$B \to \pi$ form factor with $N_f = 2$ NP $O(a)$ improved Wilson quarks

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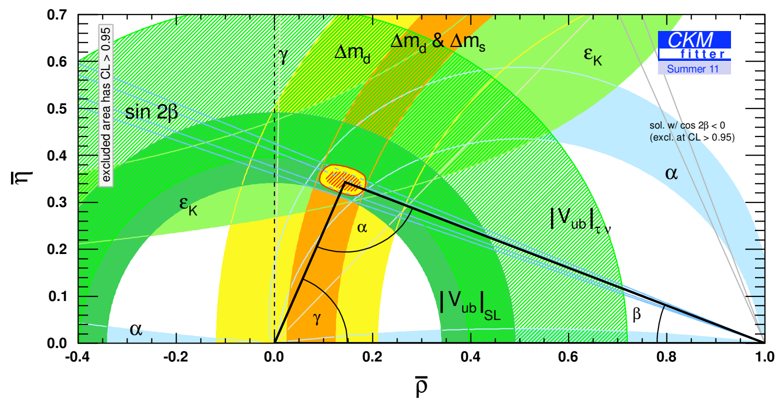
Motivation

Couplings of flavor-changing *weak interactions*:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



processes with $b \to u$ transitions

- Inclusive $B \to X_u \ell \nu$

  heavy quark and α_S expansion

Lattice input

- Exclusive $B \to \pi \ell \nu$

  hadronic formfactor $f_+(q^2)$

- Leptonic $B \to \tau \nu$ [see F. B. poster]

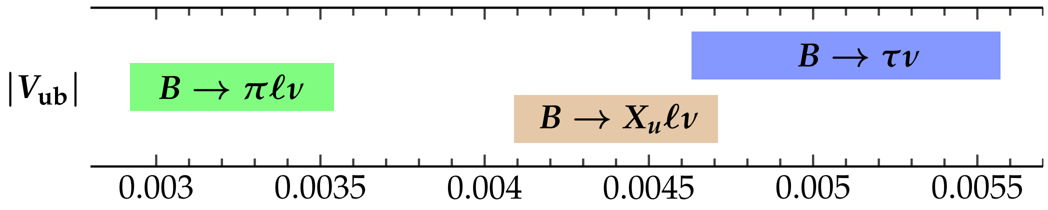
  hadronic decay constant f_B

V_{ub} *puzzle*

$+$

$(\mathcal{B}(B \to \tau\nu), \sin(2\beta))$ discrepancy

Summer 2012: [PDG'12]



$B \to \pi l \nu$ in the SM

At LO in α_{EM} and $m_l = m_\nu = 0$

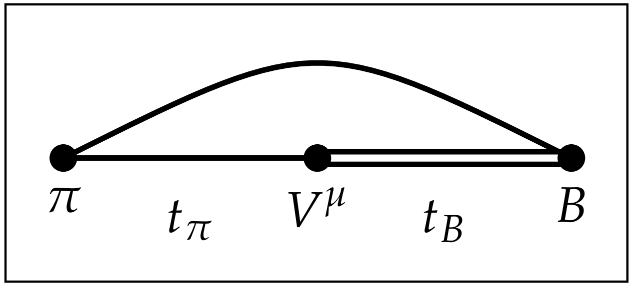
$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_B^3} \lambda^{3/2}(q^2) \left| f_+(q^2) \right|^2 \qquad q^\mu = p_B^\mu - p_\pi^\mu$$

$$\langle \pi(p_\pi) | V^\mu | B(p_B) \rangle = f_+(q^2) \left[p_B^\mu + p_\pi^\mu - \frac{m_B^2 - M_\pi^2}{q^2} q^\mu \right] + f_0(q^2) \frac{m_B^2 - M_\pi^2}{q^2} q^\mu$$

