Chiral extrapolation of matrix elements of BSM kaon operators

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Phys.Rev. D85 (2012) 074507 [arXiv:1202.1570 [hep-lat]]

Outline

- Motivation & Background
- Technical details
- Results & Outlook

Motivation

- BSM theories lead to kaon mixing operators with Dirac structures which differ from those in the SM

$$O_1 = \bar{s}^a \gamma_\mu (1-\gamma_5) d^a \bar{s}^b \gamma_\mu (1-\gamma_5) d^b$$

SM "B_K operator"

$$\left.\begin{aligned} O_2 &= \bar{s}^a (1-\gamma_5) d^a \bar{s}^b (1-\gamma_5) d^b \\ O_3 &= \bar{s}^a (1-\gamma_5) d^b \bar{s}^b (1-\gamma_5) d^a \\ O_4 &= \bar{s}^a (1-\gamma_5) d^a \bar{s}^b (1+\gamma_5) d^b \\ O_5 &= \bar{s}^a (1-\gamma_5) d^b \bar{s}^b (1+\gamma_5) d^a \end{aligned}\right\}$$

New operators induced by integrating out heavy BSM particles, e.g. SUSY partners, KK modes,...

- ΔM_K and ε_K can place strong constraints on parameters of BSM theories (complementary to LHC)
- Need non-perturbative matrix elements of operators

Background

- Matrix elements of new operators should be easier to calculate than B_K since not chirally suppressed

$$O_1 = \bar{s}^a \gamma_\mu (1-\gamma_5) d^a \bar{s}^b \gamma_\mu (1-\gamma_5) d^b$$

"LL"⇒ matrix elements ~M_K^2

$$\left.\begin{aligned}
O_2 &= \bar{s}^a (1-\gamma_5) d^a \bar{s}^b (1-\gamma_5) d^b \\
O_3 &= \bar{s}^a (1-\gamma_5) d^b \bar{s}^b (1-\gamma_5) d^a \\
O_4 &= \bar{s}^a (1-\gamma_5) d^a \bar{s}^b (1+\gamma_5) d^b \\
O_5 &= \bar{s}^a (1-\gamma_5) d^b \bar{s}^b (1+\gamma_5) d^a
\end{aligned}\right\}$$

Matrix elements finite in SU(3) chiral limit

- Level of accuracy required at present less than for B_K
- Straightforward task for lattice QCD

Background

- In fact, plan to switch soon to the basis used in most continuum calculations (e.g. Buras and collaborators)

$$O_2 = \bar{s}^a(1-\gamma_5)d^a\bar{s}^b(1-\gamma_5)d^b$$ Unchanged

$$O_3 = \bar{s}^a(1-\gamma_5)d^b\bar{s}^b(1-\gamma_5)d^a \longrightarrow [\bar{s}^a\sigma_{\mu\nu}(1-\gamma_5)d^a][\bar{s}^b\sigma_{\mu\nu}(1-\gamma_5)d^b]$$

$$O_4 = \bar{s}^a(1-\gamma_5)d^a\bar{s}^b(1+\gamma_5)d^b$$ Unchanged

$$O_5 = \bar{s}^a(1-\gamma_5)d^b\bar{s}^b(1+\gamma_5)d^a \longrightarrow [\bar{s}^a\gamma_\mu(1-\gamma_5)d^a][\bar{s}^b\gamma_\mu(1+\gamma_5)d^b]$$

- ChPT results presented here apply also to new basis

SWME calculation

- As part of our B_K project, we have also calculated the necessary bare matrix elements of the BSM operators
- Matching factors given to 1-loop by [Kim,Lee,SS PRD11]
- Need NLO chiral forms to do chiral/continuum extrapolation--these are provided here
  - ✳ Use mixed-action rooted partially quenched staggered chiral perturbation theory both for SU(3) and SU(2) cases
  - ✳ Bonus: provide continuum PQ results---only available previously in unquenched SU(3) case [Becirevic & Villadoro, PRD04]
- First numerical results presented here by H.-J. Kim

