

B-physics from the ratio method with Wilson twisted mass fermions

Andrea Shindler



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and in progress

ETMCb group

B. Blossier, N. Carrasco, P. Dimopoulos, R. Frezzotti, V. Gimenez,
G. Herdoiza, V. Lubicz, G. Martinelli, C. Michael, D. Palao,
G. C. Rossi, F. Sanfilippo, A.S., S. Simula, C. Tarantino



Ratio method

ETMCb: 2010–2011

- Use relativistic quarks (Wilson twisted mass fermions)
- For each observable b-mass point is reached through interpolation $1/\mu_h$ from c-mass region to the static point
- c-mass region computation is reliable
- the b-mass point is related to its c-mass counterpart by a chain of suitable, HQET-inspired ratios at successive values of the heavy-mass
- Ratios show a smooth chiral and continuum limit
- Static limit value of ratios is exactly known

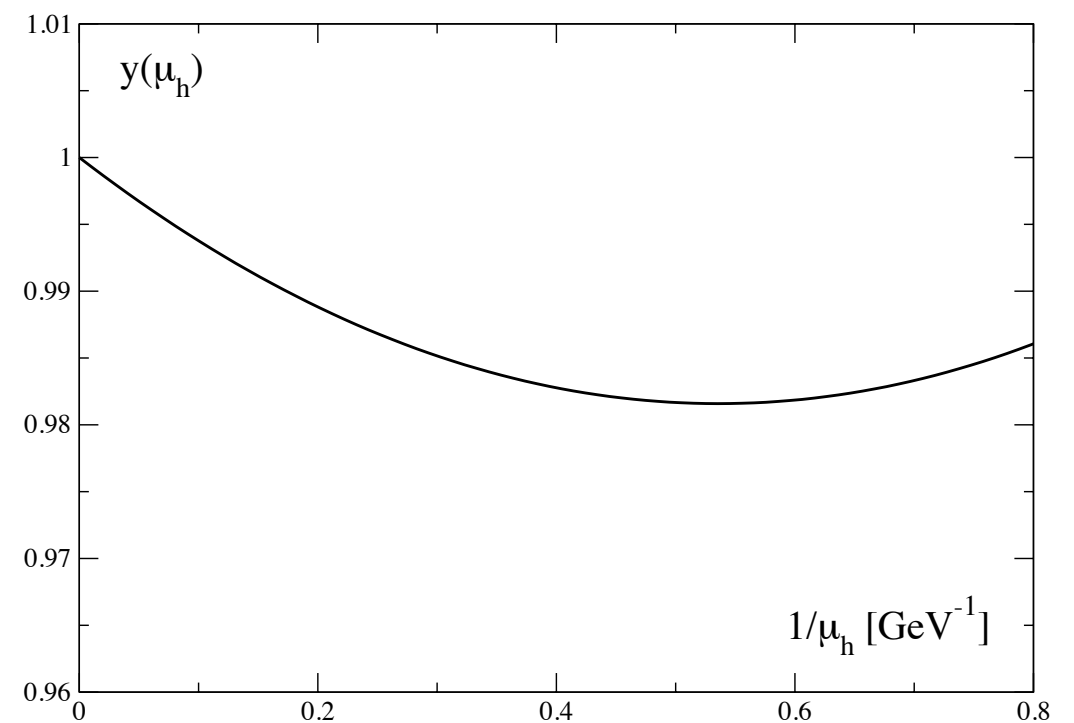
Ratio method: example

HQET expansion

$$M_{hl} = \mu_h + \bar{\Lambda} - \frac{\lambda_1 + 3\lambda_2}{2} \frac{1}{\mu_h} + O\left(\frac{1}{(\mu_h)^2}\right)$$

$$y(\mu_h, \lambda^{\text{pole}}) = \frac{1}{\lambda^{\text{pole}}} \frac{M_{hl}(\mu_h)}{M_{hl}(\mu_h/\lambda^{\text{pole}})}$$

$$y(\mu_h, \lambda) = 1 + \frac{\eta_1^{\text{pole}}}{\mu_h} + \frac{\eta_2^{\text{pole}}}{\mu_h^2}$$



$$y = 1 - \bar{\Lambda} \frac{\lambda^{\text{pole}} - 1}{\mu_h} + \left[\frac{\lambda_1 + 3\lambda_2}{2} (\lambda^{\text{pole}} + 1) + \bar{\Lambda}^2 \lambda^{\text{pole}} \right] \frac{\lambda^{\text{pole}} - 1}{(\mu_h)^2}$$

Simulation points

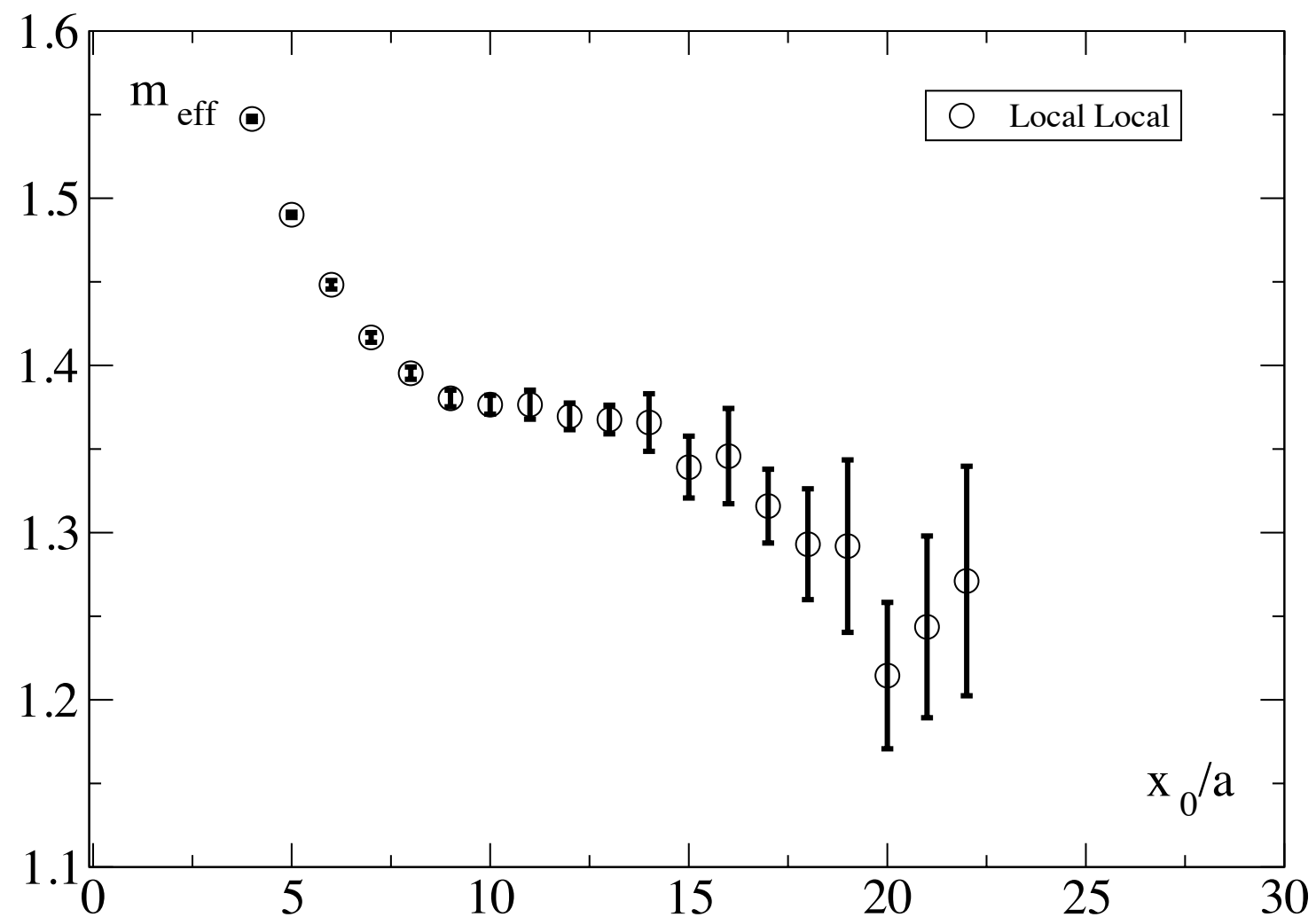
β	$a\mu_\ell$	$a\mu_s$	$a\mu_h$
3.80	0.0080, 0.0110	0.0175, 0.0194 0.0213	0.1982, 0.2331, 0.2742, 0.3225, 0.3793 0.4461 0.5246, 0.6170, 0.7257, 0.8536, 1.004
3.90	0.0030, 0.0040 0.0064, 0.0085, 0.0100	0.0159, 0.0177 0.0195	0.1828, 0.2150, 0.2529, 0.2974, 0.3498 0.4114, 0.4839, 0.5691, 0.6694, 0.7873, 0.9260
4.05	0.0030, 0.0060 0.0080	0.0139, 0.0154 0.0169	0.1572, 0.1849, 0.2175, 0.2558, 0.3008 0.3538, 0.4162, 0.4895, 0.5757, 0.6771, 0.7960
4.20	0.0020, 0.0065	0.0116, 0.0129 0.0142	0.13315, 0.1566, ,0.1842, 0.2166, 0.2548 0.2997, 0.3525, 0.4145, 0.4876, 0.5734,0.6745

$$0.098 \lesssim a \lesssim 0.054 \text{ fm} \quad m_s/6 \lesssim \bar{\mu}_l \lesssim m_s/2 \quad m_c \lesssim \bar{\mu}_h \lesssim 3 m_c$$

Computations on HLRN (Germany)
& CINECA (Italy)