Typically on the lattice one computes (B at rest):

$$\langle \pi(p_\pi) | V^\mu | B(p_B) \rangle = \lim_{T\to\infty,\, t_{B,\pi}\to\infty} R(t_\pi, t_B) e^{E_\pi t_\pi/2} e^{m_B t_B/2}$$

$$R(t_\pi, t_B) \equiv \frac{\sum_{\vec{x}_\pi, \vec{x}_B} e^{-i\vec{p}\cdot\vec{x}_\pi} \langle P_{ll}(x_\pi + x_B) V^\mu(x_B) P_{hl}(0) \rangle}{\sqrt{\sum_{\vec{x}_\pi} e^{-i\vec{p}\cdot(\vec{x}_\pi)} \langle P_{ll}(x_\pi) P_{ll}(0) \rangle \sum_{\vec{x}_B} \langle P_{hl}(x_B) P_{hl}(0) \rangle}} \, .$$



- P_{ll} and P_{hl} are interpolating operators
- other ratios are possible

Treatment of light quarks

$N_f = 2$ sea Wilson NP $O(a)$ improved

CLS based

- FV effects small by construction

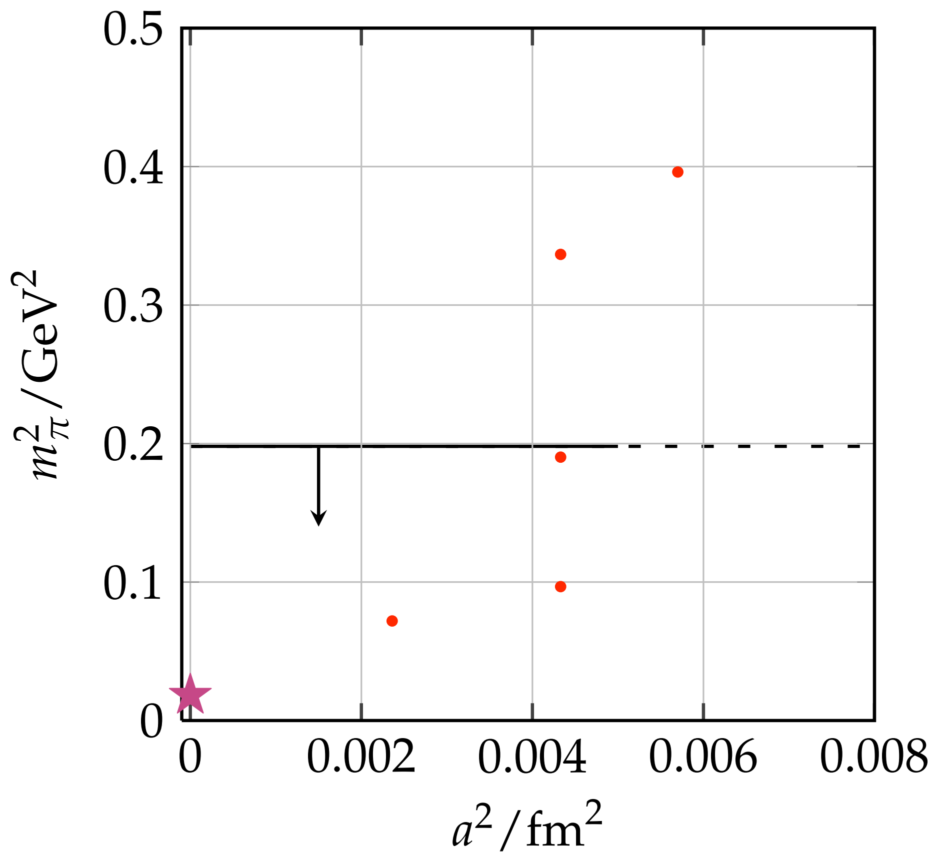
$$L m_\pi \geq 4.0$$

- data for chiral extrapolation uses

$$(250 \lesssim m_\pi \lesssim 400 - 450)\text{MeV}$$

- lattice spacings below 0.08 fm!

