Lattice 2012

Data Analysis for BSM corrections to Kaon B parameters

Brookhaven National Laboratory	Hyung-Jin Kim
Seoul National University	Boram Yoon, Jon A. Bailey, Weonjong Lee
University of Washington	Stephen Sharpe

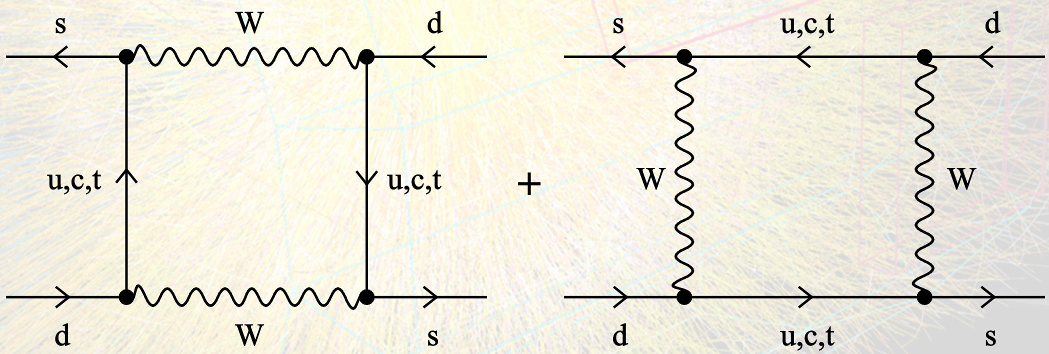
The 30th International Symposium on Lattice Field Theory, Cairns Convention Centre, Australia

Introduction

Standard Model : B_K **parameter**

⇨ Important parameter in determining indirect CP violation of neutral K meson($K^0 - \overline{K}^0$).

Kaon mixing Box diagram ⇨



$$B_K(\mu) = \frac{\sum_\nu \langle \bar{K}_0 | O_1 | K_0 \rangle}{\frac{8}{3} \langle \bar{K}_0 | \bar{s}\gamma_0\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_0\gamma_5 d | K_0 \rangle}$$

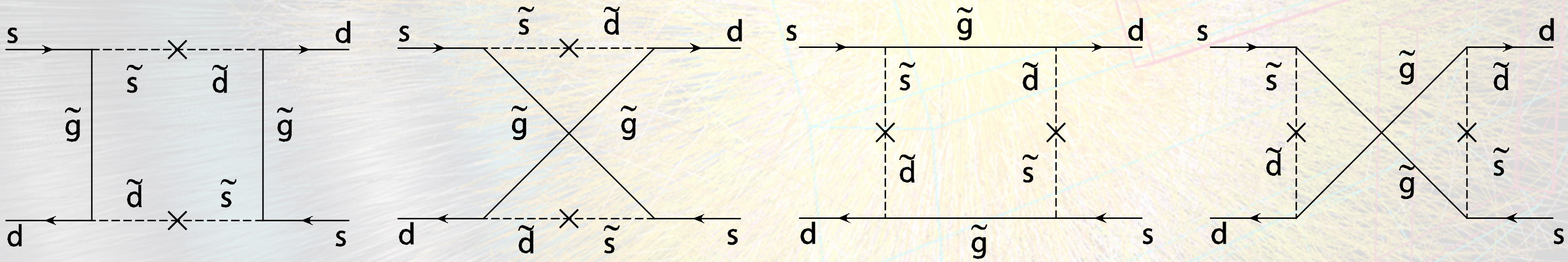
$$O_1 = \bar{s}^a \gamma_\mu (1 - \gamma_5) d^a \bar{s}^b \gamma_\mu (1 - \gamma_5) d^b$$

Introduction

Beyond Standard Model : B_i **parameter(** $i = 2, 3, 4, 5$ **)**

EX) New operators could come from light SUSY particles

(squark, gluinos in supersymmetry model are integrated out).



$$O_2 = \bar{s}^a(1-\gamma_5)d^a\bar{s}^b(1-\gamma_5)d^b$$
$$O_3 = \bar{s}^a(1-\gamma_5)d^b\bar{s}^b(1-\gamma_5)d^a$$
$$O_4 = \bar{s}^a(1-\gamma_5)d^a\bar{s}^b(1+\gamma_5)d^b$$
$$O_5 = \bar{s}^a(1-\gamma_5)d^b\bar{s}^b(1+\gamma_5)d^a$$

O_2, O_3 (or O_4, O_5) only color index difference! → Same form in χPT

$$B_j(\mu) = \frac{\langle \overline{K}_0 | O_j(\mu) | K_0 \rangle}{N_j \langle \overline{K}_0 | \bar{s}^a \gamma_5 d^a(\mu) | 0 \rangle \langle 0 | \bar{s}^b \gamma_5 d^b(\mu) | K_0 \rangle}$$

$$(N_2, N_3, N_4, N_5) = (5/3, -1/3, -2, -2/3)$$

Motivation & Background

BSM corrections to Kaon B parameters in SχPT

- In the BSM, there are additional four fermion operators which contribute to kaon mixing matrix elements.
- Experiments and calculations of hadronic matrix elements can give significant constraints on BSM.
- We calculate these matrix elements using improved staggered fermion.
- To extrapolate to physical d, s quark masses, we use staggered chiral perturbation theory.
- NLO SχPT technical details will be given here by S. Sharpe.

