Neutral meson oscillations in the Standard Model and Beyond from $N_f = 2$ tmQCD

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in collaboration with
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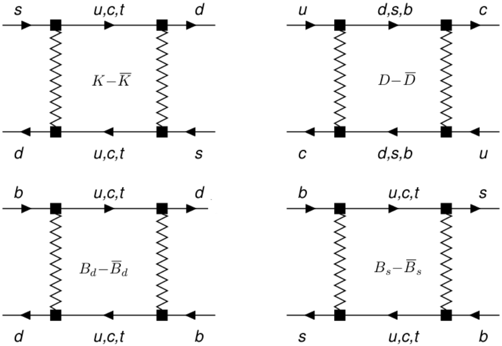
Outline

Introduction

- Flavour mixing of neutral mesons is a well known phenomenon in particle physics: K mixing (1956), B_d mixing (1987), B_s (2006)
- In 2007 BABAR and Belle first evidence of mixing in the charm sector.



- Neutral mesons oscillations are possible through a weak interaction of second order.
- $\Delta F = 2$ transitions provide some of the most stringent constraints on New Physics. The effects of NP may be noticed in the parameters describing the mixing.

Introduction

Using $N_f = 2$ twisted sea quarks we have computed:

- B_i parameters describing $K^0-\overline{K}^0$ oscillations in the Standard Model and Beyond (forthcoming paper).

$\overline{MS}(2\text{GeV})$				
B_1	B_2	B_3	B_4	B_5
0.52(04)	0.54(03)	1.13(09)	0.82(05)	0.63(07)

- B_i parameters in the D sector which controls $D^0-\overline{D}^0$ oscillations in the Standard Model and Beyond (preliminary).

$\overline{MS}(2\text{GeV})$				
B_1	B_2	B_3	B_4	B_5
0.77(05)	0.73(04)	1.37(11)	0.96(06)	1.22(13)

- B_{B_d} and B_{B_s} parameters describing $B^0_d-\overline{B}^0_d$ and $B^0_s-\overline{B}^0_s$ oscillations in the Standard Model. In order to avoid discretization errors, the **ratio method** has been applied (preliminary).

$$B_s/B_d = 1.023(18)$$

Lattice setup

Lattice Action

- Wilson Twisted Mass Action at maximal twist with $N_f=2$ sea quarks

$$S_l^{sea,Mtm}(x)=\sum_x\left\{\overline{\psi}_l(x)\left[\gamma\tilde{\nabla}-i\gamma_5\tau_3 a\frac{r_f}{2}\sum_\mu\nabla^*_\mu\nabla_\mu-i\gamma_5\tau_3 M_{cr}(r_f)+m_l\right]\psi_l(x)\right\}$$

- Osterwalder-Seiler valence quark action at maximal twist

$$S^{val,OS}=\sum_x\sum_{f=h,h',l,l'}\overline{q}_f(x)\left[\gamma\tilde{\nabla}-i\gamma_5 a\frac{r_f}{2}\sum_\mu\nabla^*_\mu\nabla_\mu-i\gamma_5 M_{cr}(r_f)+m_f\right]q_f(x)$$

$hl\sim sd$ for $K^0-\overline{K}^0$, $\sim cu$ for $D^0-\overline{D}^0$, $\sim bd$ for $B^0_d-\overline{B}^0_d$ and $\sim bs$ for $B^0_s-\overline{B}^0_s$

Chiral improved Wilson Fermions

$r_h=r'_h=r_l=-r'_l$

- $\mathcal{O}(a)$ improvement
- Continuum like renormalization pattern

R.Frezzotti and G.C. Rossi. JHEP, 10:070, 2004

Data setup for bare $K^0 - \overline{K}^0$

$\beta = 3.80$, $a \sim 0.10$ fm			
$a\mu_\ell = a\mu_{sea}$	$a^{-4}(L^3 \times T)$	$a\mu_{\text{“}s\text{”}}$	N_{stat}
0.0080	$24^3 \times 48$	0.0165, 0.0200, 0.0250	170
0.0110	“	“	180
$\beta = 3.90$, $a \sim 0.09$ fm			
0.0040	$24^3 \times 48$	0.0150, 0.0220, 0.0270	400
0.0064	“	“	200
0.0085	“	“	200
0.0100	“	“	160
0.0030	$32^3 \times 64$	“	300
0.0040	“	“	160
$\beta = 4.05$, $a \sim 0.07$ fm			
0.0030	$32^3 \times 64$	0.0120, 0.0150, 0.0180	190
0.0060	“	“	150
0.0080	“	“	220

- Stochastic propagators
- Separation between K-meson sources is $T/2$
- Smearing is not strictly necessary for strange-light mesons.

Data setup for bare $D^0 - \overline{D}^0$, $B^0_d - \overline{B}^0_d$ and $B^0_s - \overline{B}^0_s$

- Use smearing sources.
- Decrease the time separation between meson walls
  $\rightarrow$Statistical quality of the plateau is improved allowing us to reach about $3m_c$

β	3.80	3.90		4.05
$T_{sep}/a < T/2$	16	18		22
$L^3 \times T$	$24^3 \times 48$	$24^3 \times 48$	$32^3 \times 64$	$32^3 \times 64$
#confs	144	240	144	144
$a\mu_{sea} = a\mu_l$	[0.0080 0.0110]	[0.0040 0.0064 0.0085 0.0100]	[0.0030 0.0040]	[0.0030 0.0060 0.0080]
$a\mu_h \sim a\mu_s$	[0.0175 0.0194 0.0213]	[0.0159 0.0177 0.0195]		[0.0139 0.0154 0.0169]
$a\mu_h \sim a\mu_c$	[0.1982 0.2331 0.2742]	[0.1828 0.2150 0.2529]		[0.1572 0.1849 0.2175]
$a\mu_h \gtrsim a\mu_c$	[0.3225, 0.3793 0.4461 0.5246 0.6170 0.7257]	[0.2974 0.3498 0.4114 0.4839 0.5691 0.6694]		[0.25580.3008 0.3538 0.4162 0.4895 0.5757]

