Continuum Light Hadronic Observables from 2+1 flavor DWF QCD

Lattice 2014
Cairns, Australia
June 28, 2012

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Special Acknowledgement to Chris Kelly for performing most of the final data analysis
Recent RBC/UKQCD 2+1 flavor DWF ensembles

$m_\pi$ (unitary, degenerate quarks) and $a^2$ for DWF ensembles

2010 analysis
Phys.Rev. D83 (2011) 074508
Phys.Rev. D84 (2011) 014503

2008 analysis

Current analysis uses all 3 ensembles
Improving Domain Wall Fermions via DSDR

- When underlying gauge field changes topology, the DWF modes can extend farther in the fifth dimension.
- This gives a non-perturbative contribution to residual chiral symmetry breaking.
- Becomes problematic at strong coupling.
- Add ratio of determinants of twisted Wilson fermions to suppress these gauge field dislocations.
- Tune to minimize residual mass while still preserving topological ergodicity.

\[
\frac{\det[DW(-M + i\varepsilon_f \gamma^5)\dagger DW(-M + i\varepsilon_f \gamma^5)]}{\det[DW(-M + i\varepsilon_b \gamma^5)\dagger DW(-M + i\varepsilon_b \gamma^5)]} = \prod_i \frac{\lambda_i^2 + \varepsilon_f^2}{\lambda_i^2 + \varepsilon_b^2}
\]

\(\lambda_i\) are eigenvalues of the Hermitian Wilson operator.

- DSDR = Dislocation Suppressing Determinant Ratio.
Force Gradient Integrator (FGI)

- Proposed by Clark and Kennedy. Implemented (and simplified) in CPS by Hantao Yin
- For $16^3 \times 32 \times 16$ volumes, no speed-up compared to $O(\delta\tau^2)$ Omelyan

For larger volumes, where $\delta H$ grows with volume, force gradient may be helpful

Tests on $48^3 \times 64 \times 16$ with 220 Mev pions using FGI and retuning Hasenbush masses, 184 minutes/accepted configuration went down to 108 minutes/accepted configuration.

For DWF+ID ensembles analyzed here, lattice is $32^3 \times 64 \times 32$. For $m = 0.001$, FGI used with 5 intermediate Hasenbusch preconditioning masses, all at top integration level.
### Input Masses, Reweighting Range for Strange Quark

<table>
<thead>
<tr>
<th>Parameter</th>
<th>DWF+I, (2.76 fm)$^3$</th>
<th>DWF+I, (2.75 fm)$^3$</th>
<th>DWF+ID, (4.6 fm)$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$am_{\text{res}}$</td>
<td>0.000666(8)</td>
<td>0.00308(6)</td>
<td>0.001842(7)</td>
</tr>
<tr>
<td>$L_s$</td>
<td>16</td>
<td>16</td>
<td>32</td>
</tr>
<tr>
<td>Lightest input dynamical quark mass ($am_l$)</td>
<td>0.004</td>
<td>0.005</td>
<td>0.001</td>
</tr>
<tr>
<td>Input dynamical heavy quark mass</td>
<td>0.03</td>
<td>0.04</td>
<td>0.045</td>
</tr>
<tr>
<td>$am_s - am_{\text{res}}$ (from fits)</td>
<td>0.0263(9)</td>
<td>0.0336(13)</td>
<td>0.0467(6)</td>
</tr>
</tbody>
</table>
Checking Scaling at Unphysical Masses

- Total light and strange quark masses, using bare quark normalization for 32 DWF+I
- Can check scaling at unphysical quark masses by interpolating/extrapolating data to masses where $m_{ll}/m_{hhh}$ and $m_{lh}/m_{hhh}$ are identical on different ensembles.
Scaling at unphysical light quark mass

Compare
- DWF+I: $1/a = 2.28$ GeV
- DWF+I: $1/a = 1.73$ GeV
(Phys. Rev. D83 (2011) 074508)

Compare
- DWF+I: $1/a = 2.28$ GeV
- DWF+ID: $1/a = 1.37$ GeV
(RBC/UKQCD to appear)

See few percent scaling errors from $1/a = 1.73$ GeV $\rightarrow \infty$, with larger $O(5\%)$ errors from $1/a = 1.37$ GeV
Parameters in DWF+I and DWF+ID Global Fits

- Simultaneous fit to $m_{\pi^2}$, $m_{K^2}$, $f_{\pi}$, $f_{K}$, and $m_{\Omega}$, with $m_{\pi}$, $m_{K}$ and $m_{\Omega}$ chosen to be quantities without $O(a^2)$ corrections.