$$(0.048,\ 0.065,\ 0.075)\ \text{fm}$$



3 simulations fulfill our current criteria

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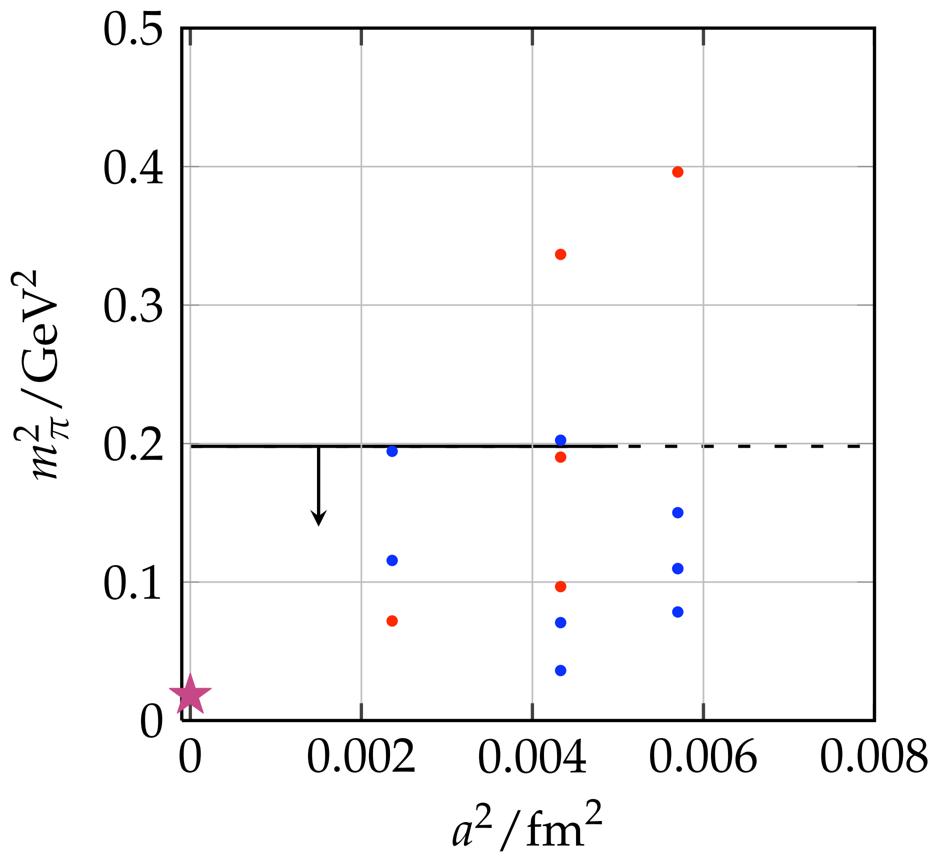


3 simulations fulfill our current criteria

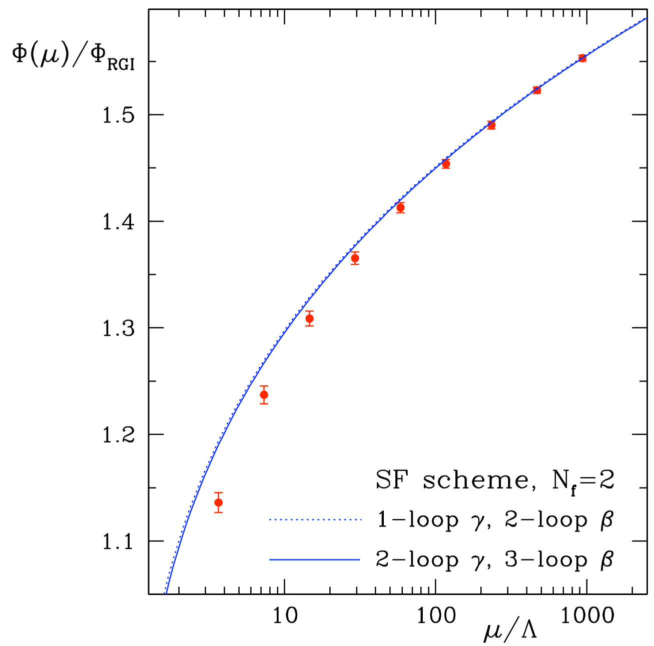
+ 8 more next year

Treatment of b quark

$m_b \gg \Lambda_{QCD}$: b treated in HQET

- expansion in $1/m_b$
  this work: static limit (LO in $1/m_b$)
- renormalizable at every order in $1/m_b$
- restrict to processes such that $p \ll m_b$
- operator mixing induces power div. in a^{-1}
  $\Rightarrow$ need NP renormalization
  [Maiani, Martinelli, Sachrajda 92]

