Lattice calculation of the K_L - K_S mass difference

Jianglei Yu

RBC and UKQCD Collaborations

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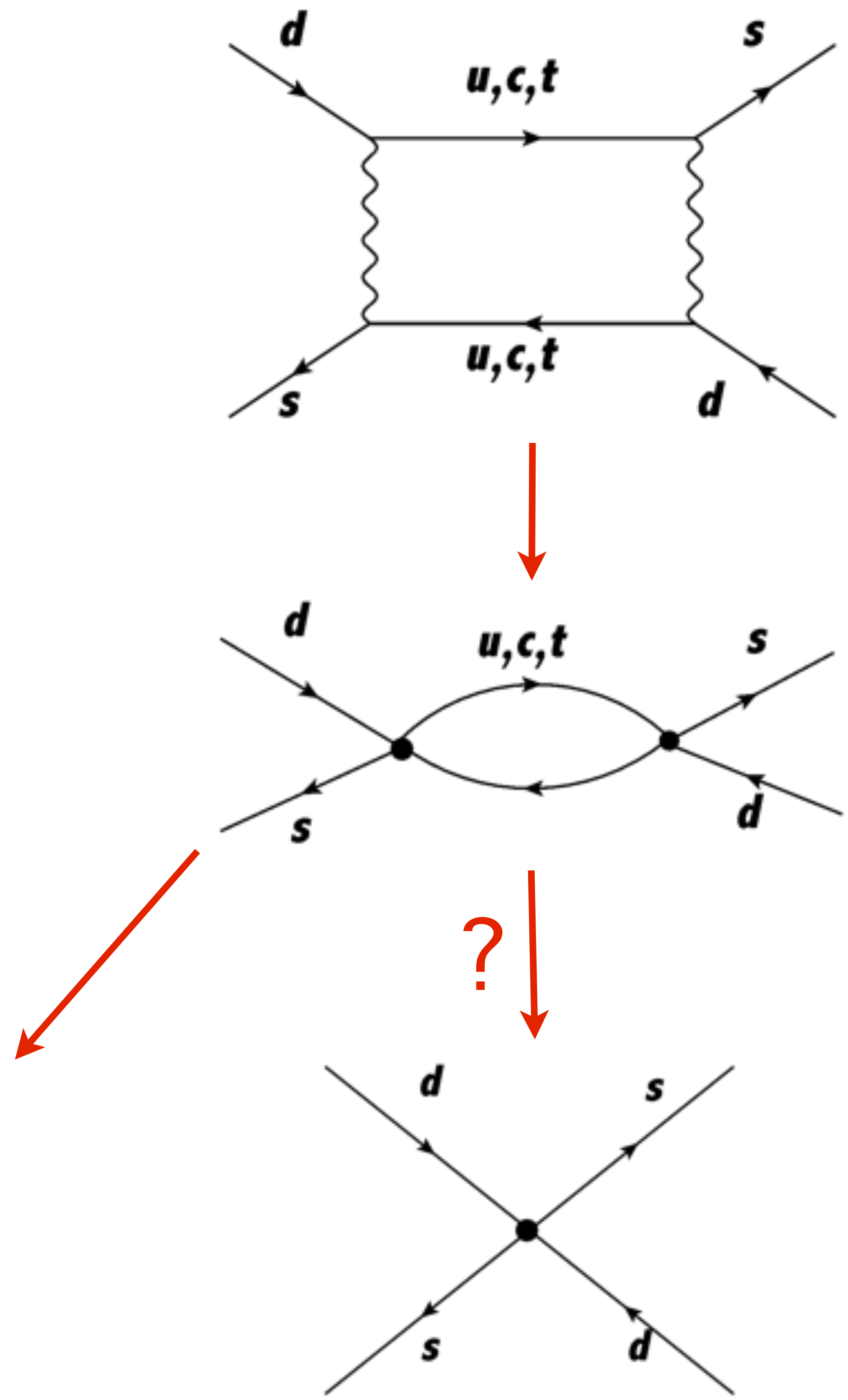
Introduction

- $K^0 - \bar{K}^0$ mixing :

  $\Delta M_K = 3.4583(6) \times 10^{-12}$ Mev

- Perturbative calculation can explain 70% of the mass difference
- Long distance effect