Bag parameters

B-parameters measure the deviation of the VIA value of the matrix elements which contribute to meson oscillation, $P^0 - \overline{P^0}$ from its physical value

$$\langle \overline{P^0} | Q_i | P^0 \rangle = B_i \langle \overline{P^0} | Q_i | P^0 \rangle_{VIA} = B_i F_i f_P^2 M_P^2$$

$$F_1 = \xi_1 = 8/3$$
$$F_i = \xi_i \left[\frac{M_P}{m_h + m_l} \right]^2 \quad i = 2, \ldots, 5 \;\; \xi_i = \{-5/3, 1/3, 2, 2/3\}$$

$$\begin{aligned}
Q_1 &= \tfrac{1}{4}[\bar{h}^a \gamma^\mu (1 - \gamma_5) l^a][\bar{h}^b \gamma_\mu (1 - \gamma_5) l^b] \\
Q_2 &= \tfrac{1}{4}[\bar{h}^a (1 - \gamma_5) l^a][\bar{h}^b (1 - \gamma_5) l^b] \\
Q_3 &= \tfrac{1}{4}[\bar{h}^a (1 - \gamma_5) l^b][\bar{h}^b (1 - \gamma_5) l^a] \\
Q_4 &= \tfrac{1}{4}[\bar{h}^a (1 - \gamma_5) l^a][\bar{h}^b (1 + \gamma_5) l^b] \\
Q_5 &= \tfrac{1}{4}[\bar{h}^a (1 - \gamma_5) l^b][\bar{h}^b (1 + \gamma_5) l^a]
\end{aligned}$$

The bare B_i can be obtained from the ratio

$$B_1 = \frac{\langle \overline{P^0} | Q_1 | P^0 \rangle}{\xi_1 \langle \overline{P^0} | A_0 | 0 \rangle \langle 0 | A_0 | P^0 \rangle} \qquad B_i = \frac{\langle \overline{P^0} | Q_i | P^0 \rangle}{\xi_i \langle \overline{P^0} | P_5 | 0 \rangle \langle 0 | P_5 | P^0 \rangle}$$

$$\rightarrow \begin{cases} B_1(t) \leftarrow \dfrac{C_3[Q_1]}{C_2[P5A0, \mathrm{LEFT}]C_2[P5A0, \mathrm{RIGHT}]} \\[2ex] B_i(t) \leftarrow \dfrac{C_3[Q_i]}{C_2[P5P5, \mathrm{LEFT}]C_2[P5P5, \mathrm{RIGHT}]} \quad i \geq 2 \end{cases}$$

Analysis procedure

1. Compute the bare B-parameters $B_i(\beta; \mu_{sea}, \mu_l^{val}, \mu_h^{val})$ by performing a fit of the correlation functions $B_i(t)$ over the range where plateau exists
2. Renormalize non perturbatively
3. Extrapolate/interpolate to the physical quark masses and extrapolate to the continuum limit $(a \to 0)$

B_K

- $\mu_l \to \mu_{u/d}$
- $\mu_{sea} \to \mu_{u/d}$
- $\mu_h \to \mu_s$

B_D

- $\mu_l \to \mu_{u/d}$
- $\mu_{sea} \to \mu_{u/d}$
- $\mu_h \to \mu_c$

B_{Bd}

- $\mu_l \to \mu_{u/d}$
- $\mu_{sea} \to \mu_{u/d}$
- $\mu_h \to \mu_b$

B_{Bs}

- $\mu_l \to \mu_s$
- $\mu_{sea} \to \mu_{u/d}$
- $\mu_h \to \mu_b$

RI-MOM

We carry out the non-perturbative calculation of RCs (step 2) adopting the RI-MOM approach, a mass independent scheme.
$\to$ 4f and 2f RCs are the same for all mesons

$K^0 - \overline{K}^0$ parameters

$B_1 \equiv B_K$ has already been presented before:
"ETM Collaboration, M.Constantinou et al. "BK- parameter from Nf=2 twisted mass lattice QCD" *Phys Rev*. **D83** (2011) 014505 [1009.5606]

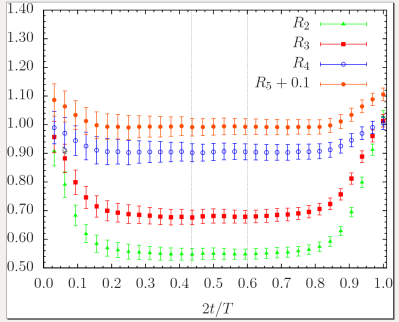
B_i fits

$B_i(\beta; \mu_{sea}, \mu_{val})$ have been computed by performing a constant fit of the ratio

$$R_i(t) = \frac{C_i^{(3)}(t)}{C_{PP}^{(2)}(t)\, C_{PP}^{(2)}(t)}$$

over the range where this function reaches a plateau.

