B and D meson decay constants from 2+1 flavor QCD with improved staggered fermions

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Motivation

- Leptonic heavy meson decays are sensitive to both strong and weak physics:

$$\Gamma(H \to \ell\nu) \propto f_H^2 G_F^2 |V_{Qq}|^2$$

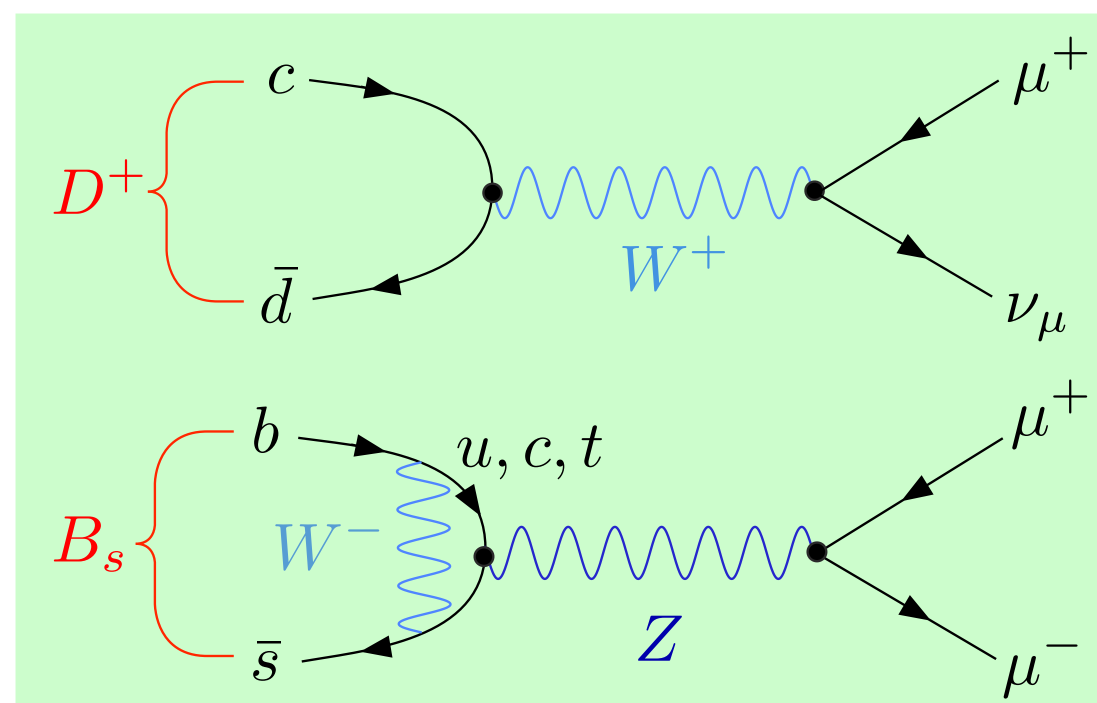
meson decay constant

weak coupling

CKM matrix element

- Accurate determination of f_H non-perturbatively (i.e. lattice) is crucial for precise CKM matrix elements

- Aside from determining CKM, decay constants are needed for rare leptonic decays - whether mediated by standard model or by new physics



Calculation overview

- Overlap of heavy meson wavefunction and axial-vector current gives f_H:

$$\langle 0|\mathscr{A}^\mu|H(p)\rangle(M_H)^{-1/2} = i(p^\mu/M_H)(f_H\sqrt{M_H}) \equiv i(p^\mu/M_H)\phi_H$$

- Extract by measuring two-point correlation function between pseudoscalar (PS) and axial-vector, normalize using PS-PS correlators

$$\begin{aligned} \Phi_2^s(t) &= \tfrac{1}{4}\textstyle\sum_{a=1}^4 \langle A_a^{4\dagger}(t,\mathbf{x})\mathscr{O}_a^{(s)}(0)\rangle, \\ C_2^{s,s'}(t) &= \tfrac{1}{4}\textstyle\sum_{a=1}^4 \langle \mathscr{O}_a^{(s)\dagger}(t,\mathbf{x})\mathscr{O}_a^{(s')}(0)\rangle, \end{aligned}$$

- Non-perturbative renormalization factors convert from bare lattice to continuum result for f_H

- Simulations at many quark masses *m* and lattice spacings *a*, fit to staggered chiPT to extrapolate to chiral/continuum limit (and to quantify discretization errors, etc.)