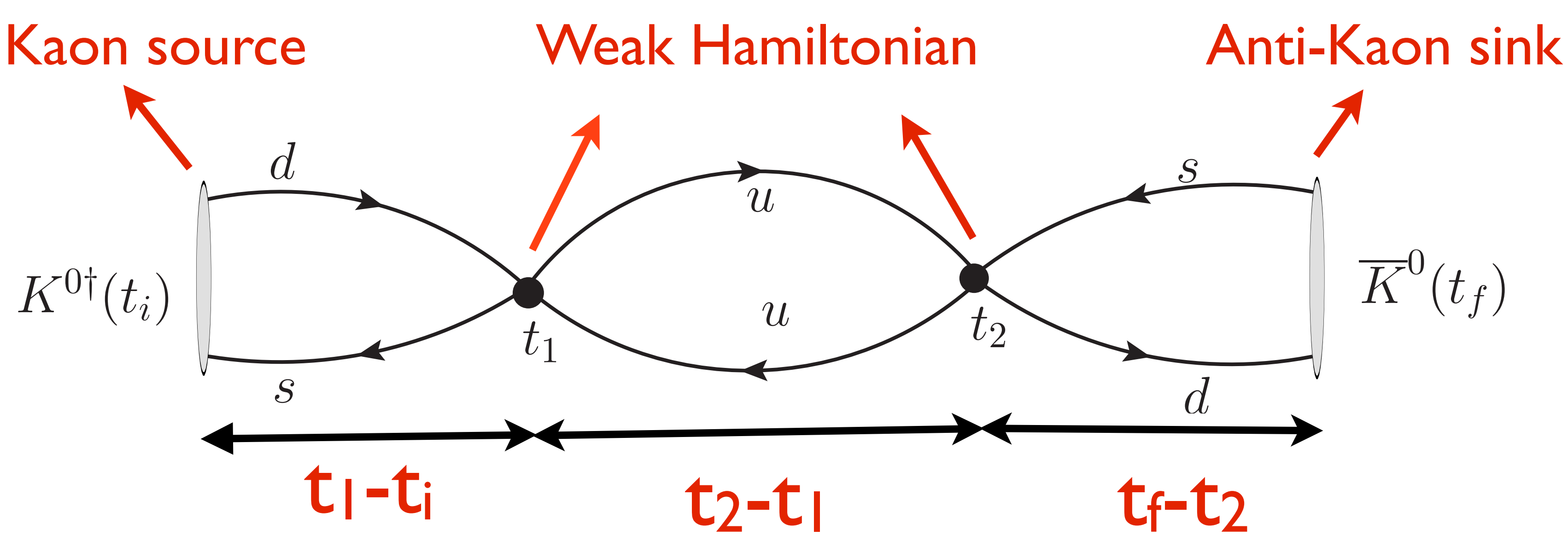


Directly evaluate second order weak process on a Lattice

Lattice four point function

Four point correlator :

$$G(t_f, t_1, t_2, t_i) = \langle \overline{K^0}(t_f) H_W(t_2) H_W(t_1) K^{0\dagger}(t_i) \rangle$$

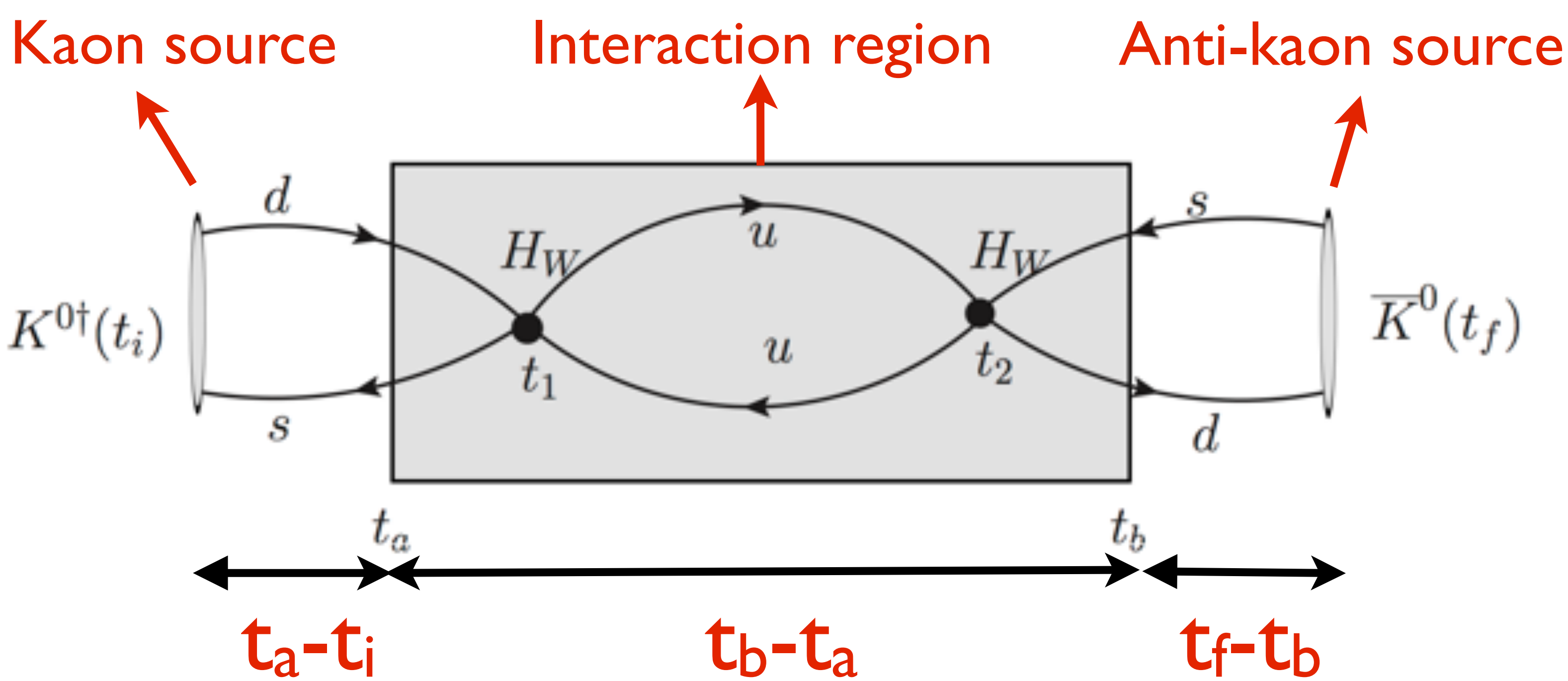


- t_1-t_i and t_f-t_2 should be sufficiently large to get a kaon
- Fix t_i and t_f , correlator depends only on t_2-t_1
- Refer to this quantity as unintegrated correlator

Integrated Correlator

Integrate the unintegrated correlator over a time interval :

$$\mathscr{A} = \frac{1}{2} \sum_{t_1=t_a}^{t_b} \sum_{t_2=t_a}^{t_b} \langle \overline{K^0}(t_f) H_W(t_2) H_W(t_1) K^{0\dagger}(t_i) \rangle$$



