Neutral *B* mixing
The Standard Model and Beyond

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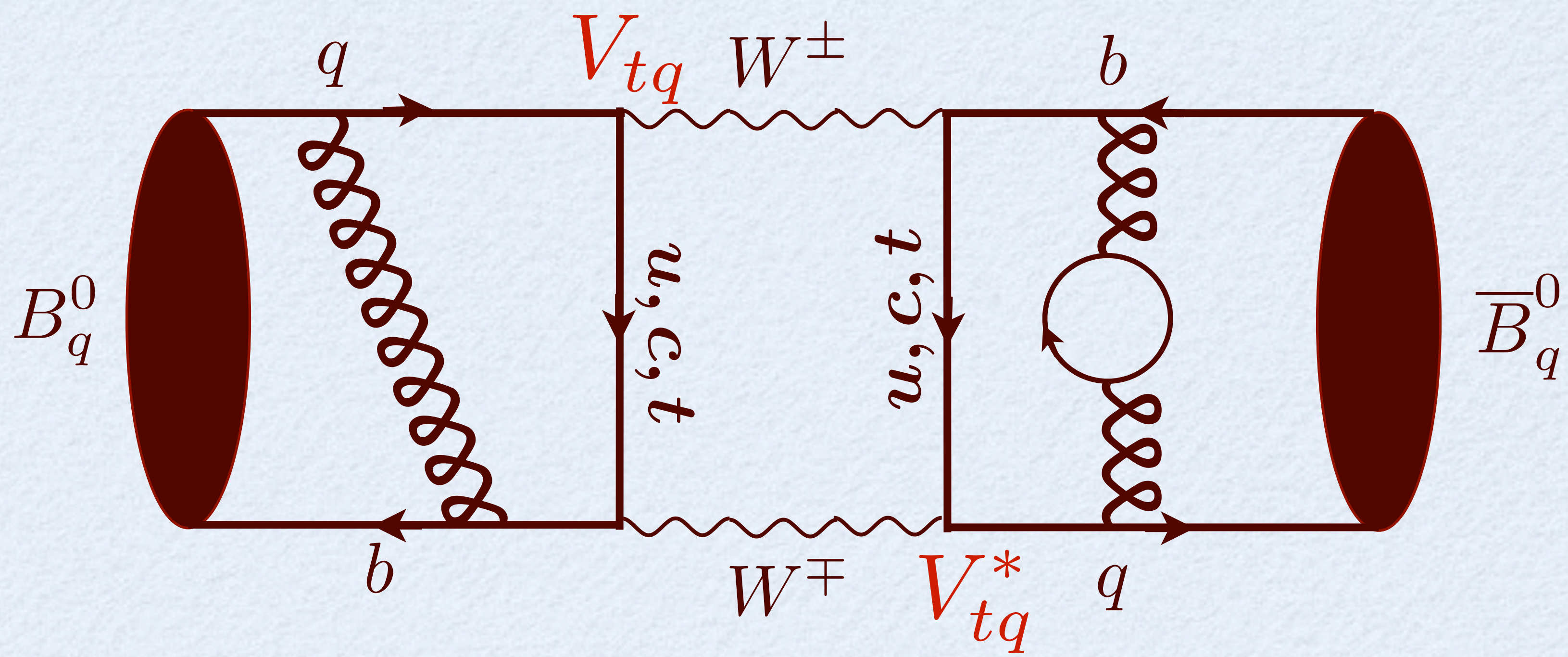
A.S. Kronfeld, J. Laiho, and R.S. Van de Water
for the Fermilab Lattice and MILC Collaborations

Neutral B mixing

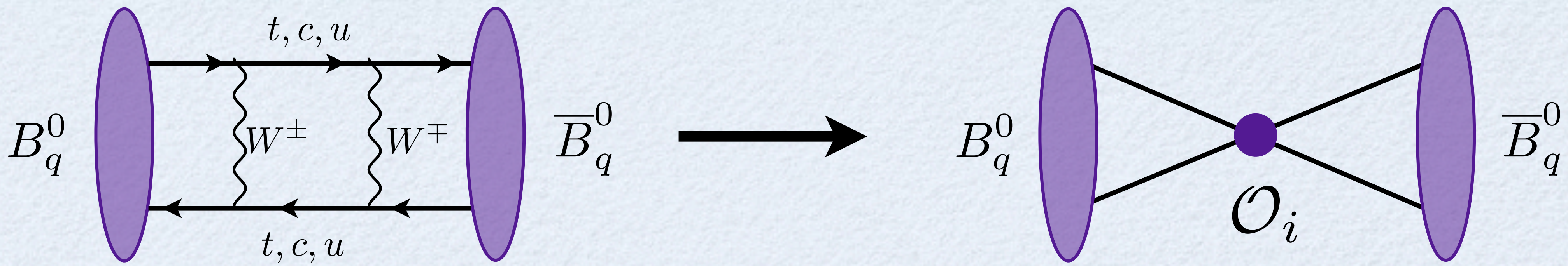


In the Standard Model, B_q^0 mixing is suppressed

- loop process,
- diagram with m_t dominates,
- but is Cabibbo suppressed

$\Rightarrow$ "Relatively" easy for new physics to cause observable effects.

Matrix Elements



$$\mathcal{H}_{\text{eff}} = \sum_{i=1}^{5} C_i \mathcal{O}_i$$

Operators are

SM

$$\mathcal{O}_1 = (\bar{b}^\alpha \gamma_\mu L q^\alpha)\, (\bar{b}^\beta \gamma_\mu L q^\beta)$$

$$\mathcal{O}_2 = (\bar{b}^\alpha L q^\alpha)\, (\bar{b}^\beta L q^\beta)$$

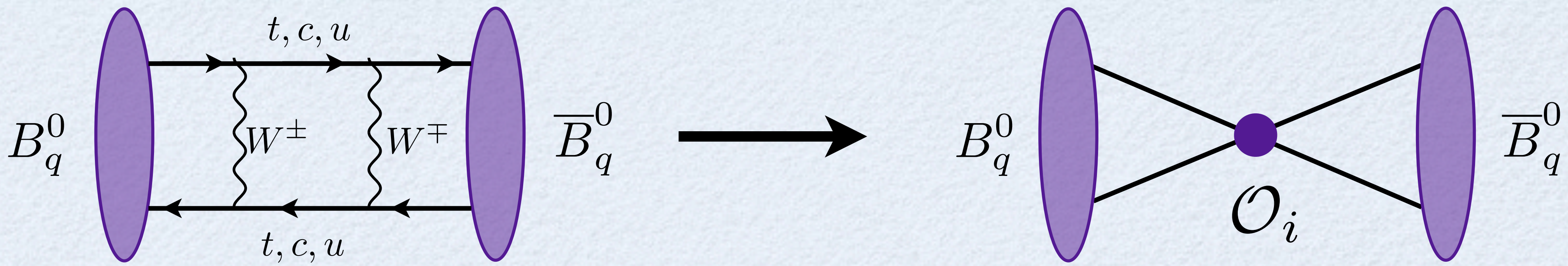
$$\mathcal{O}_3 = (\bar{b}^\alpha L q^\beta)\, (\bar{b}^\beta L q^\alpha)$$

$$\mathcal{O}_4 = (\bar{b}^\alpha L q^\alpha)\, (\bar{b}^\beta R q^\beta)$$

$$\mathcal{O}_5 = (\bar{b}^\alpha L q^\beta)\, (\bar{b}^\beta R q^\alpha)$$

BSM

Matrix Elements



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$$\mathcal{O}_5 = (\bar{b}^\alpha L q^\beta)\,(\bar{b}^\beta R q^\alpha)$$

Common parametrization

$$\langle \bar{B}_q^0 | \mathcal{O}_i(\mu) | B_q^0 \rangle \propto f_{B_q}^2 B_i(\mu)$$

$f_{B_q}^2$ $\quad$ $B_i(\mu)$

Experiment and SM: ΔM_q

$$\underbrace{\Delta M_q}_{\text{experiment}} = \underbrace{\left(\frac{G_F^2 M_W^2 S_0}{4\pi^2}\right)\eta_B(\mu)}_{\text{known}} \underbrace{|V_{tb}V_{tq}^*|^2}_{\text{want}} \underbrace{\langle \bar{B}_q^0|\mathcal{O}_1(\mu)|B_q^0\rangle}_{\text{need from lattice}}$$

experiment $< 1\%$

Our ability to constrain $|V_{tb}V_{tq}^*|^2$, is limited by $\langle \bar{B}_q^0|\mathcal{O}_1(\mu)|B_q^0\rangle$.