$\beta = 4.05 \rightarrow a \sim 0.07$fm
$M_\pi \sim 270$ MeV



$K^0 - \overline{K}^0$ parameters

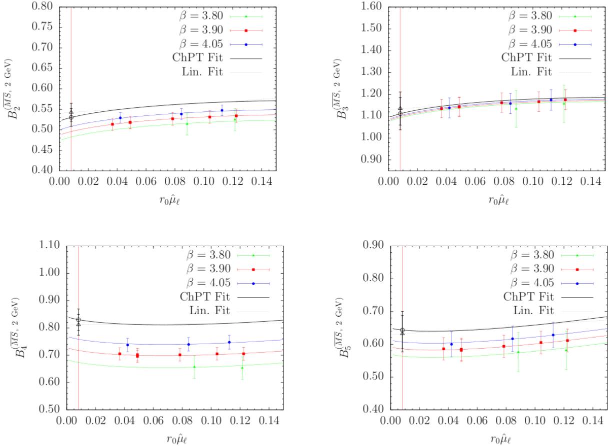
1. Linear interpolation: $\mu_h \to \mu_s$
2. Continuum and chiral extrapolation: $\mu_l = \mu_{sea} \to \mu_{u/d}$: linear, quadratic and NLO SU(2) ansatz

Lin. Fit

$$B_i = A + B(r_0\mu_l) + D\frac{a^2}{r_0^2}$$

ChPT Fit

$$B_{\substack{1,2,3\\4,5}} = B^0_{\substack{1,2,3\\4,5}}\left[1 + b(r_0\mu_l) \mp \frac{2B_0\mu_l}{32\pi^2 f_0^2}\log\frac{2B_0\mu_l}{16\pi^2 f_0^2}\right] + D\frac{a^2}{r_0^2}$$



$K^0 - \overline{K}^0$ parameters

$\overline{\mathrm{MS}}$ (2 GeV)				
B_1	B_2	B_3	B_4	B_5
0.52(02)	0.54(03)	1.13(09)	0.82(05)	0.63(07)

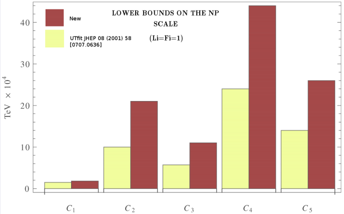
UTfit update

With these new results we are able to provide an update of the UTfit. NP generalization of the UT analysis consists in including the matrix elements of operators which are absent in the SM but may appear in some extensions.

The effective hamiltonian is parametrized by a Wilson coefficient of the form:

$$C_i(\Lambda) = \frac{F_i L_i}{\Lambda^2}$$

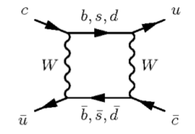
F_i= NP coupling, L_i= loop factor, Λ= scale of NP



- Phenomenollogical allowed range for each $C_i \rightarrow$lower bound of Λ
- The more stringent constraints on the NP scale come from the improved accuracy in the values of B_i parameters.

$D^0 - \overline{D}^0$ parameters

Why is D_0 mixing interesting?



It provides new information about CP violation in processes with down-type quarks in the mixing loop diagram.

Short- vs Long distance contributions

- Short-distance contributions from mixing box diagrams in the SM are expected to be small
  - b quark is supressed by the CKM matrix $\sim |V_{ub} V_{cb}^*|^2$
  - s and d quarks are GIM suppressed
- Long-distance effects ($D^0 \to KK \to \overline{D}^0, \ldots$) dominate
  Estimates are at the level of the experimental constraints $\to$prevent us to determine whether oscillations in the neutral D system arise from:
  - SM long-distance effects
  - or NP short-distance effects

In spite of that, significant constraints can be put on the NP parameter space. NP contributions are short distance and they can be computed on the lattice by considering the complete basis of four-fermion operators.

$D^0-\overline{D}^0$ parameters

Sources of systematic error

- Chiral fit ansatzs
- time fit interval

Preliminary results for B_D parameters

$B_D[\overline{MS}(2\text{ GeV})]$	
B_1	0.77(03)(04)
B_2	0.73(04)(01)
B_3	1.37(11)(01)
B_4	0.96(06)(01)
B_5	1.22(13)(02)

http://www.utfit.org/UTfit/DDbarMixing
PRELIMINARY update of the $D^0-\overline{D}^0$ mixing analysis of M.Ciuchini et al. hep-ph/0703204 including:

- preliminary B_D @ Nf=2
- new preliminary results by BaBar and Belle presented @ Charm2012

(paper in preparation)

B_{B_d} and B_{B_s}

Why B_{Bd} and B_{Bs} are important?

- Bag parameters $B_{B_{d,s}}$, together with the decay constants $f_{B_{d,s}}$(see *Andrea Shindler's talk*), encode the non perturbative QCD information about the $B^0_{d,s} - \overline{B}^0_{d,s}$ mixing amplitude in the SM.

$$\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$$

Their accurate determination would help getting information on the CKM entries V_{ts} and V_{td}

$$\left|\frac{V_{td}}{V_{ts}}\right| = \xi\sqrt{\frac{\Delta M_d M_{B_s}}{\Delta M_s M_{B_d}}}$$

- In most NP scenarios, non-SM four-fermion operators contribute to the mixing amplitude

Ratio method

Discretization errors on current lattices are expected to be large at physical m_b. We compute ratios of the bag parameters with exactly known static limit and then interpolate them to the physical b-quark mass.
(*see ETMC, JHEP01(2012)046 and Andrea Shindler's talk*)

Ratio method for bag parameters

Scaling law

The HQET asymptotic prediction for the bag paramaters is:

$$\lim_{1/\mu_h \to 0} B_i = \text{constant}$$

We construct ratios whose static limit (at physical point and in the continuum limit) shall be one

$$\vec{\omega}_B(\mu_h^{(n)}, \mu_l, a) = \frac{\mathbf{C}_B \vec{B}(\mu_h^{(n)}, \mu_l, a^2; \overline{MS}(2GeV))}{\mathbf{C}_B \vec{B}(\mu_h^{(n-1)}, \mu_l, a^2; \overline{MS}(2GeV))}$$

$$\vec{B}(\mu_h; \mu_l, a^2; \overline{MS}(2\text{GeV})) \xrightarrow{\mathbf{C}_B(\mu_h)} \vec{\tilde{B}}(\mu_h; \mu_l, a^2; \overline{MS}(\mu_b))$$

