BSM Neutral Kaon Mixing from $n_f = 2+1$ Domain Wall fermions

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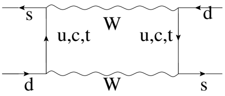
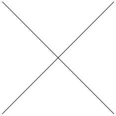
Neutral kaon mixing

In the Standard Model, $K^0 - \bar{K}^0$ mixing dominated by box diagrams with W exchange, e.g.



$\overrightarrow{\mu \ll M_W}$
OPE

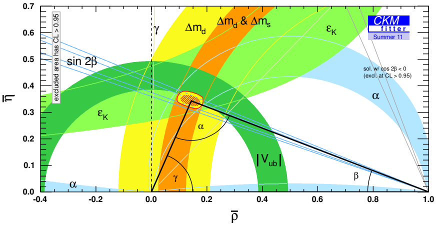


Factorise the non-perturbative contribution into

$$\langle \bar{K}^0 | \mathcal{O}_{LL}^{\Delta S=2}(\mu) | K^0 \rangle = \frac{8}{3} F_K^2 M_K^2 B_K(\mu) \qquad \text{w/ } \mathcal{O}_{LL}^{\Delta S=2} = (\bar{s}\gamma_\mu(1-\gamma_5)d)(\bar{s}\gamma^\mu(1-\gamma_5)d)$$

Related to ε via CKM parameters, schematically

$$\varepsilon \sim \text{ known factors} \times V_{\mathrm{CKM}} \times C(\mu) \times B_K(\mu)$$



[CKMfitter'11]

Standard Model and Beyond

See [F. Gabbiani et al '96]

In the SM, neutral kaon mixing occurs through W-exchanges $\rightarrow$ $(V-A)\times(V-A)$

$$O_1^{\Delta s=2} = (\bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha)\,(\bar{s}_\beta \gamma_\mu (1-\gamma_5) d_\beta)\,,$$

Beyond the SM, other Dirac structure appear in the generic Hamiltonian $H^{\Delta s=2} = \sum_{i=1}^{5} C_i(\mu)\, O_i^{\Delta s=2}(\mu)\,.$

SUSY basis

$$\begin{aligned}
O_2^{\Delta s=2} &= (\bar{s}_\alpha (1-\gamma_5) d_\alpha)\,(\bar{s}_\beta (1-\gamma_5) d_\beta)\\
O_3^{\Delta s=2} &= (\bar{s}_\alpha (1-\gamma_5) d_\beta)\,(\bar{s}_\beta (1-\gamma_5) d_\alpha)\\
O_4^{\Delta s=2} &= (\bar{s}_\alpha (1-\gamma_5) d_\alpha)\,(\bar{s}_\beta (1+\gamma_5) d_\beta)\\
O_5^{\Delta s=2} &= (\bar{s}_\alpha (1-\gamma_5) d_\beta)\,(\bar{s}_\beta (1+\gamma_5) d_\alpha)
\end{aligned}$$

This work: study of $\langle \bar{K}^0 | O_i^{\Delta s=2} | K^0 \rangle$

See also plenary talks by N. Christ, C. Tarantino

parallel talks by N. Carrasco Vela, E. Freeland, H.-J. Kim, S. Sharpe, ...

Other studies

Quenched

- Clover-Wilson fermions
  [Allton, Conti, Donini, Gimenez, Giusti, Martinelli, Talevig, Vladikas '98]
  [Donini, Gimenez, Giusti, Martinelli '99]
- Overlap fermions
  [Babich, N.G., Hoelbling, Howard, Lellouch, Rebbi '06]

$n_f > 0$, preliminary studies

- $n_f = 2 + 1$ domain wall fermions [Wennekers, RBC-UKQCD '08]
- $n_f = 2$ twisted mass [Dimopoulos, Frezzotti, Gimenez, Lubicz, Martinelli, Mescia, Papinutto, Rossi, Simula, Vladikas, ETM '10]
- $+$ current studies presented at this conference