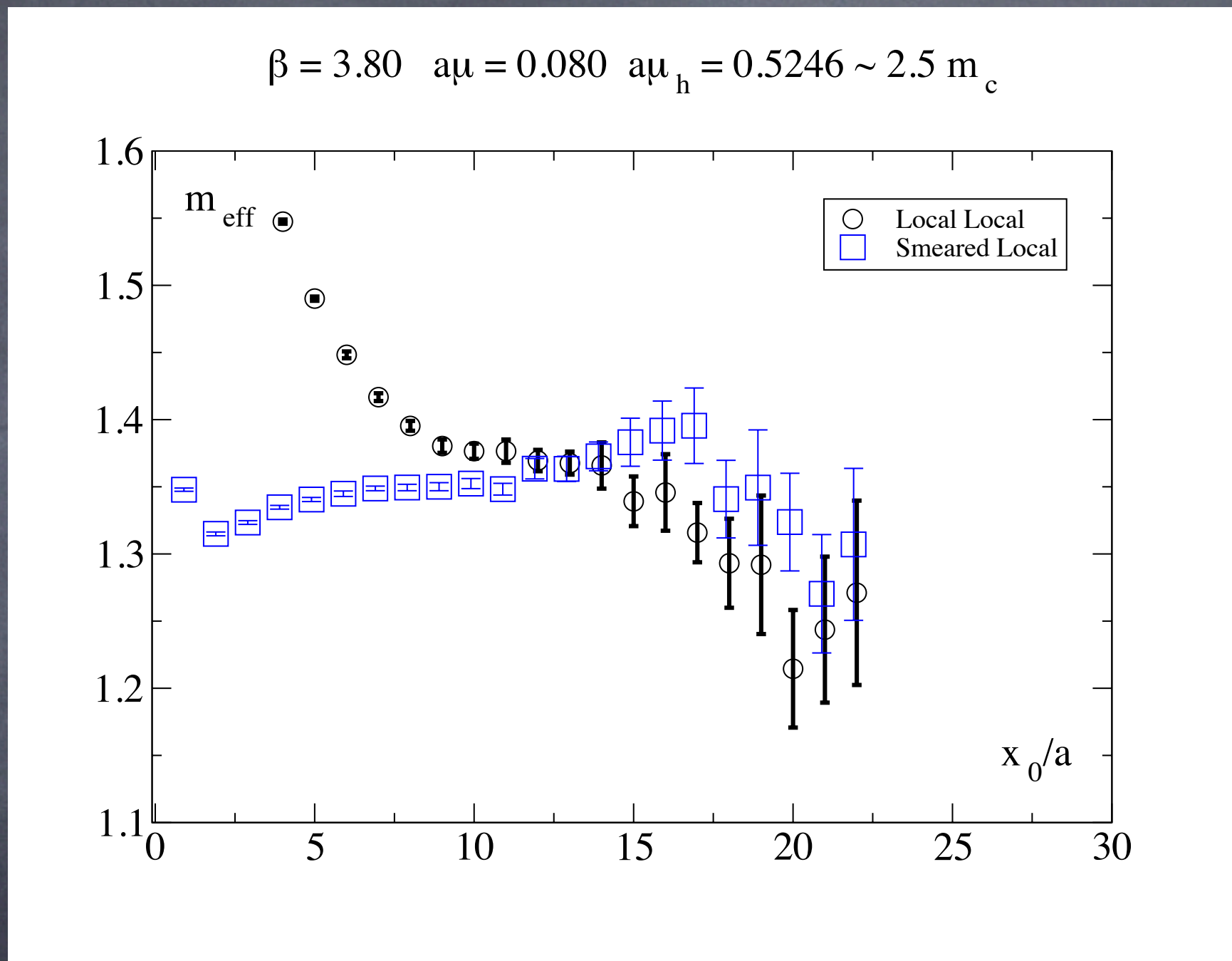
Improving projection

$$\beta = 3.80 \quad a\mu = 0.080 \quad a\mu_h = 0.5246 \sim 2.5 m_c$$



$$\frac{1}{L^3} \sum_{\mathbf{x}} \langle P_5(\mathbf{0}, 0) P_5(\mathbf{x}, x_0) \rangle$$

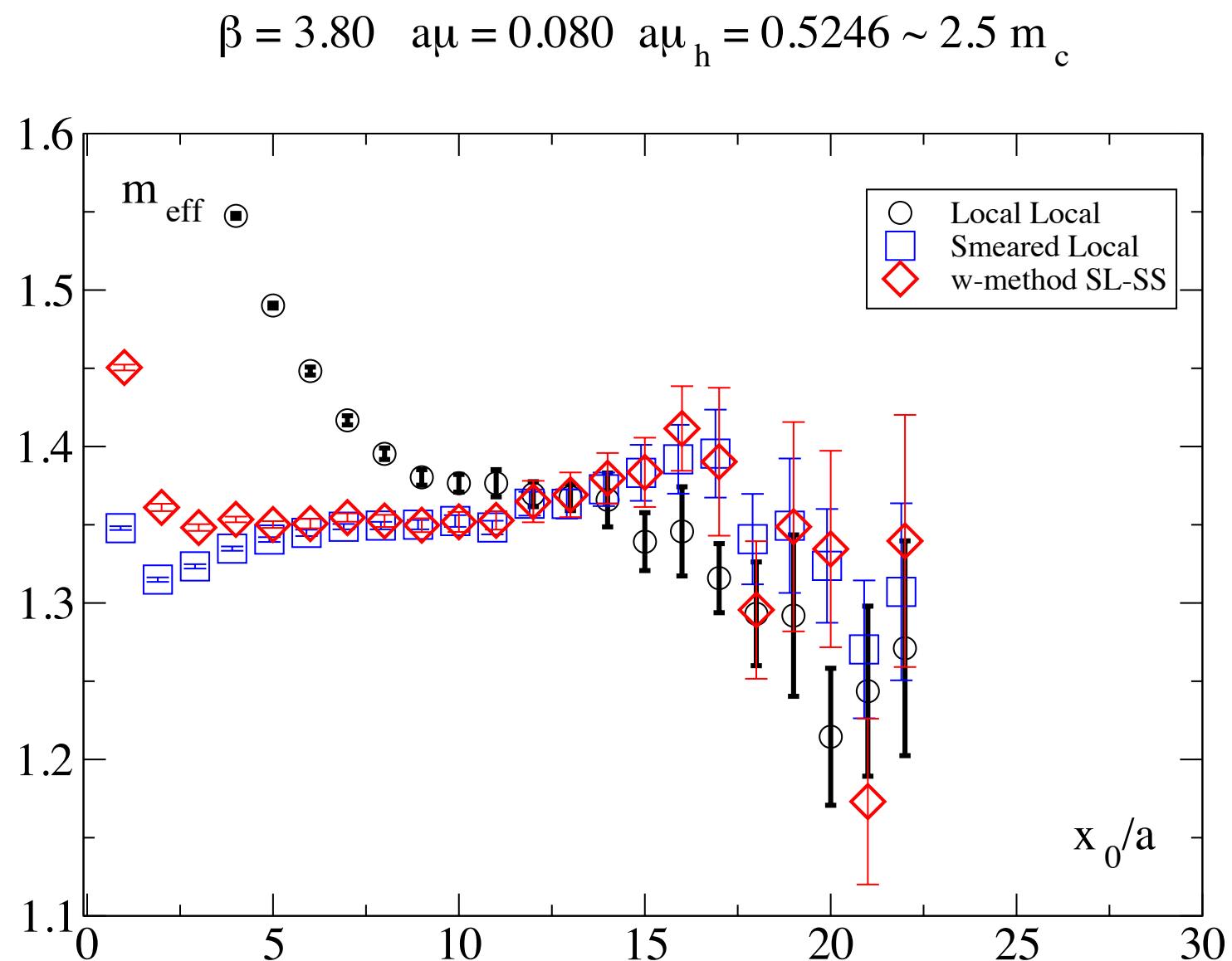
Improving projection



$$\psi^S = (1 + \kappa_G a^2 \nabla_{\text{APE}}^2)^{N_G} \psi^L \quad \kappa_G = 4 \quad N_G = 30$$

$$\alpha_{\text{APE}} = 0.5 \quad N_{\text{APE}} = 20$$

Improving projection

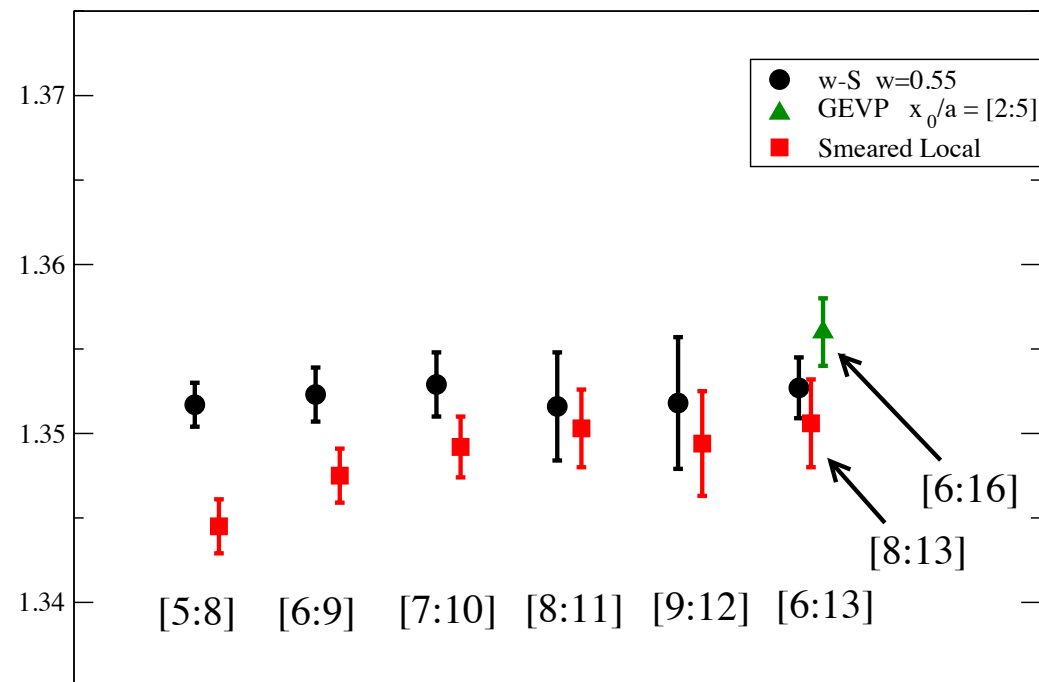


$$\Phi_{\text{source}}(w) = w\Phi^S + (1 - w)\Phi^L$$

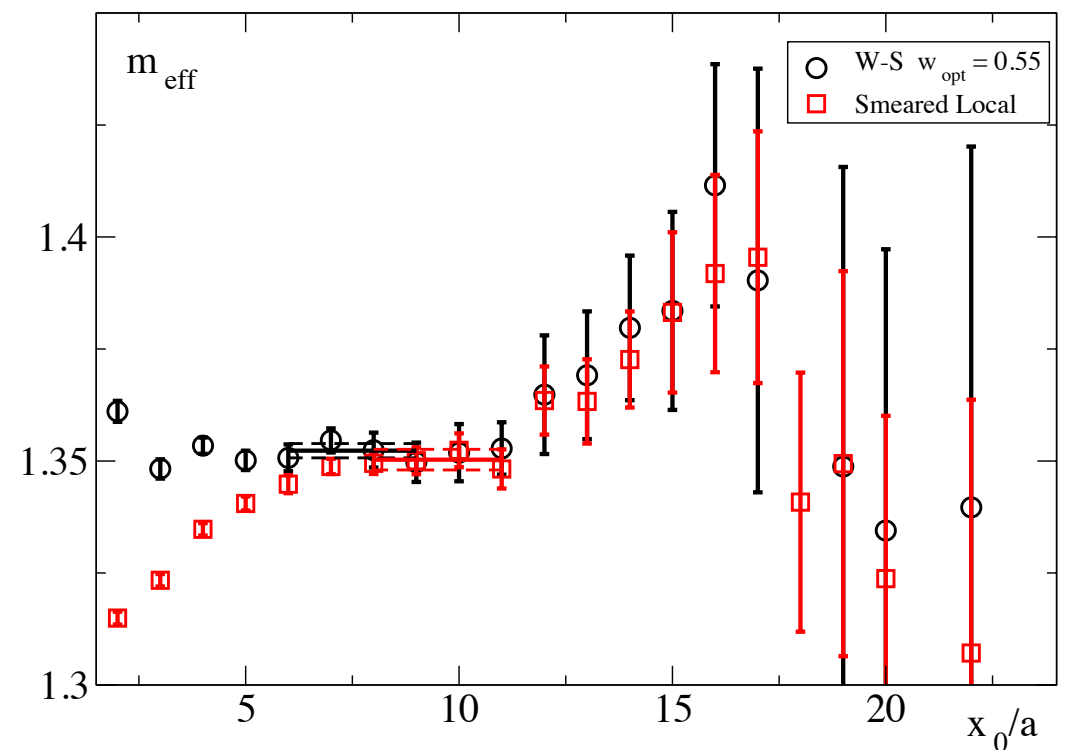
$$\max [m_{\text{eff}}(w - L)] \simeq \min [m_{\text{eff}}(w - L)]$$