Analysis Data

Lattices for data generation (Done , Progress)

a (fm)	am_l/am_s	geometry	ens×meas	ID
0.12	0.01/0.05	$20^3 \times 64$	671×9	C3
0.09	0.0062/0.031	$28^3 \times 96$	995×9	F1
0.06	0.0036/0.018	$48^3 \times 144$	749×9	S1
0.045	0.0028/0.014	$64^3 \times 192$	753×1	U1

- Improved Staggered Fermion(valence: HYP, sea: Asqtad)

Quark masses

a (fm)	am_x and am_y		phys. am_d	phys. am_s
0.12	$0.005 \times n$	with $n = 1, 2, \ldots, 10$	0.00212(2)	0.05174(5)
0.09	$0.003 \times n$	with $n = 1, 2, \ldots, 10$	0.00142(2)	0.03523(5)
0.06	$0.0018 \times n$	with $n = 1, 2, \ldots, 10$	0.00102(1)	0.02362(3)
0.045	$0.0014 \times n$	with $n = 1, 2, \ldots, 10$	0.00075(1)	0.01684(2)

Computation of $B_{2\text{-}5}$

NLO B_j parameters on the SχPT (Ref: Phys. Rev. D85, (2012), 074507)

$$B_j = B_j^{LO}[1 + \delta B_j^{anal} + \delta B_j^{\chi log}]$$

$$\delta B_j^{anal,SU(2)} = d_{j1} m_x + d_{j2}(m_u + m_d) + d_{j3} a^2 + d_{j4} a_\alpha^2 + d_{j5} \alpha^2$$

$$\delta B_j^{\chi log,SU(2)} = \pm \frac{1}{(4\pi f_2)^2}\left[\frac{-1}{16}\sum_B \ell(M_{xx,B}^2) + \frac{1}{2}\left\{\ell(M_{xx,I}^2) + (M_{\pi,I}^2 - M_{xx,I}^2)\tilde{\ell}(M_{xx,I}^2)\right\}\right]$$

$+ : j = 2,\ 3 \quad , \ - : j = 4, 5$

- General form of NLO expression.
- SU(2) analysis is in progress.
- Because $O_2, O_3 (\text{or } O_4, O_5)$ have the same form in χPT, $B_2, B_3 (\text{or } B_4, B_5)$ also have the same form.
- B_2, B_3 results are identical to NLO form for B_K .

Computation of $B_{2\text{-}5}$

1 loop matching formula (Ref: Phys. Rev. D83, (2011), 094503)

Renorm. $\mu a = 1$

$$\mathcal{O}_i^{\mathrm{Cont}'} = \sum_{j\in(A)} z_{ij}\mathcal{O}_j^{\mathrm{Lat}} - \frac{g^2}{(4\pi)^2}\sum_{k\in(B)} d_{ik}^{\mathrm{Lat}}\mathcal{O}_k^{\mathrm{Lat}}$$

→ NNLO

$$z_{ij} = b_{ij} + \frac{g^2}{(4\pi)^2}\Big(-\gamma_{ij}\log(\mu a) + d_{ij}^{\mathrm{Cont}} - d_{ij}^{\mathrm{Lat}} - C_F I_{MF} T_{ij}\Big)$$

Ex) $i = 2$ matching coefficients

Operator j	b_{2j}	γ_{2j}	d_{2j}^{Cont}	$d_{2j}^{\mathrm{Lat}}(c)$	$d_{2j}^{\mathrm{Lat}}(d)$	T_{2j}
$\mathcal{O}_{S1}^{\mathrm{Lat}}$	$-1/2$	6	$-59/12$	2.70	2.34	-1
$\mathcal{O}_{S2}^{\mathrm{Lat}}$	1	-10	$+73/12$	-17.80	-14.53	6
$\mathcal{O}_{P1}^{\mathrm{Lat}}$	$-1/2$	6	$-59/12$	3.64	3.17	-1
$\mathcal{O}_{P2}^{\mathrm{Lat}}$	1	-10	$+73/12$	5.42	4.06	-2
$\mathcal{O}_{T1}^{\mathrm{Lat}}$	$1/2$	$-14/3$	$+29/12$	-2.94	-2.52	1
$\mathcal{O}_{T2}^{\mathrm{Lat}}$	0	$2/3$	$-5/4$	0.03	0.01	0

SU(2) Chiral Fitting

Chiral Fitting function

$$B_j(NNLO) = c_1 F_0(j) + c_2 \frac{M^2_{xx,P}}{\Lambda^2} + c_3 \frac{M^4_{xx,P}}{\Lambda^4}$$

$$F_0(j) = 1 \pm \frac{1}{(4\pi f_2)^2}\left[\frac{-1}{16}\sum_B \ell(M^2_{xx,B}) + \frac{1}{2}\left\{\ell(M^2_{xx,I}) + (M^2_{\pi,I} - M^2_{xx,I})\tilde{\ell}(M^2_{xx,I})\right\}\right]$$

$\pm$ → $+ : j = 2,\ 3,\ K\ \ ,\ - : j = 4, 5$

SU(2) Chiral Fitting

Fitting Strategies

1. X-Fit

   For a given ensemble(fixed a, m_ℓ, m_s), fix m_y close to m_s^{phys}.

   Extrapolate m_x to the physical light quark mass using χ fitting function.

2. Y-Fit

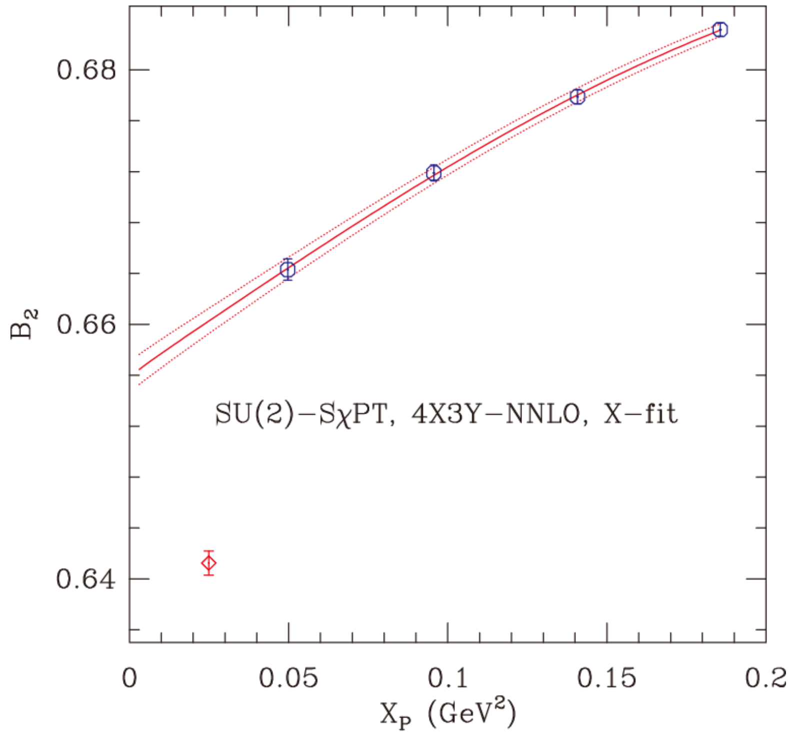
   Extrapolate m_y to the physical strange quark mass.