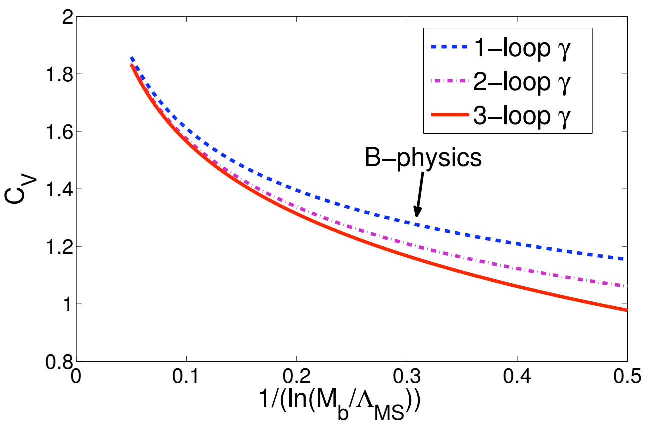
[Della Morte, Fritzsch, Heitger]



$$\langle \pi | V^i | B \rangle^{QCD} = C_V(M_b/\Lambda_{\overline{MS}}) \times Z^A_{RGI}(g_0) \langle \pi | V^i | B \rangle^{stat} + O(1/M_b)$$

$$Z^A_{RGI}(g_0) = \frac{\Phi_{RGI}}{\Phi(\mu)} \times Z^{stat}_A(g_0, \mu)$$

- $C_V(M_b/\Lambda_{\overline{MS}})$ at 3-loop (reliable? [Sommer, Les Houches 09])
- V^μ not yet O(a) improved



Form factors in HQET

HQET in B rest frame

$$\langle\pi|V^0|B\rangle = \sqrt{2m_B} f_{\|}$$
$$\langle\pi|V^j|B\rangle = \sqrt{2m_B} p_\pi^j f_\perp$$

$$f_+ = \frac{1}{\sqrt{2m_B}} f_{\|} + \frac{1}{\sqrt{2m_B}}(m_B - E_\pi) f_\perp$$
$$f_0 = \frac{\sqrt{2m_B}}{m_B^2 - m_\pi^2}[(m_B - E_\pi) f_{\|} + (E_\pi^2 - m_\pi^2) f_\perp]$$

$$V_k^{QCD} = C_V(M_b/\Lambda_{\overline{MS}}) Z^{stat}_{A,RGI}(g_0) V_k^{stat} + O(1/M_b)$$
$$V_0^{QCD} = C_{PS}(M_b/\Lambda_{\overline{MS}}) Z^{stat}_{A,RGI}(g_0) Z^{stat}_{V/A}(g_0) V_0^{stat} + O(1/M_b)$$

$C_V \times Z^A$, $C_{PS} \times Z^V$ and $O(1/m_b)$ NP determinations are under way

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$$\langle\pi|V^j|B\rangle = \sqrt{2m_B} p_\pi^j f_{\perp}$$

$$f_+ = \frac{1}{\sqrt{2m_B}} f_{\parallel} + \frac{1}{\sqrt{2m_B}} (m_B - E_\pi) f_{\perp}$$
$$f_0 = \frac{\sqrt{2m_B}}{m_B^2 - m_\pi^2} [(m_B - E_\pi) f_{\parallel} + (E_\pi^2 - m_\pi^2) f_{\perp}]$$

in static limit

expect $O(E_\pi / M_B)$ effects $10 - 20\%$

$$V_k^{QCD} = C_V(M_b/\Lambda_{\overline{MS}}) Z_{A,RGI}^{stat}(g_0) V_k^{stat} + O(1/M_b)$$
$$V_0^{QCD} = C_{PS}(M_b/\Lambda_{\overline{MS}}) Z_{A,RGI}^{stat}(g_0) Z_{V/A}^{stat}(g_0) V_0^{stat} + O(1/M_b)$$

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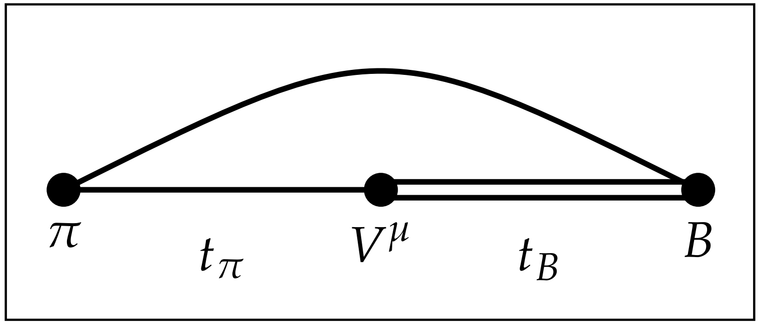
Finite T effects