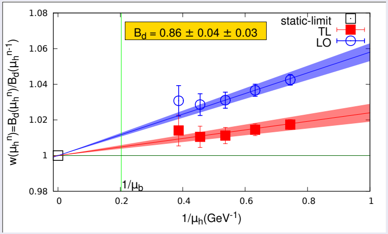
C_B provides the matching between bag parameters in QCD for a heavy quark mass μ_h and in HQET, and the running of the HQET to the renormalization scale μ_b
QCD-HQET matching is only used to construct the ratios with correct scaling law

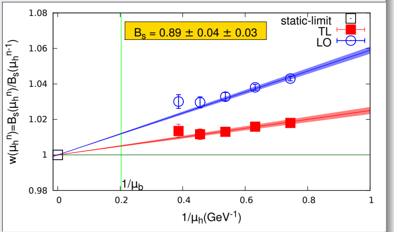
B_1 does not mix with B_i in QCD and neglecting NLO HQET scale evolution also in the static limit

$$\omega_{B_{d,s}}(\mu_h^n, \mu_l, a) = \frac{C_{B_1}(\mu_h^{(n)})}{C_{B_1}(\mu_h^{(n-1)})} \frac{B_{B_{d,s}}(\mu_h^{(n)}, \mu_l, a^2; \overline{MS}(2GeV))}{B_{B_{d,s}}(\mu_h^{(n-1)}, \mu_l, a^2; \overline{MS}(2GeV))}$$

B_{B_d} and B_{B_s}

ω dependence on $1/\mu_h$: $\omega(\mu_h)=1+b/\mu_h$





Master equation

$$\omega_{B_{d,s}}(\mu_h^{(1)})\omega_{B_{d,s}}(\mu_h^{(2)})\ldots\omega_{B_{d,s}}(\mu_h^{(K)})=\frac{C_{B_1}(\mu_h^{(K)})B_{B_{d,s}}(\mu_h^{(K)};\overline{MS}(2\text{GeV}))}{C_{B_1}(\mu_h^{(0)})B_{B_{d,s}}(\mu_h^{(0)};\overline{MS}(2\text{GeV}))}$$

Triggering point

$B_{Bd}(\mu_h^{(0)};\overline{MS}(2\text{GeV}))$=0.784(37)

$B_{Bs}(\mu_h^{(0)};\overline{MS}(2\text{GeV}))$=0.807(36)

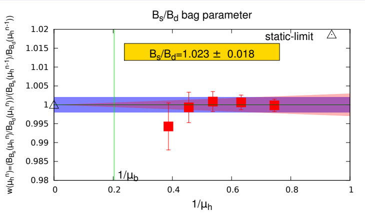
B_{B_s} / B_{B_d}

$$\omega(\mu_h^{(n)}, \mu_l, a^2) = \frac{B_{Bs}/B_{Bd}(\mu_h^{(n)}, \mu_l, a^2)}{B_{Bs}/B_{Bd}(\mu_h^{(n-1)}, \mu_l, a^2)}$$

Advantages

1. Discretization errors are under better control
2. Renormalization constants are not needed
3. $C_{B_1}(\mu_h^{(n)})$ does not depend on the spectator quark mass (d or s) $\rightarrow$ the evolution and matching from QCD(2GeV) to HQET(m_b) is cancelled.

ω dependence on $1/\mu_h$



Fit ansatz:

- $\omega(\mu_h) = a$
- $\omega(\mu_h) = 1 + b/\mu_h$
- $\omega(\mu_h) = 1 + b/\mu_h + c/\mu_h^2$

$$\frac{B_{Bs}}{B_{Bd}}(\mu_b) =$$

$$\frac{B_s}{B_d}(\mu_h^{(n=0)})\omega(\mu_h^{(1)})\omega(\mu_h^{(2)})\ldots\omega(\mu_h^{(K)})$$

triggering point: $\frac{B_{Bs}}{B_{Bd}}(\mu_h^{(n=0)}) = 1.023(10)$

$$\rightarrow \boxed{\frac{B_{Bs}}{B_{B_d}} = 1.023(18)(02)}$$

Conclusions

Using data from unquenched $N_f = 2$ simulations...

$K^0 - \overline{K}^0$ mixing

First unquenched lattice determination of B_i for $K^0 - \overline{K}^0$ relevant beyond SM.

$D^0 - \overline{D}^0$ mixing

Only quenched results existed so far. We present NEW preliminary unquenched results for the short distance NP contributions.

B_{Bd} and B_{Bs} bag parameters

Our preliminary analysis shows that ratio method can be applied successfully once

- we decrease the separation between meson sources
- we use suitable smearing techniches

Future plans

- Compute NP contributions to B_{B_d} and B_{B_s}
- Include NLO terms in the analysis of the μ_h dependence of B_{B_d} and B_{B_s}.
- **Extend the analysis to $N_f = 2+1+1$ to reduce the systematic errors due to the quenching of strange and charm in order to be competitive with the experimental results and to constrain the relevant parameters in the CKM matrix in a better way.**

Backup slides

Bag parameters

We define two "P-meson walls" at y_0 and $y_0+T_{\rm sep}$

$$W_{\rm LEFT}(y_0)=\sum_{\vec{y}}\bar{h}(\vec{y},y_0)\gamma_5 l(\vec{y},y_0)$$
$$W_{\rm RIGHT}(y_0+T/2)=\sum_{\vec{y}}\bar{h}(\vec{y},y_0+T_{\rm sep})\gamma_5 l(\vec{y},y_0+T_{\rm sep})$$