“Tension” in the CKM matrix.

Lenz et al., arXiv: 1203:0238; Laiho et al. PhysRevD. 81, 034503, and end-of-2011 update

Experiment and SM: ΔM_q

$$\frac{\Delta M_s}{\Delta M_d} = \left|\frac{V_{ts}}{V_{td}}\right|^2 \frac{\langle \bar{B}_s^0|\mathcal{O}_1(\mu)|B_s^0\rangle}{\langle \bar{B}_d^0|\mathcal{O}_1(\mu)|B_d^0\rangle} \equiv \left|\frac{V_{ts}}{V_{td}}\right|^2 \frac{M_{B_s}}{M_{B_d}}\xi^2$$

experiment want lattice

Experiment and SM: ΔM_q

SU(3)-breaking ratio

$$\frac{\Delta M_s}{\Delta M_d} = \left|\frac{V_{ts}}{V_{td}}\right|^2 \frac{\langle \bar{B}^0_s|\mathcal{O}_1(\mu)|B^0_s\rangle}{\langle \bar{B}^0_d|\mathcal{O}_1(\mu)|B^0_d\rangle} \equiv \left|\frac{V_{ts}}{V_{td}}\right|^2 \frac{M_{B_s}}{M_{B_d}} \xi^2$$

experiment

want

lattice

- Some (lattice) errors cancel in the ratio of matrix elements,

- In CKM matrix fits, use of ξ can aid in minimizing correlations between lattice inputs.

Experiment and SM: $\Delta\Gamma_q$

Recent experimental results are putting focus on $\Delta\Gamma$.

(Lenz et al., arXiv:1203.0238; Haisch, Moriond 2012)

$$\Delta\Gamma_q = \left[G_1 \langle \bar{B}_q^0 | \mathcal{O}_1(\mu) | B_q^0 \rangle + G_3 \langle \bar{B}_q^0 | \mathcal{O}_3(\mu) | B_q^0 \rangle \right] \cos\phi_q + O(1/m_b, \alpha_s)$$

dominates

also needed

Lenz, Nierste JHEP 0706:072, 2007 hep-ph/0612167
Beneke, Buchalla, Dunietz, PRD 54:4419, 1996,
Erratum-ibid.D 83 119902 (2011); hep-ph/9605259v1

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$\Delta\Gamma/\Delta M$ yields $\langle \mathcal{O}_3 \rangle / \langle \mathcal{O}_1 \rangle$

$\mathcal{O}_R \equiv \mathcal{O}_2 + \mathcal{O}_3 + (1/2)\mathcal{O}_1$ useful for estimating $1/m_b$ errors.

Experiment and BSM: ΔM_q

$$\underbrace{\Delta M_q}_{\text{experiment}} = \sum_{i=1}^{5} \underbrace{C_i(\mu)}_{\text{model dependent}} \underbrace{\langle B_q^0 | \mathcal{O}_i(\mu) | \overline{B}_q^0 \rangle}_{\text{need from lattice}}$$

experiment

< 1%

model dependent

need from lattice

Including BSM contributions, ΔM_q takes the generic form above.

Lattice values of (matrix elements of) $\mathcal{O}_1$ through $\mathcal{O}_5$ are needed to check that a given BSM model is consistent with experiment.

Status of Lattice

Status of Lattice: $\mathcal{O}_1$

FNAL-MILC

$\xi = 1.268(63) \quad 5.0\%$

arXiv: 1205.7013, submitted to PRD

Source of uncertainty	Error (%)
Statistics $\oplus$ light-quark disc. $\oplus$ chiral extrapolation	3.7
Mixing with wrong-spin operators	3.2
Heavy-quark discretization	0.3
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Light-quark masses	0.5
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Tuning κ_b	0.4
Finite volume	0.1
Mistuned coarse u_0	0.1
Total Error	5.0

Status of Lattice: $\mathcal{O}_1$

HPQCD Gamiz *et al*., Phys.Rev.D80:014503, 2009, arXiv:0902.1815

$\xi = 1.258(33)$ 2.6%

$f_{B_d}\sqrt{\hat{B}_{B_d}} = 216(15)\ \mathrm{MeV}$ 6.8%

$f_{B_s}\sqrt{\hat{B}_{B_s}} = 266(18)\ \mathrm{MeV}$ 6.9%

RBC Albertus *et al*., Phys.Rev.D82:014505, 2010, arXiv:1001.2023

$\xi = 1.13(12)$ 11%

domain-wall test calculation; one, 0.11 fm, lattice spacing

Status of Lattice: $\mathcal{O}_1$

HPQCD

0902.1815

$f_{B_d}\sqrt{}$

$f_{B_s}\sqrt{}$

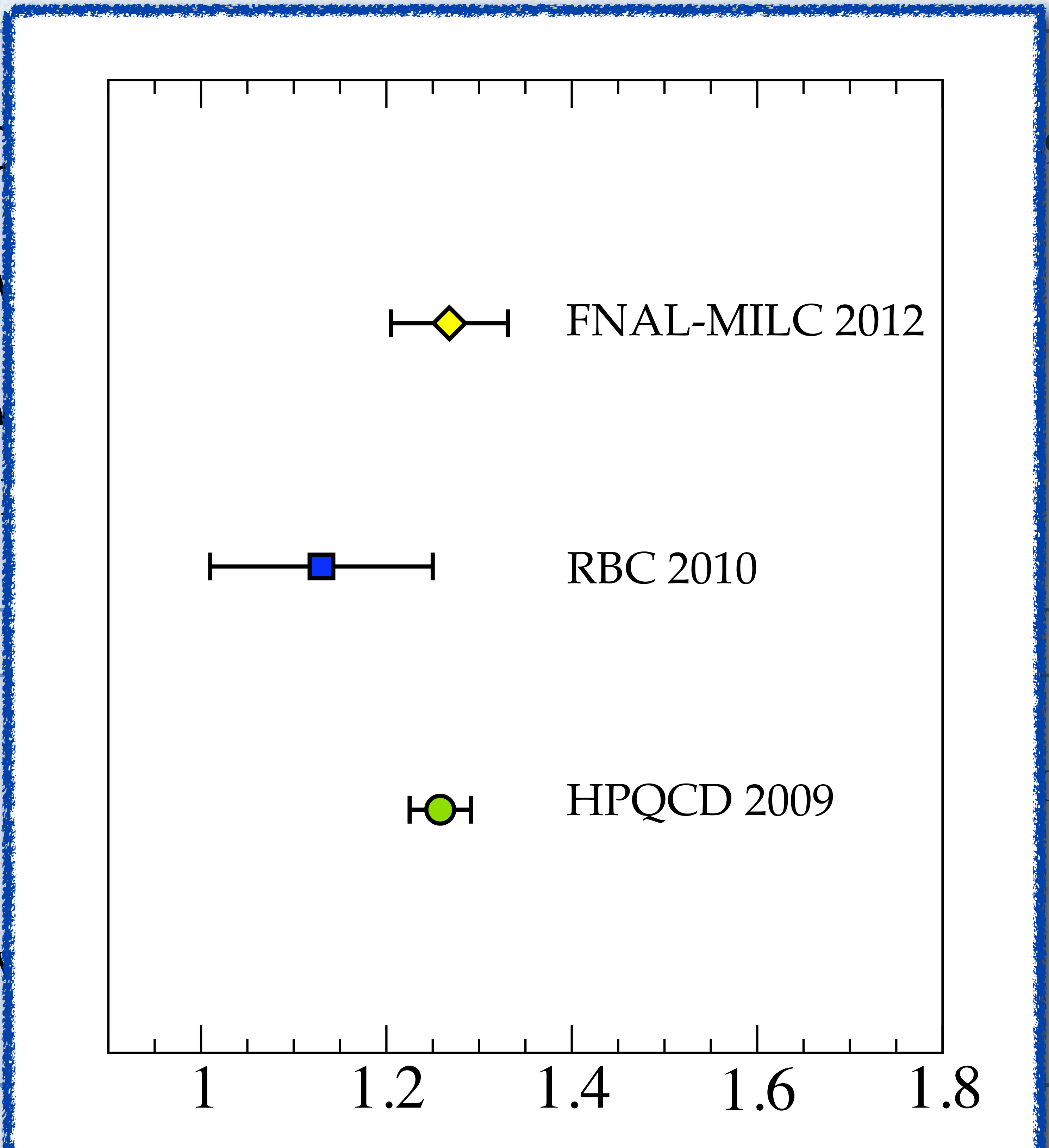
$\xi =$

RBC

01.2023

$\xi =$

domain-w



Status of Lattice: $\mathcal{O}_{1...5}$

Quenched:
Becirevic et al., JEHP 0204 (2002) 0250

Two ensembles: $\mathcal{O}_{2,3}$
E. Dalgic et al., PRD76:011501, 2007

Preliminary

FNAL-MILC: Lattice 11 Proceedings (Dec 2011), arXiv:1112:5642

	B_d^0		B_s^0	
[GeV^2]	BBGLN	BJU	BBGLN	BJU
$f_{B_q}^2 B_{B_q}^{(1)}$	0.0411(75)		0.0559(68)	
$f_{B_q}^2 B_{B_q}^{(2)}$	0.0574(92)	0.0538(87)	0.086(11)	0.080(10)
$f_{B_q}^2 B_{B_q}^{(3)}$	0.058(11)	0.058(11)	0.084(13)	0.084(13)
$f_{B_q}^2 B_{B_q}^{(4)}$	0.093(10)		0.135(15)	
$f_{B_q}^2 B_{B_q}^{(5)}$	0.127(15)		0.178(20)	

