X(3872), η_{c2} or χ'_{c1}?

Yi-Bo Yang

IHEP, CAS, China

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Background
- Candidate of X(3872)
- Transition and Form Factors

Benchmark
- charmonium spectrum at quench anisotropic lattice
- $\chi_{c2} \to J/\psi$ transition

η_{c2} as candidate of X(3872)
- Form Factor as function of $\vec{q}$
- Numeral Result

What about χ'_{c1}?
- How to pickup χ'_{c1}?

Conclusion

Outline

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Candidate of X(3872)

1. Belle, CDFII, and D0 have found a charmonium-like narrow resonance

$$M = 3871.7 \pm 0.22\mathrm{MeV}, \Gamma < 2.3\mathrm{MeV}.$$

2. Radiative decay of X(3872)
   - The measurement from BaBar and Belle is consistent for $X(3872) \to J/\psi\gamma$,

$$\mathrm{Br}(X(3872) \to J/\psi\gamma) > 0.9\%(BaBar) \text{ or } 0.6\%(Belle) \quad (1)$$

   - However,

$$\frac{\mathrm{Br}(X(3872) \to \psi'\gamma)}{\mathrm{Br}(X(3872) \to J/\psi\gamma)} = 3.4 \pm 1.4(BaBar)$$

$$\frac{\mathrm{Br}(X(3872) \to \psi'\gamma)}{\mathrm{Br}(X(3872) \to J/\psi\gamma)} < 2.1(\text{No evidence})(Belle)$$

3. J^{PC} quantum number could be 1^{++} or 2^{-+}

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$$
\begin{array}{c|cc}
 & \chi'_{c1}(1^{++}) & \eta_{c2}(2^{-+}) \\
M_{QM} & 3.925\mathrm{GeV} & 3.77-3.83\mathrm{GeV} \\
M_{Lat} & \sim 4.1\mathrm{GeV}^{*} & \sim 3.8\mathrm{GeV} \\
\Gamma(X(3872)\to J/\psi\gamma) & 21(10)\mathrm{keV}^{*} & \mathrm{unknown} \\
\Gamma(X(3872)\to \psi'\gamma) & \mathrm{unknown} & \mathrm{unknown}
\end{array}
\qquad (2)
$$

*: J.J. Dudek, R.G. Edwards, N. Mathur and C. Thomas, Phys. Rev. D **79**, 094504 (2009) [arXiv:0902.2241 (hep-lat)].

1. We focus on $\Gamma(\eta_{c2} \to J/\psi\gamma)$ firstly
2. Then consider to check the case of χ'_{c1}.

General radiative transition width

The general radiative transition width of particle i to particle f is

$$\Gamma(i \to \gamma f) = \int d\Omega_q \frac{1}{32\pi^2} \frac{|\vec{q}|}{M_i^2} \frac{1}{2J_i + 1} \times \sum_{r_i, r_f, r_\gamma} \left| M_{r_i, r_f, r_\gamma} \right|^2, \quad (3)$$

In definition, the matrix element M_{r_i, r_j, r_γ} could be expressed in terms of linear combination of form factors,

$$\begin{aligned} M_{r_i, r_f, r_\gamma = \pm} &= \sum_k \sqrt{\frac{2k+1}{2J+1}} \left[E_k \tfrac{1}{2}(1 + (-1)^k \delta P) \mp M_k \tfrac{1}{2}(1 - (-1)^k \delta P) \right] \\ &\quad \langle k \mp; J' r_f | J r_i \rangle \delta_{r_f, r_i \pm 1} \\ M_{r_i, r_f, r_\gamma = 0} &= \sum_k \sqrt{\frac{2k+1}{2J+1}} C_k \tfrac{1}{2}(1 + (-1)^k \delta P) \langle k0; J' r_f | J r_i \rangle \delta_{r_f, r_i}. \end{aligned} \quad (4)$$

With the property of CG coefficient, the decay width as a function of Q^2 could be expressed as

$$\Gamma(i \to \gamma f) \propto (2 \sum_i E_i^2(Q^2) + 2 \sum_j M_j^2(Q^2) + \sum_k C_k^2(Q^2)). \quad (5)$$

When the photon is on-shell, only electro(E) or magnetic M form factors contribute to decay width and C type vanish.