Transfer matrix for $\Delta_\pi t_\pi \gg 1$ and $\Delta_B t_B \gg 1$:

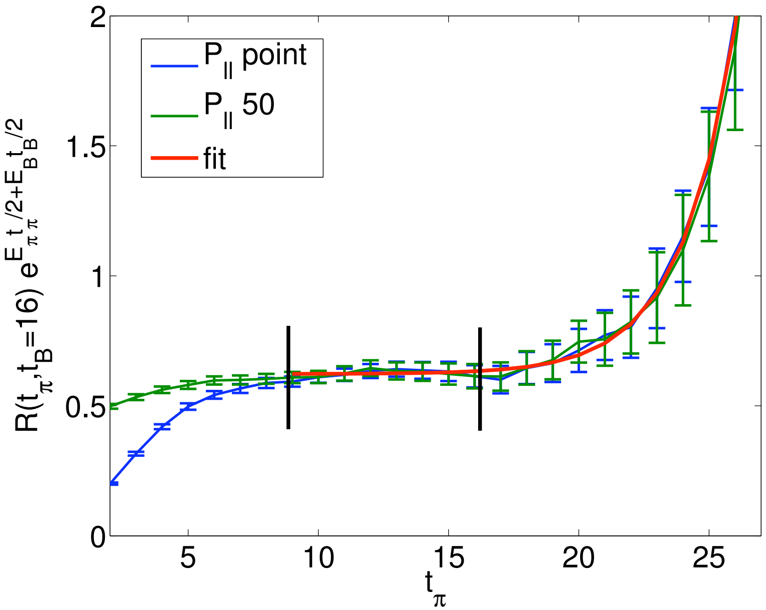
$$R(t_\pi, t_B) e^{E_\pi t_\pi / 2} e^{M_B t_B / 2} \to \frac{\langle \pi | V^\mu | B \rangle + \langle 0 | V^\mu | [\pi B] \rangle e^{-E_\pi (T - 2t_\pi) - \Delta_{\mathcal{B}} t_B}}{\sqrt{1 + e^{-E_\pi (T - 2t_\pi)}}}$$

- $\Delta_{\mathcal{B}} = E([\pi B]) - E_\pi - E_B$
- $\langle 0 | V^\mu | [\pi B] \rangle \sim \langle \pi | V^\mu | B \rangle$, $\Delta_{\mathcal{B}} \sim 0 \Rightarrow$
  - finite T effects dominate if $t_\pi > T/2$
  - finite T effects suppressed if $t_\pi < T/2 - k/E_\pi$

$\Rightarrow$ would be nice to have $T > 2L$ (we have $T = 2L$)

- or control/reduce excited states effects

Smearing, improvement of signal



id	L/a	a [fm]	m_π[MeV]	$m_\pi L$
A2	32	0.0755	630	7.7
E4	32	0.0658	580	6.2
E5			420	4.7
F6	48		310	5.0
O7	64	0.0486	270	4.2

- smear both B and π
- smearing pion earlier plateau (sig. to noise ratio deteriorates at $|p_\pi| \neq 0$)
- we use stochastic all to all (full time dilution)
- $M_\pi \sim 420$ MeV, 200cfgs, $4 \times T/a$ noise sources