Remarks

- Only the parity even part O_i^+ contribute to $\langle \bar{P}|O_i|P\rangle$
- They can be expressed in the renormalization basis

$$O_1^+ \leftrightarrow Q_1 = (\gamma_\mu \times \gamma_\mu + \gamma_\mu \times \gamma_5)_{\rm unmixed} \Rightarrow (27,1) \rightarrow m_P^2$$

$$(O_4^+, O_5^+) \leftrightarrow \begin{pmatrix} Q_2 & = & \gamma_\mu \times \gamma_\mu - \gamma_\mu \times \gamma_5 \\ Q_3 & = & 1 \times 1 - \gamma_5 \times \gamma_5 \end{pmatrix}_{\rm unmixed} \Rightarrow (8,8) \rightarrow \mathrm{Cst}$$

$$(O_2^+, O_3^+) \leftrightarrow \begin{pmatrix} Q_4 & = & 1 \times 1 + \gamma_5 \times \gamma_5 \\ Q_5 & = & \sigma_{\mu\nu} \times \sigma_{\mu\nu} \end{pmatrix}_{\rm unmixed} \Rightarrow (6,\bar{6}) \rightarrow \mathrm{Cst}$$

- O_2 and O_3 mix under renormalization, so do O_4 and O_5
- In the chiral limit $O_1 \rightarrow m_P^2$ and $O_{i\geq 2} \rightarrow \mathrm{Cst}$

Normalisation

We want to relate $\langle \bar{K}^0 | H^{\Delta s=2} | K^0 \rangle$ to experimentally measured quantity (e.g $\varepsilon_K, \Delta m_K$), and where

$$H^{\Delta s=2} = \sum_{i=1}^{5} C_i(\mu)\, O_i^{\Delta s=2}(\mu)\,.$$

$\Rightarrow$ On the lattice compute $\langle \bar{K}^0 | O_i^{\Delta s=2} | K^0 \rangle$, for $i = 1, \ldots 5$.

but the matrix elements are dimension-4 quantities

$\Rightarrow$ Normalise them to obtain dimensionless quantities

For O_1, we normalise by the VSA

$$\Rightarrow B_K = \frac{\langle \bar{K}^0 | O_1^{\Delta s=2} | K^0 \rangle}{\langle \bar{K}^0 | O_1^{\Delta s=2} | K^0 \rangle_{\mathrm{VSA}}} = \frac{\langle \bar{K}^0 | O_1^{\Delta s=2} | K^0 \rangle}{\frac{8}{3} m_K^2 f_K^2}$$

- m_K and f_K are well-measured physical quantities
- B_K is finite in the chiral limit
- (Similar functional form at NLO of χ PT in quenched and in unqueched theory [Sharpe '92])

Normalisation (II)

The VSA of the BSM WME are

$$\langle \bar{K}^0 | O_{i\geq 1} | K^0 \rangle \sim |\langle \bar{K}^0 | \bar{s}\gamma_5 d | 0 \rangle|^2 \sim (m_K f_K)^2 (m_K/(m_s + m_d))^2$$

Why introducing the quark masses in the game ?

Different normalisations have been introducde in the literature

we follow [Babich, N.G., Hoelbling, Howard, Lellouch, Rebbi '06]

$$\begin{aligned} R_i^{\mathrm{BSM}}(m_P) &= \left[\frac{f_K^2}{m_K^2}\right]_{\mathrm{expt}} \left[\frac{m_P^2}{f_P^2} \frac{\langle \bar{P} | O_i | P \rangle}{\langle \bar{P} | O_1 | P \rangle}\right]_{\mathrm{latt}} \\ &\xrightarrow{P \to K} \frac{\langle \bar{P} | O_i | P \rangle}{\langle \bar{P} | O_1 | P \rangle} \sim \mathrm{BSM/SM} \end{aligned}$$

- Finite in the chiral limit (and smooth chiral behaviour)
- Cancel systematic errors
- B_K ($\Leftrightarrow \langle \bar{P} | O_1 | P \rangle$) is well known
- No need to invoke the PCAC relation, no quark mass, etc