Estimated

9 to 6% on $\mathcal{O}_1$

$f_{B_q}\sqrt{\hat{B}_{B_q}}$

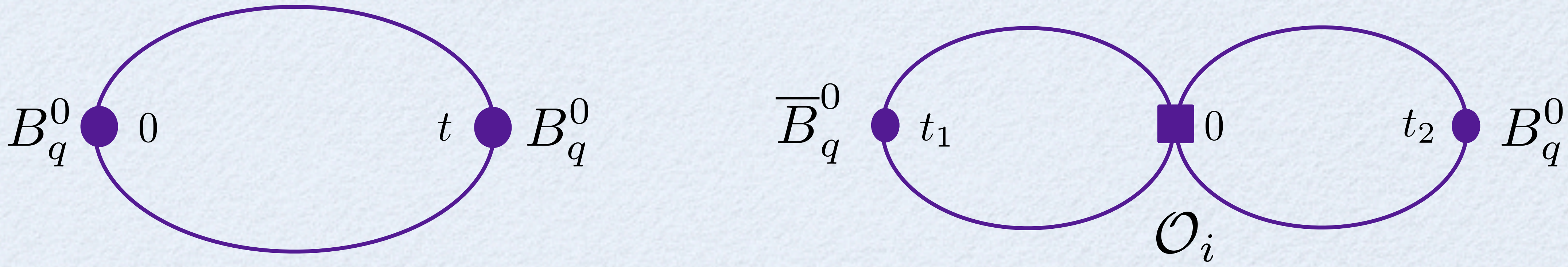
ETMC also working on this.

Details of Our Calculation

Previous vs current analysis

- Ensembles
  - more ensembles
  - higher statistics
  - smaller lattice spacing
  - smaller light-quark mass
- Results
  - full set of matrix elements
  - bag parameters in conjunction with f_B analysis
  - able to do all ratios and combinations (Ethan Neil's talk)
- Use complete ChiPT expression
  - This alone improves the error on ξ from 5.0% to 3.8%.

Analysis Overview



- Generate two- and three-points correlator data.
- Fit 2pt+3pt correlators simultaneously for each meson.
- Renormalize & match the matrix elements.
- Do a chiral and continuum extrapolation for each matrix element.

Actions and Ensembles

MILC (asqtad) gauge configurations

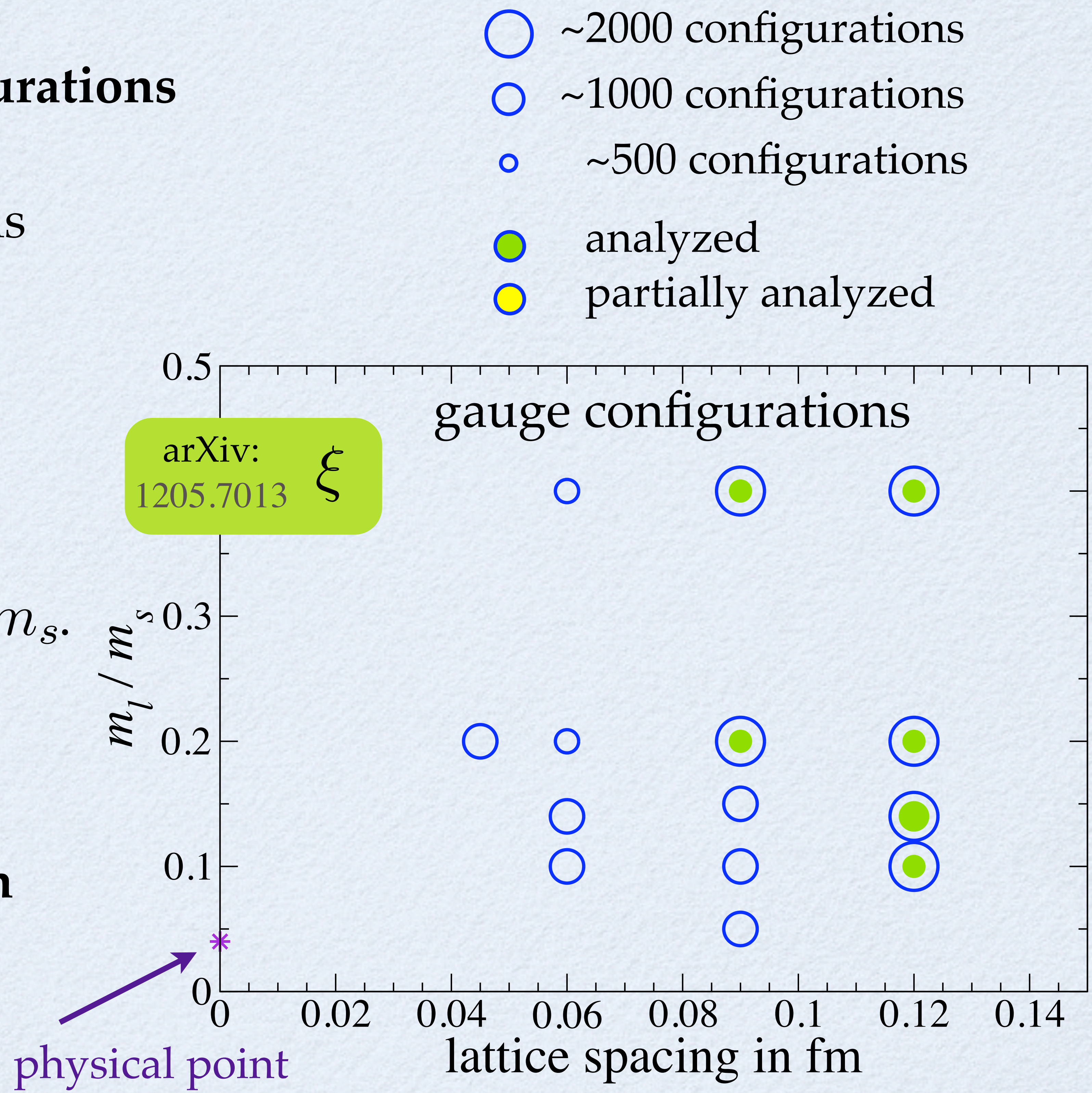
- 2+1 asqtad sea quarks,
- tadpole improved gluons
- m_l/m_s from 0.4 to 0.05

light valence quark

- **staggered action**
- mass from $>m_s$ to 0.05 m_s.