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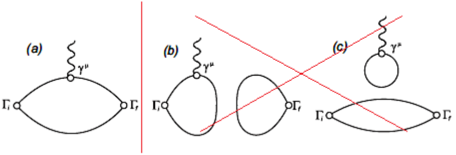
Lattice Simulation

For this purpose, it is the three point function that will be calculated on lattice



$$\Gamma^{(3),\mu}_{mn}(\vec{p}_f, \vec{q}, t, t') = \sum_{r_i, r_f} e^{-E_f t} e^{-(E_i - E_f)t'}$$
$$\times \frac{\hat{Z}^f_m(\vec{p}_f, r_f)\hat{Z}^{i*}_n(\vec{p}_i, r_i)}{2E_i\ 2E_f} \times \langle f(\vec{p}_f, r_f)|j^\mu_{\rm em}(0)|i(\vec{p}_i, r_i)\rangle \quad (t', t - t' \to \infty), \tag{6}$$

where $\hat{Z}^{i,f}_m$ are the matrix elements like $\hat{Z}^X_m(\vec{p}_X, r_X) = \langle 0|O^X_m|X(\vec{p}_X, r_X)\rangle$, which can be derived from the relevant two-point functions,

$$\begin{aligned}\Gamma^{(2)}_{X,mn}(\vec{p}_X, t) &= \sum_{\vec{x}} e^{i\vec{p}_X\cdot\vec{x}} \langle O^X_m(\vec{x}, t) O^{X\dagger}_n(\vec{0}, 0)\rangle \\ &\to \frac{1}{2E_X} e^{-E_X t} \sum_{r_X} \langle 0|O^X_m|X(\vec{p}_X, r_X)\rangle \times \langle X(\vec{p}_X, r_X)|O^{X\dagger}_n|0\rangle \quad (t \to \infty).\end{aligned} \tag{7}$$

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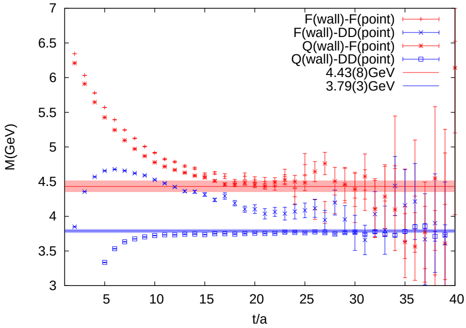
A good operator is half done

With kinds of operators of $J^{PC} = 2^{-+}$, we can calculate the correction function between them. From the diagram below, there is obvious that different sink operators would prefer to project to different states, even with similar source operator:



$$\begin{aligned} \bar{c}\gamma_5 \overleftrightarrow{D_i}\,\overleftrightarrow{D_j}\, c \ (\text{type DD}) &\quad\to\quad \text{lower state} \sim 3.8\text{GeV}, \\ \bar{c}\gamma_i \Gamma B_j c \ (\text{type F}) &\quad\to\quad \text{higher state} \sim 4.4\text{GeV} \end{aligned} \tag{8}$$

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charmonium spectrum at quench anisotropic lattice

$a_s = 5a_t$, N_{conf}=1000.

1. β=2.4
   $N_T * N_S^3 = 96 * 8^3$,
   a_s=0.222(2)fm,
2. β=2.8
   $N_T * N_S^3 = 144 * 12^3$,
   a_t=0.138(1)fm.