Checks on smearing

$$\beta = 3.80 \quad a\mu = 0.0080 \quad a\mu_h = 0.5246 \sim 2.5 m_c$$



$$\beta = 3.80 \quad a\mu = 0.0080 \quad a\mu_h = 0.5246 \sim 2.5 m_c$$



Earlier plateau essential for 3-point functions
 Checked all the masses with GEVP

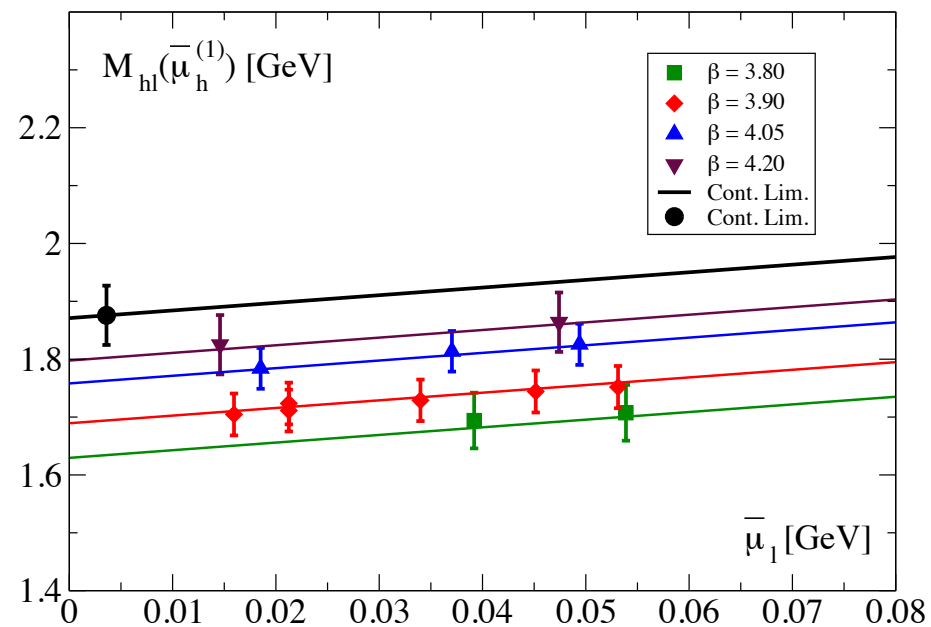
Lattice implementation

$$\begin{aligned}
 y(\bar{\mu}_h^{(n)}, \lambda; \bar{\mu}_l, a) &\equiv \frac{M_{hl}(\bar{\mu}_h^{(n)}; \bar{\mu}_l, a)}{M_{hl}(\bar{\mu}_h^{(n-1)}; \bar{\mu}_l, a)} \cdot \frac{\bar{\mu}_h^{(n-1)}}{\bar{\mu}_h^{(n)}} \cdot \frac{\rho(\bar{\mu}_h^{(n-1)}, \mu^*)}{\rho(\bar{\mu}_h^{(n)}, \mu^*)} = \\
 &= \lambda^{-1} \frac{M_{hl}(\bar{\mu}_h^{(n)}; \bar{\mu}_l, a)}{M_{hl}(\bar{\mu}_h^{(n)} / \lambda; \bar{\mu}_l, a)} \cdot \frac{\rho(\bar{\mu}_h^{(n)} / \lambda, \mu^*)}{\rho(\bar{\mu}_h^{(n)}, \mu^*)}, \quad n = 2, \dots, N
 \end{aligned}$$

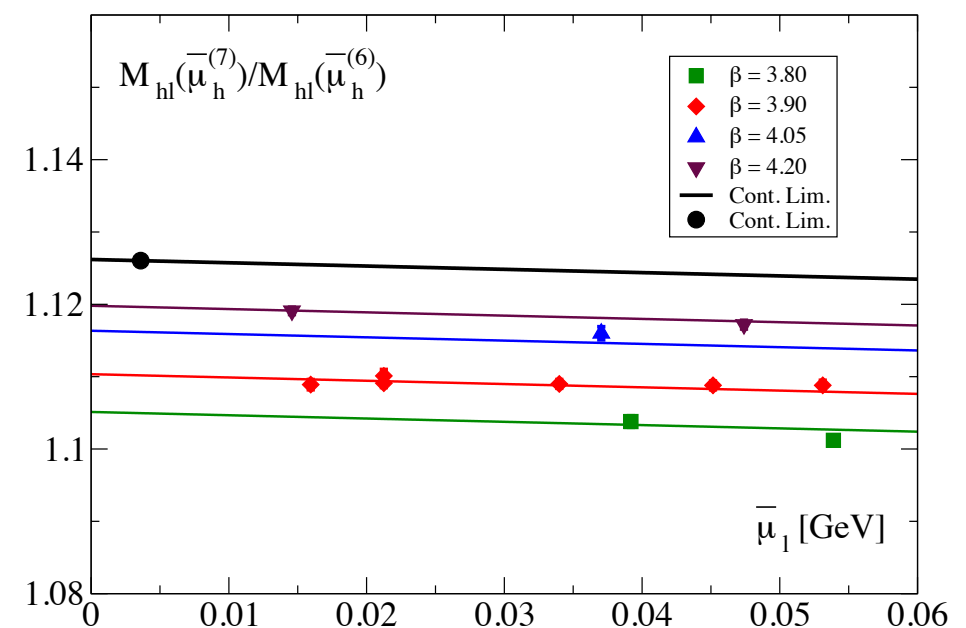
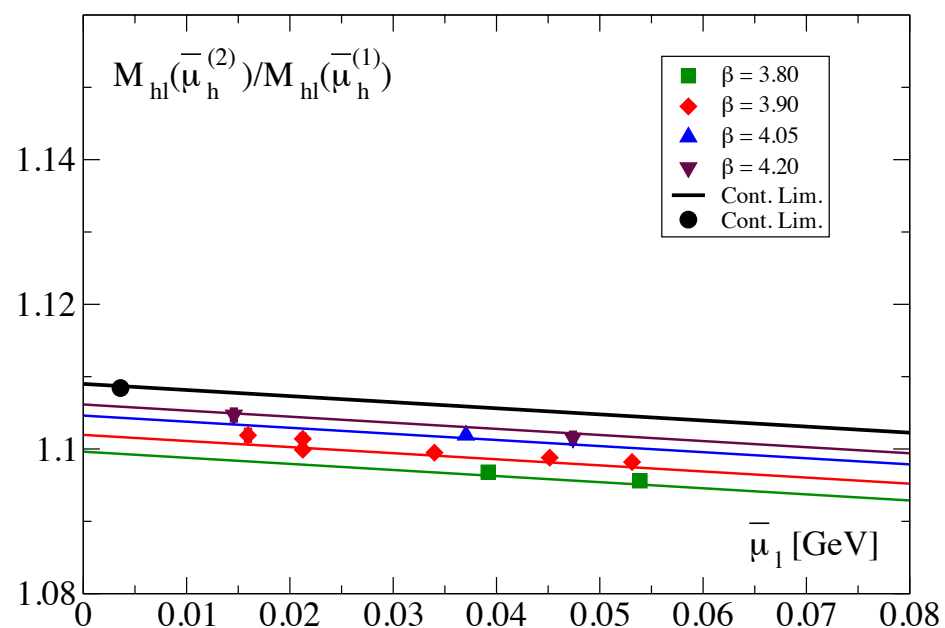
$$\frac{\bar{\mu}_h^{(n)}}{\bar{\mu}_h^{(n-1)}} = \lambda \quad \mu_h = \rho(\bar{\mu}_h, \mu^*) \bar{\mu}_h(\mu^*) \quad \bar{\mu}_h \leftarrow \overline{MS} \text{ scheme}$$