heavy valence quark

- **improved Wilson action**
- **Fermilab interpretation**



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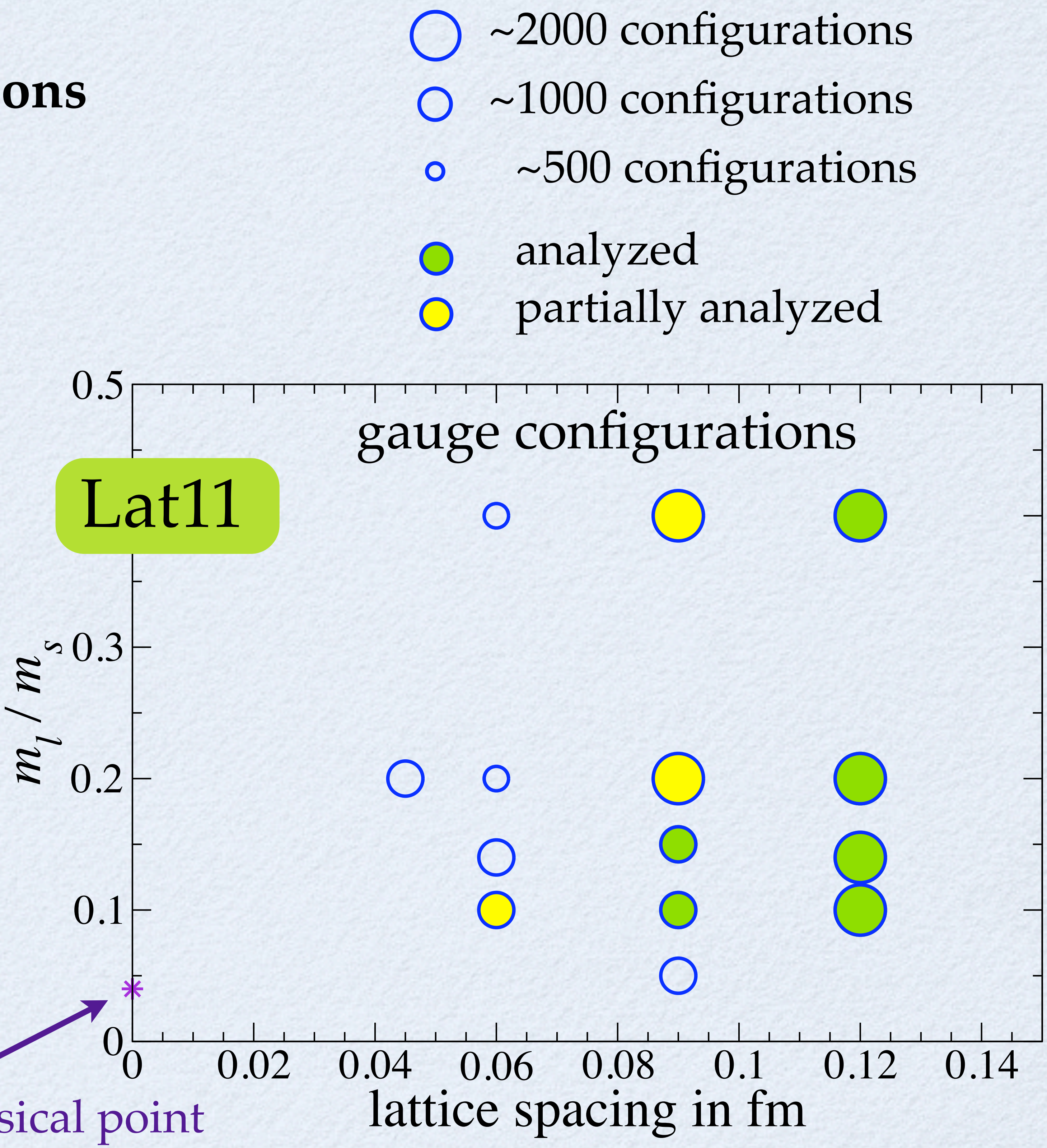

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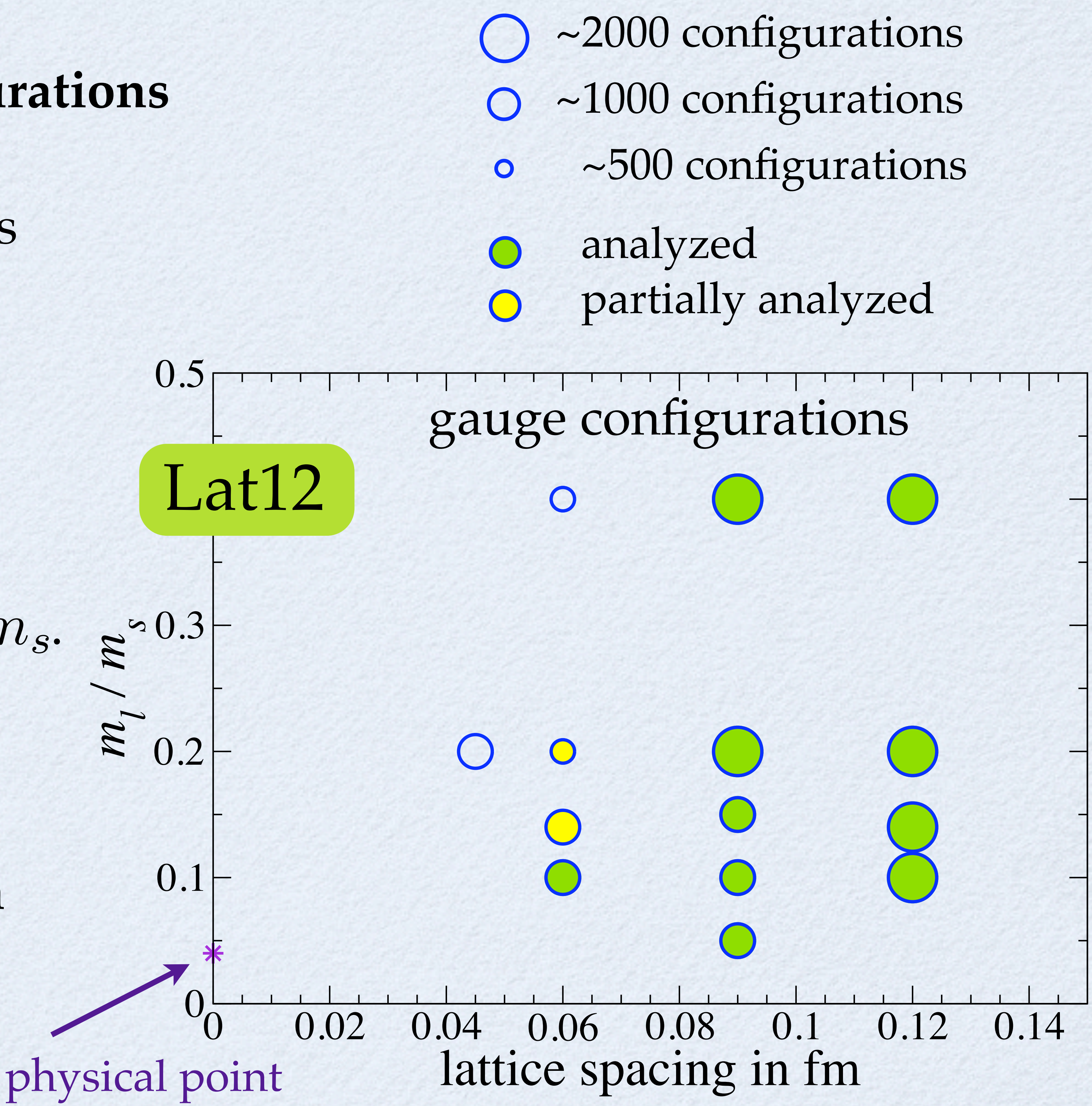


Three-points

$$C^{2\text{pt}}(t) = \sum_m \left[Z_m^2 e^{-E_m t} + (-1)^{(t+1)} (Z_m^{\text{p}})^2 e^{-E_m^{\text{p}} t} \right]$$

$$\begin{aligned} C^{3\text{pt}}(t_1, t_2) = \sum_{m,n} \Big[& Z_m Z_n \langle \mathcal{O} \rangle_{mn} e^{-E_m t_1} e^{-E_n t_2} \\ & + Z_m^p Z_n \langle \mathcal{O} \rangle_{mn}^p (-1)^{t_1} e^{-E_m^p t_1} e^{-E_n t_2} \\ & + Z_m Z_n^p \langle \mathcal{O} \rangle_{mn}^p (-1)^{t_1} e^{-E_m t_1} e^{-E_n^p t_2} \\ & + Z_m^p Z_n^p \langle \mathcal{O} \rangle_{mn}^{pp} (-1)^{t_1 + t_2} e^{-E_m^p t_1} e^{-E_n^p t_2} \Big] \end{aligned}$$

- simultaneous fit of two-point + three-point;
- constrains energies and two-point amplitudes
- use constrained curve fitting

Renormalization and Matching

Operators mix under renormalization (even in the continuum). E.g.

$$\langle \mathcal{O}_1 \rangle^R = (1 + \alpha_s \zeta_{11}) \langle \mathcal{O}_1 \rangle + \alpha_s \zeta_{12} \langle \mathcal{O}_2 \rangle$$

ζ_{ij} are calculated using 1-loop perturbation theory.