B-parameters

- B-parameters cancel many lattice systematics, renormalization scale dependence & chiral logs

$$\left.\begin{aligned}
O_2 &= \bar{s}^a(1-\gamma_5)d^a\bar{s}^b(1-\gamma_5)d^b \\
O_3 &= \bar{s}^a(1-\gamma_5)d^b\bar{s}^b(1-\gamma_5)d^a \\
O_4 &= \bar{s}^a(1-\gamma_5)d^a\bar{s}^b(1+\gamma_5)d^b \\
O_5 &= \bar{s}^a(1-\gamma_5)d^b\bar{s}^b(1+\gamma_5)d^a
\end{aligned}\right\} \quad B_j(\mu) = \frac{\langle \overline{K}_0|O_j(\mu)|K_0\rangle}{N_j\langle\overline{K}_0|\bar{s}^a\gamma_5 d^a(\mu)|0\rangle\langle 0|\bar{s}^b\gamma_5 d^b(\mu)|K_0\rangle}$$

$$(N_2, N_3, N_4, N_5) = (5/3, -1/3, -2, -2/3)$$

- Denominator does not vanish in chiral limit (unlike for B_K)

$$\langle\overline{K}_0|\bar{s}\gamma_5 d(\mu)|0\rangle\langle 0|\bar{s}\gamma_5 d(\mu)|K_0\rangle = -\left(\frac{f_K M_K^2}{m_d(\mu)+m_s(\mu)}\right)^2$$

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Staggered complications

- Valence fermions (HYP-smeared) have extra tastes
  - ✳ Fierz rearrangement differs from continuum, so have to match Wick contractions
  - ✳ Requires valence S_1, S_2, D_1, D_2 (upper-case $\Rightarrow$ fields with 4 tastes)
  - ✳ Choose operators with Goldstone taste (ξ_5)
- Sea quarks (asqtad) are rooted
  - ✳ Use standard SChPT prescription for rooting [Bernard, Aubin&Bernard]
  - ✳ Mixed action introduces additional terms in taste-singlet propagators
- Taste-breaking implies extensive operator mixing
  - ✳ Due to use of perturbative matching and through discretization effects

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All these were dealt with for B_K in [Van de Water & SS, SWME 10]

Extension to new operators relatively straightforward

Outline of steps

Continuum QCD
u,d,s + O_j

$\longleftrightarrow$ match Wick contractions keep positive parity part

PQ "continuum staggered" theory
U,D,S + D_1,D_2,S_1,S_2 + ghosts + O_j^{PQ}

Example:

$$O_2 = \bar{s}^a(1-\gamma_5)d^a\bar{s}^b(1-\gamma_5)d^b \longleftrightarrow$$

$$\bar{S}_1^a(\mathbf{1}\otimes\xi_5)D_1^a\;\bar{S}_2^b(\mathbf{1}\otimes\xi_5)D_2^b$$

$$+$$

$$\bar{S}_1^a(\gamma_5\otimes\xi_5)D_1^a\;\bar{S}_2^b(\gamma_5\otimes\xi_5)D_2^b$$

$$-\frac{1}{2}\Bigg(\bar{S}_1^a(1\otimes\xi_5)D_1^b\;\bar{S}_2^b(1\otimes\xi_5)D_2^a + \bar{S}_1^a(\gamma_5\otimes\xi_5)D_1^b\;\bar{S}_2^b(\gamma_5\otimes\xi_5)D_2^a - \sum_{\mu<\nu}\bar{S}_1^a(\gamma_\mu\gamma_\nu\otimes\xi_5)D_1^b\;\bar{S}_2^b(\gamma_\mu\gamma_\nu\otimes\xi_5)D_2^a\Bigg)$$

[Taking matrix elements between kaons with appropriate tastes]

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discretize;
use hypercube operators & 1-loop matching

PQ lattice staggered theory
$\chi_u, \chi_d, \chi_s + \chi_{d1}, \chi_{d2}, \chi_{s1}, \chi_{s2}$
+ rooting + O_j^{LAT}

GIVES DESIRED MATRIX ELEMENTS
IN CONTINUUM LIMIT

Generically: $$O_j^{\mathrm{LAT}} \cong O_j^{PQ} + \mathcal{O}\left(\frac{\alpha}{4\pi}\right)[\text{other taste ops}] + \mathcal{O}\left(\alpha^2, a^2\right)[\text{various taste ops}]$$