$$\rho(\bar{\mu}_h, \mu^*) \rightsquigarrow N^3 LO$$

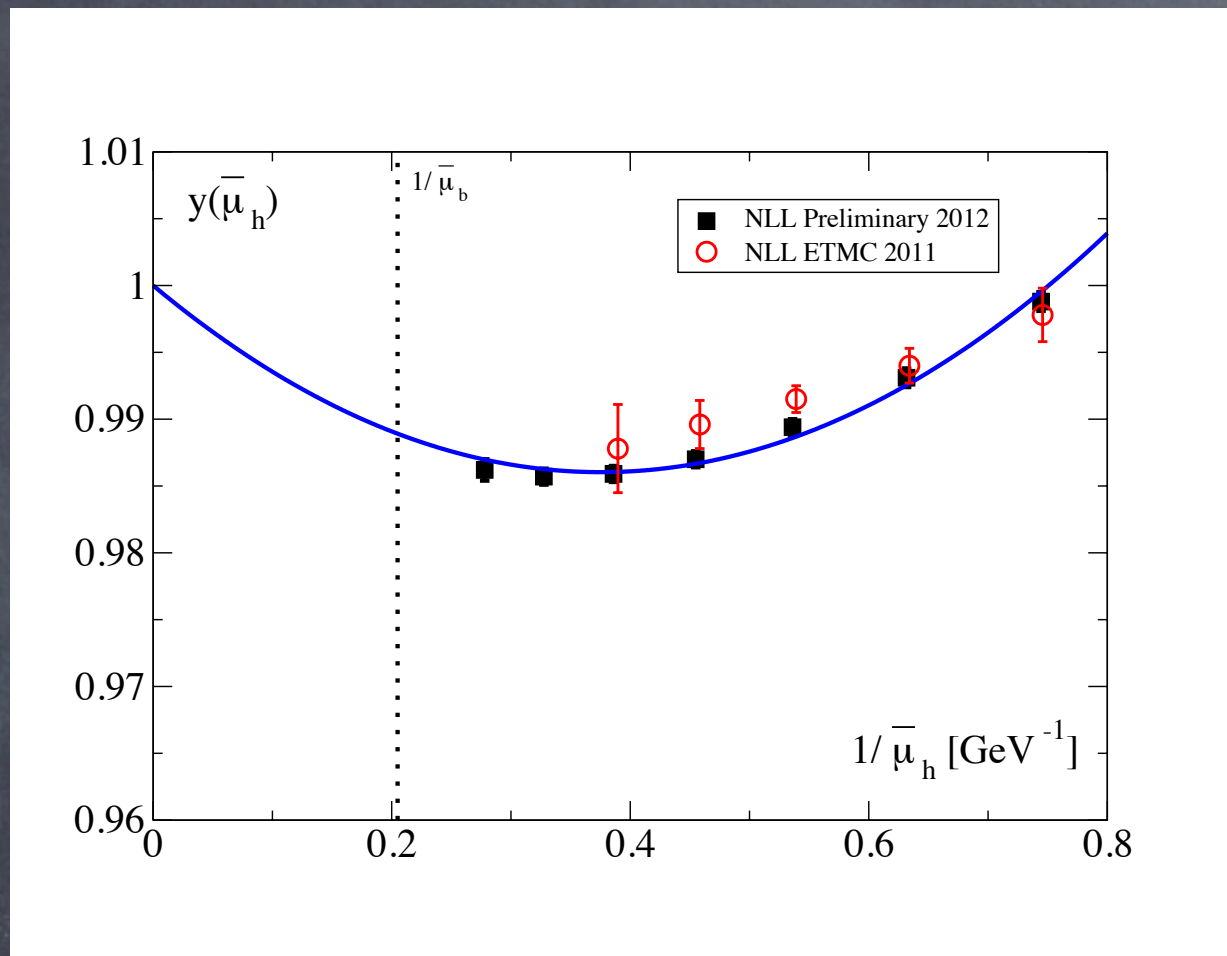
Continuum-chiral limit



- Triggering input: PS meson mass at the charm region
--> affected by tolerable cutoff effects
- Ratios have small cutoff effects



Evolution determination



Fit ansatz valid at NP level

$$y(\bar{\mu}_h) = 1 + \frac{\eta_1}{\bar{\mu}_h} + \frac{\eta_2}{\bar{\mu}_h^2}$$

Strong cancellation of perturbative factors

- Resolve recursion

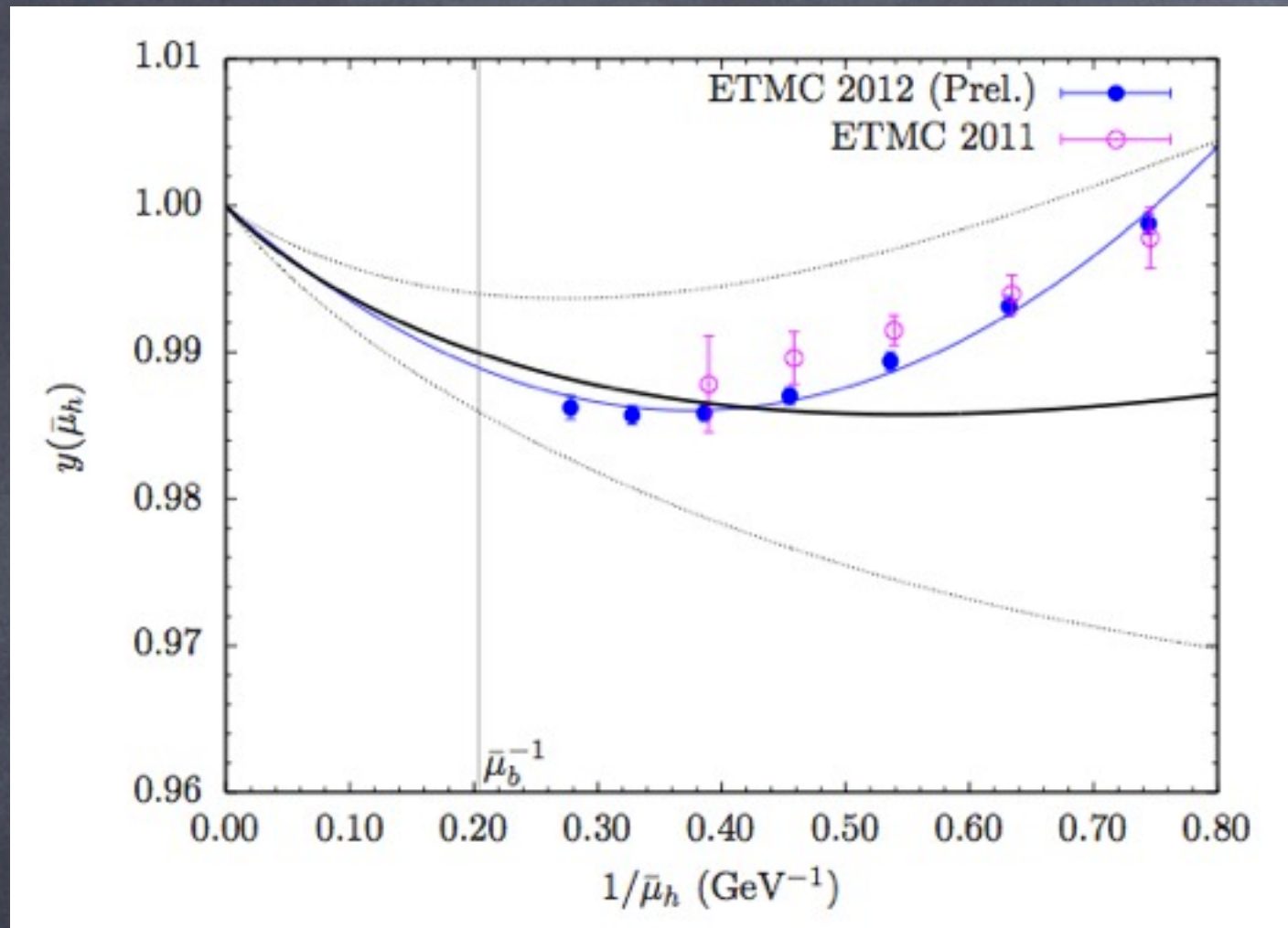
$$y(\bar{\mu}_h^{(2)})y(\bar{\mu}_h^{(3)}) \cdots y(\bar{\mu}_h^{(K+1)}) = \lambda^{-K} \frac{M_{hu/d}(\bar{\mu}_h^{(K+1)})}{M_{hu/d}(\bar{\mu}_h^{(1)})} \cdot \left[\frac{\rho(\bar{\mu}_h^{(1)}, \mu^*)}{\rho(\bar{\mu}_h^{(K+1)}, \mu^*)} \right]$$

- Adjust $(\lambda, \bar{\mu}_h^{(1)})$ such that $M_{hu/d}(\bar{\mu}_h^{(K+1)}) \equiv M_B^{\text{expt}}$ for K integer

- Our calculation

$$\lambda = 1.1784, \quad \bar{\mu}_h^{(1)} = 1.14 \text{ GeV}(\overline{MS}, 2 \text{ GeV}) \rightsquigarrow \bar{\mu}_b = \lambda^K \bar{\mu}_h^{(1)} (K = 9)$$

Phenomenological check

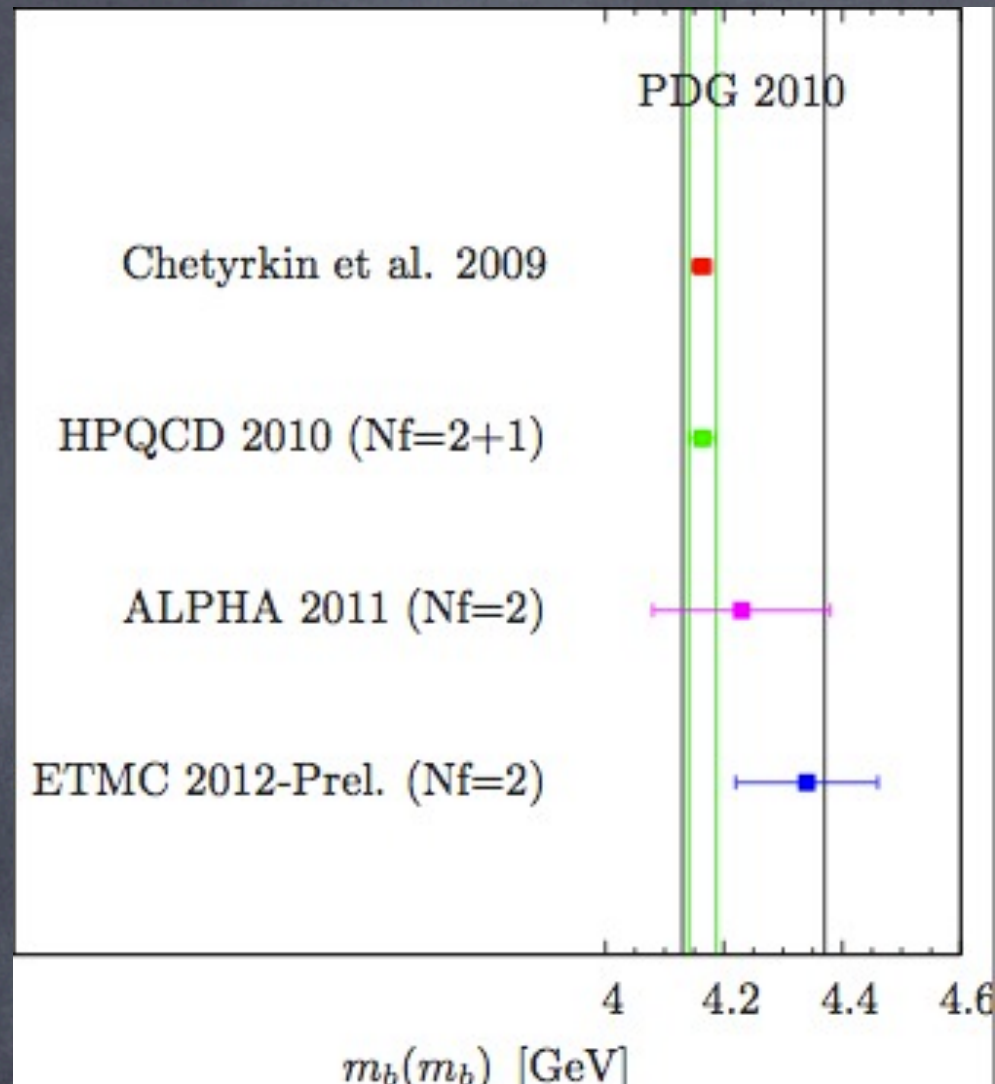


$$y(\bar{\mu}_h) = 1 + \frac{\eta_1}{\bar{\mu}_h} + \frac{\eta_2}{\bar{\mu}_h^2}$$

$$y = 1 - \bar{\Lambda} \frac{\lambda^{\text{pole}} - 1}{\bar{\mu}_h} + \left[\frac{\lambda_1 + 3\lambda_2}{2} (\lambda^{\text{pole}} + 1) + \bar{\Lambda}^2 \lambda^{\text{pole}} \right] \frac{\lambda^{\text{pole}} - 1}{(\bar{\mu}_h)^2}$$

$$\bar{\Lambda} = 0.39(11) \text{ GeV}, \quad \lambda_1 = -0.19(10) \text{ GeV}^2, \quad \lambda_2 = 0.12(2) \text{ GeV}^2$$

b-quark mass summary



- stat. $\rightarrow$ 0.2 %
- ZP + scale setting 2.1-2.8%
- y-ratios negligible

$$m_b^{\overline{MS}}(m_b)|_{N_f=2} = 4.35(12) \text{ GeV} \quad (NLL \ \& \ LL) \qquad m_b^{\overline{MS}}(m_b)|_{N_f=2} = 4.32(12) \text{ GeV} \quad (TL)$$