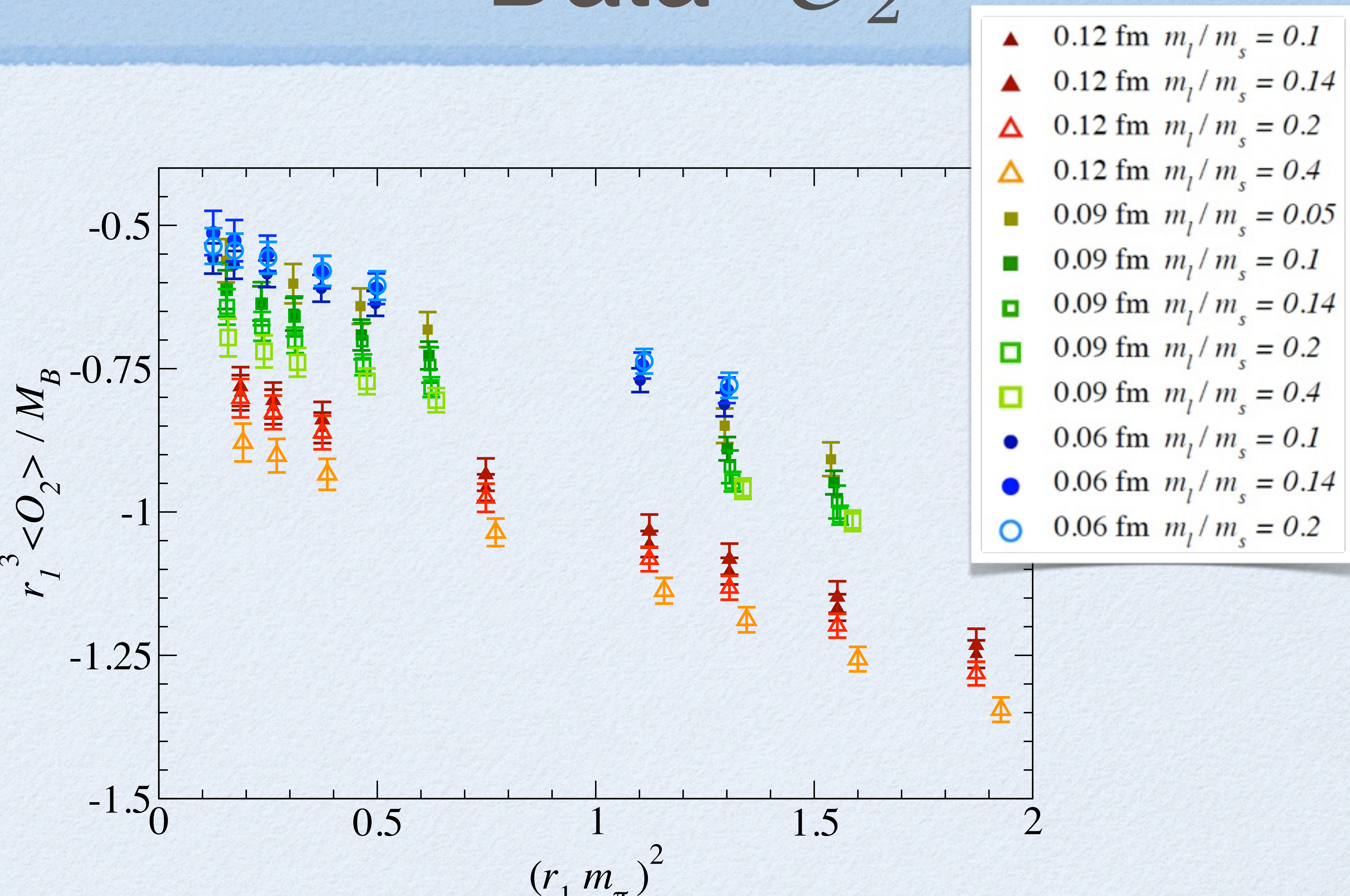
We use the "V" scheme as implemented by Q. Mason et al. with 4-loop running.

$$\alpha_s = \alpha_{\mathrm{V}}(2/a)$$

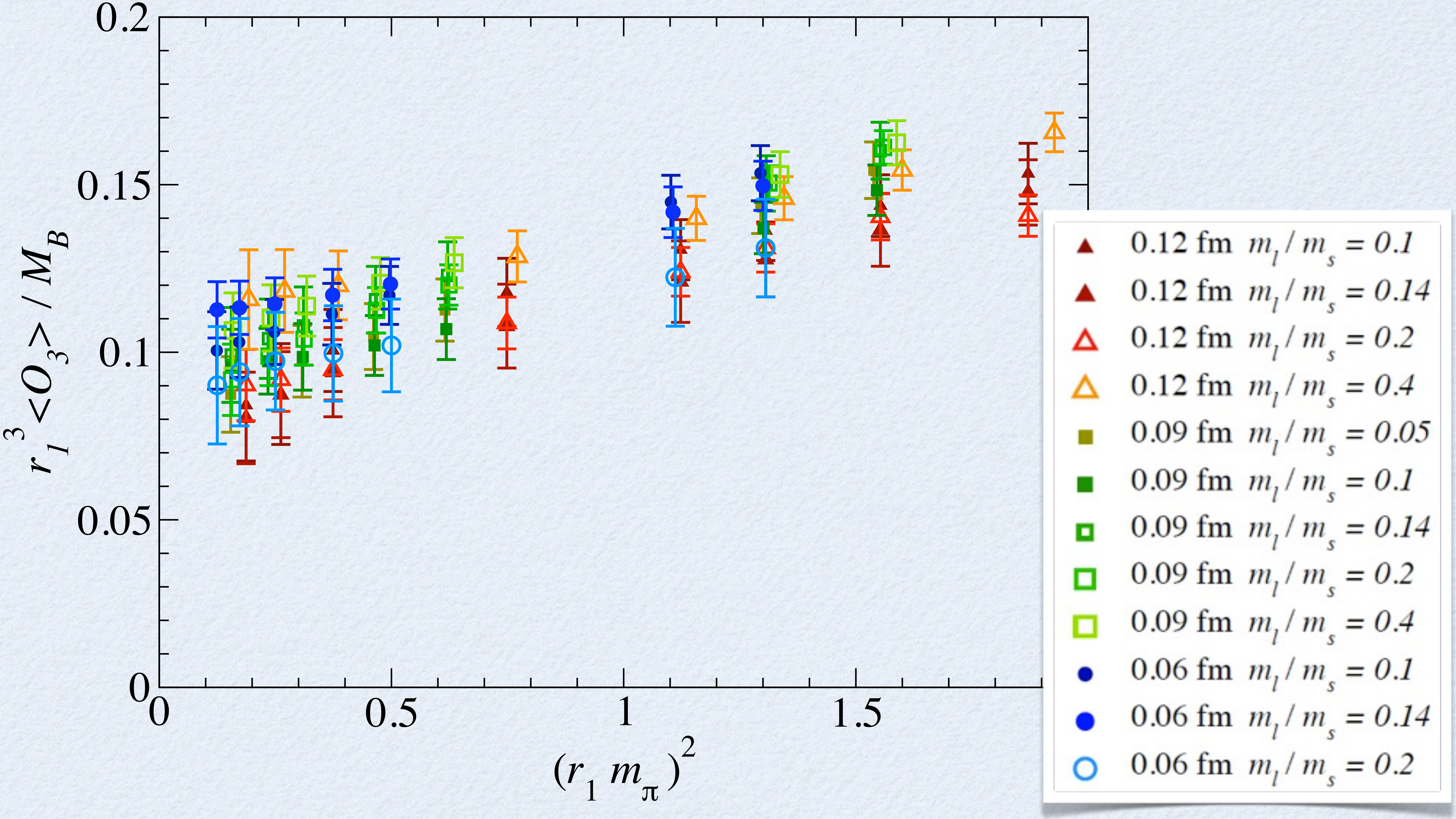
Q. Mason et al. [HPQCD Collaboration],
Phys. Rev. Lett. **95**, 052002 (2005) hep-lat/0503005

T. van Ritbergen et al., Phys. Lett. B 400, 379 (1997) hep-ph/9701390

Data $\mathcal{O}_2$



Data $\mathcal{O}_3$



Status: ChiPT

We use SU(3), partially-quenched, heavy-meson, staggered ChiPT

continuum, PQ: Detmold and Lin, aXiv:0612028, hep-lat, 2006

Claude Bernard's talk

With staggered light quarks, matrix elements of wrong-spin operators appear in the ChiPT.

Because the five matrix elements $\langle \mathcal{O}_{1\ldots5} \rangle$ form a complete basis, wrong-spin contributions can be written in terms of them.

- Mixing occurs: $\langle \mathcal{O}_1 \rangle$ $\langle \mathcal{O}_2 \rangle$ $\langle \mathcal{O}_3 \rangle$ and $\langle \mathcal{O}_4 \rangle$ $\langle \mathcal{O}_5 \rangle$
- No new LEC's are introduced.