Simulation details

- Fermions: $n_f = 2 + 1$ Domain-Wall. Glue: Iwasaki
- Single lattice spacing $a \sim 0.086\ \mathrm{fm} \leftrightarrow a^{-1} \sim 2.28\ \mathrm{GeV}$, $32^3 \times 64 \times 16$ ($\Rightarrow L \sim 2.8\ \mathrm{fm}$)
- Three unitary light quark masses $am = 0.004, 0.006, 0.008 \quad \Rightarrow m_\pi \sim 290, 340, 390\ \mathrm{MeV}$
- $am_s = 0.03$ fixed (physical value ~ 0.028)
- Residual mass $am_{\mathrm{res}} \sim 0.00067$

More details can be found in [Aoki et al, RBC-UKQCD '10, arXiv:1012.4178, 1011.0892]

Extraction of the bare matrix elements

Follow [ArXiv:1012.4178]

Create /Annihilate kaon/antikaon with wall-source at fixed time t_i, t_f

Insert the four-quark operator at time t, and let $t \in [t_i, t_f]$

$\Rightarrow$ Define

$$
\begin{aligned}
c_i(t) &= \langle \bar{P}(t_f) O_i(t) \bar{P}(t_i) \rangle \\
&\longrightarrow \langle 0|P|\bar{P}\rangle \langle \bar{P}|O_i|P\rangle \langle P|P|0\rangle \times \exp(-m_P(t_f - t_i))
\end{aligned}
$$

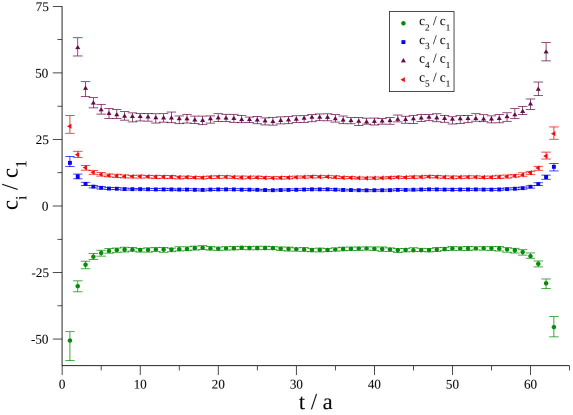
And compute the ratios

$$
\frac{c_i(t)}{c_1(t)} \longrightarrow \frac{\langle \bar{P}|O_i|P\rangle}{\langle \bar{P}|O_1|P\rangle}
$$

Extraction of m_P, f_P from 2-pt functions, done as in [arXiv:1011.0892]

Plateaux

$$\frac{c_i(t)}{c_1(t)} \quad \longrightarrow \quad \frac{\langle \bar{P}|O_i|P\rangle}{\langle \bar{P}|O_1|P\rangle} \qquad \text{fit in the range } t/a = [12, 52]$$



Example for our lightest unitary kaon $am^{val}_{\mathrm{light}} = am^{sea}_{\mathrm{light}} = 0.004$, $am^{val}_s = 0.03$

Extraction of the Z-factors

- Mixing pattern given by the $SU(3)_L \otimes SU(3)_R$ decomposition of the operators

  If chiral symmetry is respected the Z matrix is of the form

$$Z = \begin{pmatrix} Z_{11} & & & & \\ & Z_{22} & Z_{23} & & \\ & Z_{32} & Z_{33} & & \\ & & & Z_{44} & Z_{45} \\ & & & Z_{54} & Z_{55} \end{pmatrix}$$

  $\Rightarrow$ Domain-Wall disctretisation is a natural candidate ("almost exact" chiral symmetry)

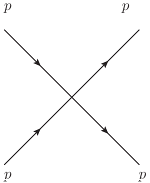
- We perform the NP renormalization à la Rome-Southampton [Martinelli et al '94]
- We use momentum sources [Göckeler et al, QCDSF '98] $\Rightarrow$ tiny statistical errors
- And twisted boundary condition $\Rightarrow$ lie on a scaling trajectory [Arthur and Boyle '10]