Introducing the 4-fermion operators at $x=(\vec{x},x_0)$ we construct the 2- and 3-point correlation functions

$$C_3[Q_i](x_0)=\sum_{\vec{x}}\langle W_{\rm RIGHT}(y_0+T_{\rm sep})Q_i(x)W_{\rm LEFT}(y_0)\rangle$$

$$C_2[\text{RIGHT,P5A0}](x_0)=\sum_{\vec{x}}\langle A_0(x))W_{\rm RIGHT}(y_0)\rangle \qquad C_2[\text{RIGHT,P5P5}](x_0)=\sum_{\vec{x}}\langle P_5(x))W_{\rm RIGHT}(y_0)\rangle$$
$$C_2[\text{LEFT,P5A0}](x_0)=\sum_{\vec{x}}\langle W_{\rm LEFT}(y_0+T_{\rm sep})A_0(x)\rangle \qquad C_2[\text{LEFT,P5P5}](x_0)=\sum_{\vec{x}}\langle W_{\rm LEFT}(y_0+T_{\rm sep})P_5(x)\rangle$$

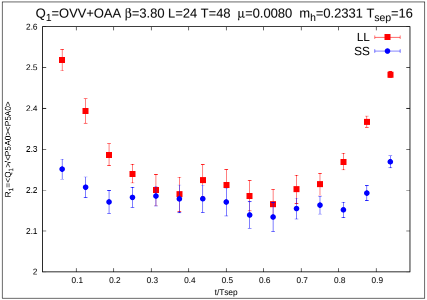
$$\rightarrow\begin{cases} B_1(x_0)) \xleftarrow[y_0<<x_0<<y_0+T_{\rm sep}]{} \dfrac{C_3[Q_1,SS]}{C_2[P5A0,\text{LEFT},SL]C_2[P5A0,\text{RIGHT},SL]} \\[2ex] B_i(x_0) \xleftarrow[y_0<<x_0<<y_0+T_{\rm sep}]{} \dfrac{C_3[Q_i,SS]}{C_2[P5P5,\text{LEFT},SL]C_2[P5P5,\text{RIGHT},SL]}\; i\geq 2 \end{cases}$$

Improvements for $\mu_h > \mu_s$

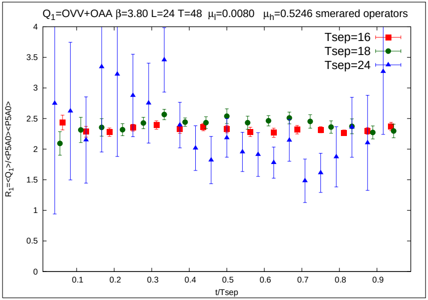
Smearing:

- Gaussian smeared fermions: $\psi^S = (1 + k_G a^2 \Delta^F)^{N_G} \psi^L$, $k_G = 4$ and $N_G = 30$
- APE smeared links: $\alpha_{APE} = 0.5$ and $N_{APE} = 20$

Exploratory tests on T_{sep}:



smearing tests:



$D^0 - \overline{D}^0$ parameters

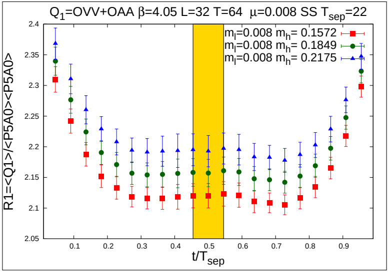
B_i fits

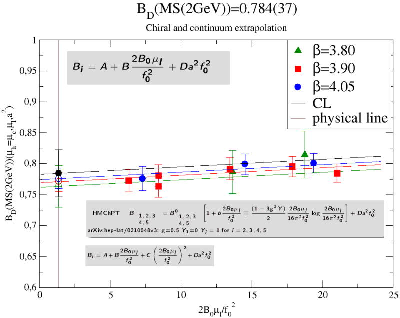
$$R_1(t) = \frac{C_1^{(3)}(t)}{C_{AP}^{(2)}(t)C_{AP}^{(2)}(t)}$$

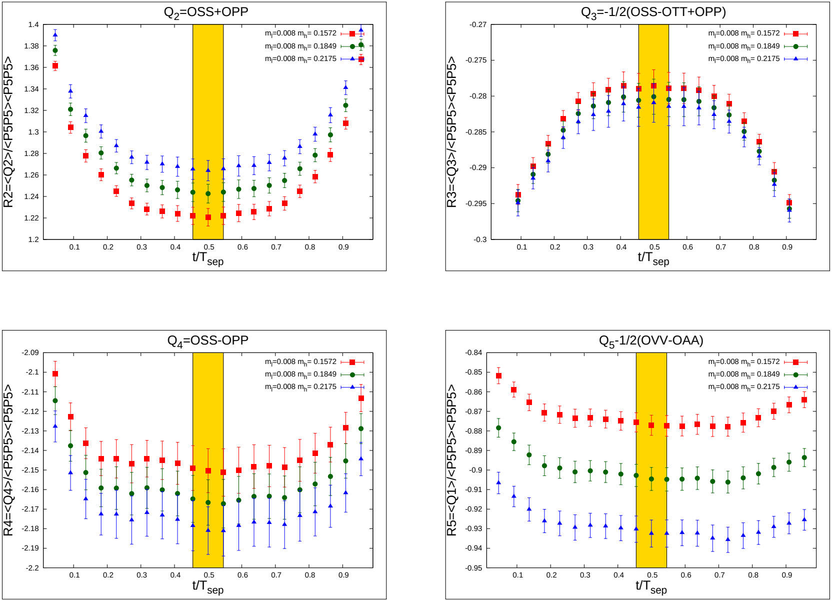
$$R_i(t) = \frac{C_i^{(3)}(t)}{C_{PP}^{(2)}(t)C_{PP}^{(2)}(t)}$$

1. Linear interpolation: $\mu_h \to \mu_c$
2. Continuum ($a \to 0$) and chiral extrapolation: $\mu_l = \mu_{sea} \to \mu_{u/d}$

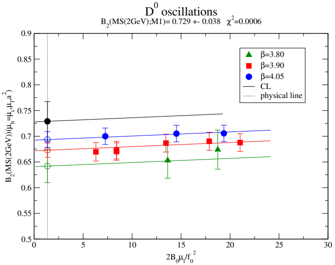
- linear
- quadratic
- SU(2) NLO HMChPT ansatz.