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match using lattice symmetries

Symanzik effective theory
U,D,S + D_1,D_2,S_1,S_2 + rooting
a^2 corrections explicit

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Chiral effective theory (rSChPT)
Approx. $SU(28|16)_L \times SU(28|16)_R$
Operators in chiral form
a^2, α corrections explicit

CORRESPONDS TO SU(3) ChPT
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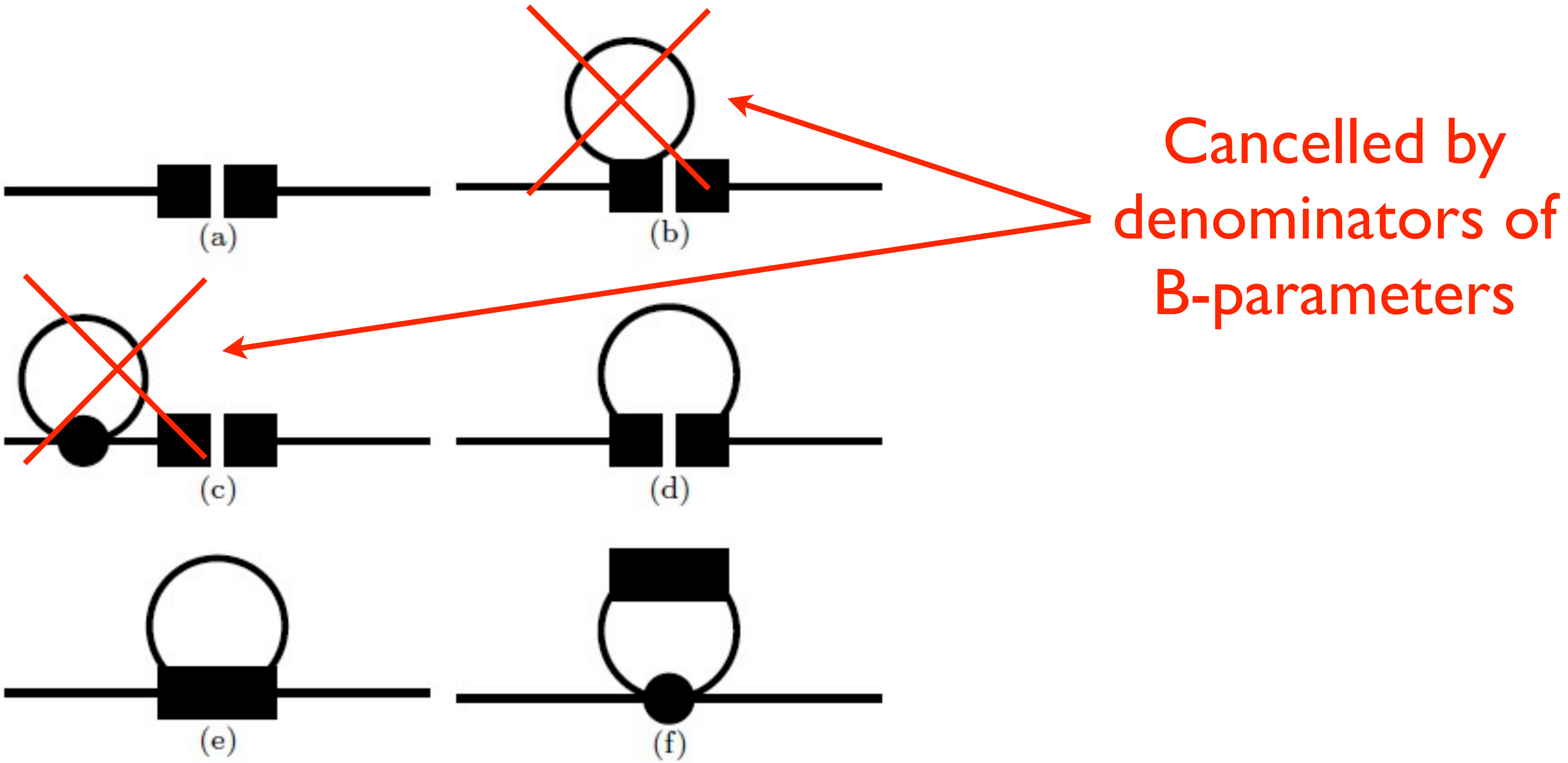
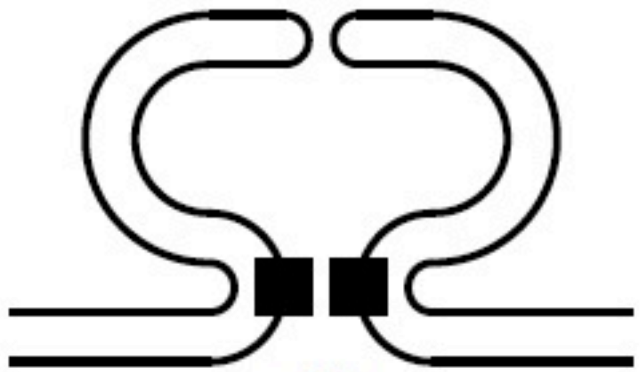
CORRESPONDS TO SU(3) ChPT
IN CONTINUUM