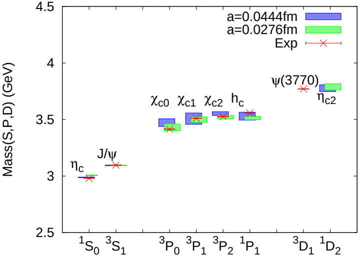


Table 1: The results show good agreement with the experimental data. The systematic uncertainties are seemingly small.

meson	J^{PC}	$M(2.4)$	$M(2.8)$	Expt.	QM
$\eta_c(^1S_0)$	0^{-+}	2.989(2)	3.007(3)	2.979	2.982
$J/\psi(^3S_1)^*$	1^{--}	3.097	3.097	3.097	3.090
$h_c(^1P_0)$	1^{+-}	3.530(35)	3.513(14)	3.526	3.516
$\chi_{c0}(^3P_0)$	0^{++}	3.472(34)	3.431(30)	3.415	3.424
$\chi_{c1}(^3P_1)$	1^{++}	3.508(50)	3.499(25)	3.511	3.505
$\chi_{c2}(^3P_2)$	2^{++}	3.552(17)	3.520(15)	3.556	3.556
$\psi''(^3D_1)$	1^{--}	-	-	3.770	3.785
$\eta_{c2}(^1D_2)$	2^{-+}	3.777(30)	3.789(28)	-	3.799

*:input to set $a\ m_c$.

$\chi_{c2} \to J/\psi$ transition

Momenta of two charmoninum involved here are:

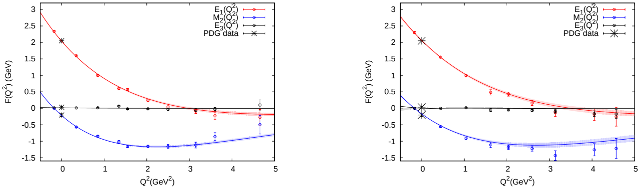
$$\chi_{c2} \quad : \quad \vec{p} = (0,0,0)$$
$$J/\psi \quad : \quad \vec{p}' = (0,0,0) \sim (2,2,2)$$
$$\vec{q} = \vec{p} - \vec{p'}. \tag{9}$$

$$\langle V(\vec{p}_V, \lambda_V)|j^{\mu}(0)|T(\vec{p}_T, \lambda_T)\rangle = \alpha_1^{\mu} E_1(Q^2)$$
$$+\alpha_2^{\mu} M_2(Q^2) + \alpha_3^{\mu} E_3(Q^2) + \alpha_4^{\mu} C_1(Q^2) + \alpha_5^{\mu} C_2(Q^2) \tag{10}$$

$$\Gamma(\chi_{c2} \to \gamma J/\psi) = \frac{16\alpha|\vec{q}|}{45 M_{\chi_{c2}}^2}(|E_1(0)|^2 + |M_2(0)|^2 + |E_3(0)|^2)$$

with fit function inspired by non-relativistic quark model

$$F_k(Q^2) = F_k(0)(1 + \lambda_k Q^2)e^{-\frac{Q^2}{16\beta_k^2}} \tag{11}$$



The left for β=2.4, the right for β=2.8.

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$\chi_{c2} \to J/\psi\gamma$

The numerical result is:

$$
\begin{array}{c|cccc}
 & M_1(\mathrm{GeV}) & E_2(\mathrm{GeV}) & M_3(\mathrm{GeV}) & \Gamma(\mathrm{keV}) \\
\beta = 2.4 & 2.038(18) & -0.218(4) & 0.014(3) & 347 \pm 20 \\
\beta = 2.8 & 2.080(21) & -0.171(10) & 0.005(8) & 352 \pm 11 \\
Cont. & 2.106(20) & -0.141(15) & -0.007(12) & 361 \pm 9
\end{array} \quad (12)
$$

The PDG average for transition width is 384(38)keV, and
$a_2 = \frac{M_2(0)}{\sqrt{E_1(0)^2 + M_2(0)^2 + E_3(0)^2}}$=-0.100(15),
$a_3 = \frac{M_2(0)}{\sqrt{E_1(0)^2 + M_2(0)^2 + E_3(0)^2}}$=0.016(13).
our prediction for transition width agrees with experimental data, while $a_{2,3}$ are slightly smaller.