List of ensembles used

$\approx a$ [fm]	am_h	am_l	β	r_1/a	$N_{\rm conf}$ (previous)	$N_{\rm conf}$ (updated)	label
0.045	0.014	0.0028	7.81	7.21	—	800	A
0.06	0.018	0.0018	7.46	5.31	—	825	B
		0.0025	7.465	5.33	—	800	C
		0.0036	7.47	5.35	—	631	D
		0.0072	7.48	5.40	—	591	E
0.09	0.031	0.00155	7.075	3.74	—	790	F
		0.0031	7.08	3.75	435	1012	G
		0.00465	7.085	3.77	—	983	H
		0.0062	7.09	3.79	557	1934	I
		0.0124	7.11	3.86	518	1994	J
0.12	0.050	0.005	6.76	2.74	678	2097	K
		0.007	6.76	2.74	833	2107	L
		0.010	6.76	2.74	592	2256	M
		0.020	6.79	2.82	460	2097	N
0.15	0.0484	0.0097	6.572	2.22	631	631	O

- MILC gauge ensembles, N_f=2+1 asqtad in the sea, clover heavy quarks w/ Fermilab interpretation
- “Previous” shows data analyzed for previous published result [arXiv: 1112.3051; Phys. Rev. **D**85 (114506).]
- Four time-sources per configuration, evenly spaced

Two-point correlators

- Functional form:

$$C_{ij}(t) = \sum_{n=0}^{N_X} \Big[A_{i,n} A_{j,n} \left(e^{-E_n t} + e^{-E_n (N_t - t)} \right) - (-1)^t A'_{i,n} A'_{j,n} \left(e^{-E'_n t} + e^{-E'_n (N_t - t)} \right) \Big]$$

- Note mixing with opposite-parity state

- Point (“**d**”) or smeared (“**1S**”) source/sink for pseudoscalars, “**rot**” denotes axial vector sink

- Mixed correlators d1S/1Sd to be used for cross-validation

this analysis (→ 3)

$C_{\text{src,sink}}$	1	2	3	4	5	6	7
$C_{d,d}$	•		•		•		•
$C_{1S,1S}$		•	•			•	•
$C_{d,1S}$				•	•	•	•
$C_{1S,d}$				•	•	•	•
$C_{d,rot}$	•		•	•	•	•	•
$C_{1S,rot}$		•	•	•	•	•	•

Two-point correlators, continued

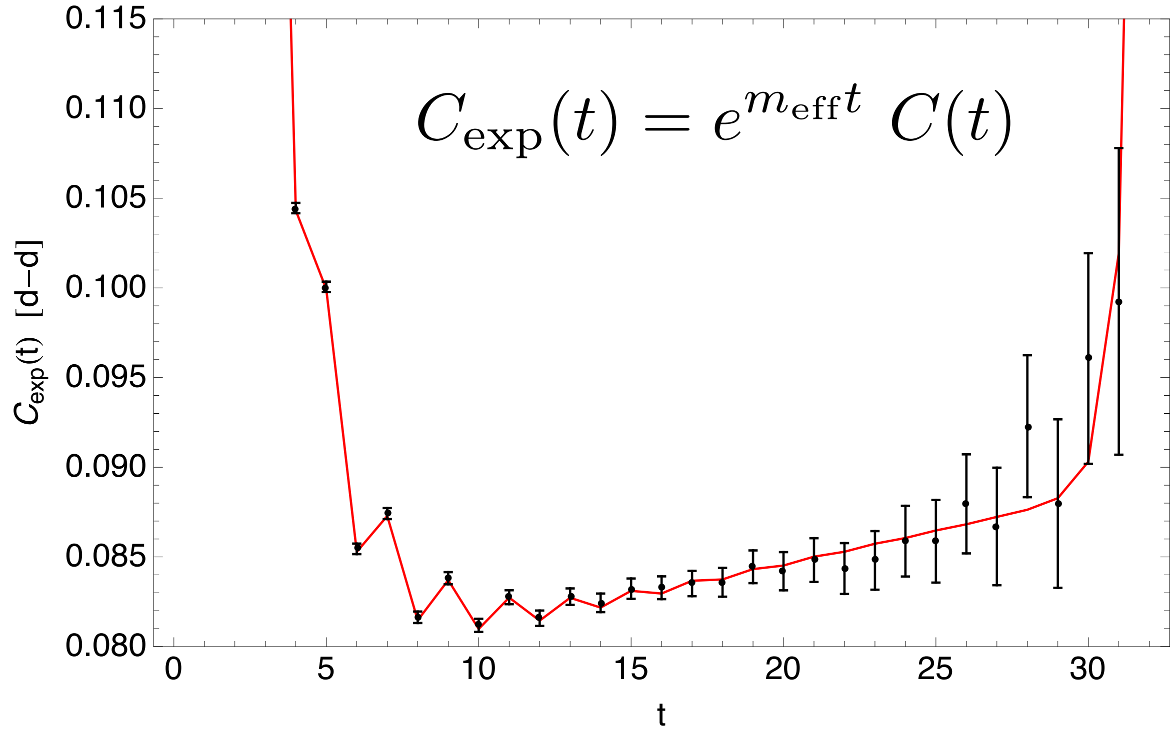
- Standard bag of tricks to yield numerically stable two-point fits:
  - Log mapping, to keep fit parameters in physically allowed domain:

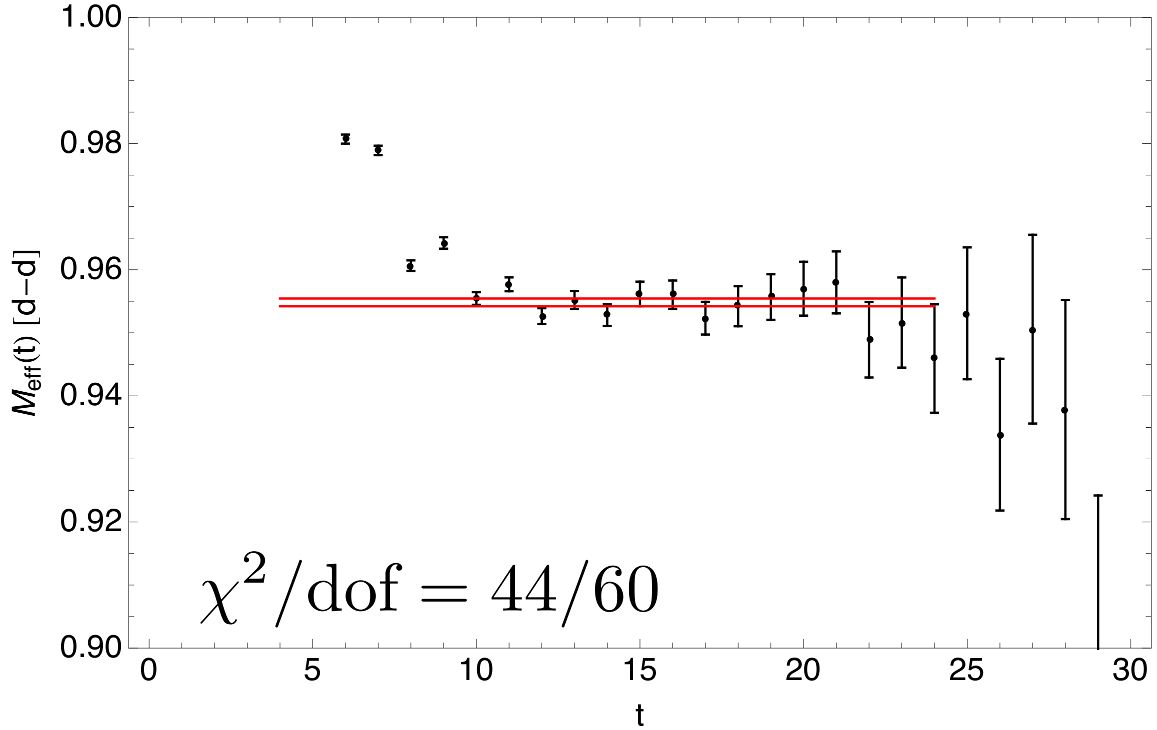
$$A_i = \exp(\log(A_i)) \qquad E_i = E_0 + \sum_{k=1}^{i} \exp(\log(\Delta E_k))$$

  - Ground state projection: fit to $C_{\exp}(t) = e^{m_{\rm eff} t}\ C(t)$
  - Bayesian priors on all fit parameters, included as augmented chi-square:

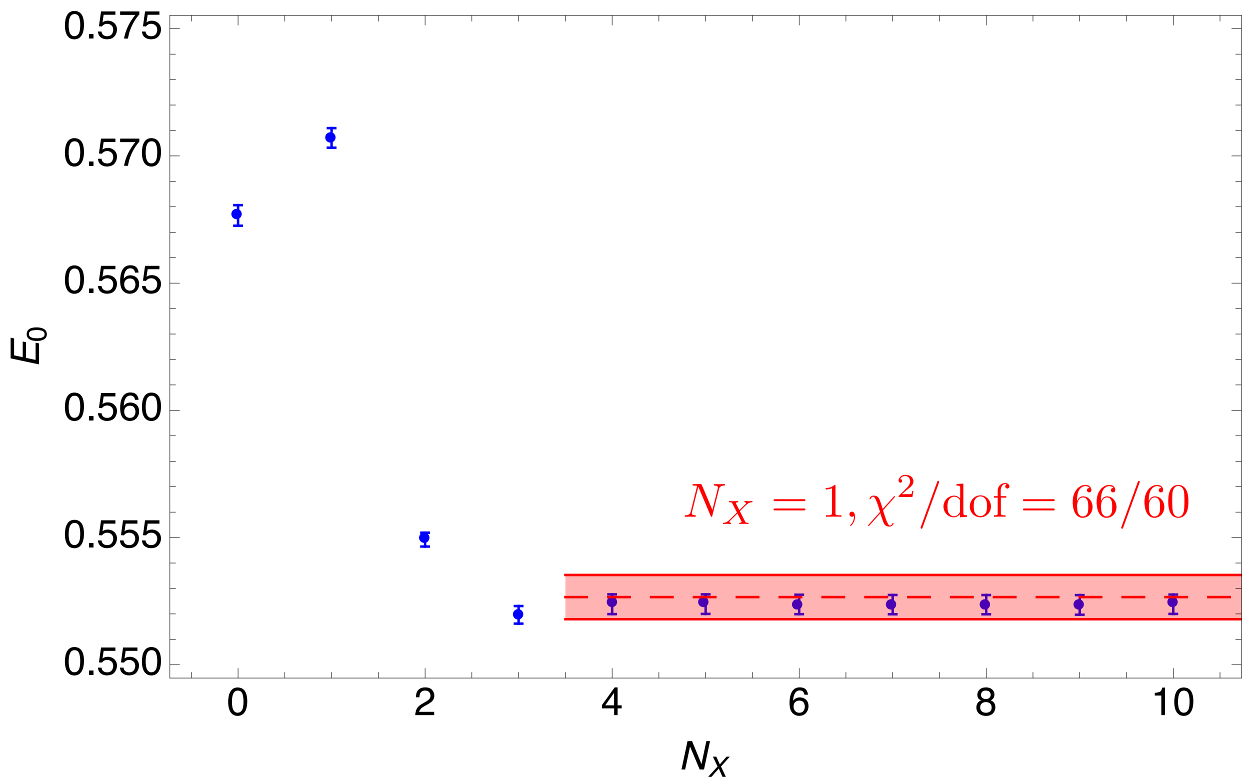
$$\chi^2_{aug} = \chi^2 + \sum_{j=1}^{N_{par}} \frac{(a_j - \hat{a}_j)^2}{\hat{\sigma}_j^2}$$

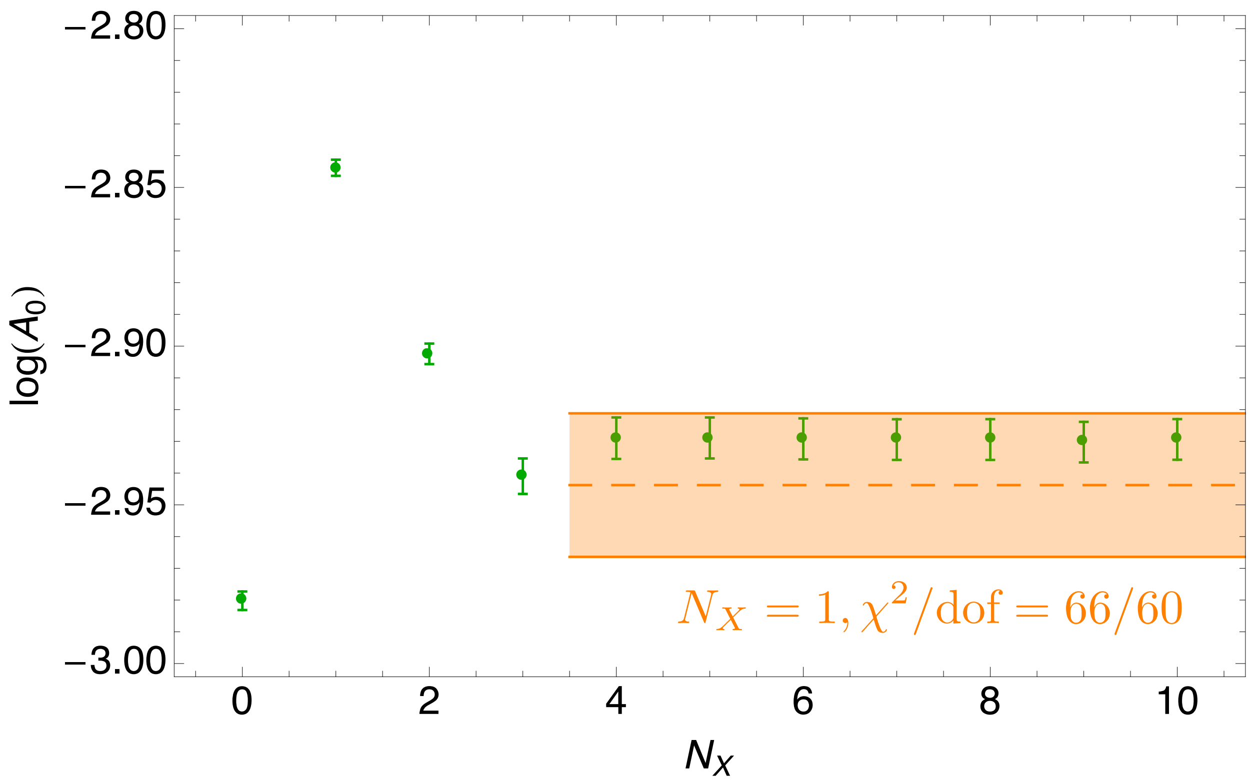
- Example fit shown below (left), full range from [2, $N_t/2$]. 3 excited states used ("4+4"). Good agreement with standard "plateau" ground-state (right).