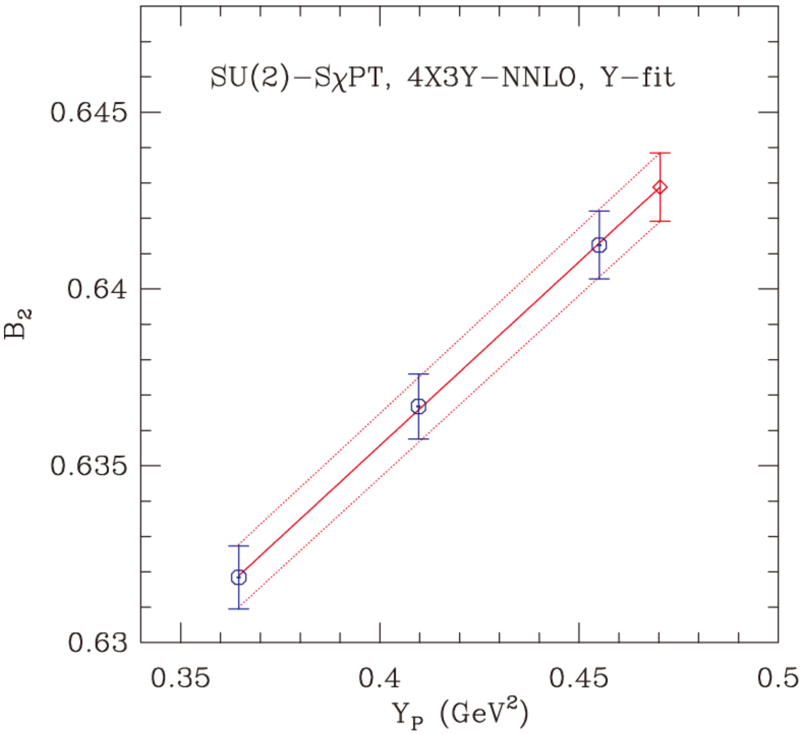
3. Continuum extrapolation

   Extrapolate a to continuum limit.

Fitting Result

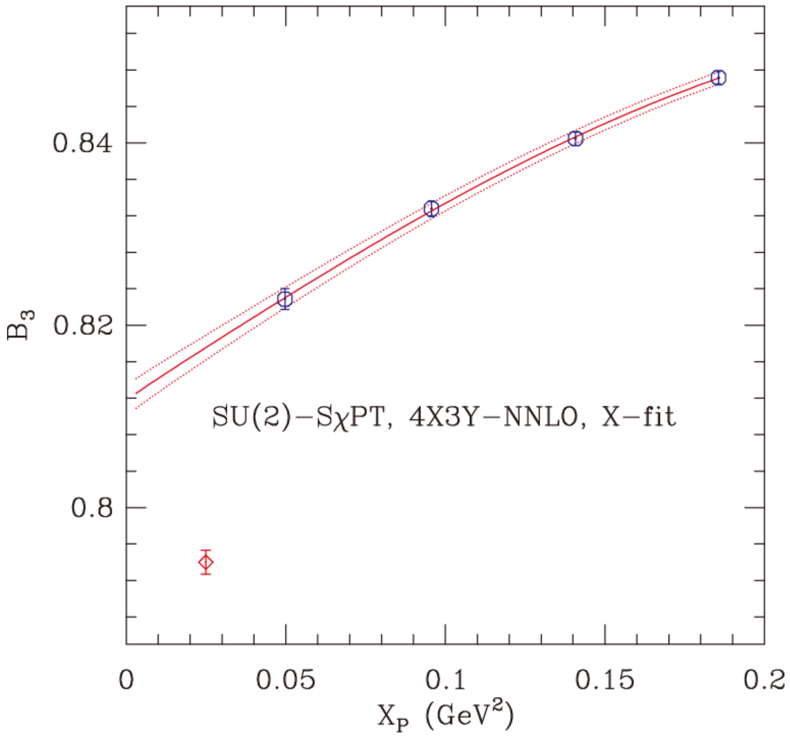
SU(2) Chiral Fitting: B_2 ($20^3 \times 64$, Diag. Approx.)