Do NLO calculation
Obtain SU(2) result by "trick"

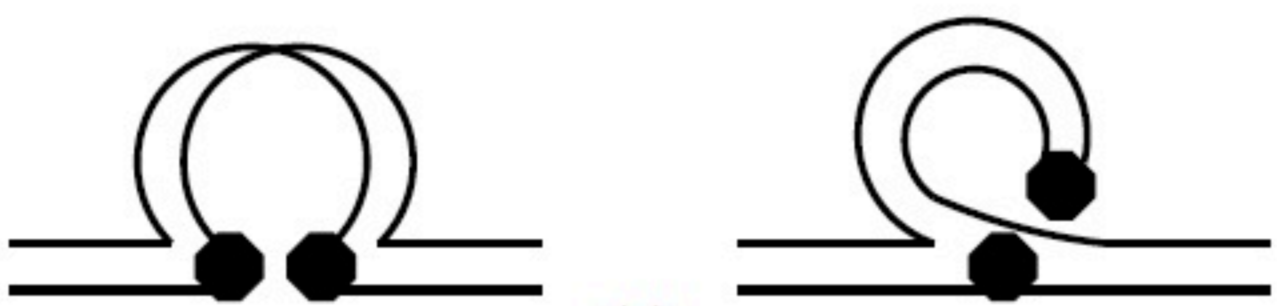
Contributing diagrams



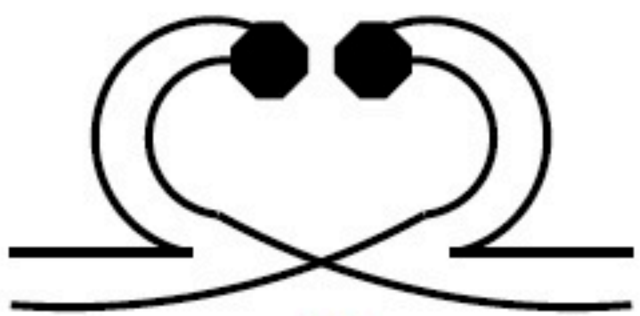
Quark-line diagrams



(a)



(b)



(c)

No sea-quarks (except implicitly in hairpin diagram)

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Results

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$$O_3 = \bar{s}^a(1-\gamma_5)d^b\bar{s}^b(1-\gamma_5)d^a$$

Differ only in color structure ⇒ same form in ChPT

$$O_4 = \bar{s}^a(1-\gamma_5)d^a\bar{s}^b(1+\gamma_5)d^b$$
$$O_5 = \bar{s}^a(1-\gamma_5)d^b\bar{s}^b(1+\gamma_5)d^a$$

Differ only in color structure ⇒ same form in ChPT

General form of NLO result (for SU(3) and SU(2) cases):

$$B_j = B_j^{\mathrm{LO}}\left[1 + \delta B_j^{\mathrm{anal}} + \delta B_j^{\log}\right]$$

m_d(valence) m_s(valence) sea-quark masses

SU(3):

$$\delta B_j^{\mathrm{anal}} = c_{j1}(m_x + m_y) + c_{j2}(m_u + m_d + m_s) + c_{j3}a^2 + c_{j4}a_\alpha^2 + c_{j5}\alpha^2.$$