- t_f-t_b and t_a-t_i should be sufficiently large to get a kaon
- Fix t_i and t_f, correlator depends only on t_b-t_a
- Refer to this quantity as integrated correlator

After inserting a sum over intermediate states one obtains correlator as a function of integration time interval T :

$$
\begin{aligned}
\mathscr{A} = N_K^2 e^{-M_K(t_f-t_i)} \Bigg\{ & \sum_{n\neq n_0} \underline{\frac{\langle \overline{K}^0|H_W|n\rangle\langle n|H_W|K^0\rangle}{M_K-E_n}} \left(\underbrace{-T}_{1} - \underbrace{\frac{1}{M_K-E_n}}_{2} + \underbrace{\frac{e^{(M_K-E_n)T}}{M_K-E_n}}_{3,4} \right) \\
& + \underbrace{\frac{1}{2}\langle \overline{K}^0|H_W|n_0\rangle\langle n_0|H_W|K^0\rangle T^2}_{5} \Bigg\}
\end{aligned}
$$

1.Linear term, the coefficient gives finite volume approximation to ΔM_K

2. Constant term, which is trival

3. Exponential decreasing term, come from states $E_n > M_K$

4. Exponential increasing term, come from states $E_n < M_K$

5. Quadratic term, come from state $E_n = M_K$

We need to do subtraction for 4 and 5 terms

$$\mathscr{A} = N_K^2 e^{-M_K(t_f - t_i)} \left\{ \sum_{n \neq n_0} \frac{\langle \overline{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{M_K - E_n} \left(-T - \frac{1}{M_K - E_n} + \frac{e^{(M_K - E_n)T}}{M_K - E_n} \right) \right.$$
$$\left. + \frac{1}{2} \langle \overline{K}^0 | H_W | n_0 \rangle \langle n_0 | H_W | K^0 \rangle T^2 \right\}$$

1. Evaluate integrated correlator
2. Subtract the exponentially increasing π^0 term
3. Make $\pi\pi$ state degenerate with Kaon, remove T^2 term
4. Calculate mass difference
5. Add back finite-volume correction term

We need step 3 and 5 to correct finite volume effect

Mass difference from all possible intermediate states are encoded in the coefficient of linear term. In 3 flavor, there will be quadratic divergence short distance contribution to ΔM_K. Must introduce charm quark, GIM will remove this divergence.

Effective weak Hamiltonian

The ΔS=1 effective weak Hamiltonian in a 4 flavor theory :

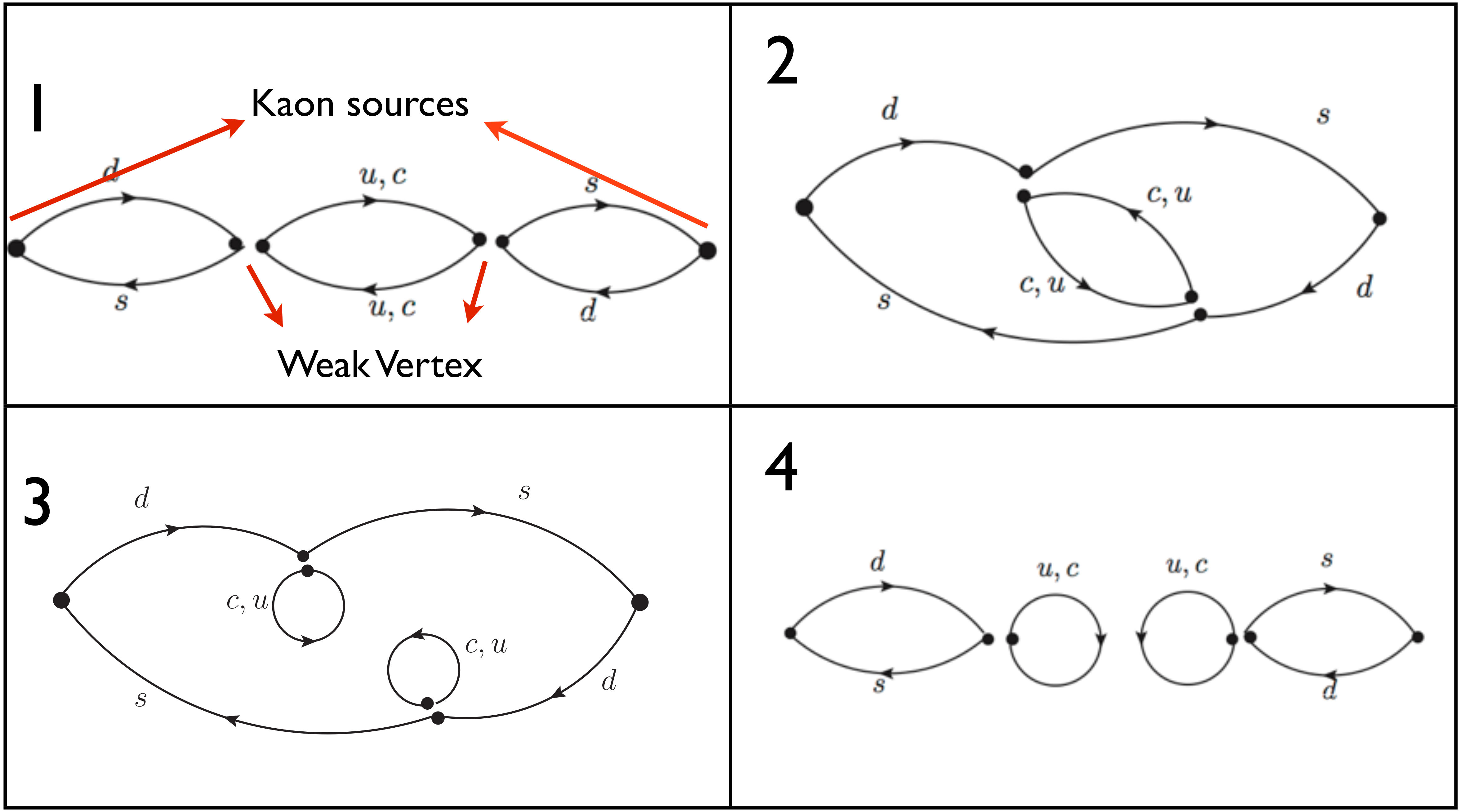
$$H_W = \frac{G_F}{\sqrt{2}} \sum_{q,q'=u,c} V_{qd} V^*_{q's} (C_1 Q_1^{qq'} + C_2 Q_2^{qq'})$$