- 18 Parameters in SU(2) chiral expansion:
  
  - $m_{\pi^2}$ and $f_{\pi}$: 8 parameters – 2 LO, 4 NLO, 2O($a^2$)
  - $m_{K^2}$ and $f_{K}$: 6 parameters – 2 LO, 4 NLO, 2O($a^2$)
  - $m_{\Omega}$: 1 LO, 1 NLO

- Fits also determine
  
  - 3 lattice spacings
  - 2 ratios of light quark mass renormalization factors
  - 2 ratios of strange quark mass renormalization factors
  - $m_s$

- Only use SU(2) ChPT to NLO

- Also do analytic fits to compare with ChPT and to help estimate chiral extrapolation errors
Global Fits to Multiple Ensembles

- Fit $m_{\pi}^2, f_{\pi}, m_{K}^2, f_{K}$ and $m_{\Omega}$ to an expansion in powers of $a^2$ and $m_l$, including SU(2) logs where appropriate. Examples are

\[
m_{ll}^2 = \chi_l \left[ 1 + \frac{c_B a^2}{f^2} \right] + \chi_l \cdot \left\{ \frac{16}{f^2} \left( 2L_8^{(2)} - L_5^{(2)} \right) + 2\left( 2L_6^{(2)} - L_4^{(2)} \right) \right\} \chi_l + \frac{1}{16\pi^2 f^2} \chi_l \log \frac{\chi_l}{\Lambda^2_{\chi}} \right\} \]

\[
f_{ll} = f \left[ 1 + \frac{c_f a^2}{f^2} \right] + f \cdot \left\{ \frac{8}{f^2} \left( 2L_4^{(2)} + L_5^{(2)} \right) \chi_l - \frac{\chi_l}{8\pi^2 f^2} \log \frac{\chi_l}{\Lambda^2_{\chi}} \right\} \right\} \]

- Note different $O(a^2)$ coefficients used for DWF+I and DWF+ID
- Fit all partially quenched data, including SU(2) ChPT finite volume corrections in fit
- Reweight data from simulation $m_h$ to self-consistently determined $m_s$ (Jung)
- Interpolate valence propagators to self-consistently determined $m_s$
- Use $m_{\pi} m_{K}$ and $m_{\Omega}$ set scale.
Degenerate $m^2_{\pi}/m_x$ versus $m_x = m_y$  
NLO + FVglobal fit, physical match point  
NLO + FV curves plotted, original data

Degenerate $m^2_{\pi}/m_x$ versus $m_x = m_y$  
NLO + FVglobal fit, physical match point  
NLO + FV curves plotted, original data

• Early fits from partial DWF+ID dataset  
• Data consistent with chiral logarithms
$m_\pi^2/m_f$ versus $m_f$
Chiral Extrapolation for $f_\pi$

- DWF+ID ensemble gives results for much smaller quark masses.
- Can drop pion masses above 350 MeV for ChPT and still do fits.
- Can drop pion masses above 260 MeV for analytic fits and still do them.
- ChPT and analytic agree if pion masses below 260 MeV are used.
- $f_\pi$ now much closer to physical value than with 2010 analysis
Chiral Extrapolation for $B_K$

- DWF+ID data rise slightly for light quarks
- Factor of 0.906(3) between normalization in graphs of 2010 and current analysis
Some physical results

<table>
<thead>
<tr>
<th>DWF+I (2010 Analysis)</th>
<th>DWF+I and DWF+ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_\pi^{\text{continuum}} = 124(2)(5)\text{ MeV}$</td>
<td>$f_\pi = 127.1(2.7)(0.7)(2.5)\text{ MeV}$,</td>
</tr>
<tr>
<td>$f_K^{\text{continuum}} = 149(2)(4)\text{ MeV}$</td>
<td>$f_K = 152.4(3.0)(0.1)(1.5)\text{ MeV}$,</td>
</tr>
<tr>
<td>$m_{\text{ud}}^{\overline{\text{MS}}}(2\text{ GeV}) = (3.59 \pm 0.21)\text{ MeV}$</td>
<td>$m_{u/d}^{\overline{\text{MS}}, 3\text{ GeV}} = 3.05(8)(6)(1)(4)\text{ MeV}$,</td>
</tr>
<tr>
<td>$m_s^{\overline{\text{MS}}}(2\text{ GeV}) = (96.2 \pm 2.7)\text{ MeV}$</td>
<td>$m_s^{\overline{\text{MS}}, 3\text{ GeV}} = 83.6(1.7)(0.7)(0.4)(1.0)\text{ MeV}$,</td>
</tr>
<tr>
<td>$m_s/m_{\text{ud}} = 26.8(0.8)<em>{\text{stat}}(1.1)</em>{\text{sys}}$</td>
<td>$m_s/m_{u/d} = 27.36(39)(30)(22)(0)$ .</td>
</tr>
<tr>
<td>$\hat{m}_{\text{ud}} = 9.34(34)(31)(16)(21)\text{ MeV}$,</td>
<td>$\hat{m}_{\text{ud}} = 8.77(23)(17)(3)(12)\text{ MeV}$,</td>
</tr>
<tr>
<td>$\hat{m}_s = 250.2(3.9)(0.5)(0.3)(5.5)\text{ MeV}$</td>
<td>$\hat{m}_s = 240.5(4.9)(2.0)(1.2)(2.9)\text{ MeV}$,</td>
</tr>
<tr>
<td>$B_K^{\overline{\text{MS}}, 3\text{ GeV}} = 0.529(5)(15)(2)(11)$</td>
<td>$B_K^{\overline{\text{MS}}, 3\text{ GeV}} = 0.535(8)(7)(3)(11)$ (stat, chiral, finite V, pert. theory)</td>
</tr>
</tbody>
</table>
RBC/UKQCD 2+1 flavor DWF ensembles

Thermalizing on BNL BGQ

T=0 ensemble as part of finite T DWF with HotQCD

Proposed

Thermalizing on BNL BGQ