Perfect agreement using M_{hs} as observable

$$m_b(m_b, \overline{MS})|_{N_f=2} = 4.29(14) \text{ GeV}$$

ETMC 2011

Ratio method: f_{B_s}

$$\Phi_{hs}(\bar{\mu}_h, \mu_b^*) = [C_A^{stat}(\mu_b^*, \bar{\mu}_h)]^{-1} \cdot \Phi_{hs}^{QCD}(\bar{\mu}_h) \quad C_A^{stat}(\mu_b^*, \bar{\mu}_h) \rightsquigarrow N^2 LO$$

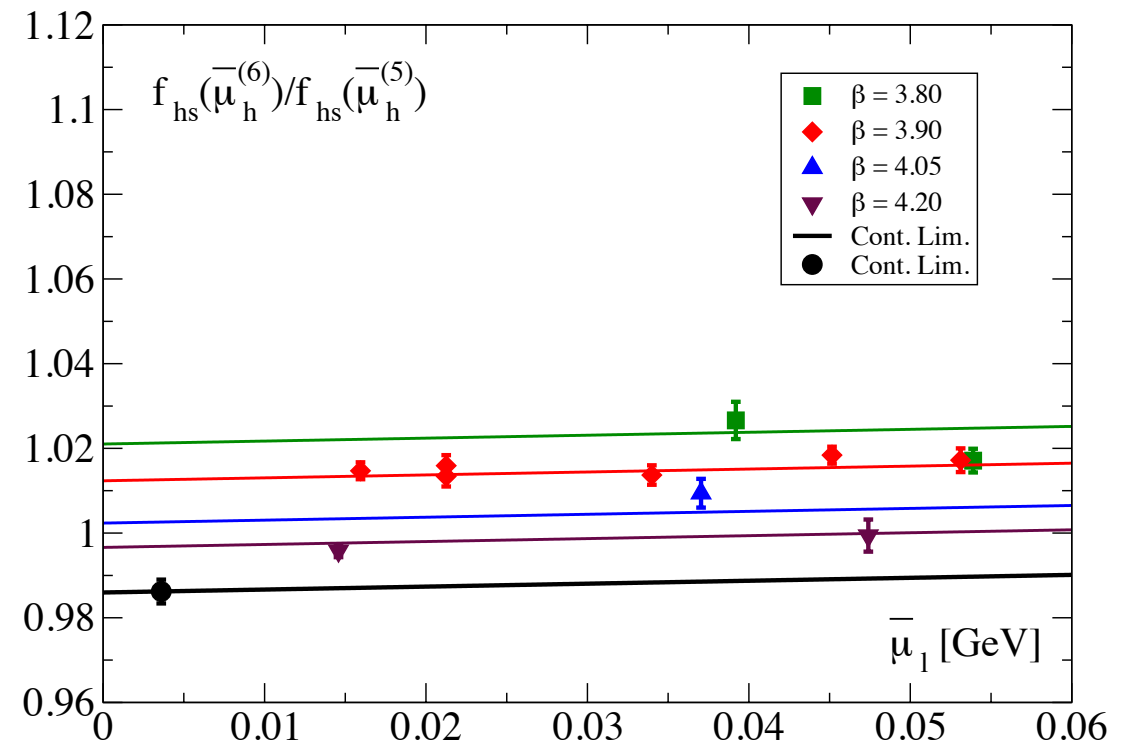
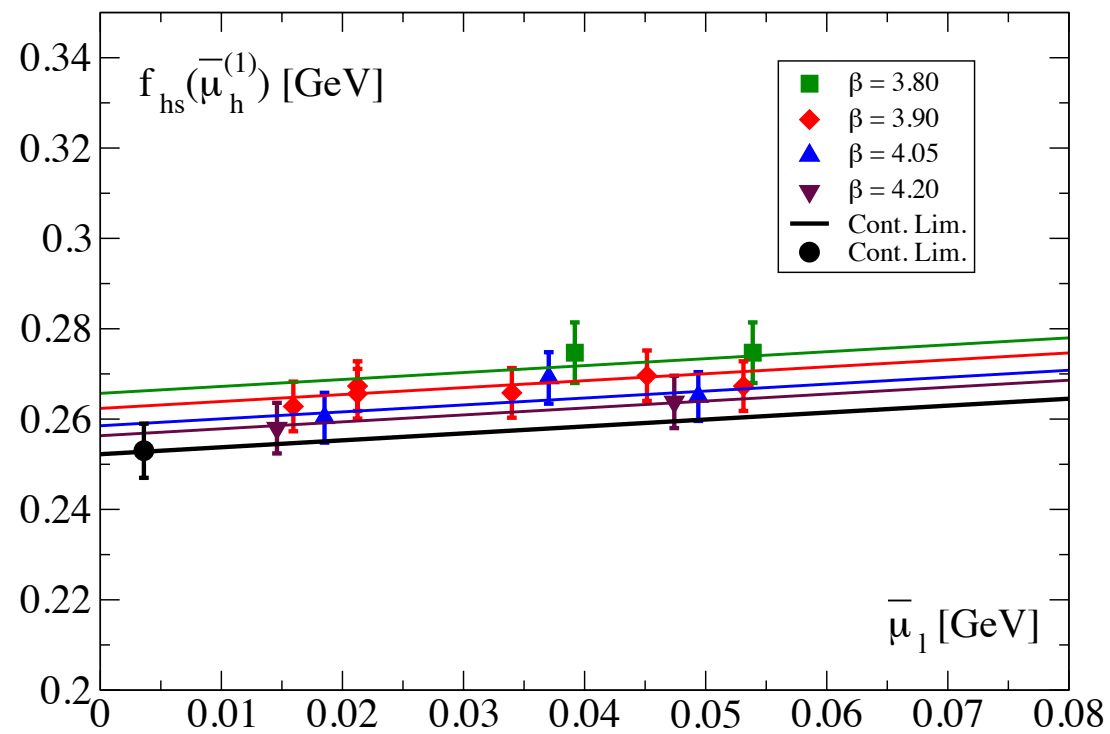
$$\Phi_{hs}(\bar{\mu}_h, \mu_b^*) = \frac{(f_{hs}\sqrt{M_{hs}})^{QCD}}{C_A^{stat}(\bar{\mu}_h, \mu_b^*)} = \Phi_0(\mu_b^*) \left(1 + \frac{\Phi_1(\mu_b^*)}{\bar{\mu}_h} + \frac{\Phi_2(\mu_b^*)}{(\bar{\mu}_h)^2} \right) + \mathcal{O}\left(\frac{1}{(\bar{\mu}_h)^3}\right)$$

$$z(\bar{\mu}_h, \lambda; \bar{\mu}_l, a) \equiv \lambda^{1/2} \frac{f_{hl}(\bar{\mu}_h, \bar{\mu}_l, a)}{f_{hl}(\bar{\mu}_h/\lambda, \bar{\mu}_l, a)} \cdot \frac{C_A^{stat}(\mu_b^*, \bar{\mu}_h/\lambda)}{C_A^{stat}(\mu_b^*, \bar{\mu}_h)} \frac{[\rho(\bar{\mu}_h, \mu^*)]^{1/2}}{[\rho(\bar{\mu}_h/\lambda, \mu^*)]^{1/2}}$$

$$z_s(\bar{\mu}_h, \lambda; \bar{\mu}_l, \bar{\mu}_s, a) \equiv \lambda^{1/2} \frac{f_{hs}(\bar{\mu}_h, \bar{\mu}_l, \bar{\mu}_s, a)}{f_{hs}(\bar{\mu}_h/\lambda, \bar{\mu}_l, \bar{\mu}_s, a)} \cdot \frac{C_A^{stat}(\mu_b^*, \bar{\mu}_h/\lambda)}{C_A^{stat}(\mu_b^*, \bar{\mu}_h)} \frac{[\rho(\bar{\mu}_h, \mu^*)]^{1/2}}{[\rho(\bar{\mu}_h/\lambda, \mu^*)]^{1/2}}$$