Our result is in good agreement with PDG data. And then, let us turn to $\eta_{c2} \to J/\psi$ transition.

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$\eta_{c2} \to J/\psi$ transition

1. The momentum modes used here are similar to those for χ_{c2},

$$\vec{p} = (0,0,0),\ \vec{p}' = (0,0,0) \sim (2,2,2),\ \vec{q} = \vec{p} - \vec{p'}. \qquad (13)$$

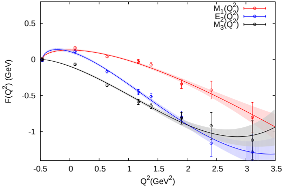
2. But the fit function is different.

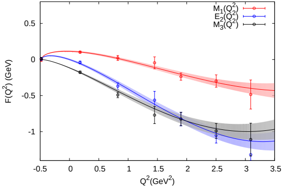
$$\Gamma(\chi_{c2} \to \gamma J/\psi) = \frac{16\alpha|\vec{q}|}{45 M^2_{\chi_{c2}}}(|M_1(0)|^2 + |E_2(0)|^2 + |M_3(0)|^2)$$

with

$$\begin{aligned} F_i(v) &= Av + Bv^3 + Cv^5 + O(v^7)(F_i \to M_1,\ E_2) \\ F_i(v) &= Bv^3 + Cv^5 + Dv^7 + O(v^9)(F_i \to M_3), \end{aligned} \qquad (14)$$

Form factor M_1, E_2 and M_3 are expanded as series of $v = \sqrt{\Omega/(m_V^2 m_T^2)} = |\vec{q}|/m_{J/\psi}$ in order to describe lattice data, instead of series of Q^2.





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Why we need v but not Q^2?

The most general Lorentz covariant decomposition with P and C parity invariance is:

$$\langle V(\vec{p}_V, \lambda_V)|j^\mu(Q^2)|\eta_2(\vec{p}_T, \lambda_T)\rangle = a(Q^2)A^\mu + b(Q^2)B^\mu + c(Q^2)C^\mu + d(Q^2)D^\mu + e(Q^2)E^\mu \tag{15}$$

with

$$\begin{aligned} A^\mu &= \epsilon^{\mu\nu\rho\sigma}\epsilon^*_{\nu,V} p^V_\rho \epsilon^\beta_{\sigma,T} p^V_\beta, \;\; B^\mu = \epsilon^{\beta\nu\rho\sigma} p^T_\beta \epsilon^*_{\nu,V} p^V_\rho \epsilon^\mu_{\sigma,T}, \\ C^\mu &= \epsilon^{\mu\nu\rho\sigma} p^T_\nu p^V_\rho \epsilon^\beta_{\sigma,T}\epsilon^*_{\beta,V}, \\ D^\mu &= \epsilon^{\mu\nu\rho\sigma} p^V_\nu p^T_\rho \epsilon^*_{\sigma,V}\epsilon^{\alpha\beta}_T p^V_\alpha p^V_\beta, \;\; E^\mu = \epsilon^{\mu\nu\rho\sigma} p^V_\nu p^T_\rho \epsilon^\beta_{\sigma,T} p^V_\beta \epsilon^{*\alpha}_V p^T_\alpha. \end{aligned} \tag{16}$$

We can make Taylor expansion for form factors as function of variable $v = \sqrt{\Omega/(m_T^2 m_V^2)} = \sqrt{\frac{(m_T^2-m_V^2)^2}{4m_T^2 m_V^2} + \frac{(m_T^2+m_V^2)}{2m_T^2 m_V^2}Q^2 + \frac{1}{4m_T^2 m_V^2}Q^4}$ (When $\vec{p_T} = 0$, $v = \vec{q}/m_V$),