Status: ChiPT

E.g.

$$\langle \overline{B}^0_q | \mathcal{O}^q_1 | B^0_q \rangle = \beta_1 \left(1 + \frac{\mathcal{W}_{q\bar{b}} + \mathcal{W}_{b\bar{q}}}{2} + \mathcal{T}_q + \mathcal{Q}_q + \tilde{\mathcal{T}}^{(\mathrm{a})}_q + \tilde{\mathcal{Q}}^{(\mathrm{a})}_q \right)$$

$$+ (2\beta_2 + 2\beta_3)\tilde{\mathcal{T}}^{(\mathrm{b})}_q + (2\beta'_2 + 2\beta'_3)\tilde{\mathcal{Q}}^{(\mathrm{b})}_q$$

$$+\text{analytic terms}$$

$\mathcal{O}_1$

wrong-spin contributions

- ✦ Mixing occurs.
- ✦ No new LEC's are introduced.

We will do a simultaneous fit for each set of mixed operators.

Status of Lattice: $\mathcal{O}_1$

FNAL-MILC

$\xi = 1.268(63) \quad 5.0\%$

arXiv: 1205.7013, submitted to PRD

Source of uncertainty	Error (%)
Statistics $\oplus$ light-quark disc. $\oplus$ chiral extrapolation	3.7
Mixing with wrong-spin operators	3.2
Heavy-quark discretization	0.3
Scale uncertainty (r_1)	0.2
Light-quark masses	0.5
One-loop matching	0.5
Tuning κ_b	0.4
Finite volume	0.1
Mistuned coarse u_0	0.1
Total Error	5.0

Conclusion

Finished

- $\xi = 1.268(63)$ arXiv: 1205.7013

In progress

- nearly done with three-point analysis
  - fourteen ensembles across four lattice spacings
- corrected chiral form exists; extrapolation remains to be done
- results will include:
  - complete set (5) of matrix elements and bag parameters
  - ratios ξ and $\langle\mathcal{O}_3\rangle/\langle\mathcal{O}_1\rangle$ and the combination $\langle\mathcal{O}_R\rangle$
- "half-way point" (Lattice 11, arXiv:1112:5642):
  - error on $f_{B_q}\sqrt{\hat{B}_{B_q}}$ 9 to 6% (perturbation theory, continuum-ChiPT extrap.)
  - Errors will be smaller for full-data set analysis.

Backup Slides

Status of Lattice: $\mathcal{O}_{1...5}$

FNAL-MILC: Lattice 11 Proceedings (Dec 2011), arXiv:1112:5642

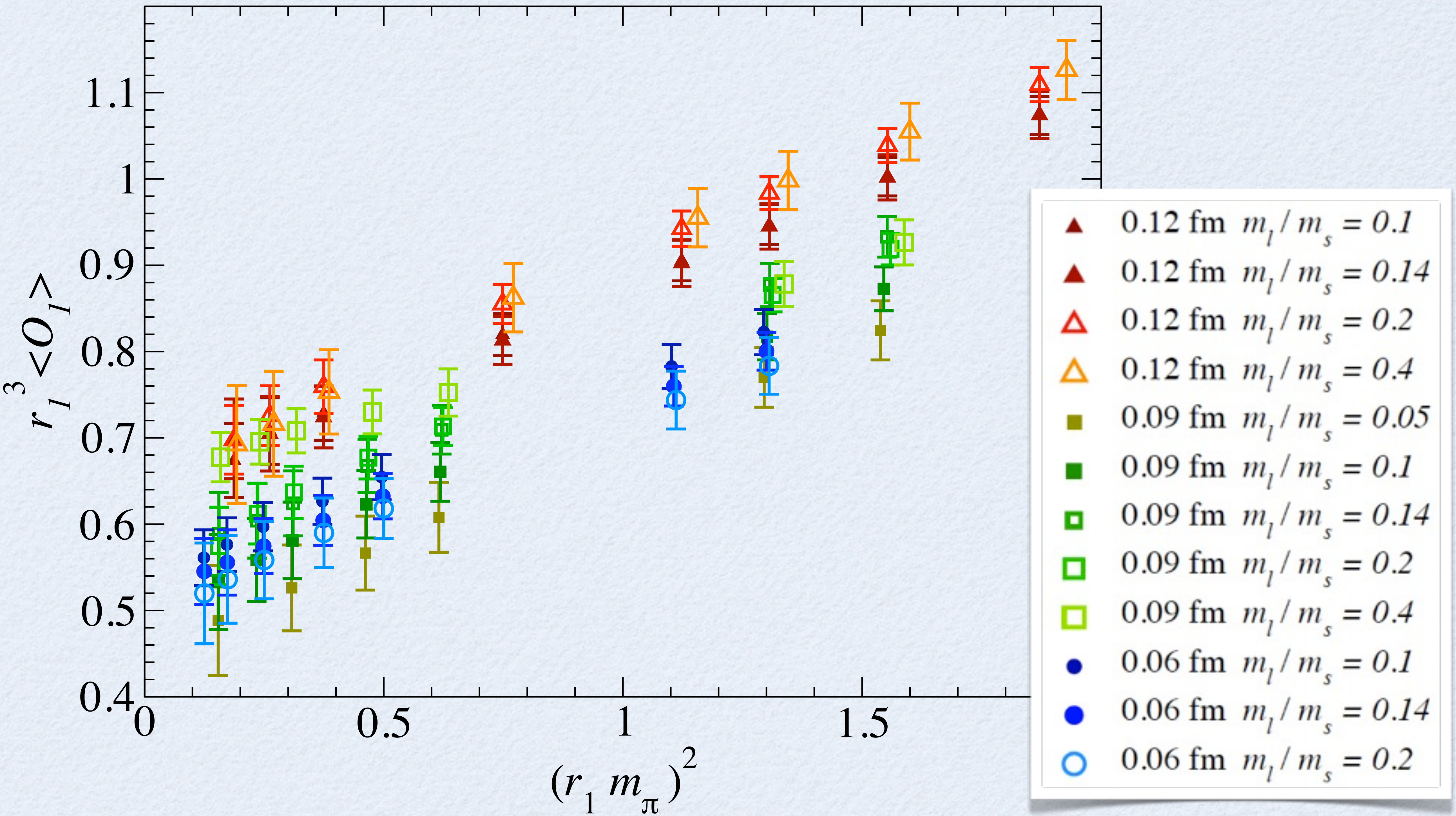
Source of Error [%]	$\langle \mathcal{O}_i \rangle$
scale (r_1)	3
κ_b tuning	4
light-quark masses	1
heavy-quark discretization	4
one-loop matching	8
finite-volume effects	1
subtotal	10

	Source of Error [%]	$\langle \mathcal{O}_1 \rangle$	$\langle \mathcal{O}_2 \rangle$	$\langle \mathcal{O}_3 \rangle$	$\langle \mathcal{O}_4 \rangle$	$\langle \mathcal{O}_5 \rangle$
B_d^0	statistical + some systematic	8.6	6.8	16	4.3	5.5
	chiral-continuum truncation	12	11	3.3	0.2	4.4
B_s^0	statistical + some systematic	6.7	4.6	10	2.5	3.4
	chiral-continuum truncation	1.8	6.6	4.5	1.6	3.7