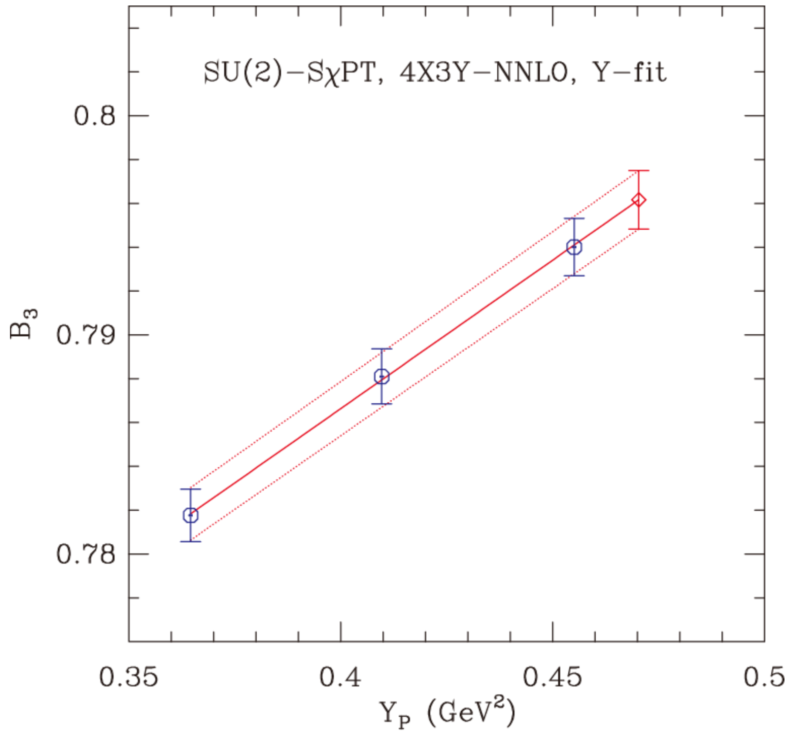




	c_1	c_2	c_3	χ^2/dof	B_2
m10	0.6276(12)	0.2667(107)	-0.5034(310)	0.1881(518)	0.6412(10)
m9	0.6229(11)	0.2711(099)	-0.5085(287)	0.1889(476)	0.6367(09)
m8	0.6180(11)	0.2759(092)	-0.5139(265)	0.1873(432)	0.6318(09)

SU(2) Chiral Fitting: B_3 ($20^3 \times 64$, Diag. Approx.)

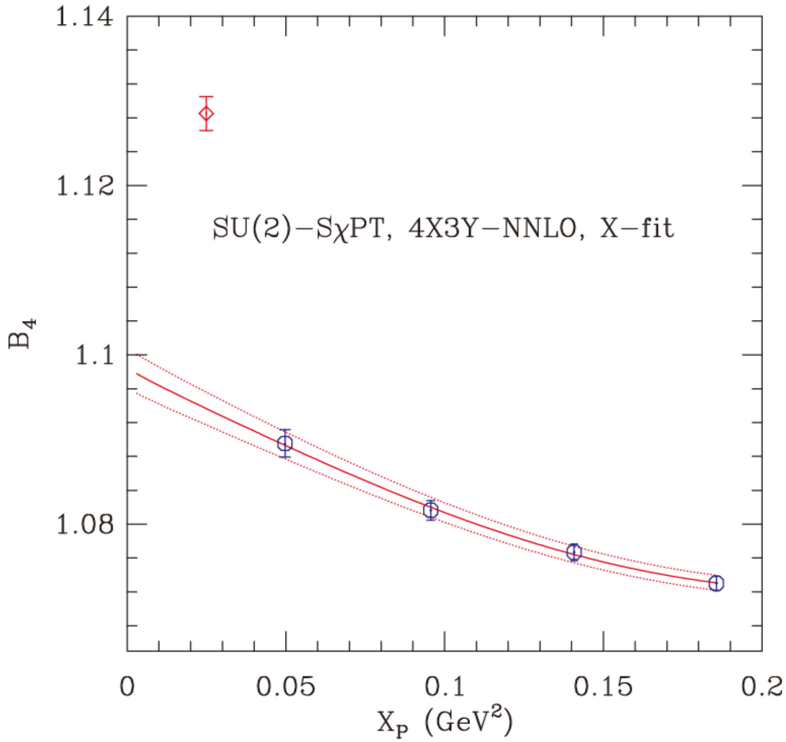


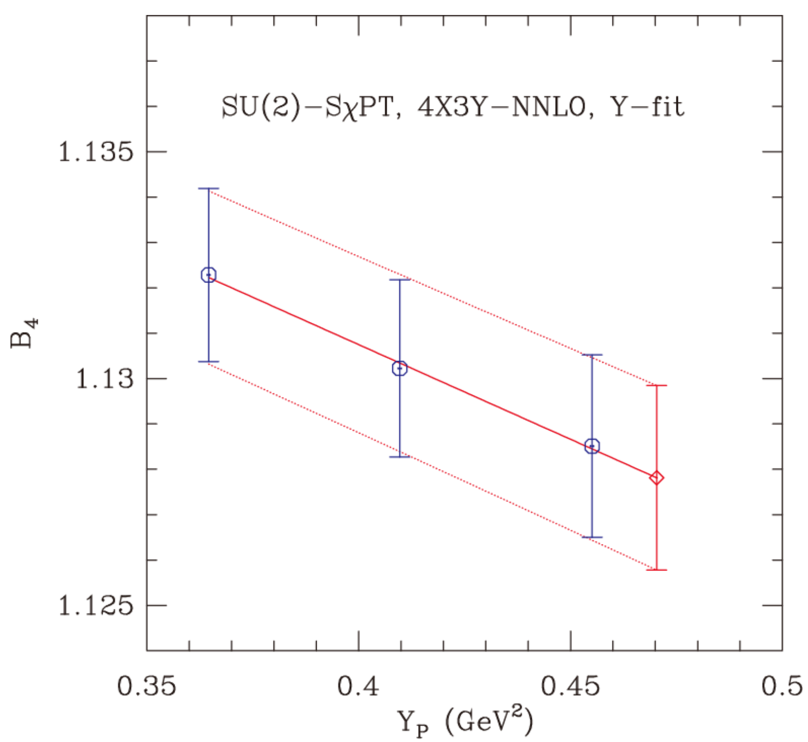


	c_1	c_2	c_3	χ^2/dof	B_3
m10	0.7767(16)	0.3464(151)	-0.6632(442)	0.1867(547)	0.7940(13)
m9	0.7707(15)	0.3530(139)	-0.6692(406)	0.1853(497)	0.7881(13)
m8	0.7642(14)	0.3605(128)	-0.6760(371)	0.1818(446)	0.7818(12)

Fitting Result

SU(2) Chiral Fitting: B_4 ($20^3 \times 64$, Diag. Approx.)