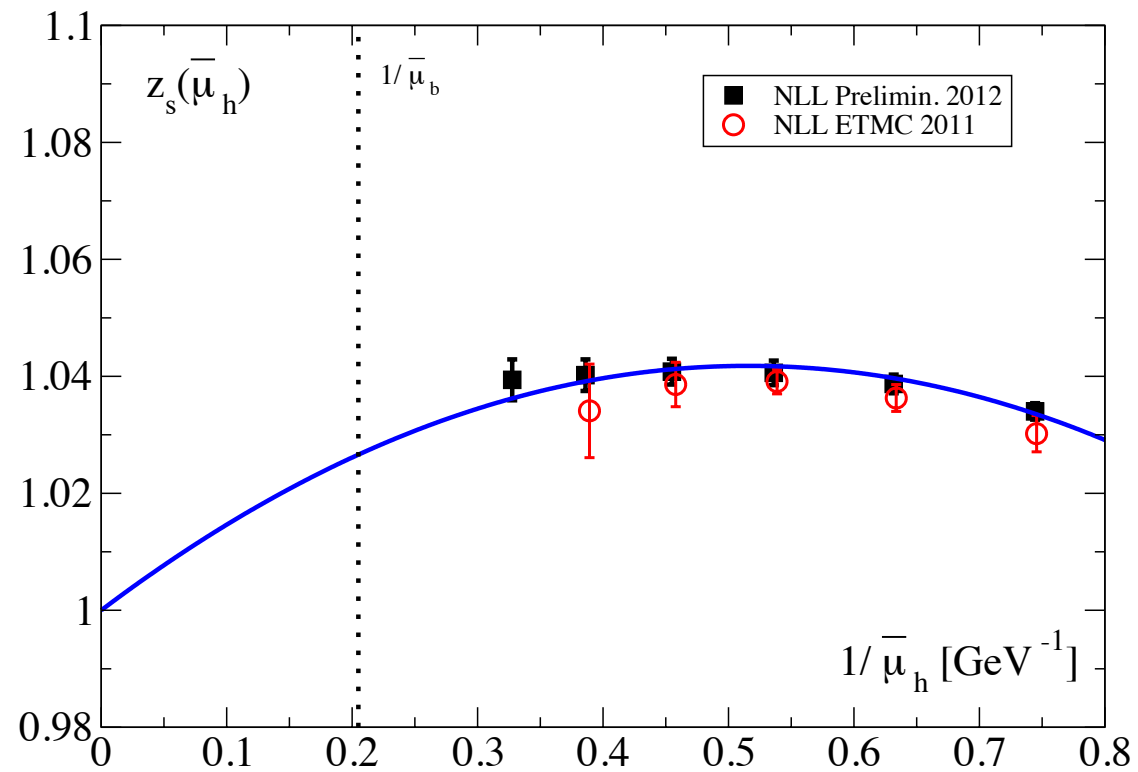
$$\zeta(\bar{\mu}_h, \lambda; \bar{\mu}_l, \bar{\mu}_s, a) \equiv \frac{z_s}{z} = \frac{f_{hs}(\bar{\mu}_h, \bar{\mu}_l, \bar{\mu}_s, a)}{f_{hs}(\bar{\mu}_h/\lambda, \bar{\mu}_l, \bar{\mu}_s, a)} \cdot \frac{f_{hl}(\bar{\mu}_h/\lambda, \bar{\mu}_l, \bar{\mu}_s, a)}{f_{hl}(\bar{\mu}_h, \bar{\mu}_l, \bar{\mu}_s, a)}$$

Continuum-chiral extrapolation



- Ratios have small discretization effects 2%–3%
- Triggering pseudoscalar decay constant affected by tolerable cutoff effects

Decay constant ratio



$$z_s(\bar{\mu}_h) = 1 + \frac{\zeta_1}{\bar{\mu}_h} + \frac{\zeta_1}{\bar{\mu}_h^2}$$

- stat. $\rightarrow$ 0.2 %
- ZP + scale setting 2.1-2.8%
- z-ratios negligible

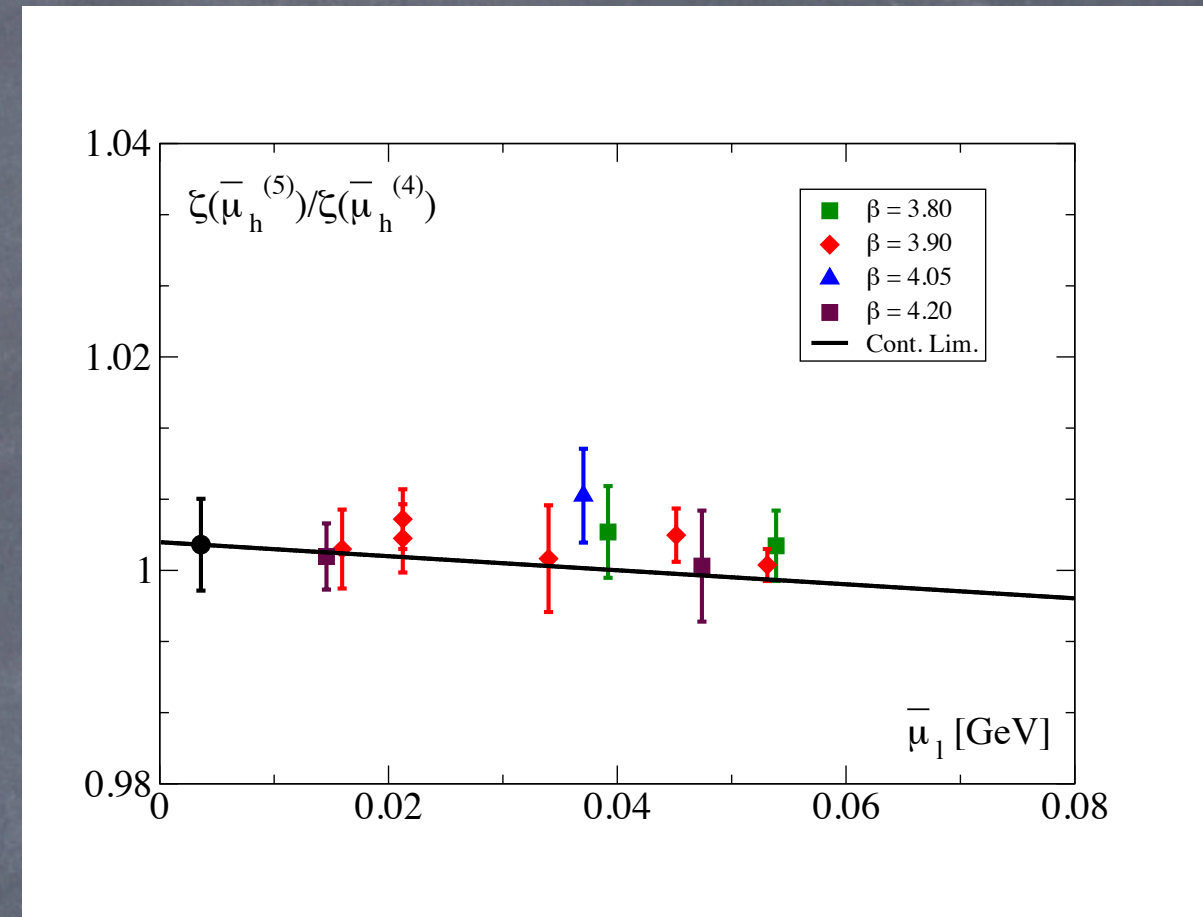
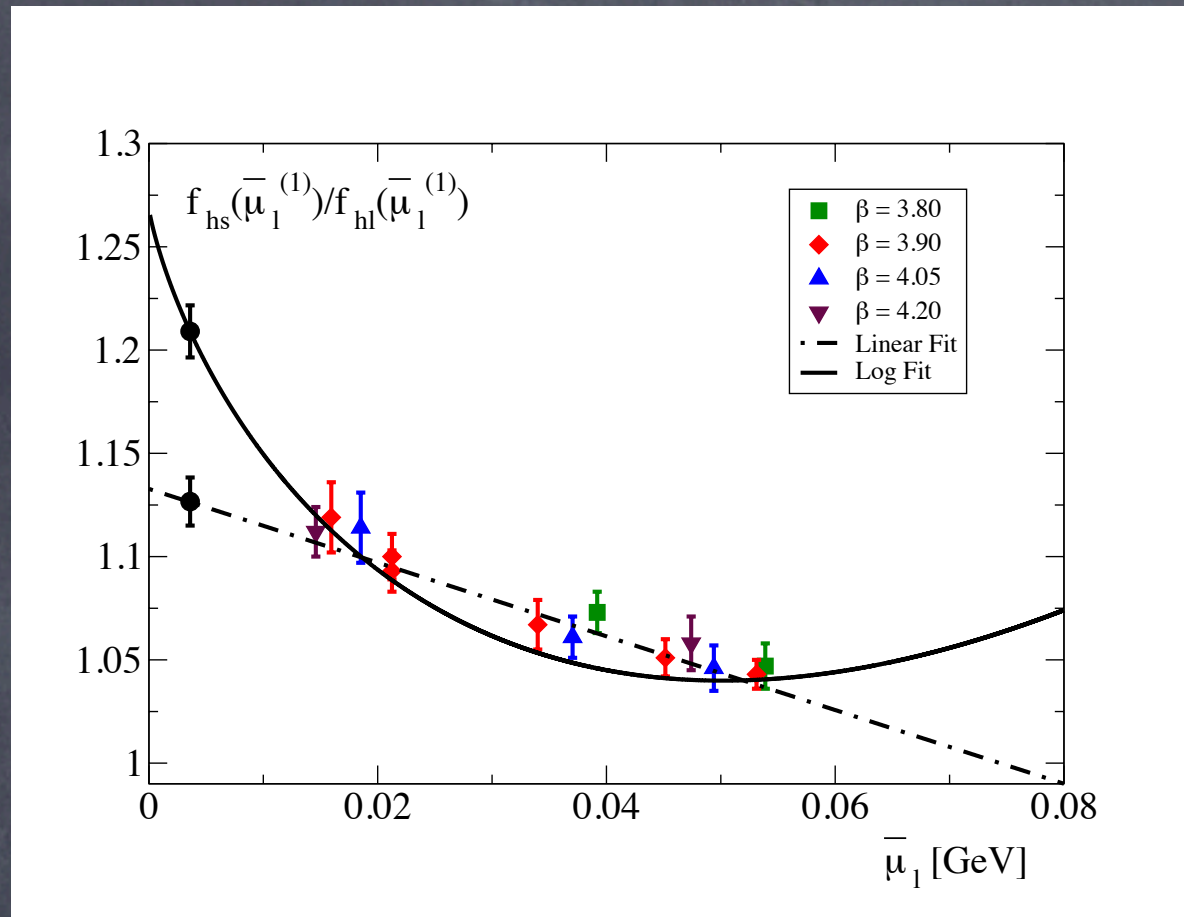
$$z_s(\bar{\mu}_h^{(2)}) z_s(\bar{\mu}_h^{(3)}) \cdots z_s(\bar{\mu}_h^{(K+1)}) =$$

$$\lambda^{K/2} \frac{f_{hs}(\bar{\mu}_h^{(K+1)})}{f_{hs}(\bar{\mu}_h^{(1)})} \cdot \left[\frac{C_A^{\text{stat}}(\bar{\mu}_h^{(1)}, \mu^*)}{C_A^{\text{stat}}(\bar{\mu}_h^{(K+1)}, \mu^*)} \right] \cdot \left[\frac{\rho(\bar{\mu}_h^{(K+1)}, \mu^*)}{\rho(\bar{\mu}_h^{(1)}, \mu^*)} \right]^{1/2}$$

$$\bar{\mu}_h^{(K+1)} \equiv m_b \implies f_{hs}(\bar{\mu}_h^{(K+1)}) = f_{B_s}$$

