{\parallel}$ 

 $f_{ll}/f_{lh}$ 

Parameters in DWF+I and DWF+ID Global Fits

- Simultaneous fit to  $m_{\pi}^2$ ,  $m_K^2$ ,  $f_{\pi}$ ,  $f_K$ , and  $m_{\Omega}$ , with  $m_{\pi}$ ,  $m_K$  and  $m_{\Omega}$  chosen to be quantities without O(a<sup>2</sup>) corrections
- 18 Parameters in SU(2) chiral expansion:
  - \*  $m_{\pi}^2$  and  $f_{\pi}$ : 8 parameters 2 LO, 4 NLO, 2O(a<sup>2</sup>)
  - \*  $m_{K}^{2}$  and  $f_{K}$ : 6 parameters 2 LO, 4 NLO, 2O(a<sup>2</sup>)
  - \*  $m_{\Omega}$ : 1 LO, 1 NLO
- Fits also determine
  - \* 3 lattice spacings
  - \* 2 ratios of light quark mass renormalization factors
  - \* 2 ratios of strange quark mass renormalization factors
  - \* m<sub>s</sub>
- Only use SU(2) ChPT to NLO
- Also do analytic fits to compare with ChPT and to help estimate chiral extrapolation errors

#### Global Fits to Multiple Ensembles

• Fit  $m_{\pi}^2$ ,  $f_{\pi}$ ,  $m_K^2$ ,  $f_K$  and  $m_{\Omega}$  to an expansion in powers of  $a^2$  and  $m_{l_1}$  including SU(2) logs where appropriate. Examples are

$$m_{ll}^{2} = \chi_{l} \left[ 1 + c_{B}a^{2} \right] + \chi_{l} \cdot \left\{ \frac{16}{f^{2}} \left( (2L_{8}^{(2)} - L_{5}^{(2)}) + 2(2L_{6}^{(2)} - L_{4}^{(2)}) \right) \chi_{l} + \frac{1}{16\pi^{2}f^{2}} \chi_{l} \log \frac{\chi_{l}}{\Lambda_{\chi}^{2}} \right\}$$
$$f_{ll} = f \left[ 1 + c_{f}a^{2} \right] + f \cdot \left\{ \frac{8}{f^{2}} (2L_{4}^{(2)} + L_{5}^{(2)}) \chi_{l} - \frac{\chi_{l}}{8\pi^{2}f^{2}} \log \frac{\chi_{l}}{\Lambda_{\chi}^{2}} \right\}.$$

- Note different  $O(a^2)$  coefficients used for DWF+I and DWF+ID
- Fit all partially quenched data, including SU(2) ChPT finite volume corrections in fit
- Reweight data from simulation  $m_h$  to self-consistently determined  $m_s$  (Jung)
- Interpolate valence propagators to self-consistently determined m<sub>s</sub>
- Use  $m_{\pi} m_{K}$  and  $m_{\Omega}$  set scale.

 $m_{\pi}^2/m_f$  versus  $m_f$ 



- Early fits from partial DWF+ID dataset
- Data consistent with chiral logarithms

# $m_{\pi}^2/m_f$ versus $m_f$









# Chiral Extrapolation for $f_{\pi}$

- DWF+ID ensemble gives results for much smaller quark masses.
- Can drop pion masses above 350 MeV for ChPT and still do fits.
- Can drop pion masses above 260 MeV for analytic fits and still do them.
- ChPT and analytic agree if pion masses below 260 MeV are used.
- $f_{\pi}$  now much closer to physical value than with 2010 analysis



# Chiral Extrapolation for $B_{K}$

- DWF+ID data rise slightly for light quarks
- Factor of 0.906(3) between normalization ۲ in graphs of 2010 and current analysis





2010 analysis

# Some physical results

DWF+I (2010 Analysis)	DWF+I and DWF+ID
$f_{\pi}^{\text{continuum}} = 124(2)(5) \text{MeV}$	$f_{\pi} = 127.1(2.7)(0.7)(2.5)$ MeV,
$f_K^{\text{continuum}} = 149(2)(4) \mathrm{MeV}$	$f_K = 152.4(3.0)(0.1)(1.5)$ MeV,
$(f_K/f_\pi)^{\text{continuum}} = 1.204(7)(25),$	$f_K/f_{\pi} = 1.1991(116)(69)(116).$
$m_{ud}^{\overline{\text{MS}}}(2\text{GeV}) = (3.59\pm0.21)\text{MeV}$	$m_{u/d}(\overline{\text{MS}}, 3 \text{ GeV}) = 3.05(8)(6)(1)(4) \text{ MeV}$
$m_s^{\overline{\mathrm{MS}}}(2\mathrm{GeV}) = (96.2\pm2.7)\mathrm{MeV}$	$m_s(\overline{\text{MS}}, 3 \text{ GeV}) = 83.6(1.7)(0.7)(0.4)(1.0) \text{ MeV}$
$\frac{m_s}{m_{ud}} = 26.8(0.8)_{\text{stat}}(1.1)_{\text{sys}}.$	$\frac{m_s}{m_{u/d}} = 27.36(39)(30)(22)(0)$
$\hat{m}_{ud} = 9.34(34)(31)(16)(21) \mathrm{MeV},$	$\hat{m}_{ud} = 8.77(23)(17)(3)(12) \mathrm{MeV},$
$\hat{m}_s = 250.2(3.9)(0.5)(0.3)(5.5) \mathrm{MeV}.$	$\hat{m}_s = 240.5(4.9)(2.0)(1.2)(2.9) \mathrm{MeV},$
$B_K(\overline{\text{MS}}, 3 \text{ GeV}) = 0.529(5)(15)(2)(11)$	$B_K(\overline{\text{MS}}, 3 \text{ GeV}) = 0.535(8)(7)(3)(11)$ (stat, chiral, finite V, pert. theory)

#### RBC/UKQCD 2+1 flavor DWF ensembles