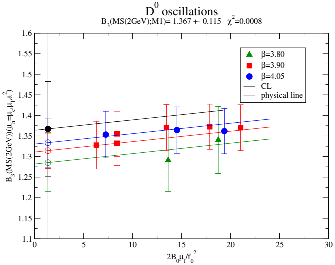


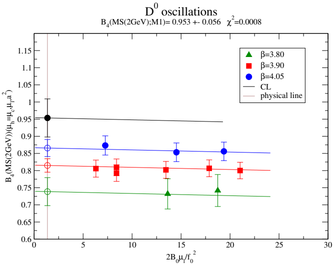


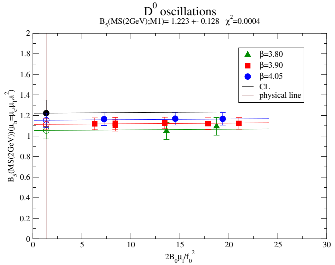

Q2=OSS+OPP
R2=<Q2>/<P5P5><P5P5>
t/Tsep
ml=0.008 mh= 0.1572
ml=0.008 mh= 0.1849
ml=0.008 mh= 0.2175
Q3=-1/2(OSS-OTT+OPP)
R3=<Q3>/<P5P5><P5P5>
t/Tsep
ml=0.008 mh= 0.1572
ml=0.008 mh= 0.1849
ml=0.008 mh= 0.2175
Q4=OSS-OPP
R4=<Q4>/<P5P5><P5P5>
t/Tsep
ml=0.008 mh= 0.1572
ml=0.008 mh= 0.1849
ml=0.008 mh= 0.2175
Q5-1/2(OVV-OAA)
R5=<Q1>/<P5P5><P5P5>
t/Tsep
ml=0.008 mh= 0.1572
ml=0.008 mh= 0.1849
ml=0.008 mh= 0.2175