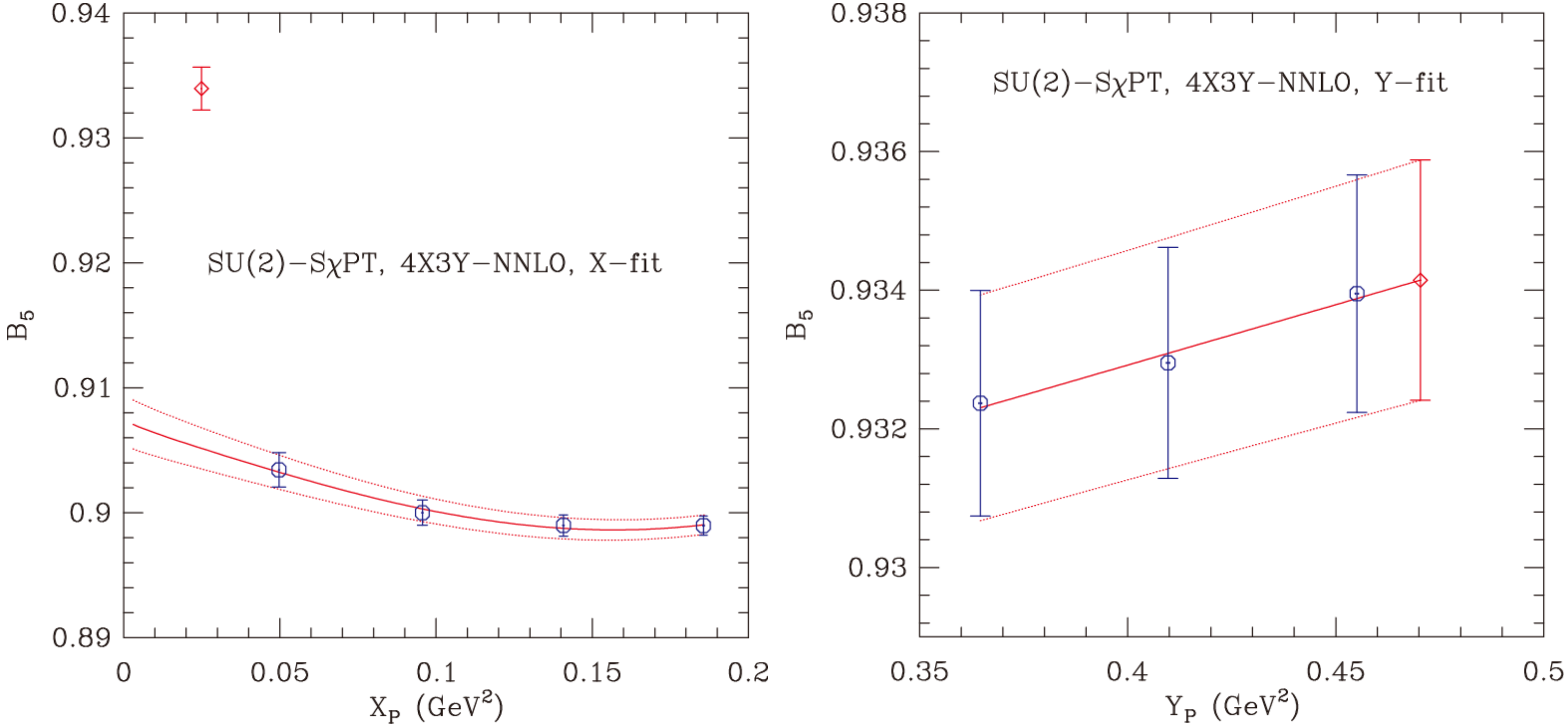




	c_1	c_2	c_3	χ^2/dof	B_4
m10	1.1503(25)	-0.3581(212)	0.9319(611)	0.1944(558)	1.1285(20)
m9	1.1522(24)	-0.3620(199)	0.9408(569)	0.1966(510)	1.1302(20)
m8	1.1543(23)	-0.3668(185)	0.9512(529)	0.1975(460)	1.1323(19)

Fitting Result

SU(2) Chiral Fitting: B_5 ($20^3 \times 64$, Diag. Approx.)



	c_1	c_2	c_3	χ^2/dof	B_5
m10	0.9503(21)	-0.2281(181)	0.7698(521)	0.1843(534)	0.9340(17)
m9	0.9494(20)	-0.2334(169)	0.7769(485)	0.1832(483)	0.9330(17)
m8	0.9490(20)	-0.2398(157)	0.7857(449)	0.1806(430)	0.9324(16)

Golden Combinations

Golden combinations for canceling the leading χ logs

$$R_j \equiv \frac{B_2}{B_3}, \frac{B_4}{B_5}, \frac{B_K}{B_2}, \left(B_{2(\text{or }3)} \cdot B_{4(\text{or }5)}\right) \quad \text{in SU(2) } \chi \text{ fitting}$$