Here we only include current-current operators :

$$Q_1^{qq\prime} = (\bar{s}_i d_i)_{V-A} (\bar{q}_j q'_j)_{V-A}$$
$$Q_2^{qq\prime} = (\bar{s}_i d_j)_{V-A} (\bar{q}_j q'_i)_{V-A}$$

All the penguin operators are neglected, since they are highly suppressed because of GIM cancellation

Four types of diagrams

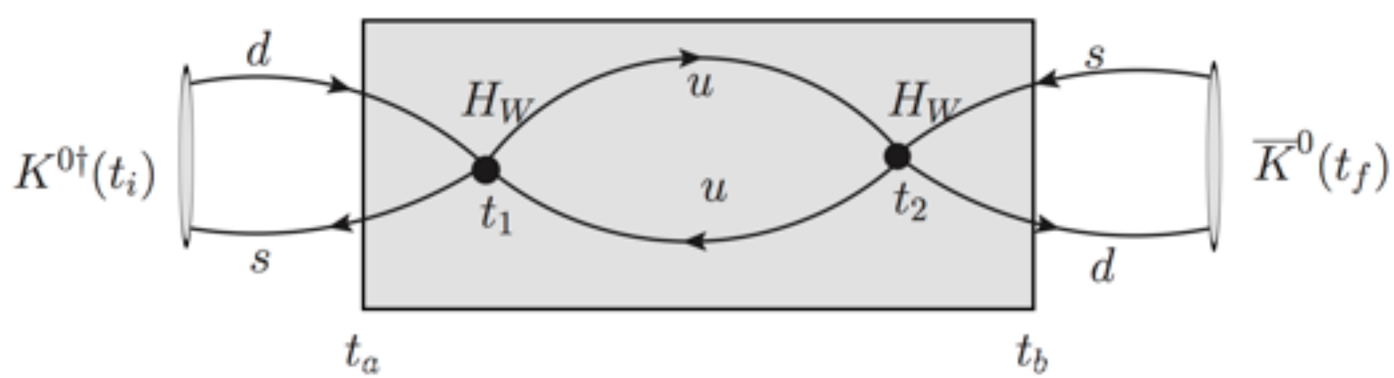


Type 3 and 4 diagrams not included in current calculation

Setup of the calculation

Lattice ensemble :

- $16^3 \times 32 \times 16$, 2+1 flavor DWF
- Inverse lattice spacing 1.73(3) Gev
- M_π = 421 Mev, M_K = 559Mev
- 800 configurations, each separated by 10 time units

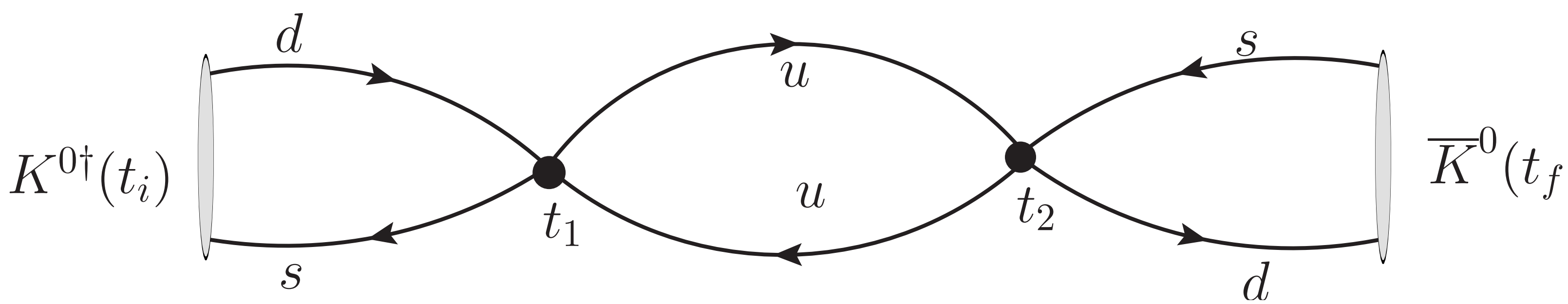


- Kaon wall sources at t_i = 0 at t_f = 27
- Weak Hamiltonian act between t_a=4 and t_b=23

Unintegrated correlator

$$G(T;t_i,t_f) = N_K^2 e^{-M_K(t_f-t_i)} \sum_n \langle \overline{K^0}|H_W|n\rangle\langle n|H_W|K^0\rangle e^{-(E_n-M_K)T}$$