$$f_{B_s} = 232(6) \text{ MeV}$$

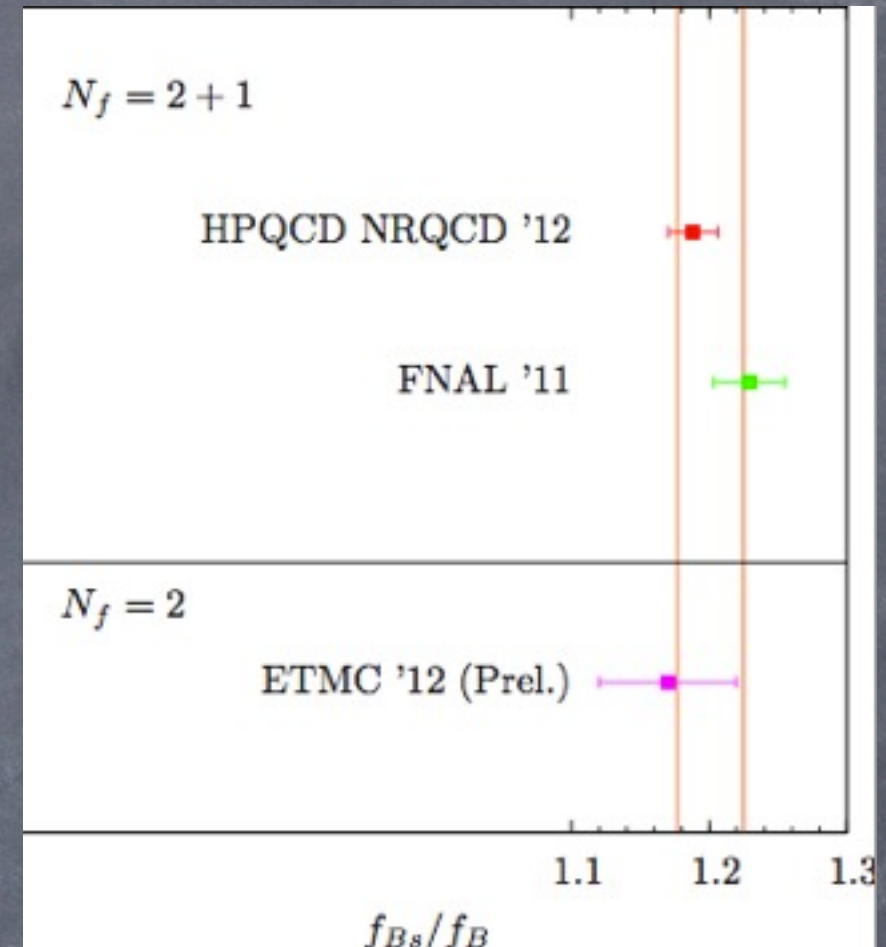
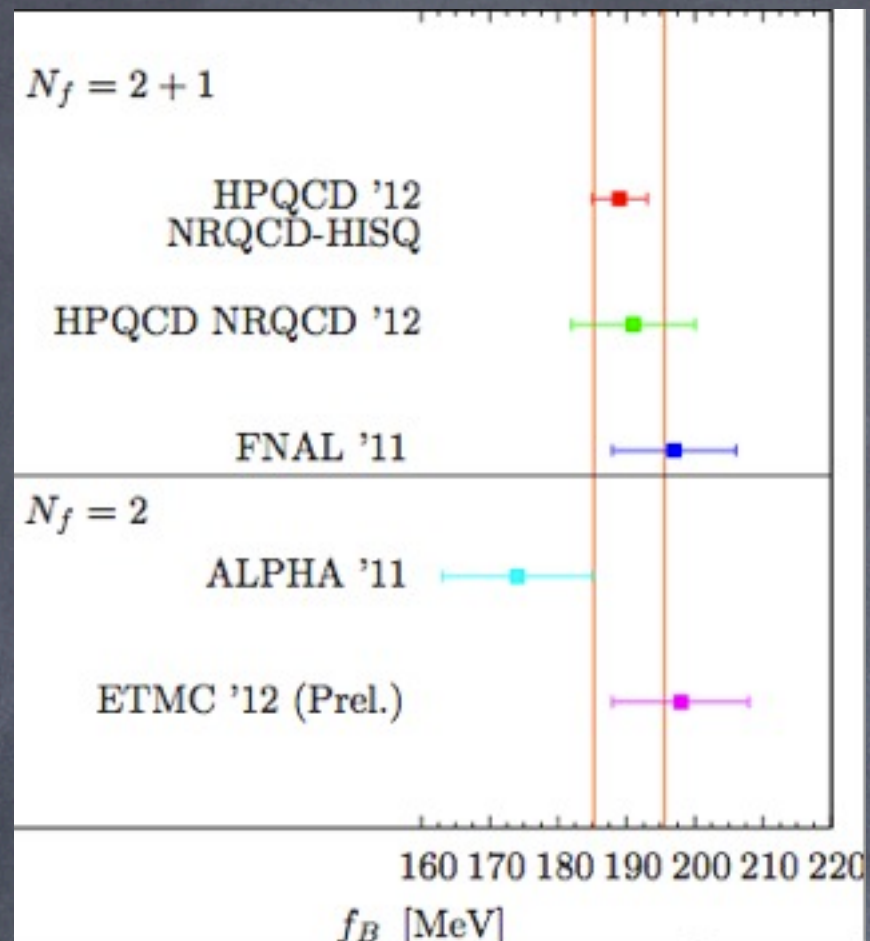
$$f_{Bs}/f_B$$



$$f_{hs}/f_{hl} = A \left[1 - \frac{3(1 + 3\hat{g}^2)}{4} \frac{2B_0\bar{\mu}_l}{(4\pi f_0)^2} \log \left(\frac{2B_0\bar{\mu}_l}{(4\pi f_0)^2} \right) + B\bar{\mu}_l + Ca^2 \right]$$

- HMCHPT and linear fits increase systematic error
- Ratios show very weak dependence on the light and heavy quark masses
- Apply recursion for the ratio

Comparison



$$f_{B_s} = 232(6) \text{ MeV} \quad (NLL \ \& \ LL)$$

$$f_{B_s} = 229(6) \text{ MeV} \quad (TL)$$

$$\frac{f_{B_s}}{f_B} = 1.17(2)(4) \Rightarrow f_B = 198(7)(7) \text{ MeV}$$

$$f_B = 195(12) \text{ MeV} \quad f_{B_s} = 232(10) \text{ MeV} \quad f_{B_s}/f_B = 1.19(5) \quad ETMC \ 2011$$

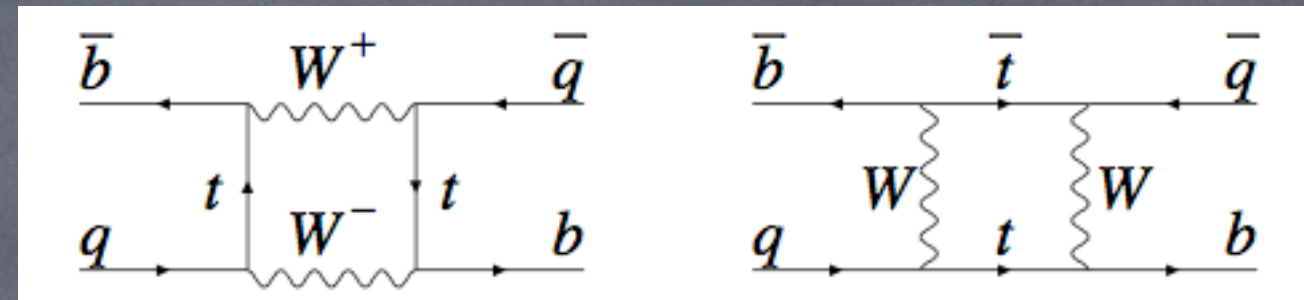
$$f_b = 194(9) \text{ MeV} \quad PDG$$

B parameters

See talk of
N. Carrasco Vela

$$\Delta M_q = \frac{G_F^2 M_W^2}{6\pi^2} |V_{tq} V_{tb}^*|^2 \eta_2^B S_0(x_t) M_{B_q} f_{B_q}^2 \hat{B}_{B_q}$$

$$\langle \bar{B}_q | O | B_q \rangle^{\overline{MS}} \equiv \frac{8}{3} f_{B_q}^2 B_{B_q}^2(\mu) M_{B_q}^2$$



$$O \equiv [\bar{b}^A \gamma_\mu (1 - \gamma_5) q^A] [\bar{b}^B \gamma_\mu (1 - \gamma_5) q^B]$$

$$\tilde{B}_{B_{d/s}}(\bar{\mu}_h, \mu_b^*) = [C(\bar{\mu}_h, \mu_b^*, \mu)]^{-1} B_{B_{d/s}}(\bar{\mu}_h, \mu)$$

$$\omega_{d/s}(\bar{\mu}_h, \lambda; \bar{\mu}_l, a) = \frac{B_{B_{d/s}}(\bar{\mu}_h, \bar{\mu}_l, a; \mu)}{B_{B_{d/s}}(\bar{\mu}_h/\lambda, \bar{\mu}_l, a; \mu)} \cdot \frac{C(\bar{\mu}_h/\lambda; \mu_b^*, \mu)}{C(\bar{\mu}_h; \mu_b^*, \mu)}$$