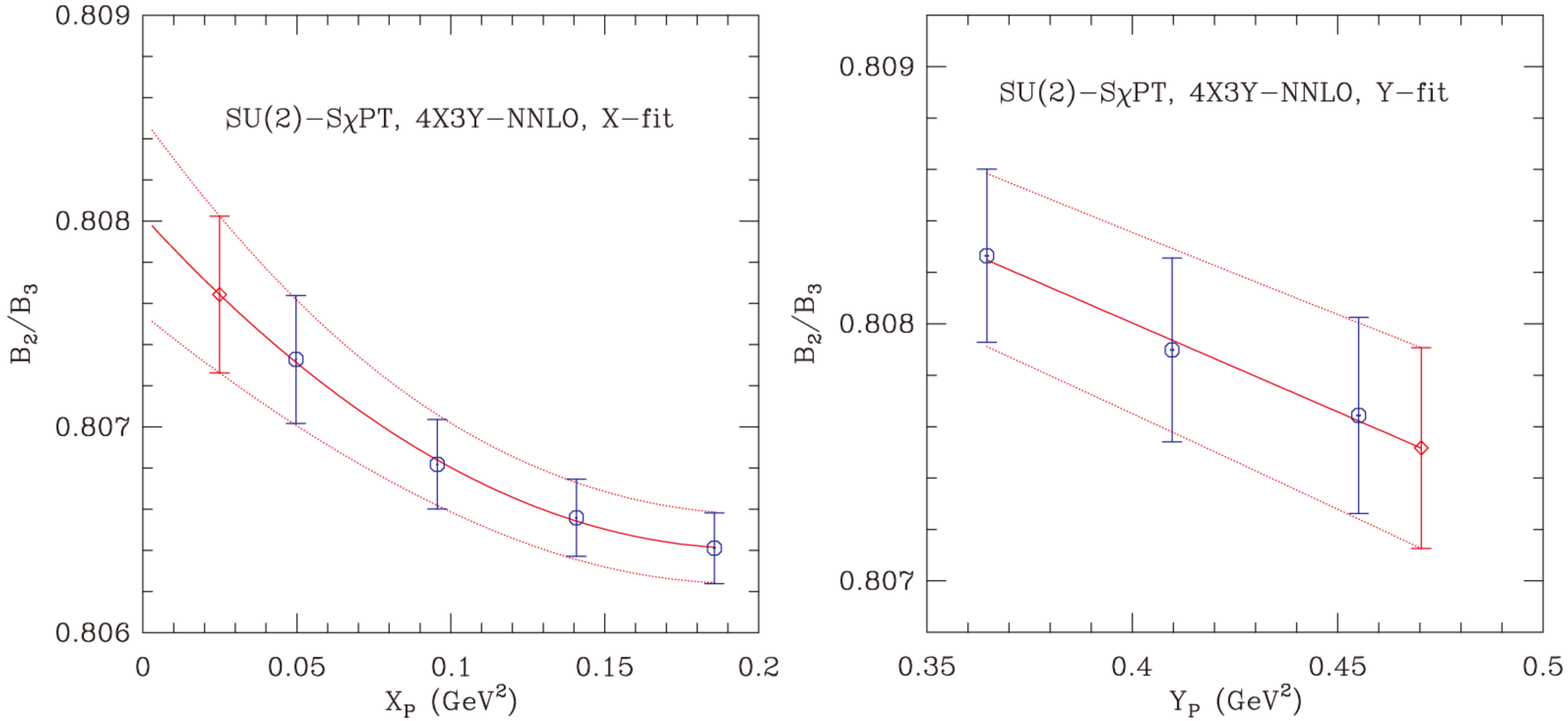
SU(2) fitting function for the gold combination

$$R_j^{NNLO} = c_1 + c_2 \frac{M_{xx,P}^2}{\Lambda^2} + c_3 \frac{M_{xx,P}^4}{\Lambda^4}$$

Gold combination of fitting function is especially important in Staggered χPT calculation, because it cancels the major effect of taste symmetry breaking!

※ more about the SU(3) and SU(2) cases will be explained in the S. Sharpe`s talk!

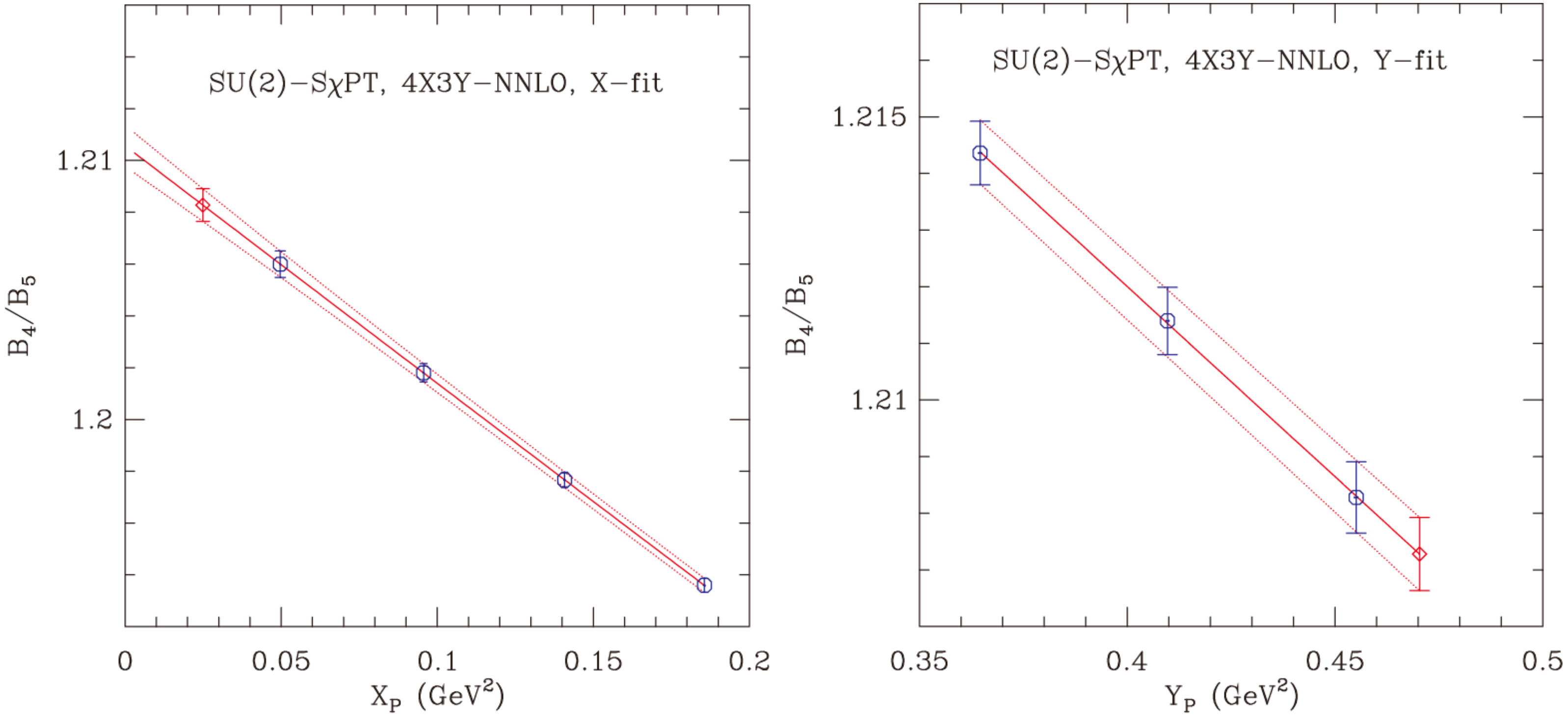
SU(2) Chiral Fitting: B_2/B_3 ($20^3 \times 64$, Diag. Approx.)