Here T is the time separation between to weak Hamiltonian

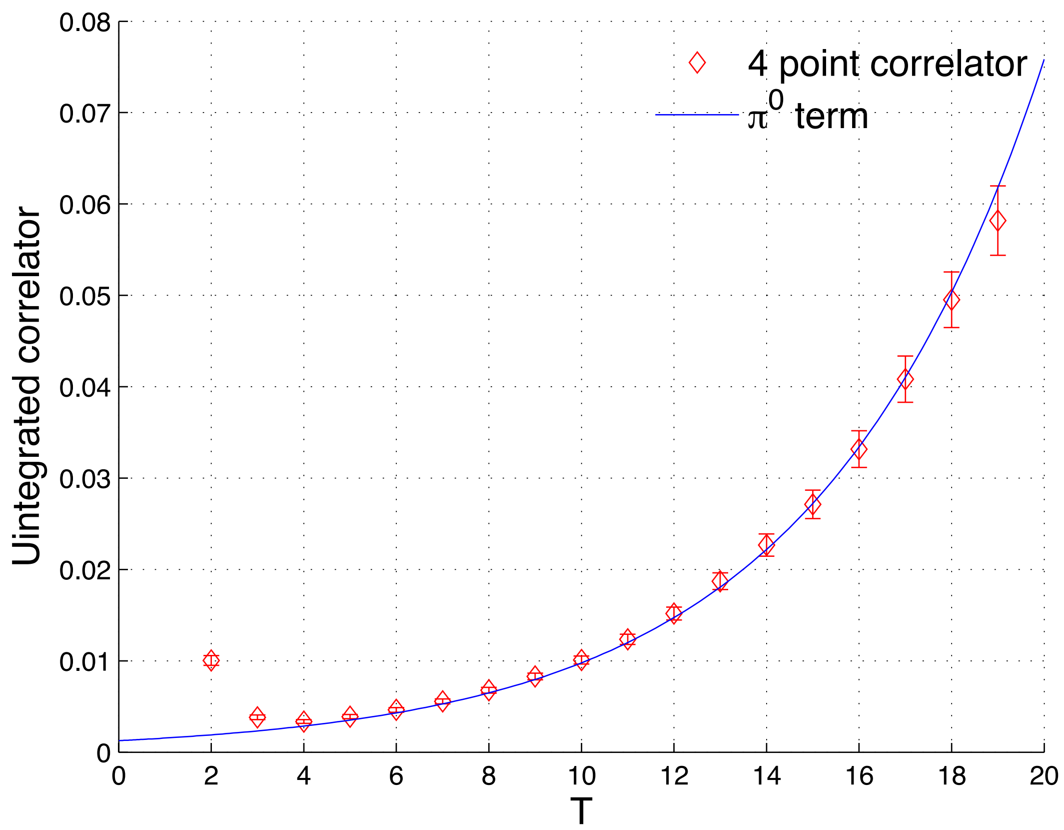


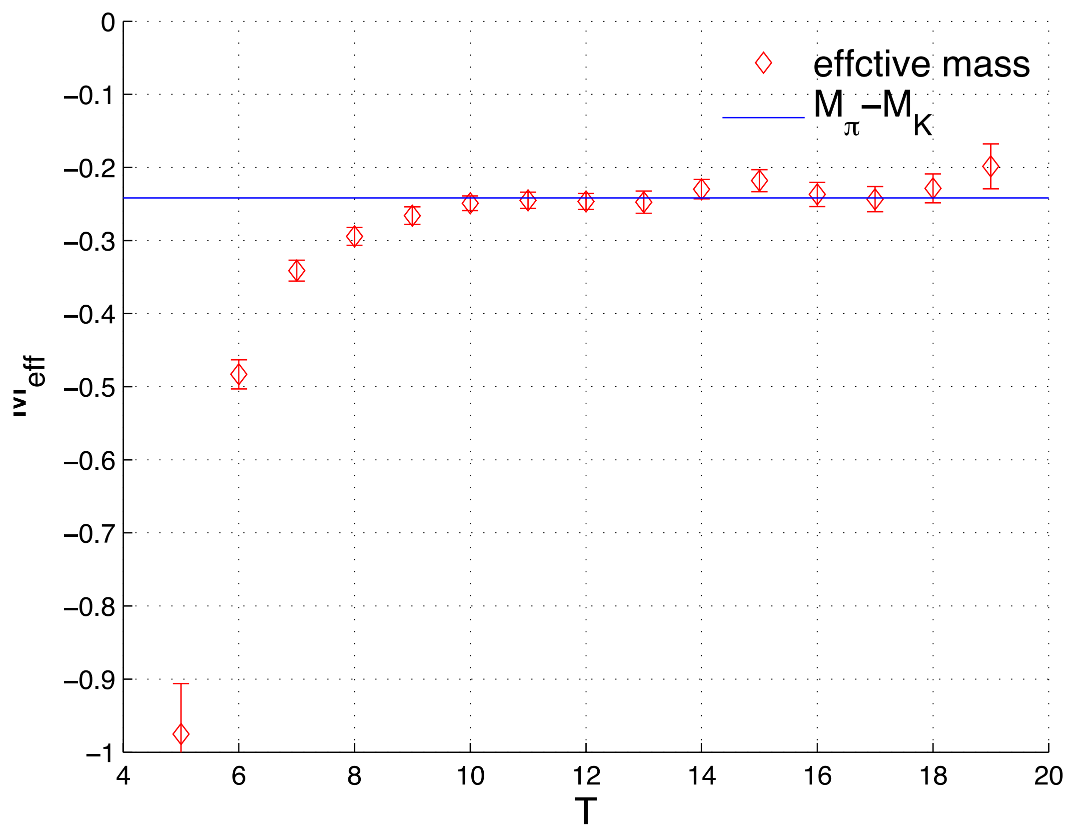
- Separate the Hamiltonian into two parity channel :
  - Parity conserving channel, long distance effect dominate by π^0 intermediate state
  - Parity violating channel, long distance effect dominate by $\pi\pi$ intermediate state
- Use various kaon masses