- Another example, showing stability with addition of more states. Comparison to "plateau" fit with t_{min}=20 (may not be optimal.)





Renormalization and mass tuning

- From two-point fits, convert to

$$\phi_Q = \sqrt{2} Z_{A^4_{Qq}} A_{A^4_{Qq}} = \sqrt{2} \left(\rho_{A^4_{Qq}} \sqrt{Z_{V^4_{qq}} Z_{V^4_{QQ}}} \right) A_{A^4_{Qq}}$$

- "Mostly non-perturbative" renormalization: factorize axial-current renormalization, determine Z_V non-perturbatively, leftover piece ρ computed in lattice PT. Large cancellation ensures ρ~1.

- Heavy-fermion mass parameters K tuned by matching kinetic mass M_2 to physical value. Slight difference between simulated K and tuned K, so we have to adjust:

$$\phi_Q \to \phi_Q + \Delta\phi_Q = \phi_Q + \left(\frac{d\phi_Q}{d\kappa} \right) (\kappa_{\rm sim} - \kappa_{\rm tune})$$

- Derivatives are O(10), but mistunings O(10^{-3}), so shifts are fairly small.

Chiral/continuum extrapolation

- Fit to rooted staggered chiPT (rSxPT) to extrapolate to continuum and physical quark masses:

$$\phi = \phi^0[1 + (\text{chiral logs}) + (\text{NLO analytic}) + (\text{NNLO analytic}) \\ +(\text{LQ discretization}) + (\text{HQ discretization})]$$

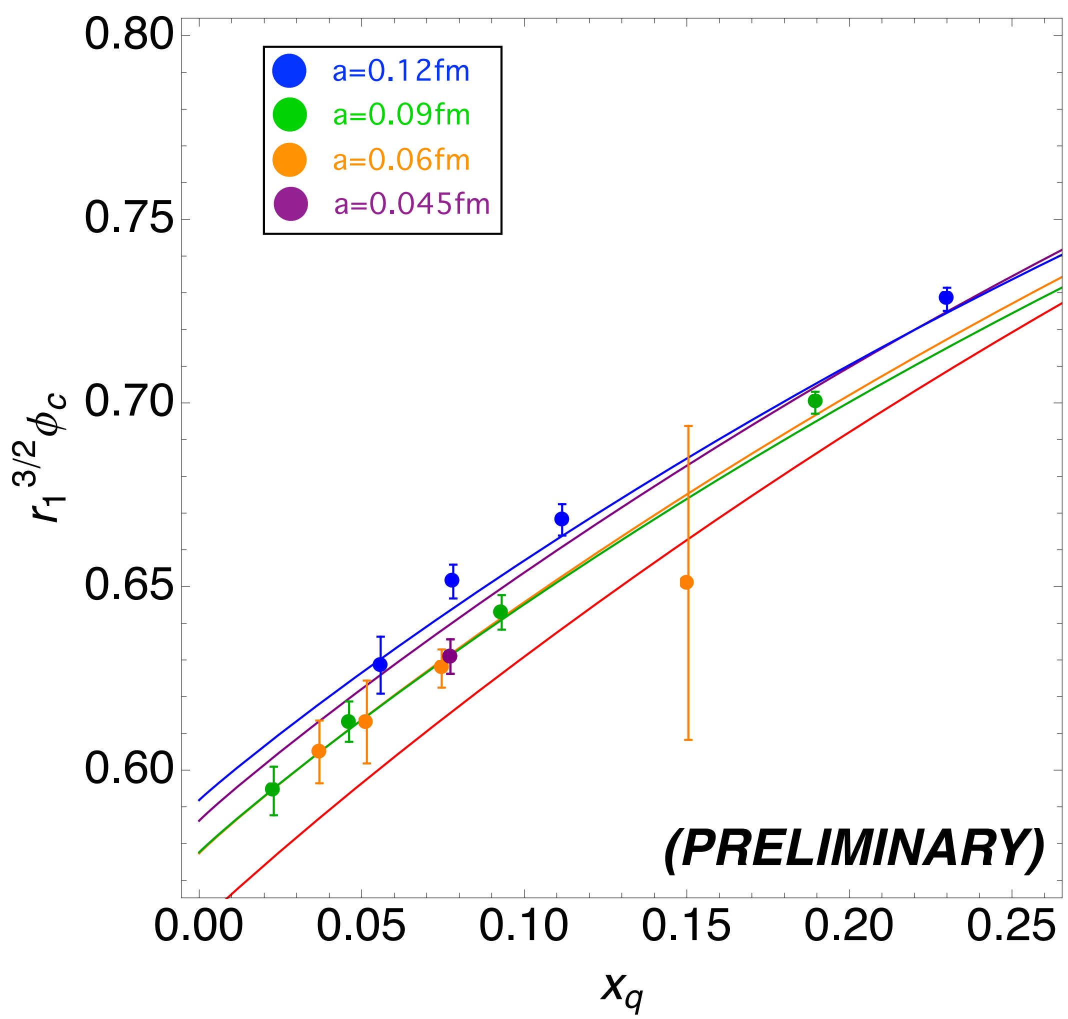
- Terms included for taste-breaking effects, finite-volume corrections in chiral logs, hyperfine/LQ flavor splitting of heavy-light mesons (details in arXiv: 1112.3051)