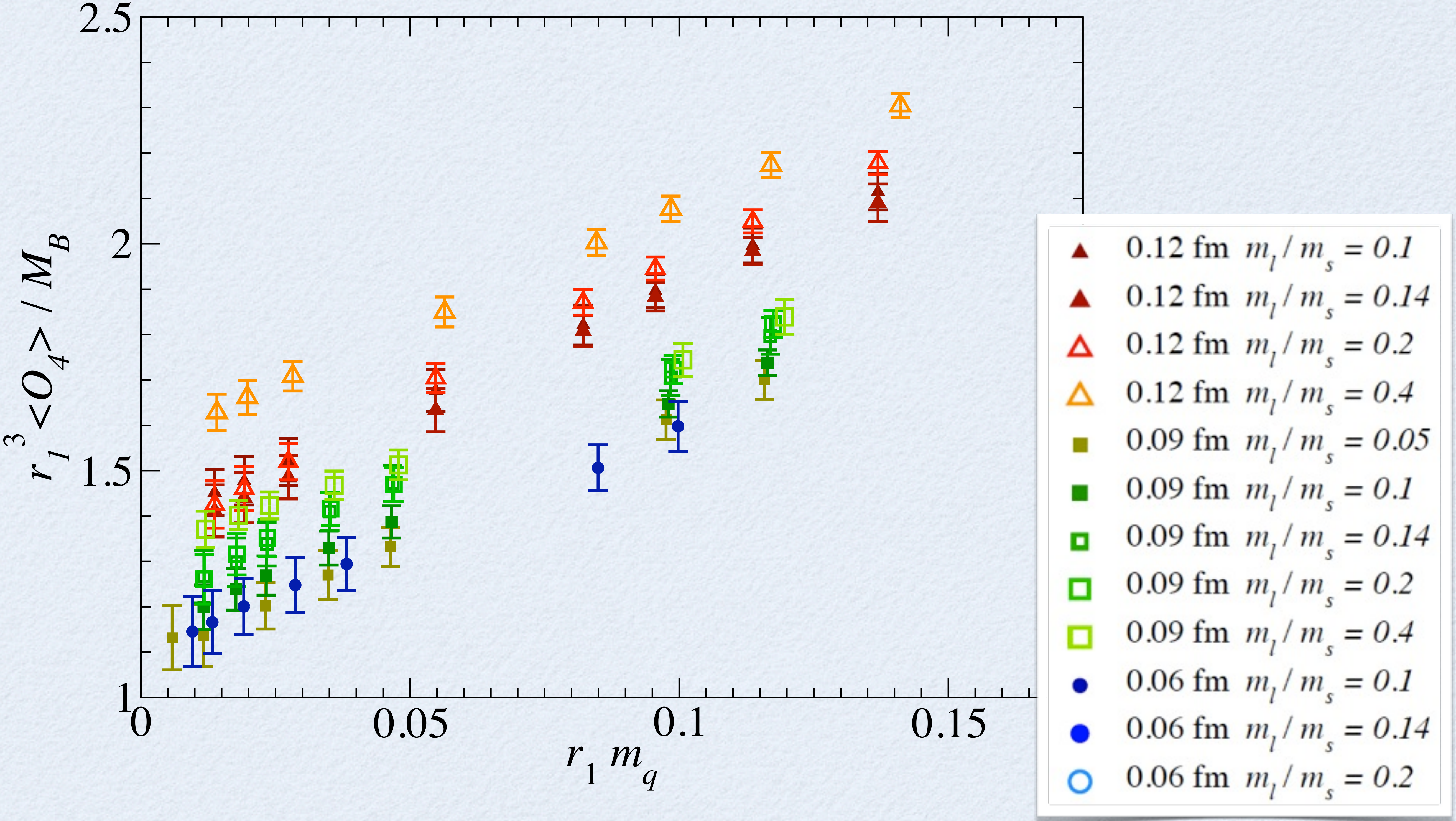
9 to 6% on $\mathcal{O}_1$

$$f_{B_q}\sqrt{\hat{B}_{B_q}}$$

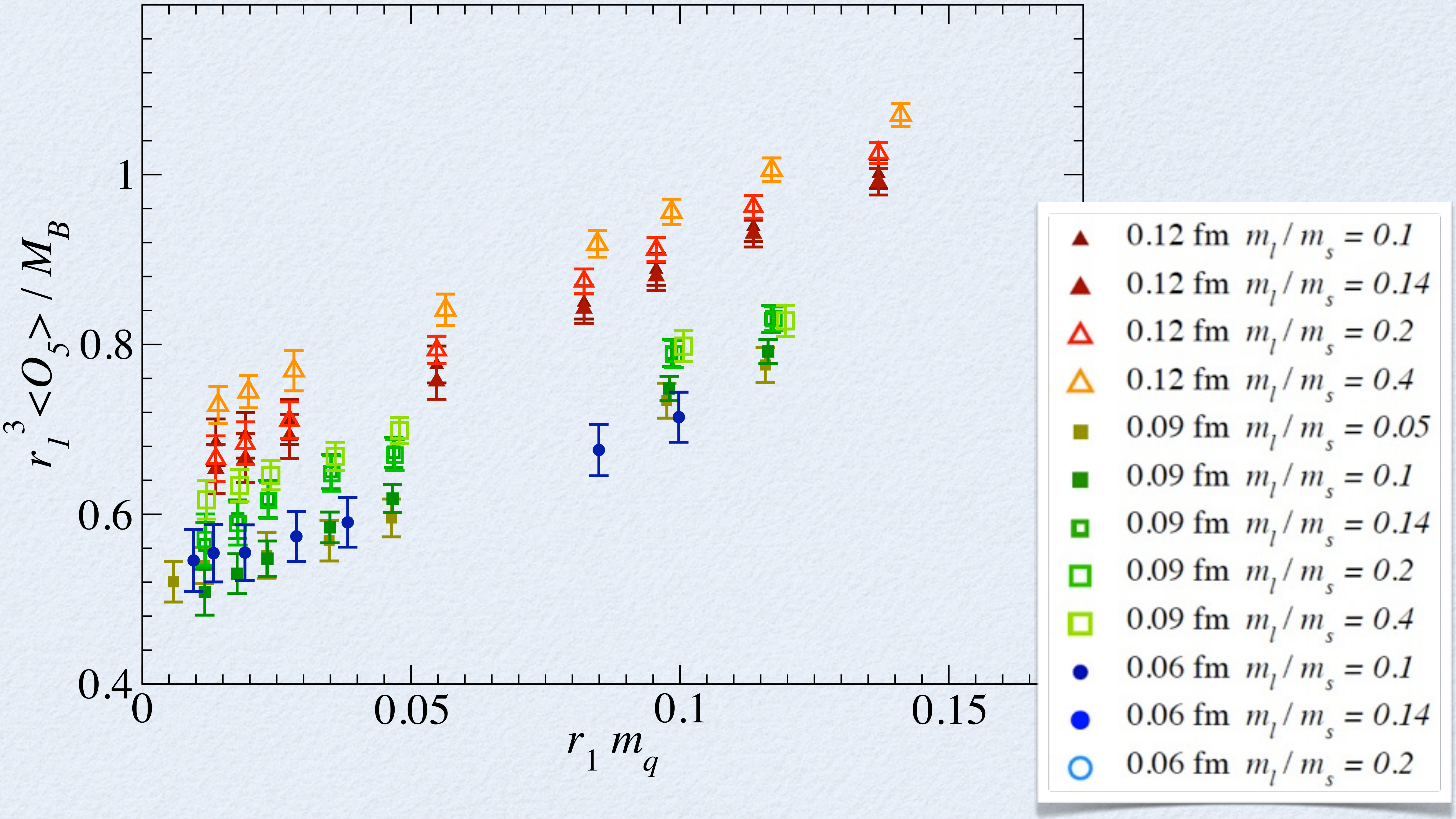
Data $\mathcal{O}_1$



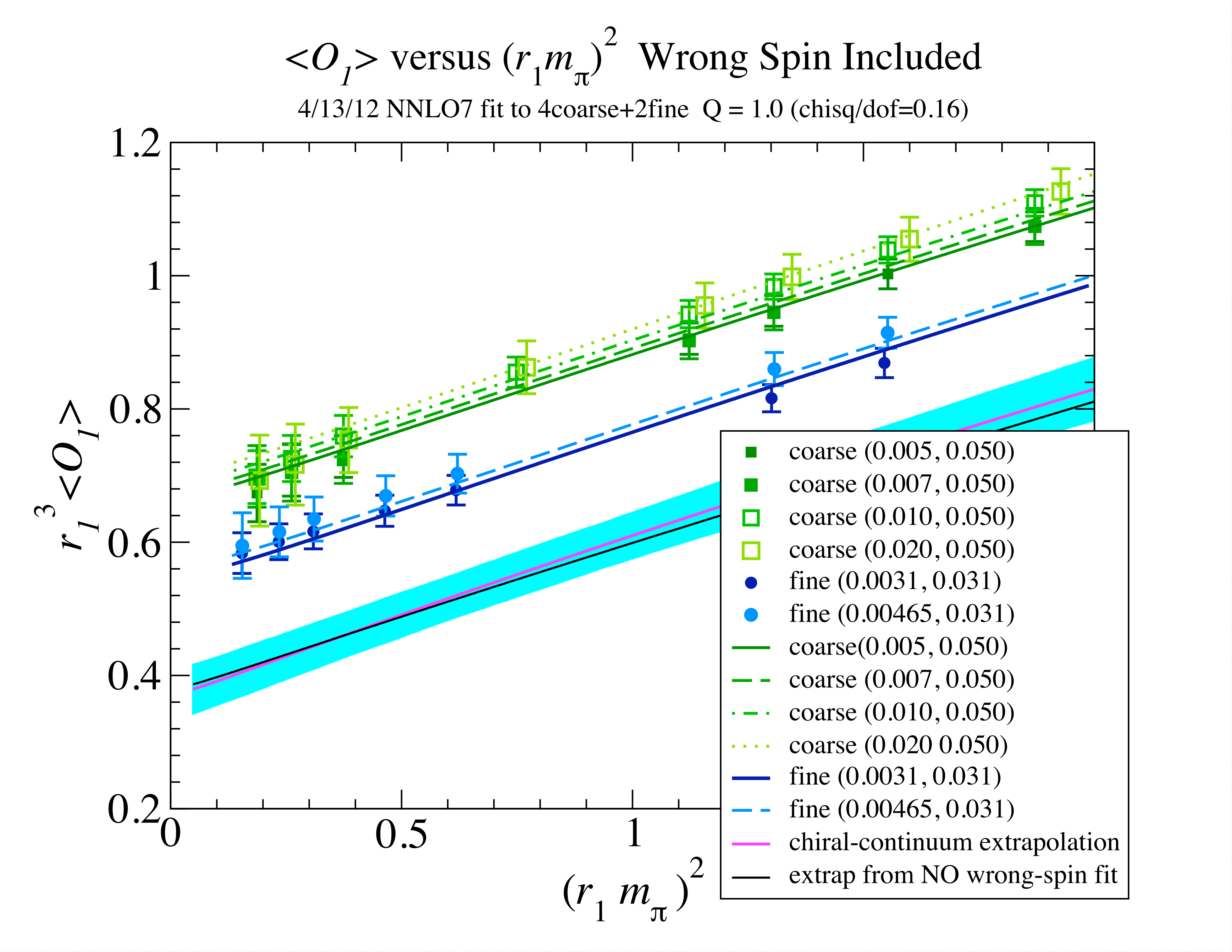
Data $\mathcal{O}_4$



Data $\mathcal{O}_5$



ChiPT systematic



Experiment and Lattice: ΔM_q

$$\underbrace{\Delta M_q}_{<1\%} = \underbrace{\left(\frac{G_F^2 M_W^2 S_0}{4\pi^2}\right) \eta_B(\mu)}_{\text{known}} \underbrace{|V_{tb}V_{tq}^*|^2}_{\text{want}} \underbrace{\langle \bar{B}_q^0 | \mathcal{O}_1(\mu) | B_q^0 \rangle}_{\text{need from lattice}}$$

$$\Delta M_d = 0.507 \pm 0.003(\text{stat}) \pm 0.003(\text{sys})\ \text{ps}^{-1}$$ PDG, J.Phys G37, 1 (2010)

$$\Delta M_s = 17.77 \pm 0.10(\text{stat}) \pm 0.07(\text{sys})\ \text{ps}^{-1}$$ CDF, PRL 97, 242003 (2006)

(LHCb-CONF-2011-050: 17.73(5))

Our ability to constrain $|V_{tb}V_{tq}^*|^2$, is limited by $\langle \bar{B}_q^0 | \mathcal{O}_1(\mu) | B_q^0 \rangle$.

"Tension" in the CKM matrix.

Lenz et al., arXiv: 1203:0238; Laiho et al. PhysRevD. 81, 034503, and end-of-2011 update

Experiment and Lattice: $\Delta\Gamma_q$

$$\frac{\Delta\Gamma_d}{\Gamma_d} = 0.010 \pm 0.037$$ HFAG/PDG 2011; BaBar, DELPHI