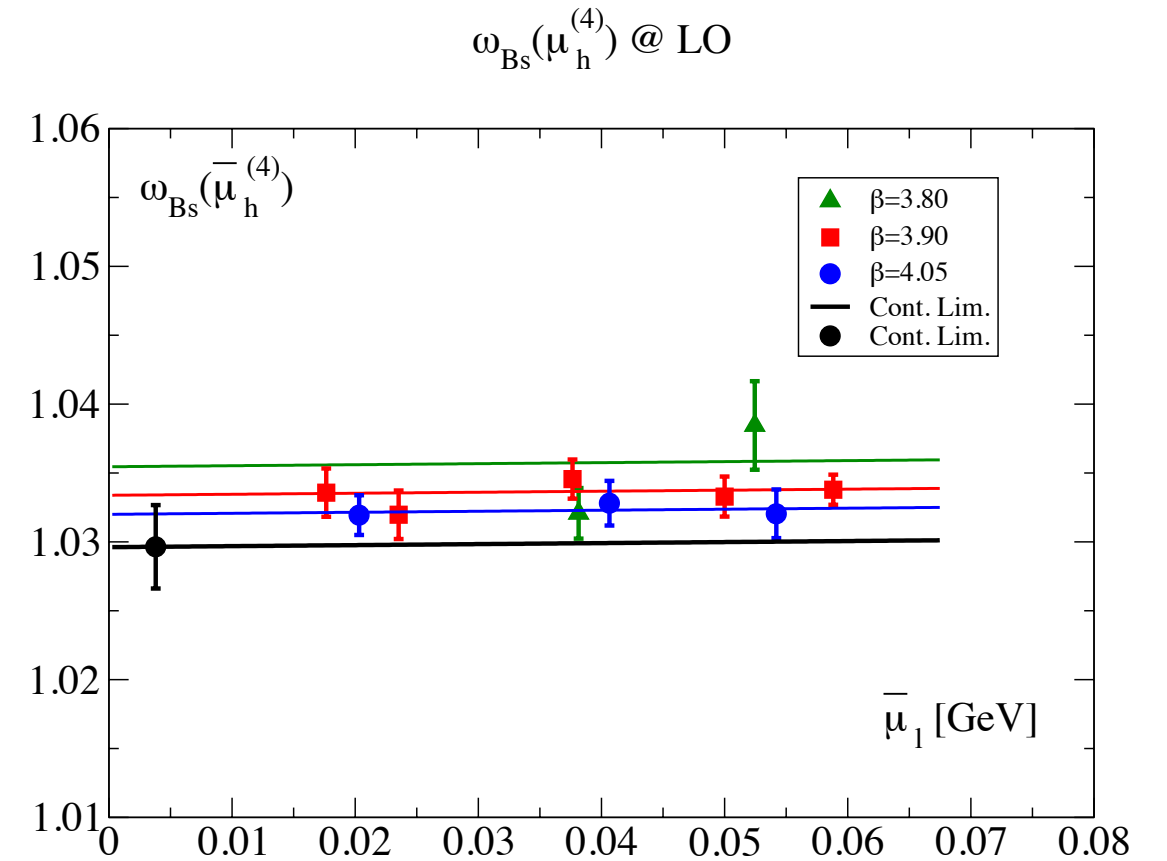
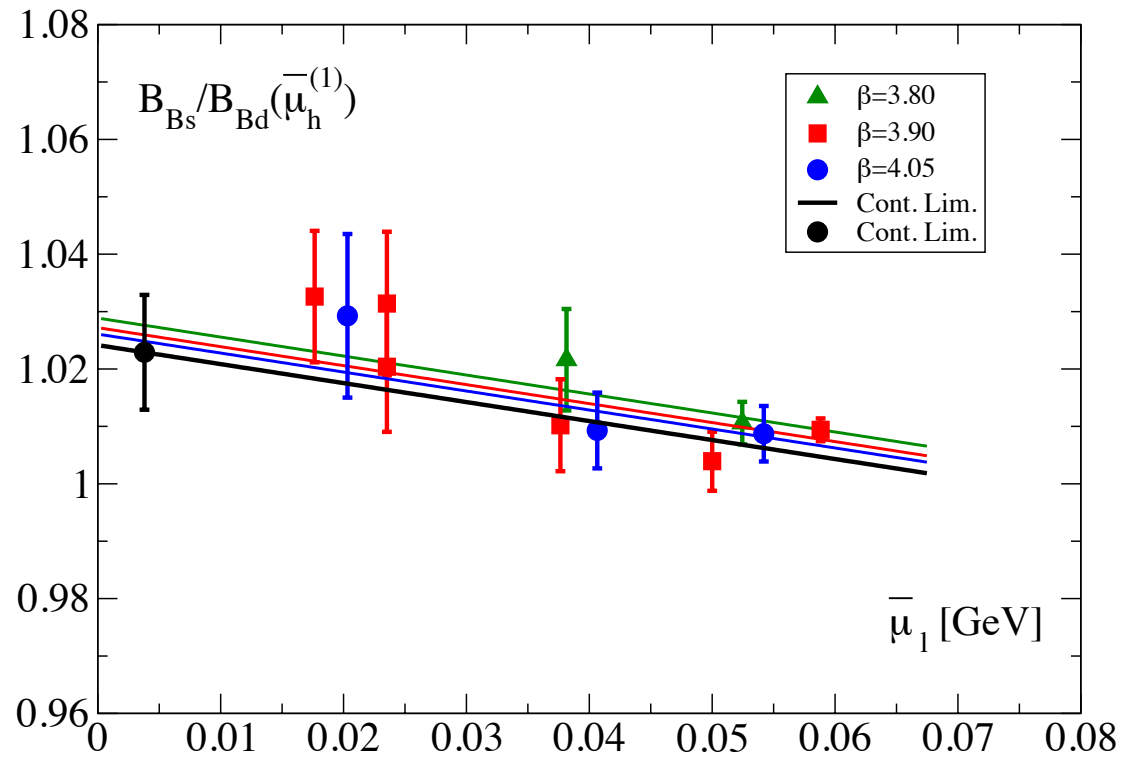
@ TL and LO there is no mixing

$$\omega(\bar{\mu}_h) = 1 + \frac{c_1}{\bar{\mu}_h}$$

$$\omega(\bar{\mu}_h, \lambda; \bar{\mu}_l, a) = \frac{B_{B_s}/B_{B_d}(\bar{\mu}_h, \bar{\mu}_l, a)}{B_{B_s}/B_{B_d}(\bar{\mu}_h/\lambda, \bar{\mu}_l, a)}$$

@TL and LO No renormalization or matching needed

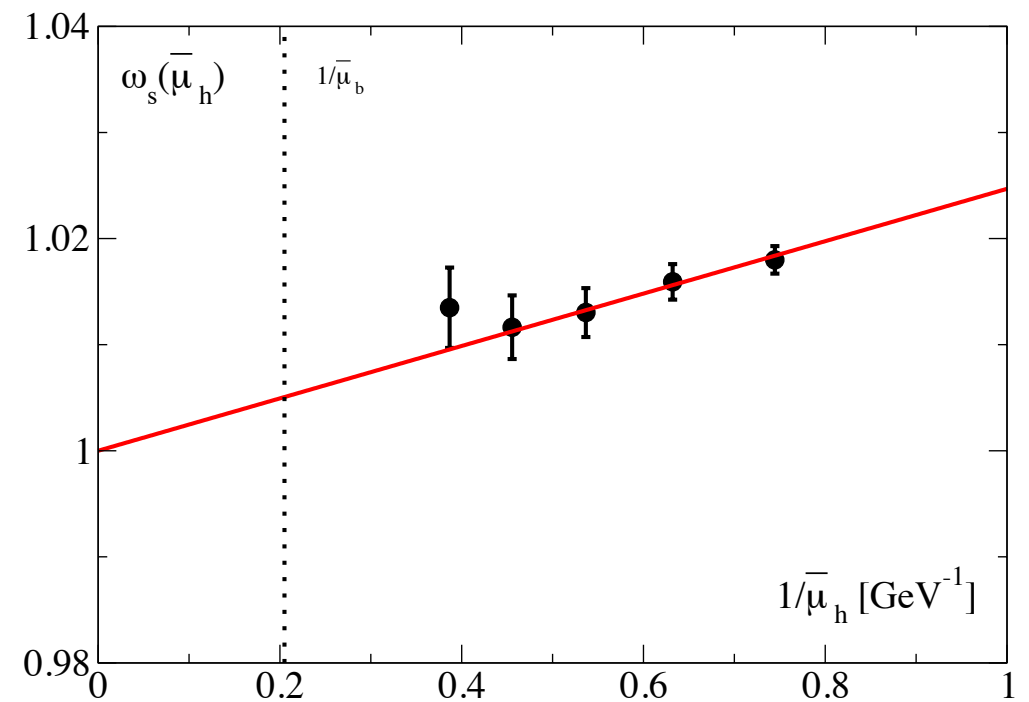
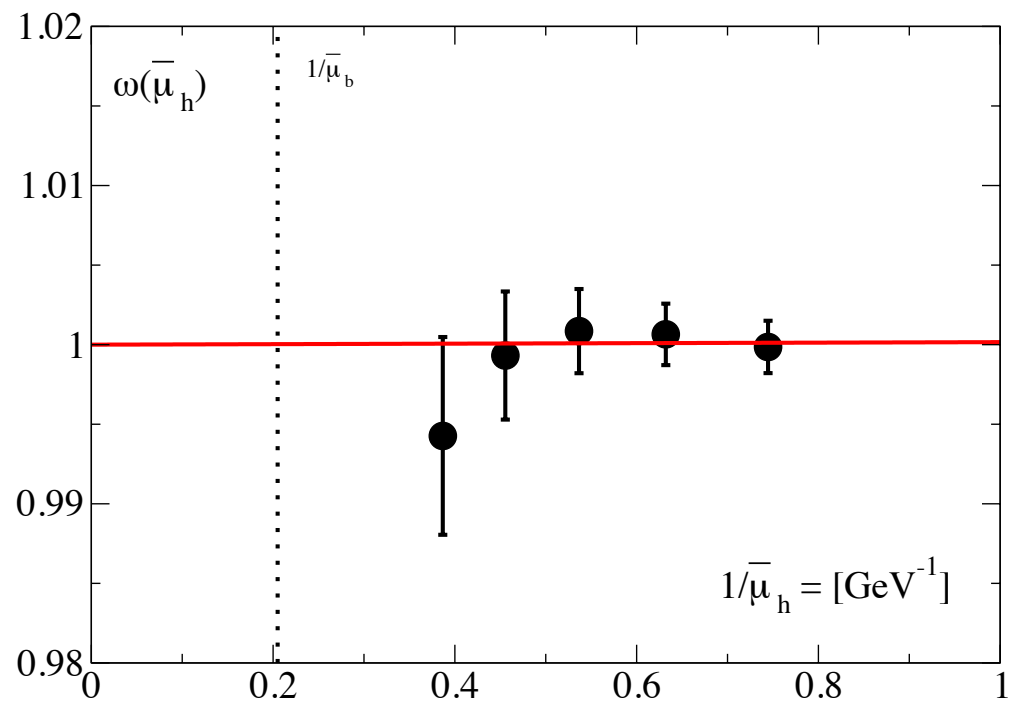
Chiral continuum extrapolation



$$\omega_s(\bar{\mu}_h^{(2)})\omega_s(\bar{\mu}_h^{(3)})\cdots\omega_s(\bar{\mu}_h^{(K+1)}) = \frac{B_{B_s}(\bar{\mu}_h^{(K+1)}; \bar{\mu}_b)}{B_{B_s}(\bar{\mu}_h^{(1)}; \mu)} \cdot \left[\frac{C(\bar{\mu}_h^{(1)}; \bar{\mu}_b, \mu)}{C(\bar{\mu}_h^{(K+1)}; \bar{\mu}_b, \mu)} \right]$$

Cutoff effects well under control

Ratios interpolation



$$B_{B_d}^{\overline{M}^S}(\bar{\mu}_b) = 0.86(4)(3)$$

$$B_{B_s}^{\overline{M}^S}(\bar{\mu}_b) = 0.89(4)(3)$$

$$\frac{B_{B_s}}{B_{B_d}} = 1.023(18)(2)$$

Very Preliminary

Conclusions

- Better interpolating operators for heavy-light 2-point functions
=> sub-percent statistical error
- Very small statistical errors --> strong check on the validity of the ratio method
- Precise determination of m_b , f_B , f_{B_s} for $N_f=2$
- Final error dominated by error in the scale setting and renormalization factor
- Ratio method extended to the B parameters
- First indications that ratio method is valid also in this case
- Preliminary estimate of B parameters