$$\begin{aligned} M_1(\xi) &= i\, m_V \left[\sqrt{\frac{5}{12}}\left(a\, m_V + a\, m_T - 2c\, m_T\right) v + \frac{2a\, m_V - 3c\, m_T - 4d\, m_V^2 m_T + 6e\, m_V m_T^2}{4\sqrt{15}} v^3 \right. \\ &\left. + \frac{-2a\, m_V + 3c\, m_T}{16\sqrt{15}} v^5 + O\left(v^7\right)\right], \\ E_2(v) &= i\, m_V \left[-\sqrt{\frac{3}{4}}\left(a\, m_T - a\, m_V\right) v + \frac{2a\, m_V - c\, m_T - 4d\, m_V^2 m_T + 2e\, m_V m_T^2}{4\sqrt{3}} v^3 \right. \\ &\left. + \frac{-2a\, m_V + c\, m_T}{16\sqrt{3}} v^5 + O\left(v^7\right)\right], \\ M_3(\xi) &= i\, m_V \left[-\frac{-a\, m_V - c\, m_T + 2d\, m_V^2 m_T + 2e\, m_V m_T^2}{\sqrt{15}} v^3 - \frac{a\, m_V + c\, m_T}{4\sqrt{15}} v^5 + O\left(v^7\right)\right]. \end{aligned} \tag{17}$$

The numerical result is:

$$\begin{array}{c|cccc} & M_1(\mathrm{GeV}) & E_2(\mathrm{GeV}) & M_3(\mathrm{GeV}) & \Gamma(\mathrm{keV}) \\ \beta=2.4 & 0.133(13) & 0.111(17) & -0.093(9) & 4.4\pm0.9 \\ \beta=2.8 & 0.115(11) & -0.0007(14) & -0.117(9) & 3.1\pm0.6 \\ Cont. & 0.104(10) & -0.071(20) & -0.132(10) & 3.8\pm0.9 \end{array} \quad (18)$$

For the case of $Q^2 = 0$, the corresponding $|\vec{q}|$ for the $\psi(\psi')$ transition are

$$|\vec{q}| = 0.65GeV, \ v = 0.2 \qquad \text{for J}/\psi$$
$$|\vec{q}| = 0.11GeV, \ v = 0.03 \qquad \text{for } \psi'$$

So the partial width of η_{c2} to ψ' should be suppressed by a kinematic factor

$$(0.11/0.65)^3 \sim 1/200 \quad (19)$$

Suppose the kinematic kernel $a, b, c..$ of those two transition modes is of same order, the decay width of $\eta_{c2} \to \psi'$ should be much smaller than the one of $\eta_{c2} \to J/\psi$.

$$\frac{\mathrm{Br}(X(3872) \to \psi'\gamma)}{\mathrm{Br}(X(3872) \to J/\psi\gamma)} = 3.4 \pm 1.4(BaBar)$$
$$\frac{\mathrm{Br}(X(3872) \to \psi'\gamma)}{\mathrm{Br}(X(3872) \to J/\psi\gamma)} < 2.1(\text{No evidence})(Belle)$$

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There have also several phenomenological studies on this transition.

1. Light front quark model results (H.W. Ke and X.Q. Li, Phys. Rev. D **84**, 114026 (2011))

$$M_1 = 0.079(2)\mathrm{GeV}, E_2 = -0.086(2)\mathrm{GeV}, M_3 = -0.125(3)\mathrm{GeV}, \qquad (20)$$

   Which gives a width Γ=3.54(12)keV.

2. non-relativistic quark model (Y. Jia, W.L. Sang and J. Xu, arXiv:1007.4541)

$$M_1 \sim 0.026 - 0.045\mathrm{GeV}, E_2 \simeq M_3 \simeq -0.13\mathrm{GeV}, \qquad (21)$$

   and Γ ~4keV.

Both of them are in reasonable agreement with our results, 3.8(9)keV. Please turn to our e-print arXiv:1206.2086 for more details.