Parity conserving channel

In long distance, correlator dominate by π^0 term :

$$G(T;t_i,t_f) = N_K^2 e^{-M_K(t_f-t_i)} \langle \overline{K^0}|H_W|\pi^0\rangle\langle\pi^0|H_W|K^0\rangle e^{-(E_\pi - M_K)T}$$

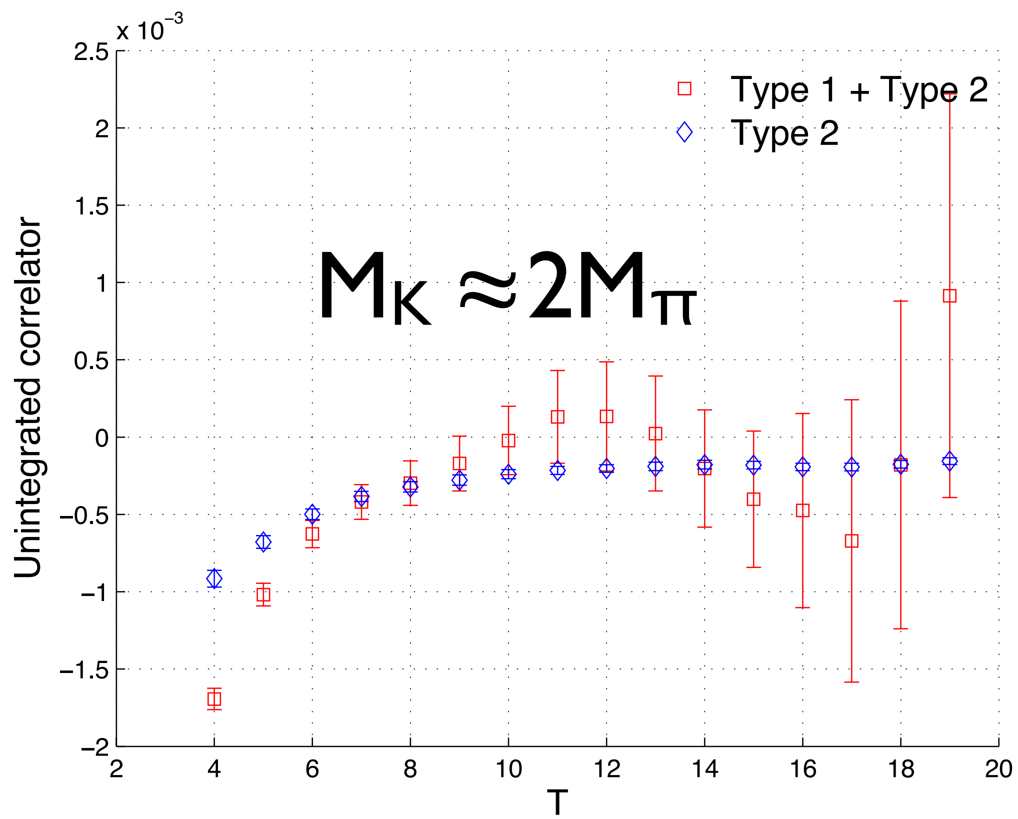




Parity violating channel

In long distance, correlator dominate by ππ term :

$$G(T; t_i, t_f) = N_K^2 e^{-M_K(t_f - t_i)} \langle \overline{K^0} | H_W | \pi\pi \rangle \langle \pi\pi | H_W | K^0 \rangle e^{-(E_{\pi\pi} - M_K)T}$$

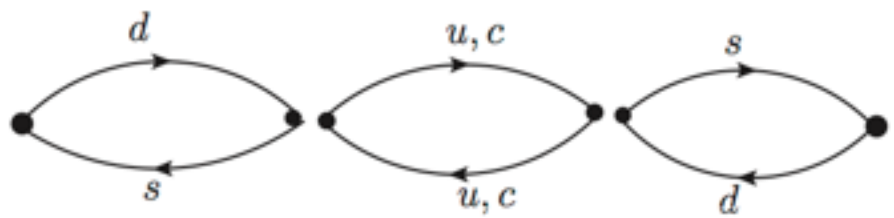


We expect to see plateau at long distance :