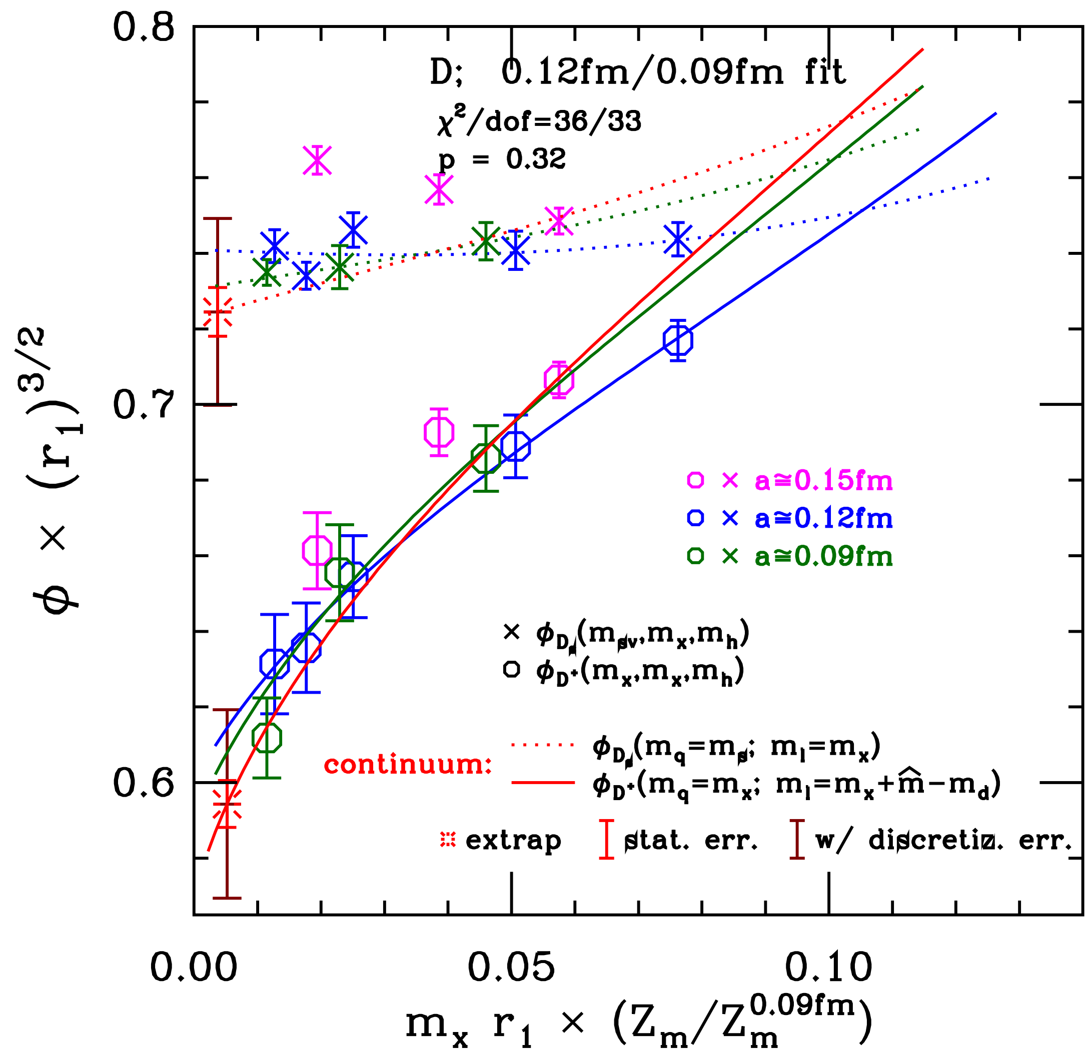
- Continuum extrapolation just requires setting all discretization effects (including taste breaking) to zero in the fit. Can use fits to split out and quantify each discretization error.

- NNLO terms found necessary to fit points with valence mass near strange

Chiral fit results

(New results are **blind** - unknown offset included!)