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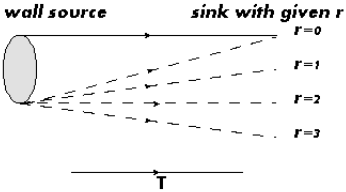
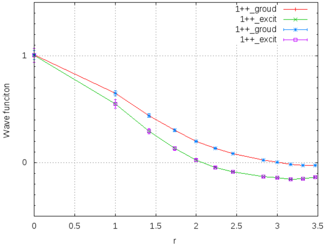
How to pickup χ'_{c1}?

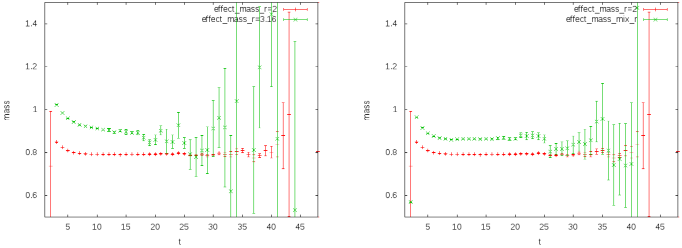
1. Former lattice result given a mass prediction of χ'_{c1} much heavier than X(3872)
2. Is there exist any method to pickup χ'_{c1} besides smearing or variations?

We use the operator $\bar{\psi}_c(0)\psi_c(r)$ on gauge fixed configuration and consider the matrix element of χ_{c1} and χ'_{c1}(by two-term fit with different r),





$< 0|\bar{c}(0)\gamma_i\gamma_5 c(r)|\chi_{c1}(\chi'_{c1}) >$ with different r. When we choose r carefully, we could found a proper r where the matrix element of the ground state is near zero.



The effective mass of χ_{c1} and χ'_{c1}. The left one are base on special chosen operators. The right one..is more or less like some kind of variation.

1. Larger lattice would be more easier to found proper r.
2. Only could be used to pickup the exited state of p wave at present, since the wave function of the ground state should have a zero at finite r.
3. The signal-to-noise ratio of non-zero r operator is much poorer than the point case(r=0). But at least, we could use the plateau got here to guild variation.

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1. We choose DD type operator for $\eta_{c2} \to J/\psi$ transition, and get the decay width as 4keV, very small and agree with phenomenological studies.
2. The decay width of $\eta_{c2} \to \psi'\gamma$ might be much smaller than the one of $\eta_{c2} \to J/\psi\gamma$.
3. We are trying to find a new way to confirm the decay width of $\chi_{c1} \to J/\psi\gamma$ transition.

Thanks!

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Why we need v but not Q^2?

The most general Lorentz covariant decomposition with P and C parity invariance is:

$$\langle V(\vec{p}_V,\lambda_V)|j^\mu(Q^2)|\eta_2(\vec{p}_T,\lambda_T)\rangle = a(Q^2)A^\mu + b(Q^2)B^\mu + c(Q^2)C^\mu + d(Q^2)D^\mu + e(Q^2)E^\mu \tag{22}$$

with

$$\begin{aligned} A^\mu &= \epsilon^{\mu\nu\rho\sigma}\epsilon^*_{\nu,V}p^V_\rho\epsilon^\beta_{\sigma,T}p^V_\beta, \quad B^\mu = \epsilon^{\beta\nu\rho\sigma}p^T_\beta\epsilon^*_{\nu,V}p^V_\rho\epsilon^\mu_{\sigma,T}, \\ C^\mu &= \epsilon^{\mu\nu\rho\sigma}p^T_\nu p^V_\rho\epsilon^\beta_{\sigma,T}\epsilon^*_{\beta,V}, \\ D^\mu &= \epsilon^{\mu\nu\rho\sigma}p^V_\nu p^T_\rho\epsilon^*_{\sigma,V}\epsilon^{\alpha\beta}_T p^V_\alpha p^V_\beta, \quad E^\mu = \epsilon^{\mu\nu\rho\sigma}p^V_\nu p^T_\rho\epsilon^\beta_{\sigma,T}p^V_\beta\epsilon^{*\alpha}_V p^T_\alpha. \end{aligned} \tag{23}$$