- No signal at long distance
- Good signal from type 2 diagrams only

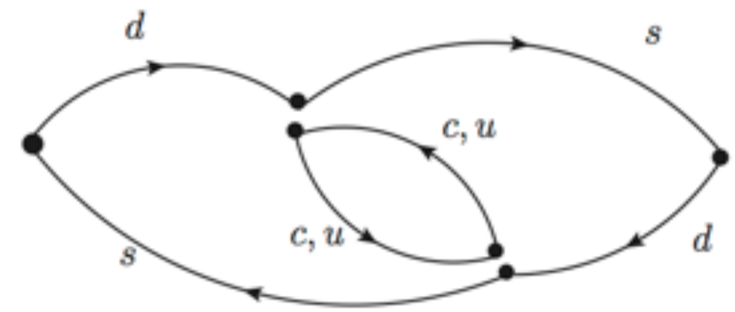
Parity violating channel

Type 1



Noise behave like π ,exponentially increasing noise to signal ratio

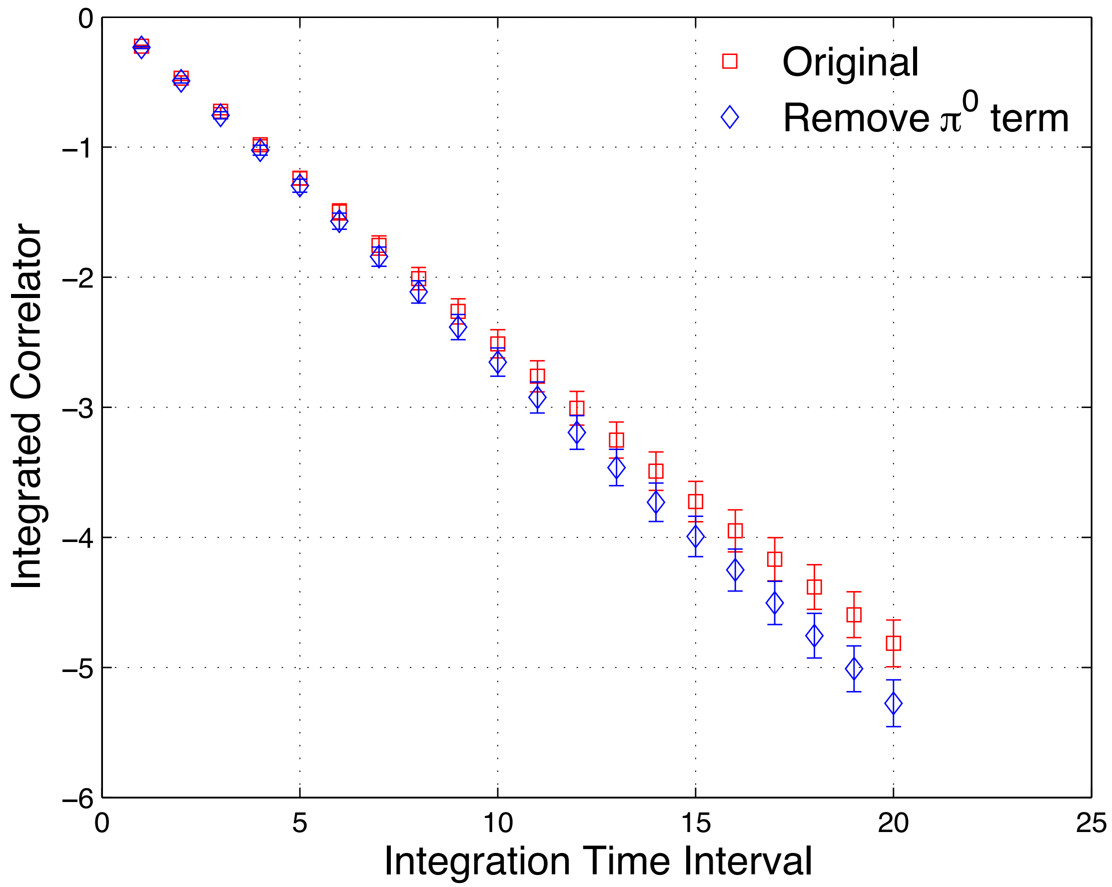
Type 2



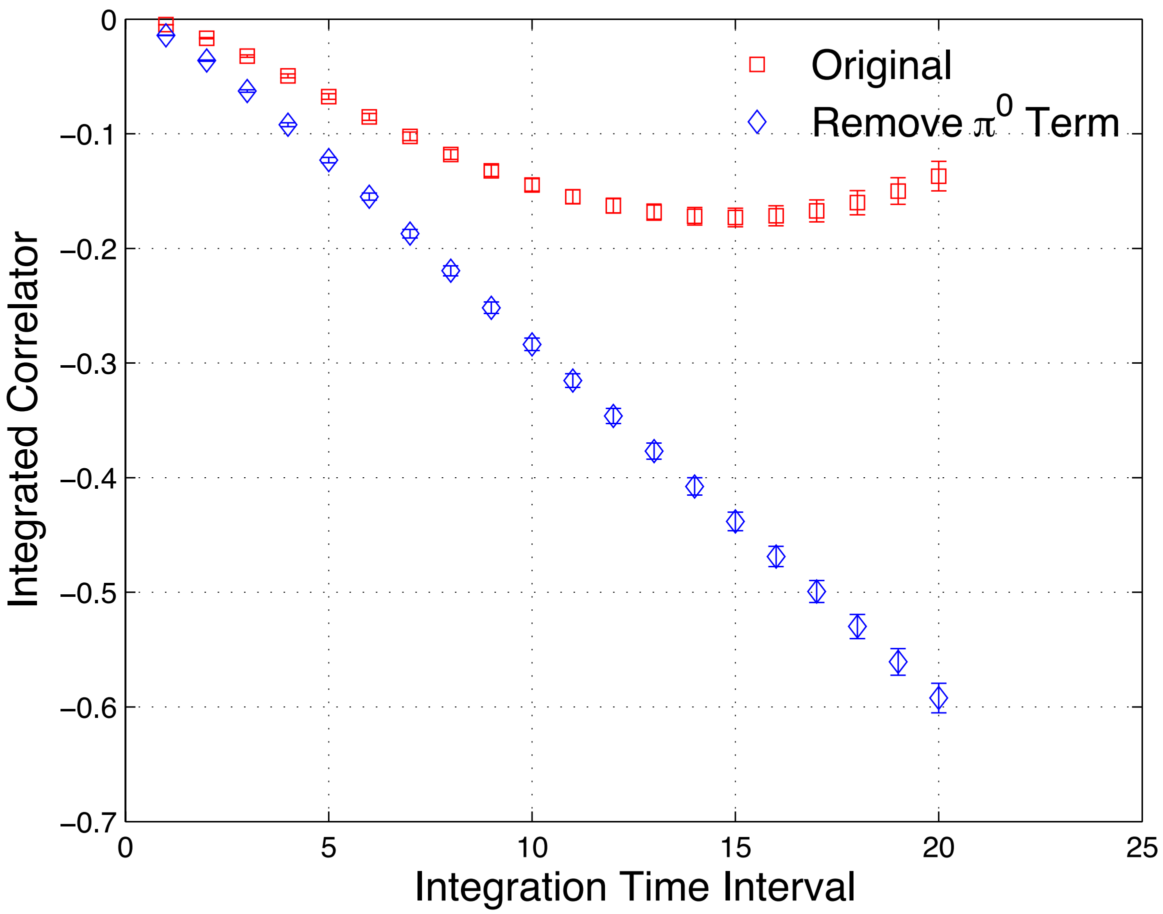
The signal from type 2 contractions don't have such noise