Kinematics

Original choice of kinematics contains exceptional channels

$\Rightarrow$ Potentially important IR effects $\sim 1/p^2$

$\Rightarrow$ can obscure the good chiral properties of the DW fermions



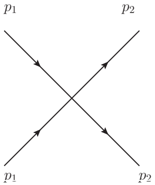
Nice solution: use non-exceptional schemes

for example SMOM $p_1^2 = p_2^2 = (p_1 - p_2)^2$

$\Rightarrow$ Better IR behaviour

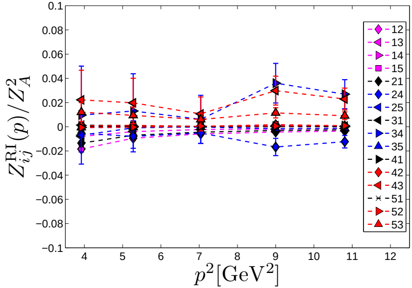
$\Rightarrow$ In many cases the perturbative expansion converges faster

See for example [Aoki et al '07, '10, Sturm et al '09, Lehner and Sturm '11]

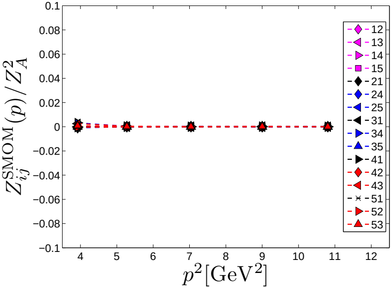


Exceptional vs non-exceptional

Chirally forbidden mixing matrix elements



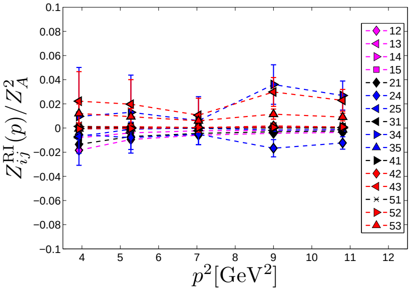
RI-MOM (Exceptional)



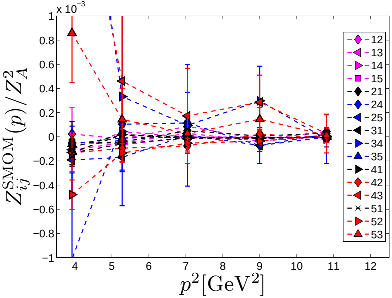
SMOM (γ_μ, γ_μ) (non-Exceptional)

Results given in the renormalization basis and in the MOM schemes

Chirally forbidden mixing matrix elements



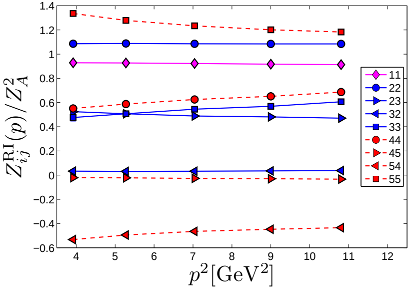
RI-MOM (Exceptional)



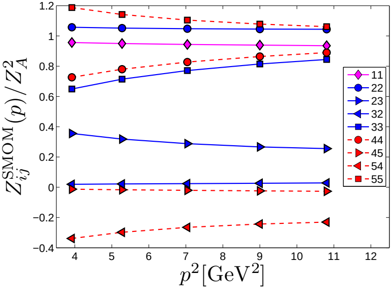
SMOM (γ_μ, γ_μ) (non-Exceptional)

Results given in the renormalization basis and in the MOM schemes

Physical mixing matrix elements



RI-MOM (Exceptional)



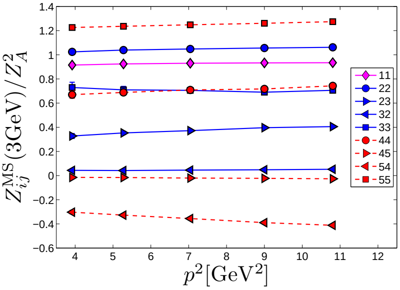
SMOM (γ_μ, γ_μ) (non-Exceptional)