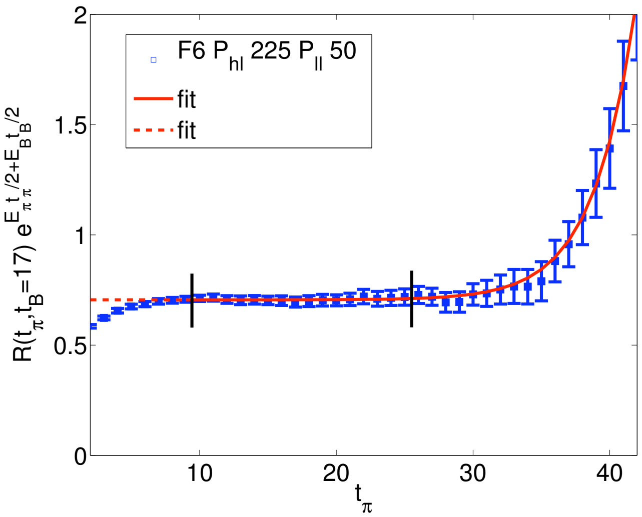
Data analysis

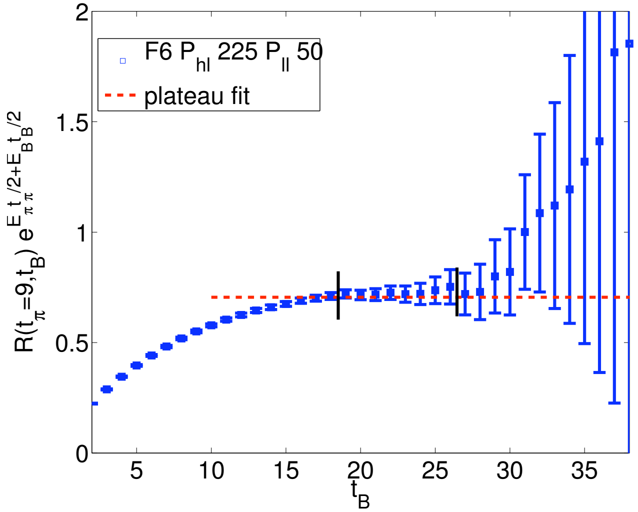
$$R(t_\pi, t_B) e^{E_\pi t_\pi/2} e^{M_B t_B/2} \to \frac{\langle \pi | V^\mu | B \rangle + \langle 0 | V^\mu | [\pi B] \rangle e^{-\Delta_B t_B} e^{-E_\pi (T - 2t_\pi)}}{\sqrt{1 + e^{-E_\pi (T-2t_\pi)}}}$$

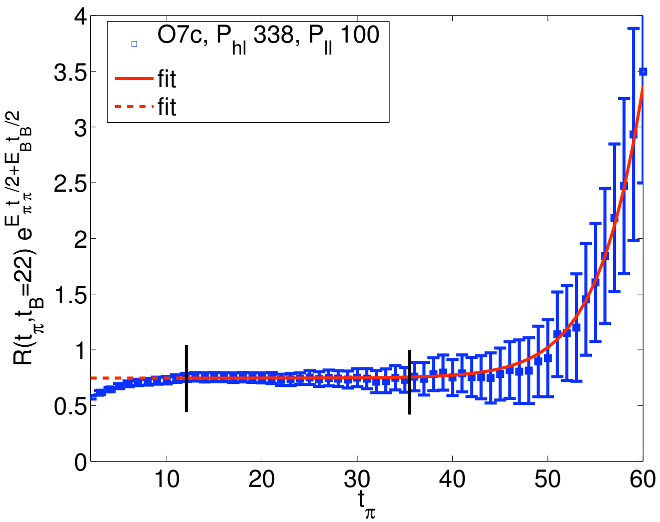
Fit to: $$\frac{A + B_{t_B} e^{-E_\pi (T - 2t_\pi)}}{\sqrt{1 + e^{-E_\pi (T-2t_\pi)}}}$$

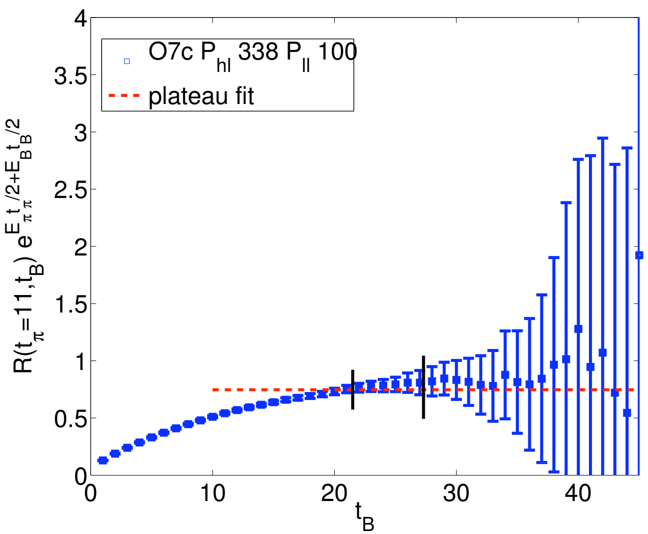
- A,B_{t_B} parameters
- E_π from 2pt-functions
- checked independendence from range (by amount $r_0/2$)
- cross-checked with plateau result
- correlations and autocorrelations conservatively taken into account
  [Schaefer Sommer Virotta]