Integrated Correlator

No charm quark

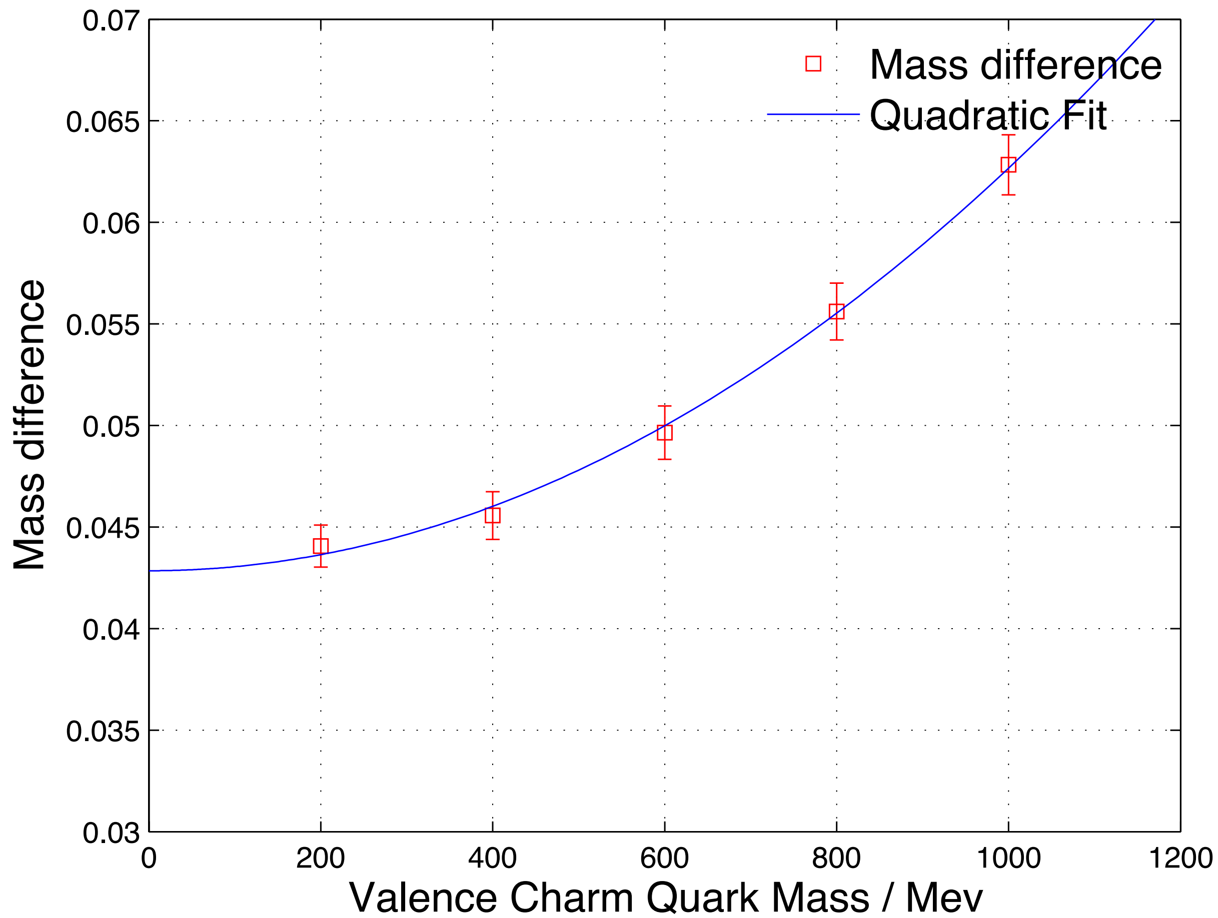


1 Gev charm quark



Quadratic Fit :

$$\Delta M_K(m_c) = a\ m_c^2 + b$$



Is 1 Gev charm too heavy ?
Quadratic dependence on m_c will be cutoff buy lattice spacing if charm is too heavy.
The fitting result suggest we haven't reach that region

Mass difference

M_π = 421 Mev
M_c = 1 Gev

- Only included statistical error
- Finite volume effect not corrected

$$\Delta M_K^{exp} = 3.483(6) \times 10^{-12} \text{ Mev}$$

M_K (Mev)	ΔM_K^{11}	ΔM_K^{12}	ΔM_K^{22}	ΔM_K ($\times 10^{-12}$ Mev)
563	6.38(14)	-2.64(14)	1.47(8)	5.52(24)
707	8.90(21)	-2.96(23)	2.10(12)	7.38(37)
775	10.63(27)	-3.18(30)	2.48(15)	8.61(49)
839	12.56(34)	-3.62(40)	2.89(20)	9.93(65)

$\approx 2M_\pi$

Conclusions and future plans

- Lattice calculation of ΔM_K is possible :
  - ✓ Evaluate four point correlators
  - ✓ Use GIM to remove divergence in short distance
  - ✓ Remove π exponentially term
  - Use on-shell $K \to \pi\pi$ kinematics, remove quadratic term from integrated correlator
  - Add finite volume correction term
- Include type 3 and type 4 diagrams in future
- Use Low mode averaging or A2A to collect statistics more efficiently