	c_1	c_2	c_3	χ^2/dof	B_2/B_3
m10	0.8080(5)	-0.0164(48)	-0.0414(141)	0.0188(222)	0.8076(4)
m9	0.8083(4)	-0.0177(43)	-0.0416(128)	0.0160(183)	0.8079(4)
m8	0.8087(4)	-0.0194(39)	-0.0423(114)	0.0134(149)	0.8083(3)

Fitting Result

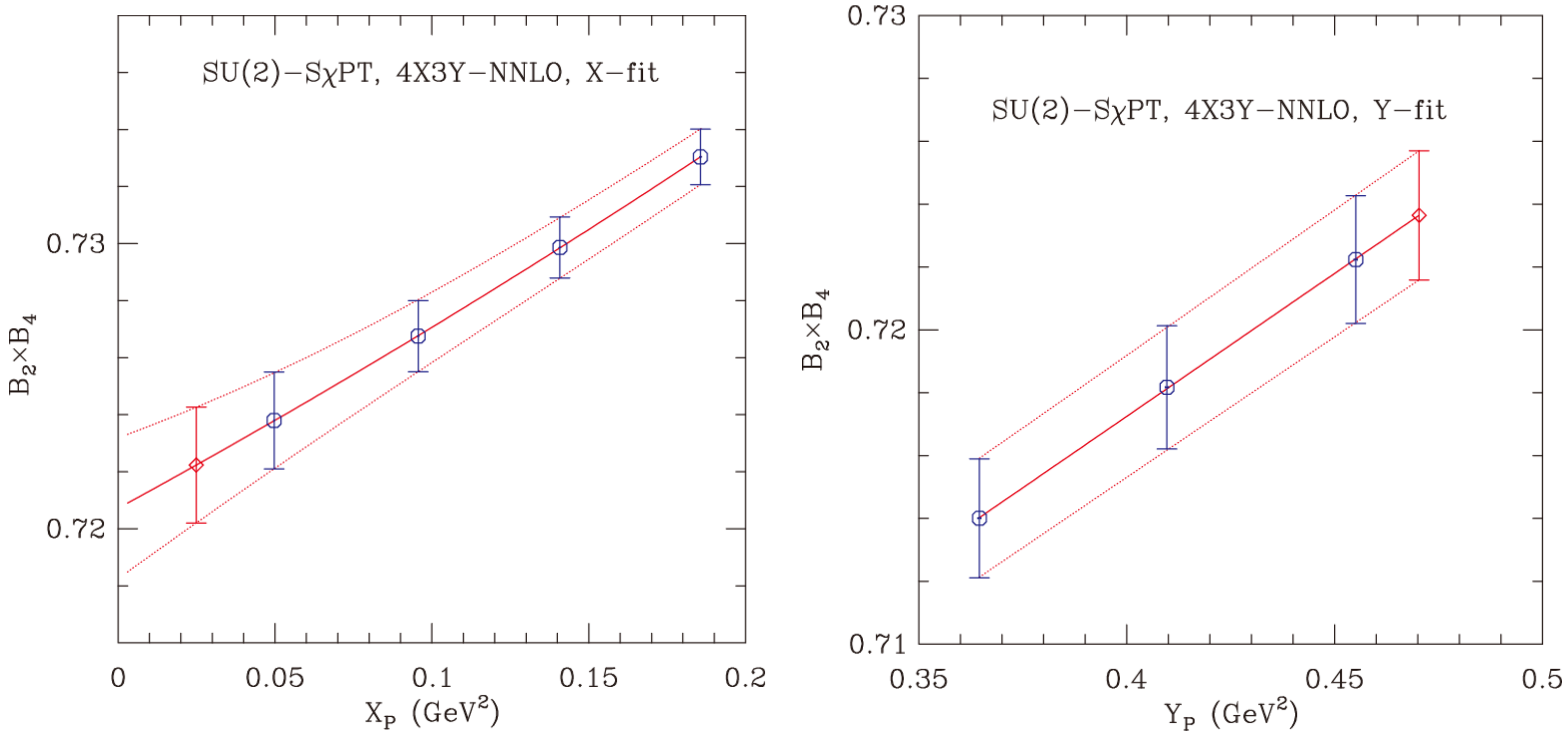
SU(2) Chiral Fitting: B_4/B_5 ($20^3 \times 64$, Diag. Approx.)



	c_1	c_2	c_3	χ^2/dof	B_4/B_5
m10	1.2106(8)	-0.0920(81)	0.0029(242)	0.0014(66)	1.2083(6)
m9	1.2136(7)	-0.0883(74)	-0.0006(221)	0.0009(49)	1.2114(6)
m8	1.2165(7)	-0.0840(68)	-0.0045(201)	0.0005(31)	1.2144(6)