Standard disc. errors

Taste-breaking $a_\alpha^2 = a^2\alpha_V(\pi/a)^2$

Matching errors (Absent if use NPR)

Results

$$O_2 = \bar{s}^a(1-\gamma_5)d^a\bar{s}^b(1-\gamma_5)d^b$$
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Differ only in color structure ⇒ same form in ChPT

General form of NLO result (for SU(3) and SU(2) cases):

$$B_j = B_j^{\mathrm{LO}}\left[1 + \delta B_j^{\mathrm{anal}} + \delta B_j^{\log}\right]$$

m_d(valence)

LECs now depend on m_s

~~SU(3)~~:

SU(2):

$$\delta B_j^{\mathrm{anal}} = d_{j1}(m_x + \cancel{m_y}) + d_{j2}(m_u + m_d + \cancel{m_s})$$
$$+ d_{j3}a^2 + d_{j4}a_\alpha^2 + d_{j5}\alpha^2 .$$

Standard disc. errors

Taste-breaking
$a_\alpha^2 = a^2\alpha_V(\pi/a)^2$

Matching errors
(Absent if use NPR)

SU(3) Chiral logs ($N_f=2+1$)

average over tastes, B

$$\delta B_j^{\log} = \frac{1}{(4\pi f)^2}\left[\frac{1}{16}\sum_B \left\{2\ell(M_{K,B}^2) - 2M_{K,B}^2\tilde{\ell}(M_{K,B}^2) \pm \ell(M_{xx,B}^2) \pm \ell(M_{yy,B}^2)\right\} \pm \frac{I_\eta + I_x + I_y}{3}\right]$$

+ sign for B_2 & B_3
− sign for B_4 & B_5

Chiral log functions: $\ell(X) = X \ln\left(\frac{X}{\mu^2}\right)\,, \qquad \tilde{\ell}(X) = -\ln\left(\frac{X}{\mu^2}\right) - 1$

I_η, I_x & I_y depend on taste-singlet masses: $M_{xx,I}\,, M_{yy,I}\,, M_{uu,I}\,, M_{ss,I}$ & $M_{\eta,I}$

- Completely predicted in terms of f and masses of valence-valence and sea-sea "pions"
  - Great simplification compared to B_K where 13 additional (unknown) LECs enter into SU(3) chiral logs, because LO matrix element is chirally suppressed
- Use of mixed action has NO impact on form

SU(2) Chiral logs (N_f=2+1)

$$\delta B_j^{\log} = \pm\frac{1}{(4\pi f)^2}\left[-\frac{1}{16}\sum_B \ell(M^2_{xx,B}) + \frac{1}{2}\left\{\ell(M^2_{xx,I}) + (M^2_{\pi,I} - M^2_{xx,I})\tilde{\ell}(M^2_{xx,I})\right\}\right]$$