- Updated results (left) vs. previous (right). Horiz. axis not identical but roughly M_π^2 in both cases.
- All results N_X=1, large t_{min} chosen by eye - starting point and basis for comparison with full-range fits

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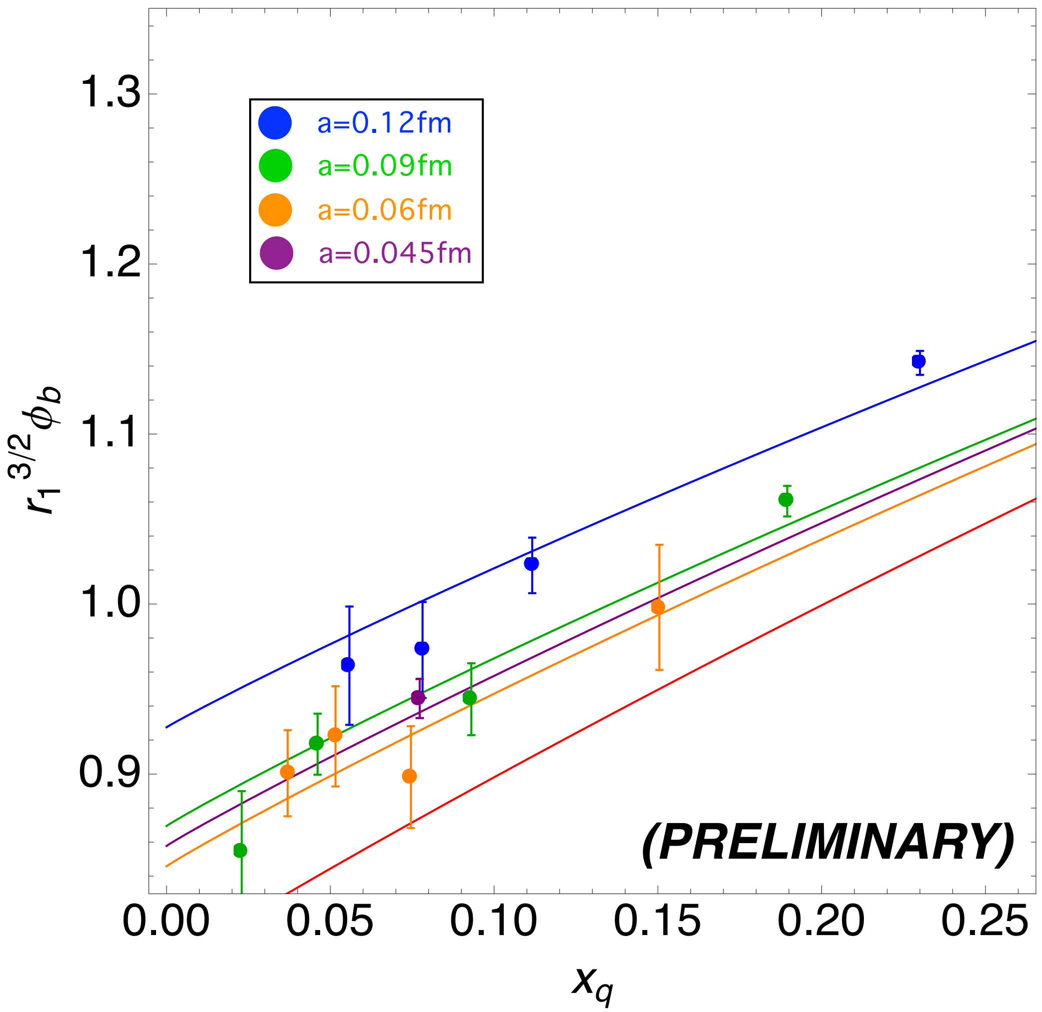