With some mathematica works, we could also write the current matrix element as multipole expansion.

$$\begin{aligned} &\langle V(\vec{p}_V,\lambda_V)|j^\mu(Q^2)|\eta_2(\vec{p}_T,\lambda_T)\rangle = \\ &\frac{i\,M_1(Q^2)}{5\Omega^{1/2}}\Big[-\sqrt{15}C^\mu + \frac{1}{\Omega}\sqrt{15}E^\mu(-m_V m_T + p_T.p_V)\Big] \\ &+\frac{i\,E_2(Q^2)}{3\Omega^{1/2}}\Big[\sqrt{3}C^\mu + \frac{1}{\Omega}\Big(2\sqrt{3}D^\mu m_T^2 - \sqrt{3}E^\mu(m_V m_T + p_T.p_V)\Big)\Big] \\ &+\frac{i\,M_3(Q^2)}{30\Omega^{1/2}}\Big[-2\sqrt{15}C^\mu + \frac{1}{\Omega}\Big(5\sqrt{15}D^\mu m_T^2 + 2\sqrt{15}E^\mu(4m_V m_T + p_T.p_V)\Big)\Big] \\ &-\frac{i\,C_2(Q^2)}{\sqrt{q^2}\Omega^{1/2}}\Big[A^\mu m_T + B^\mu m_T + C^\mu m_T + \\ &\qquad \frac{1}{\Omega}\Big(D^\mu m_T(m_T^2 - p_T.p_V) + E^\mu m_T(m_V^2 - p_T.p_V)\Big)\Big] \end{aligned} \tag{24}$$

with $\Omega \equiv (p_T.p_V)^2 - m_T^2 m_V^2$.

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What about χ'_{c1}?
How to pickup χ'_{c1}?
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Transition form factors are always expressed as Lorentz scalar functions, $Q^2 = -(p_i - p_f)^2$. However, if one looks into the Lorentz decomposition, the quantity $\Omega \equiv (p_T.p_V)^2 - m_T^2 m_V^2$ would also be an interesting Lorentz invariant kinematic variable. We could express the $\eta_{c2} \to J/\psi$ transition form factor of multipole expansion (M_1,E_2,M_3) in terms of linear combination of the ones in Lorentz covariant decomposition($a(Q^2)$,$b(Q^2)$,$c(Q^2)$..), and series the coefficient as polynomial of variable $v = \sqrt{\Omega/(m_T^2 m_V^2)}$(When $\vec{p_T} = 0$, $v = \vec{q}/m_V$),

$$
\begin{aligned}
M_1(\xi) &= i\, m_V \left[\sqrt{\frac{5}{12}} \left(a\, m_V + a\, m_T - 2c\, m_T\right) v + \frac{2a\, m_V - 3c\, m_T - 4d\, m_V^2 m_T + 6e\, m_V m_T^2}{4\sqrt{15}} v^3 \right. \\
&\left. + \frac{-2a\, m_V + 3c\, m_T}{16\sqrt{15}} v^5 + O\left(v^7\right) \right], \\
E_2(v) &= i\, m_V \left[-\sqrt{\frac{3}{4}} \left(a\, m_T - a\, m_V\right) v + \frac{2a\, m_V - c\, m_T - 4d\, m_V^2 m_T + 2e\, m_V m_T^2}{4\sqrt{3}} v^3 \right. \\
&\left. + \frac{-2a\, m_V + c\, m_T}{16\sqrt{3}} v^5 + O\left(v^7\right) \right], \\
M_3(\xi) &= i\, m_V \left[-\frac{-a\, m_V - c\, m_T + 2d\, m_V^2 m_T + 2e\, m_V m_T^2}{\sqrt{15}} v^3 - \frac{a\, m_V + c\, m_T}{4\sqrt{15}} v^5 + O\left(v^7\right) \right].
\end{aligned} \tag{25}
$$