Plateaux



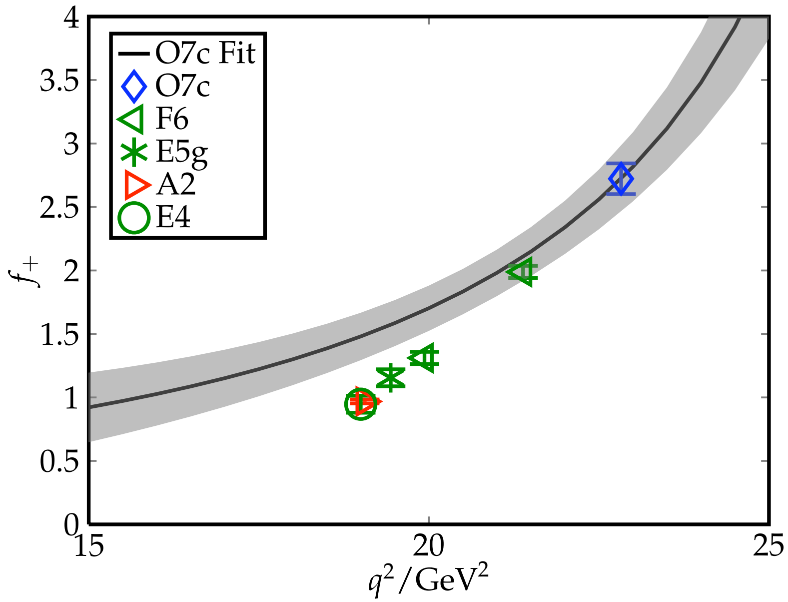


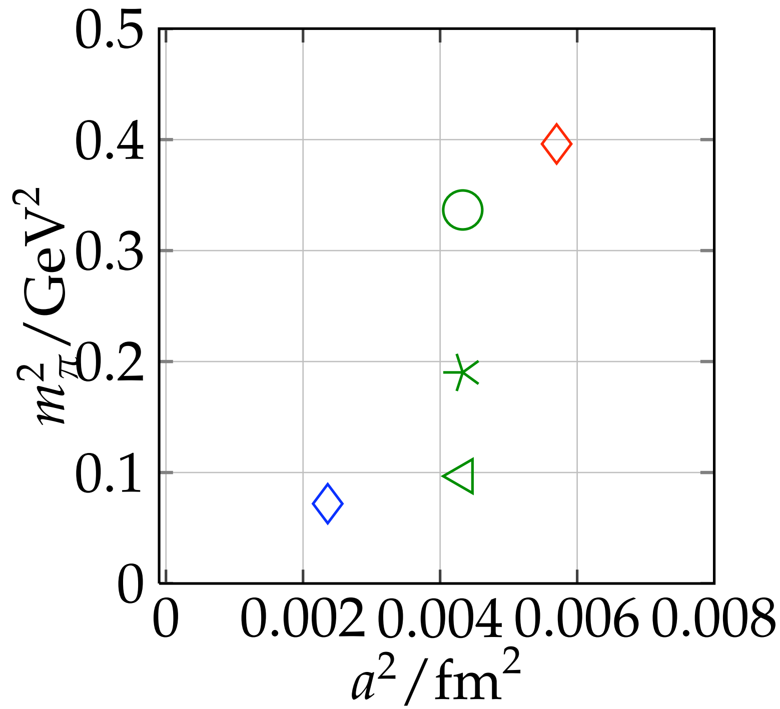




investigate further techniques to improve plateaux

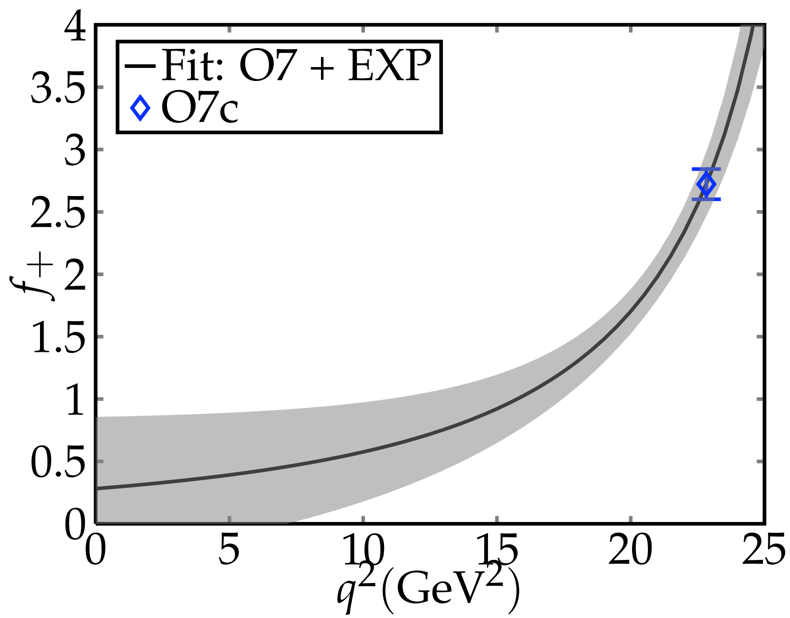
Results

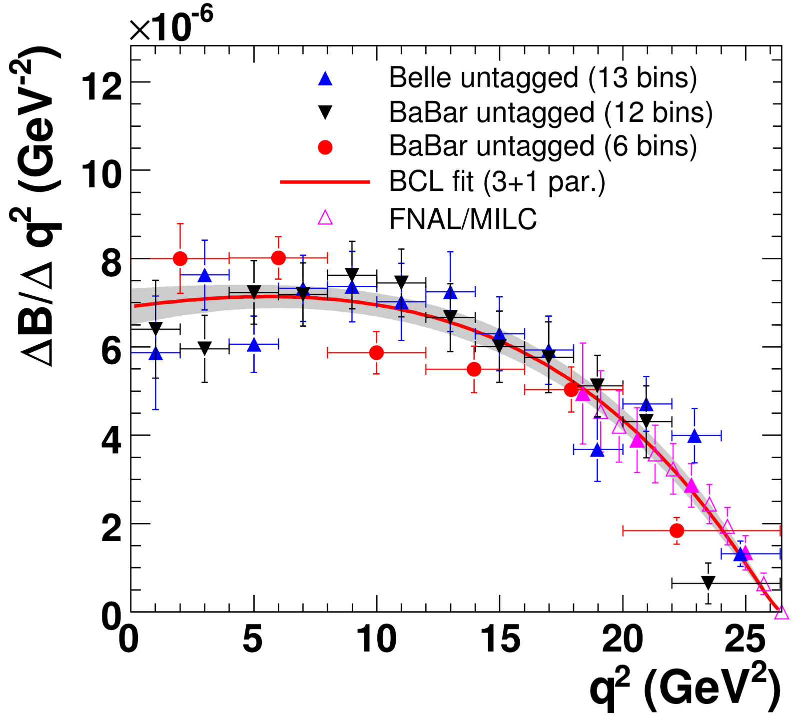




- static limit, no continuum extrapolation
- points at lower q^2 larger truncation errors $\sim E_\pi/m_B \sim 10 - 20\%$
- possibly significant $O(a) = O(a\Lambda\alpha_S)$ effect $\Rightarrow$ improve V^μ

Expected error, next year





Using one point to $\sim 6\%$ ($q^2 \sim 22$ GeV^2):

- z-expansion (analiticity, unitarity, QCD properties, kinematics)
  [Bourrely, Caprini, Lellouch 09]
- experimental data

$\Rightarrow$ V_{ub} to 15%

Next year:

- include all $1/m_b$ terms
- full NP renormalization and matching
- $O(a\Lambda)$ improvement; but not $O(\Lambda^2 a/m_b)$
- continuum limit, χ extrapolation