+ sign for B_2 & B_3
− sign for B_4 & B_5

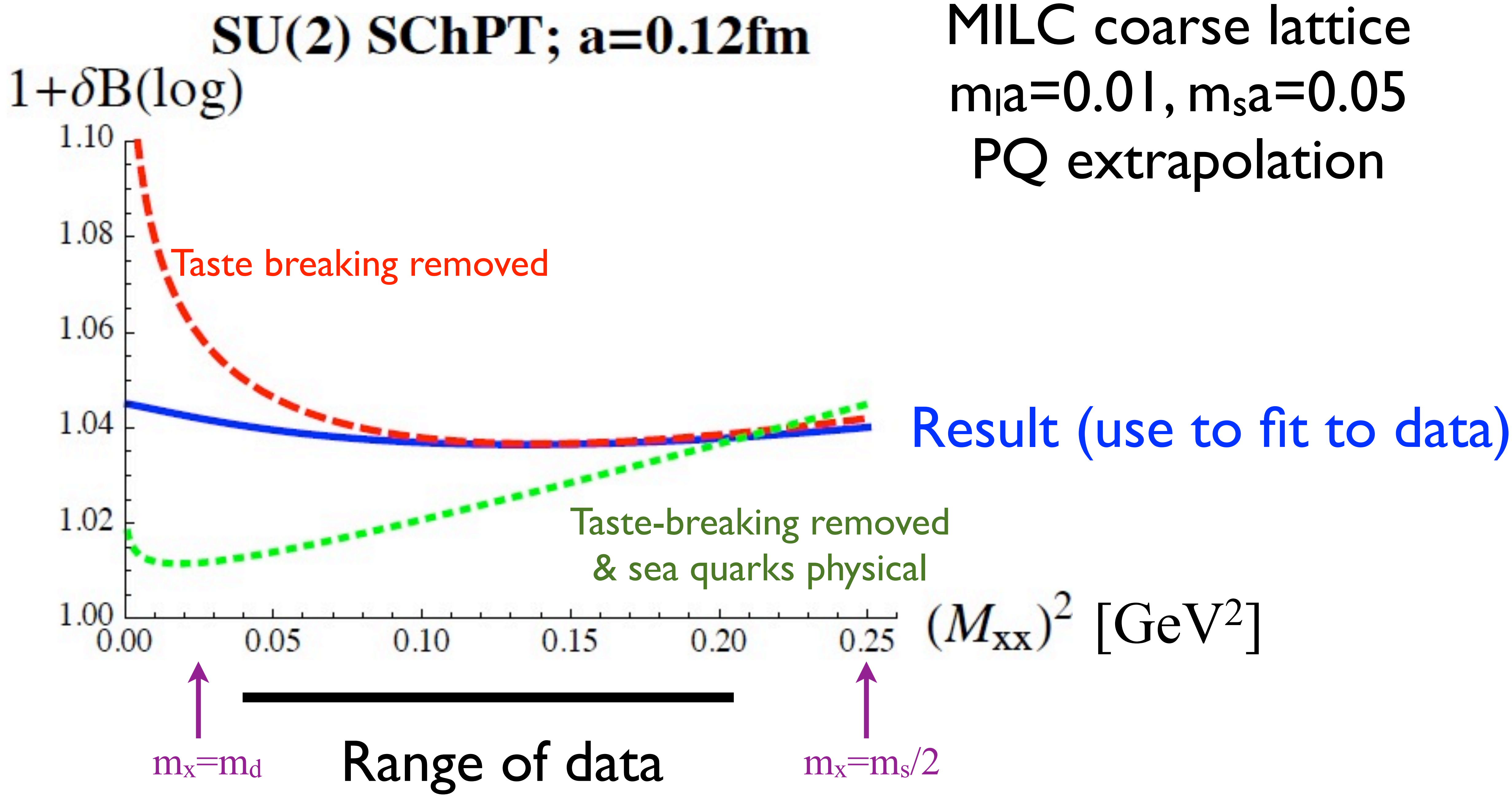
average over tastes, B

- Completely predicted in terms of f and masses of valence-valence and sea-sea "pions"
- Use of mixed action has NO impact on form
- Overall sign of chiral log flips between $B_{2,3}$ and $B_{4,5}$
- Result for $B_{2,3}$ identical to that for B_K !

Silver & Gold

- [Becirevic & Villadoro]: B_2/B_3 and B_4/B_5 have no NLO chiral logs--“golden” for chiral extrapolation
  - ✶ Terminology from continuum SU(3) ChPT
  - ✶ Holds also for PQ SChPT and for both SU(3) and SU(2) cases
- New: $B_2 x B_4$ is golden in SU(2) ChPT (only “silver” in SU(3) case [partial cancellation of logs])
- New: B_2/B_K is golden in SU(2) ChPT
- Using golden combinations is particularly important for a staggered calculation, since it removes an important source of taste breaking
  - ✶ Will use in SWME numerical calculation

Numerical example ($B_{2,3,K}$)