Fitting Result

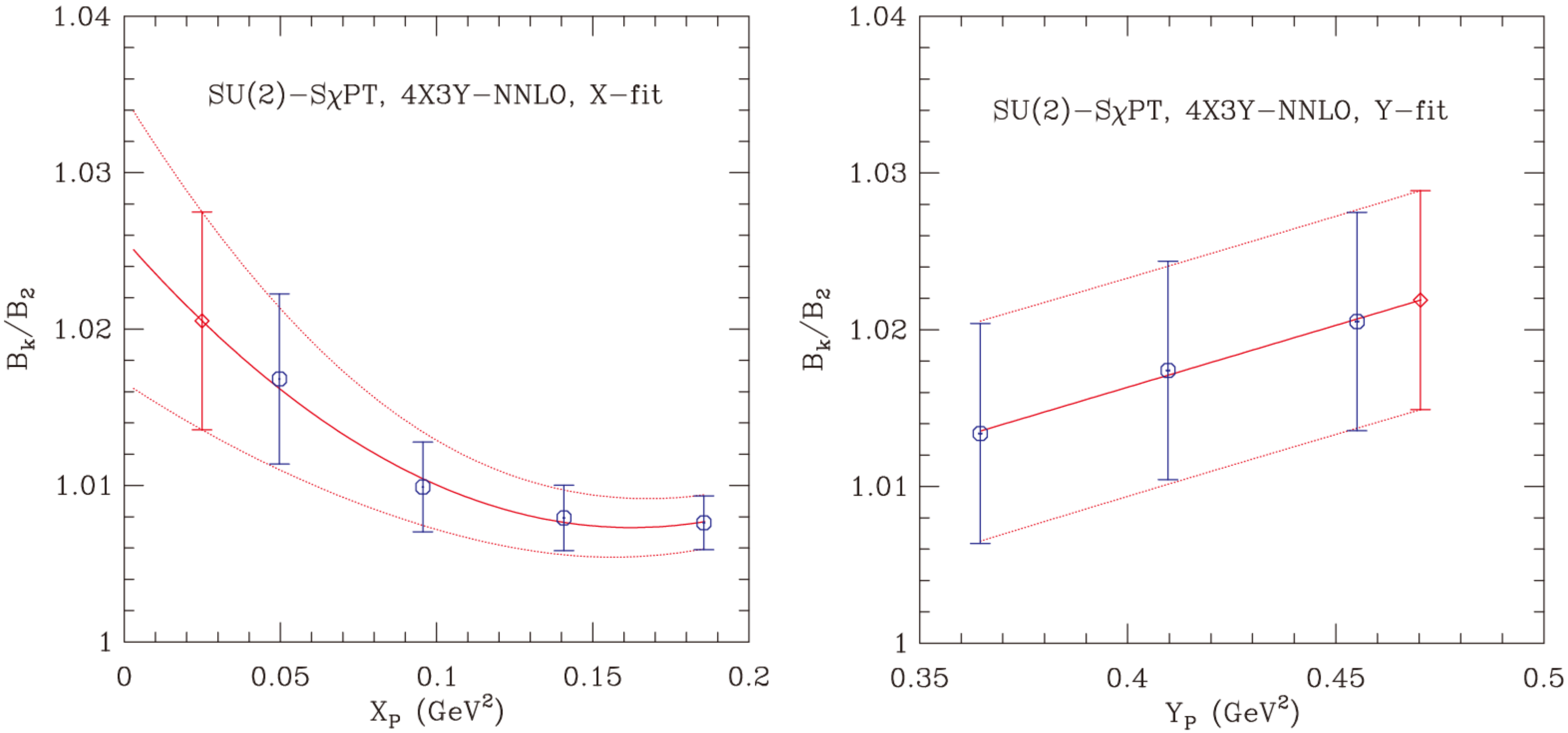
SU(2) Chiral Fitting: $B_2 \times B_4$ ($20^3 \times 64$, Diag. Approx.)



	c_1	c_2	c_3	χ^2/dof	$B_2 \times B_4$
m10	0.7207(25)	0.0600(219)	0.0346(630)	0.0004(25)	0.7222(20)
m9	0.7165(24)	0.0643(204)	0.0292(585)	0.0005(25)	0.7182(20)
m8	0.7123(23)	0.0688(189)	0.0240(542)	0.0006(25)	0.7140(19)

Fitting Result

SU(2) Chiral Fitting: B_K/B_2 ($20^3 \times 64$, Diag. Approx.)



	c_1	c_2	c_3	χ^2/dof	B_K/B_2
m10	1.0258(92)	-0.2268(994)	0.6977(2935)	0.0668(777)	1.0205(70)
m9	1.0232(91)	-0.2512(990)	0.8034(2926)	0.0856(851)	1.0174(70)
m8	1.0197(92)	-0.2750(991)	0.9200(2934)	0.1092(926)	1.0134(70)

Preliminary result of B parameters

	Coarse(C3)	Fine(F1)
$B_2 =$	$0.6429(10)$	$0.5820(10)$
$B_3 =$	$0.7962(13)$	$0.7271(14)$
$B_4 =$	$1.1278(20)$	$1.0999(20)$
$B_5 =$	$0.9341(17)$	$0.9310(18)$
$B_2/B_3 =$	$0.8075(04)$	$0.8004(03)$
$B_4/B_5 =$	$1.2073(06)$	$1.1815(06)$
$B_2 \cdot B_4 =$	$0.7236(21)$	$0.6381(17)$
$B_K/B_2 =$	$1.0219(70)$	$1.0119(63)$
$B_K =$	$0.5789(53)$	$0.5182(42)$

Statistical error only

Renorm. $\mu a = 1$

Future Plan

- We have fitting results for the BSM B-parameters on MILC coarse, fine and superfine lattice.($20^3 \times 64$, $28^3 \times 96$, $48^3 \times 144$)
- Efficiency of golden combinations is not clear yet.
- Need to do continuum extrapolation, study finite-volume effect.
- Plan to use correlated fits.
- Plan to do alternative analysis based on SU(3) SχPT.