$$\Delta\Gamma_s = (0.116 \pm 0.019)\ \mathrm{ps}^{-1}$$ LHCb 1 fb^{-1}, Moriond 2012 (0.37 fb^{-1} arXiv:1112.3183)

$$\Delta\Gamma_s = (0.163 \pm 0.065)\ \mathrm{ps}^{-1}$$ D0 8 fb^{-1}, arXiv 1109.3166

Recent LHCb results are putting focus on $\Delta\Gamma$!

A scenario with new physics in $\Delta\Gamma_s$ yields a better fit to current data than a scenario with new physics in ΔM_s .[‡] (Haisch, Moriond 2012)

[‡]Specifically, NP in Γ_{12} versus M_{12} .

"We introduce a fourth scenario with NP in both $M_{12}^{d,s}$ and $\Gamma_{12}^{d,s}$, which can accommodate all data."

Lenz et al., arXiv:1203.0238

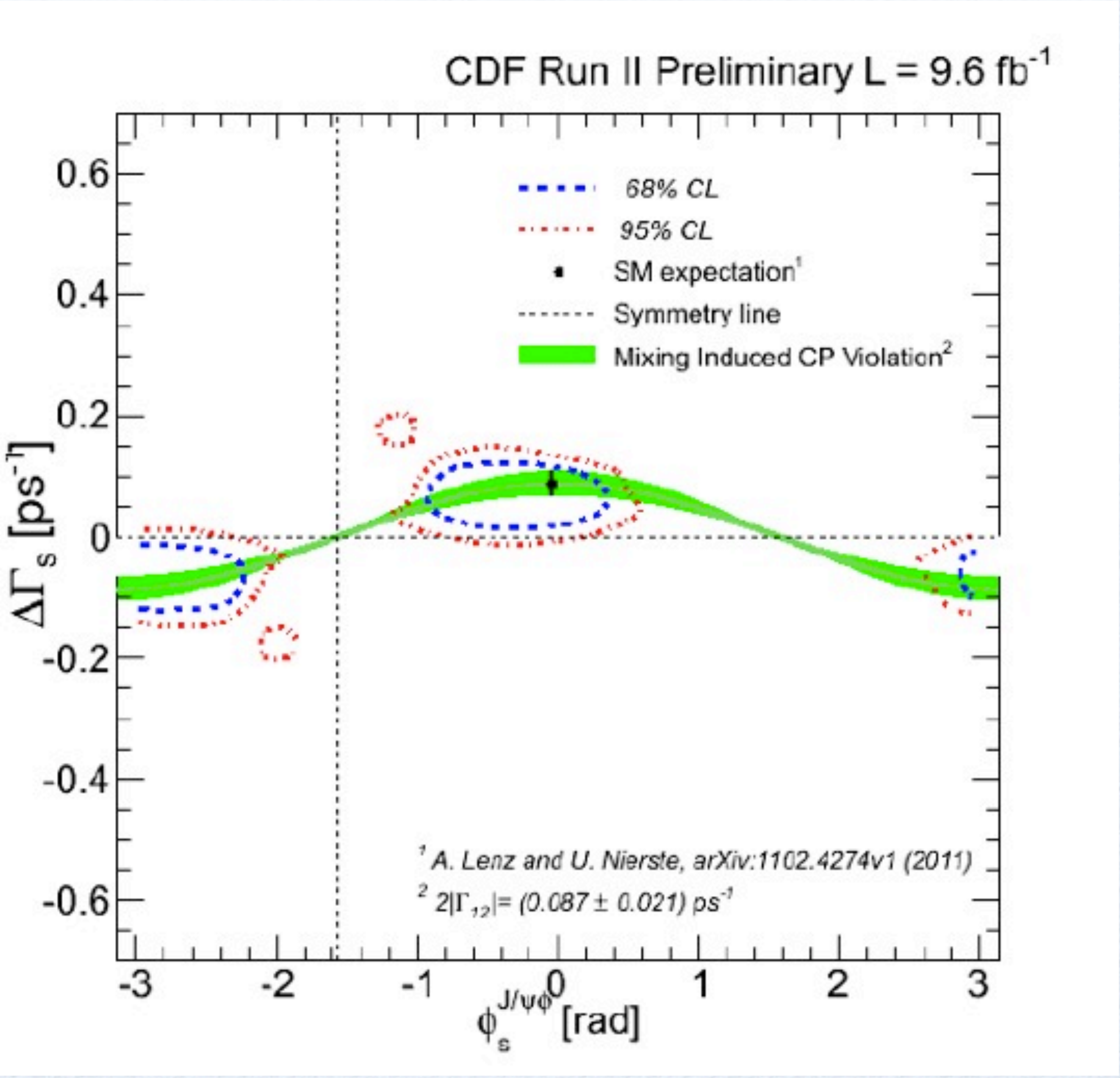
Experiment and Lattice: $\Delta\Gamma_q$

$$\Delta\Gamma_q = f_{B_q}^2 \left[G_1 B_{1,q} + G_3 B_{3,q}\right] \cos\phi_q + O(1/m_b, \alpha_s)$$

green band = theory constraint on new physics

Improved matrix elements may improve this band.

L. Sabato, Lake Louise 2012



Cross-checks

- We compare energies computed from 2pts to energies from the 2pt+3pt fits.
- Two people fit $\langle \mathcal{O}_3 \rangle$.
- We use N=2,4,6 results to verify the plateau.