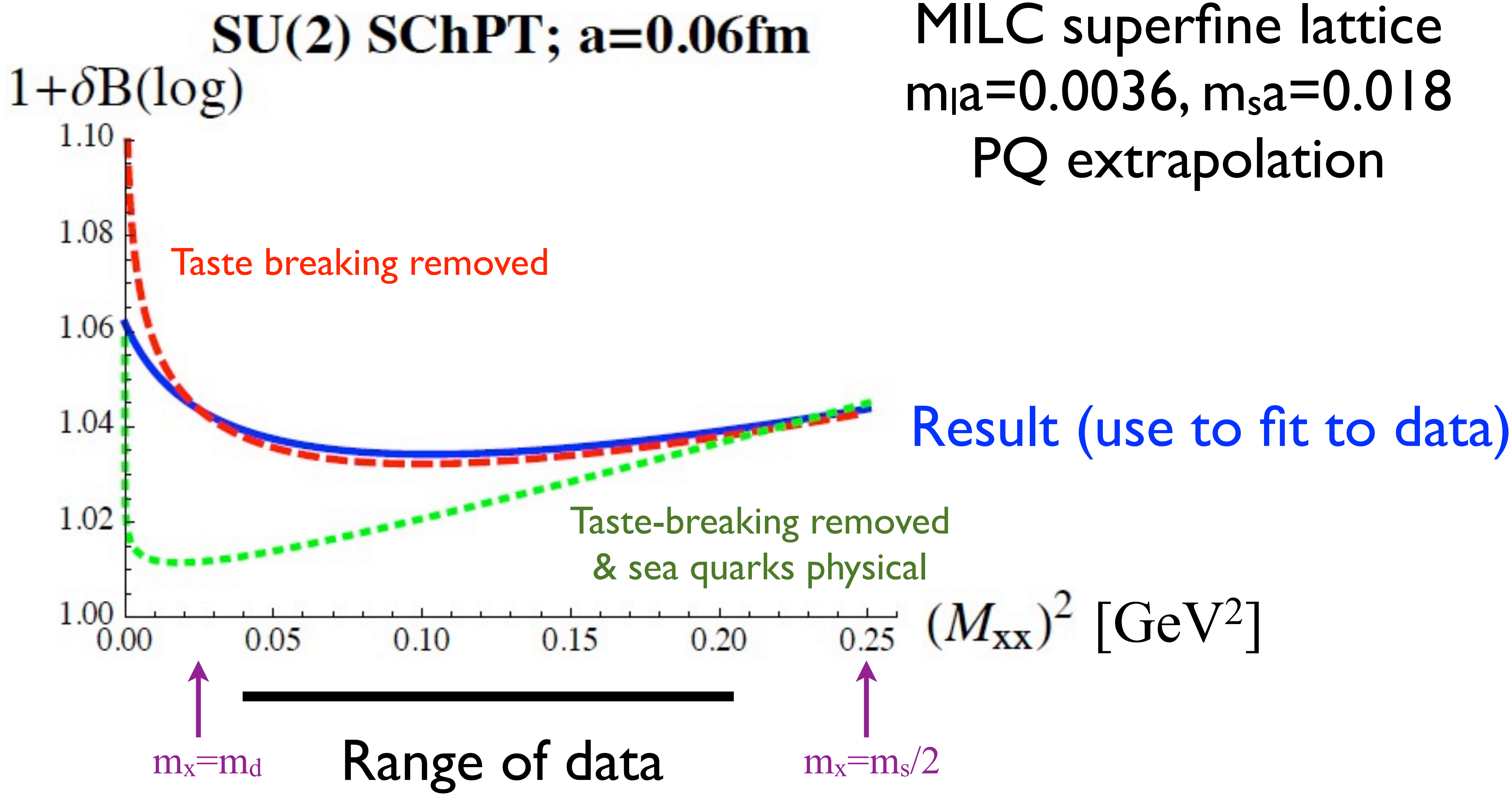


MILC coarse lattice
$m_l a=0.01$, $m_s a=0.05$
PQ extrapolation

SChPT predicts significantly less curvature than in continuum

Numerical example ($B_{2,3,K}$)

MILC superfine lattice
$m_l a=0.0036$, $m_s a=0.018$
PQ extrapolation



More curvature predicted on the finer lattice

Summary

- SChPT NLO calculation extended to BSM $\Delta S=2$ ops
- In SU(2) case, all chiral logs are related to those in B_K
- Appropriate ratios/products cancel chiral logs, which should simplify chiral extrapolations (particularly for staggered fermions, since taste-breaking reduced)
- In continuum limit, results give previously unavailable PQ SU(3) and SU(2) expressions (useful for DWF)