B_i from M1 RCs

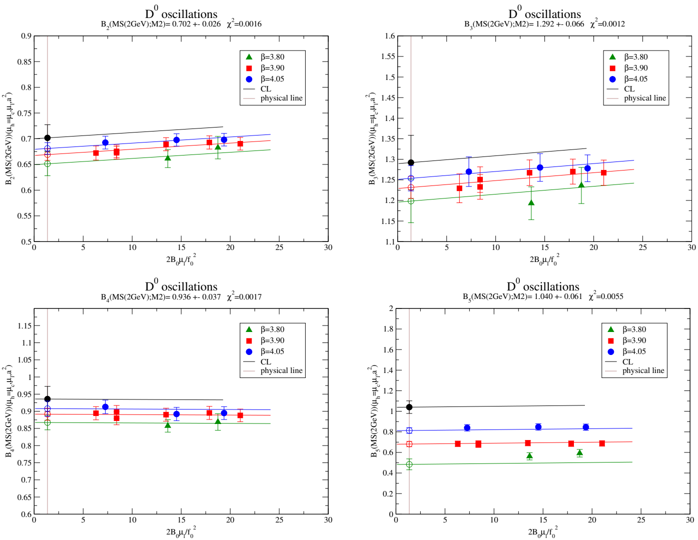




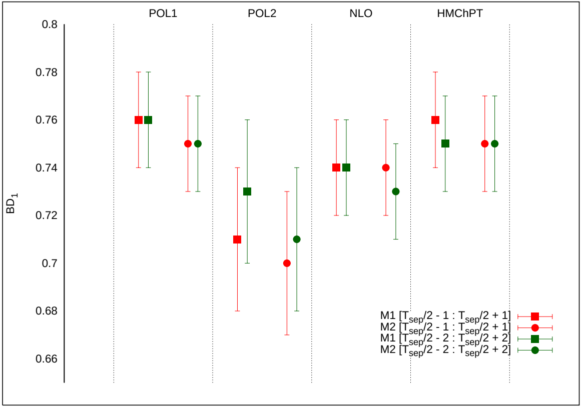




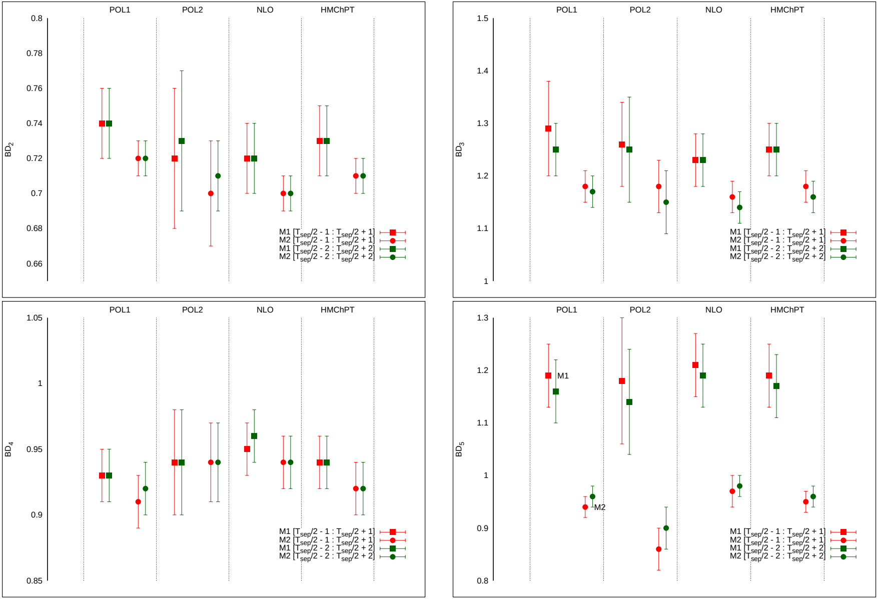
B_i from M2 RCs

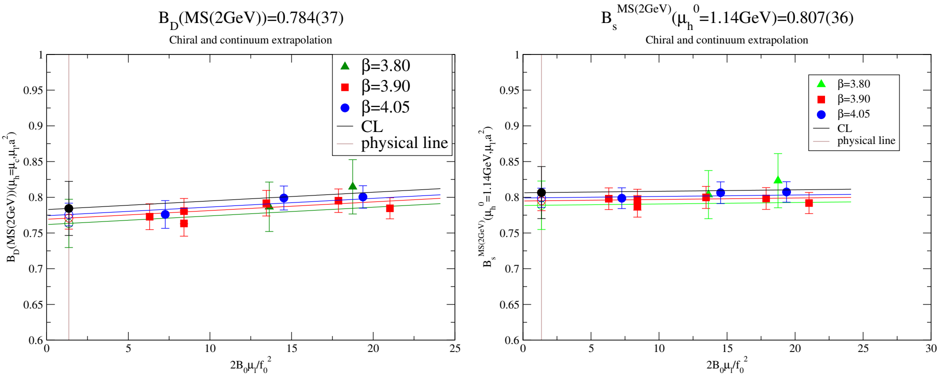


Sistematic error on B_D



Sistematic error on B_i $i \geq 2$ for $D^0-\overline{D}^0$





B_D(MS(2GeV))=0.784(37)
Chiral and continuum extrapolation
B_D(MS(2GeV))(μ_h=μ_c,μ_l,a^2)
2B_0μ_l/f_0^2
β=3.80
β=3.90
β=4.05
CL
physical line
B_s^MS(2GeV)(μ_h^0=1.14GeV)=0.807(36)
Chiral and continuum extrapolation
B_s^MS(2GeV)(μ_h^0=1.14GeV,μ_l,a^2)
2B_0μ_l/f_0^2
β=3.80
β=3.90
β=4.05
CL
physical line

Optimized B parameters

Optimized three point correlator

$$C_3[WW, w_{TW}, w_{OS}] = \tilde{w}_{1,TW}\tilde{w}_{1.OS}C_3[SS] + \tilde{w}_{1,TW}\tilde{w}_{2,OS}C_3[SL] + \tilde{w}_{2,TW}\tilde{w}_{1,OS}C_3[LS] + \tilde{w}_{2,TW}\tilde{w}_{2,OS}C_3[LL]$$

$$\tilde{w}_1 = \frac{w}{C_2[SL, t=1]} \qquad \tilde{w}_2 = \frac{1-w}{C[LL, t=1]}$$

Optimized B parameter

$$B_1[WW](t) = \frac{C_3[Q_1, WW, w_{TW}, w_{OS}]}{C_2[P5A0, \mathrm{LEFT}, WL, w_{TW}^{P5A0}]C_2[P5A0, \mathrm{RIGHT}, WL, w_{OS}^{P5A0}]}$$

$$B_i[WW](t) = \frac{C_3[Q_i, WW, w_{TW}, w_{OS}]}{C_2[P5P5, \mathrm{LEFT}, WL, w_{TW}^{P5P5}]C_2[P5P5, \mathrm{RIGHT}, WL, w_{OS}^{P5P5}]} i \geq 2$$

Optimized B parameters

$$B[WW](t) = \frac{C_3[WW, w_{r_1,r_2}, w_{r_3,r_4}]}{C_2[\text{LEFT}, WL, w_{r_1,r_2}]C_2[\text{RIGHT}, WL, w_{r_3,r_4}]}$$

$$\begin{cases} \text{if } r_i = r_j \rightarrow w_{(r_i,r_j)} = w_{OS} \\ \text{if } r_i \neq r_j \rightarrow w_{(r_i,r_j)} = w_{TW} \end{cases}$$

P5A0 TW		
m_1	m_2	w_{opt}
0.0080	0.2331	1
0.0080	0.5246	0.7

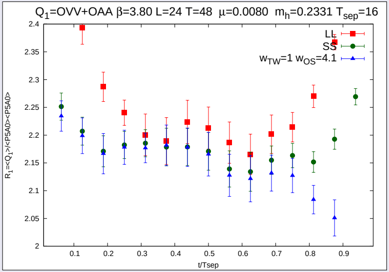
P5A0 OS		
m_1	m_2	w_{opt}
0.0080	0.2331	4.1
0.0080	0.5246	1.7

P5P5 TW		
m_1	m_2	w_{opt}
0.0080	0.2331	1.1
0.0080	0.5246	0.7

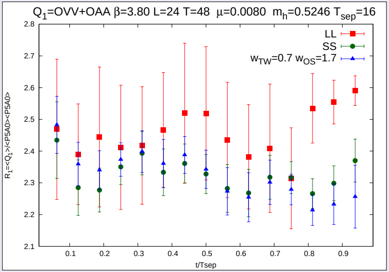
P5P5 OS		
m_1	m_2	w_{opt}
0.0080	0.2331	3.3
0.0080	0.5246	1

Optimized B parameters

$m_h \sim m_c$



$m_h \sim 2m_c$



- $B_i[WW](t) \simeq B_i[SS](t)$ for $t \subset [T_{sep}/2 - 2 : T_{sep}/2 + 2]$ within statistical errors
- For $t \subset [T_{sep}/2 - 2 : T_{sep}/2 + 2]$:
  - $C_2[WL]$ has plateau
  - $C_2[SL]$ has no plateau yet

$\rightarrow$excited state contributions in B_i are negligible within statistical errors.