Results given in the renormalization basis and in the MOM schemes

Conversion to $\overline{\mathrm{MS}}$

- Conversion factors RI-MOM $\to \overline{\mathrm{MS}}$ are known at one-loop for the whole basis [Ciuchini et al '93]
- Conversion factors SMOM's $\to \overline{\mathrm{MS}}$ are known at one-loop for the $(27,1)$ and the $(8,8)$, for four different SMOM-schemes [Aoki et al '10, Lehner and Sturm '11]
- Bad news: conversion factors for the $(6,\bar{6})$ are not available for non-exceptional scheme

Therefore, for consistency, we use only the RI-MOM scheme.



Z/Z_A^2 in the renormalization basis

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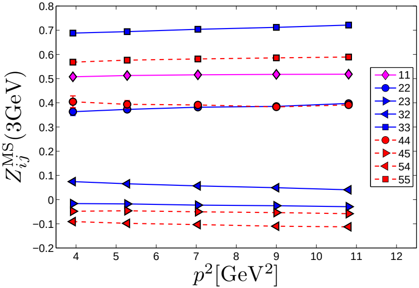
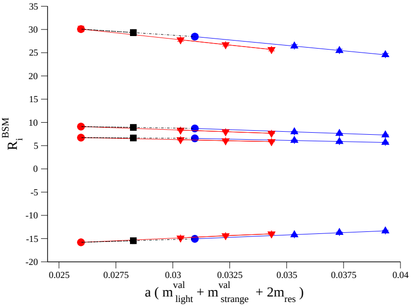


Z in the SUSY basis

$$R_i^{\mathrm{BSM}}(m_P) = \left[\frac{f_K^2}{m_K^2}\right]_{\mathrm{expt}} \left[\frac{m_P^2}{f_P^2}\,\frac{\langle \bar{P}|O_i|P\rangle}{\langle \bar{P}|O_1|P\rangle}\right]_{\mathrm{latt}}$$



Keep $m^{val}_{\mathrm{srange}} = m^{sea}_{\mathrm{strange}} = 0.03$ fixed, and extrapolate light quark masses to physical value

Keep $m^{val}_{\mathrm{strange}} = 0.025$ fixed, and extrapolate the light quark masses to physical value

Interpolate to the physical strange $m^{val}_{\mathrm{strange}} = 0.0273(7)$

Other possibility: use NLO Heavy-meson χPT [Detmold and Lin '06] (for staggered, see [Bailey, Kim, Lee, Sharpe '12])

Physical results and error budget

Preliminary results in $\overline{\mathrm{MS}}$ at 3 GeV and error budget (in %)

i	R_i^{BSM}	B_i	stat.	discr.	extr.	NPR	PT	total
2	$-15.5(1.7)$	0.43 (5)	1.3	1.5	3.8	9.2	4.0	10.9
3	8.9(0.8)	1.25 (11)	1.3	1.5	3.6	4.1	6.2	8.5
4	29.3(2.9)	0.69 (7)	1.6	1.5	4.2	3.0	8.1	9.9
5	6.7(0.9)	0.47 (6)	1.8	1.5	2.8	3.2	12.5	13.3

and we found $B_1 = B_K = 0.517(4)_{\mathrm{stat}}$

(compared to the continuum value $0.529(5)_{\mathrm{stat}}(19)_{\mathrm{syst}}$ with SMOM-($\not{q}, \not{q}$) [Aoki at al, RBC-UKQCD '10])

N.B. Discretisation errors computed only for B_K

Conclusions and outlook

- We are computing BSM contributions to neutral kaon mixing with $n_f = 2 + 1$ Domain-Wall fermions
- Confirm previous quenched study where large ratios non-SM/SM were found
- Errors $\sim 10\%$
- Two dominant sources of errors come from the NPR with exceptional scheme
- Plan to compute the matching factors and add another lattice spacing

  in collaboration with C.Lehner, A.T.Lytle, C.T.Sachrajda ...
- Expect to reduce the error below $\sim 5\%$

Acknowledgements:

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