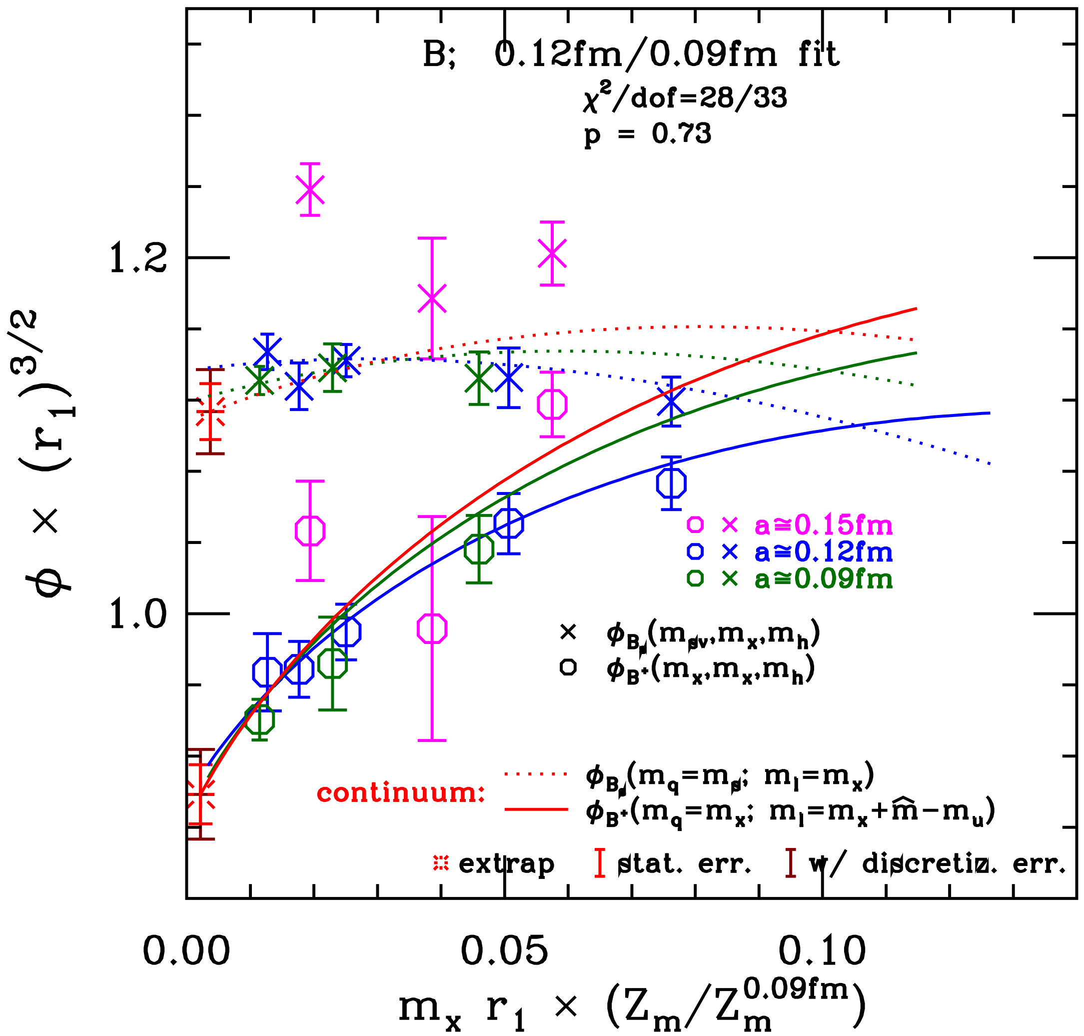

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Error budget projection

Source	f_{D^+} (MeV)	f_{D_s} (MeV)	f_{B^+} (MeV)	f_{B_s} (MeV)
Statistics	2.3 *[1.1]*	2.3 *[1.1]*	3.6 *[1.8]*	3.4 *[1.7]*
Heavy-quark disc.	8.2 *[3.6]*	8.3 *[3.6]*	3.7 *[1.9]*	3.8 *[2.0]*
Light-quark disc.	2.9 *[0.7]*	1.5 *[0.3]*	2.5 *[0.6]*	2.1 *[0.5]*
Chiral extrapolation	3.2 *[1.6]*	2.2 *[1.1]*	2.9 *[1.5]*	2.8 *[1.4]*
Heavy-quark tuning	2.8 *[2.0]*	2.8 *[2.0]*	3.9 *[2.4]*	3.9 *[2.4]*
$Z_{V^4_{QQ}}$ and $Z_{V^4_{qq}}$	2.8 *[1.4]*	3.4 *[1.7]*	2.6 *[1.5]*	3.1 *[1.9]*
u_0 adjustment	1.8 *[0]*	2.0 *[0]*	2.5 *[0]*	2.8 *[0]*
Other sources	3.8 *[3.8]*	3.0 *[3.0]*	3.5 *[3.5]*	4.8 *[4.8]*
Total *[projected]* error	11.3 *[6.1]*	10.8 *[5.6]*	8.9 *[5.5]*	9.5 *[6.4]*

- Error breakdown from previous published analysis, along with projected improvements here based on known scaling of discretization errors, mass dependence, etc. Overall projected uncertainty ~2-3%.

Conclusion

- Update of previous asqtad B/D decay constant calculation will improve in several directions - ensembles with smaller *a*, lighter mass, more statistics. Projected errors roughly 5-6 MeV for the various f_B/f_D.

- In addition, planning determinations of ratios: f_{Bs}/f_B, f_{Ds}/f_D, which are more precise due to cancellations. Cross-ratios f_{Bs}/f_{Ds}, f_B/f_D as well, which can then be combined with e.g. measurement of D decay constants on HISQ (see talk by D. Toussaint, Friday afternoon)

- Finalizing estimation of systematic errors, implementing remaining chiral fit terms (hyperfine splitting mostly), then analysis will be unblinded - stay tuned