Four fermion Renormalizations constants

1. Computation of $D_{ij}(p^2, \beta, \mu_{sea}, \mu_{val})$:

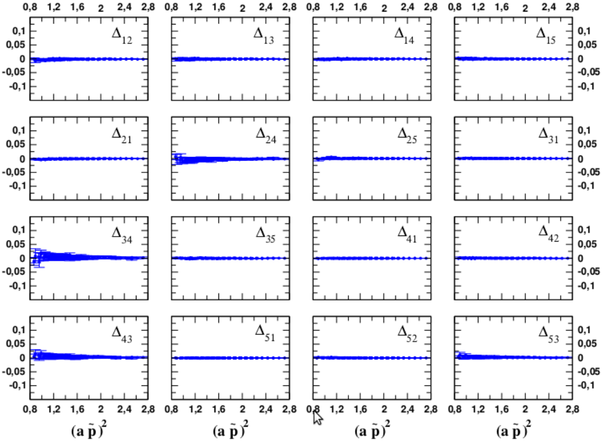
$$\Lambda^{\pm}_{\Gamma_1\Gamma_2}(p) = \overbrace{S^{-1}S^{-1}G_{\Gamma_1\Gamma_2}S^{-1}S^{-1}}^{\text{amputed GF}}$$

$$\underbrace{D^{\pm}}_{\text{dynamical matrix}} = \underbrace{P^{\pm}}_{\text{tree-level projector}} \overbrace{\Lambda^{\pm}}$$

$$P^{\pm}\Lambda^{\pm(0)} = I$$

2. Valence Chiral limit with substraction of the Goldstone Pole
$\rightarrow D_{ij}(p^2, \beta, \mu_{sea}, \mu_{val} = 0,)$
3. Computation of $Z_{ij}(p^2, \beta, \mu_{sea}, \mu_{val} = 0)$ with removal of $O(a^2 g^2)$ cuttof effects
$Z^{\pm} = Z_{\psi}^{-2}[D^{\pm T}]^{-1}$
4. Sea Chiral Limit$\rightarrow Z_{ij}(p^2, \beta, \mu_{val} = \mu_{sea} = 0)$
5. Evolution to a common reference scale using NLO QCD: RI'-MOM$\rightarrow$RGI
6. $a^2 p^2$ fit
M1: p^2 linear fit method
M2: p^2 window method

Absence of wrong chirality mixing



The behaviour of the mixing coefficients Δ_{ij}, as a function of $a^2\tilde{p}^2$ for $\beta = 4.05$.

Ratio method for bag parameters

$$\vec{B}(\mu_h;\mu_l,a^2;\overline{MS}(2\text{GeV})) \xrightarrow{U^{QCD}(2\text{GeV},\mu_h)} \vec{B}(\mu_h;\mu_l,a^2;\overline{MS}(\mu_h))$$

$\downarrow C_B(\mu_h)$ $\qquad\qquad$ $\downarrow$ QCD-HQET matching

$$\tilde{\vec{B}}(\mu_h;\mu_l,a^2;\overline{MS}(\mu_b)) \xleftarrow{U^{HQET}(\mu_h,\mu_b)} \tilde{\vec{B}}(\mu_h;\mu_l,a^2;\overline{MS}(\mu_h))$$

$$\omega_{B_{d,s}}(\mu_h^n,\mu_l,a) = \frac{\tilde{B}_{d,s}(\mu_h^{(n)},\mu_l,a^2;\overline{MS}(\mu_b))}{\tilde{B}_{d,s}(\mu_h^{(n-1)},\mu_l,a^2;\overline{MS}(\mu_b))} = \frac{C_{B_1}(\mu_h^{(n)})}{C_{B_1}(\mu_h^{(n-1)})}\frac{B_{d,s}(\mu_h^{(n)},\mu_l,a^2;\overline{MS}(2\text{GeV}))}{B_{d,s}(\mu_h^{(n-1)},\mu_l,a^2;\overline{MS}(2\text{GeV}))}$$

$$C_B(\mu_h) = U^{HQET}(\mu_b,\mu_h)c(\mu_h)U^{HQET}(\mu_h,2\text{GeV})$$

$$U(\mu_1,\mu_2) = \omega(\mu_1)\omega^{-1}(\mu_2), \qquad U(\mu_1,\mu_2) = \left(\frac{\alpha_S(\mu_2)}{\alpha_S(\mu_1)}\right)^{-\gamma_0^T/2\beta_0}$$

$$\omega(\mu) = [1+J\frac{\alpha_S(\mu)}{4\pi}]\alpha_S(\mu)^{-\gamma_0^T/2\beta_0} \quad \xrightarrow{@TL}$$

$$c(\mu_h) = 1+\sum_n c^{(n)}\frac{\alpha_S(\mu)}{4\pi} \qquad c(\mu_h)=1$$

γ_0 is block-diagonal $\rightarrow B_1$ evolve without mixing with B_2 and B_3 @LO

$$C_{B_1}(\mu_h^{(n)}) = \frac{\left[\frac{\alpha(\mu_b)}{\alpha(\mu_h^{(n)})}\right]^{-(\gamma_0^{11})^{HQET}/2\beta_0}\left[\frac{\alpha(\mu_h^{(n)})}{\alpha(2\text{GeV})}\right]^{-(\gamma_0^{11})^{QCD}/2\beta_0}}{\left[\frac{\alpha(\mu_b)}{\alpha(\mu_h^{(n)})}\right]^{-\gamma_A^{HQET}/\beta_0}}$$

B_{B_d} and B_{B_s}

- Interpolation of the original set of heavy masses to the set $\mu_h^{(n)}=\lambda^n\mu_h^{(0)}$ (see *Andrea Shindler talk*)
- For each $\mu_h^{(n)}$:
  - $\omega_{B_d}(\mu_h^{(n)},\mu_l=\mu_{sea},a^2)$: chiral and continuum extrapolation
  - $\omega_{B_s}(\mu_h^{(n)},\mu_l,\mu_{sea},a^2)$:interpolation in the valence "light" quark $\mu_l\to\mu_s$ + chiral (in the sea) and continuum extrapolation

$\to\omega(\mu_h^{(n)})$