Contribution of the charm quark to the $\Delta I = 1/2$ rule

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$\Delta I = 1/2$ rule (I)

$T(K \to (\pi\pi)_I) = A_I e^{i\delta_I}$, $\quad I = 0, 2$, $\quad |A_0|/|A_2| \simeq 22.1$ $\quad$ "$\Delta I = 1/2$ rule"

T: Transition amplitude, A: Amplitude, δ: Scattering phase shift
Isospin I of K,π and $\pi\pi$: $1/2$, 1 and $0, 2$

- Weak interaction does not distinguish different isospin final states
  $\Longrightarrow$ Origin: Strong interaction effects
- Short-distance QCD effects and large N$_c$ arguments $\longrightarrow$ only small enhancement
  $\Longrightarrow$ Study long-distance contributions non-perturbatively via LQCD
- Bypass direct computation of $K \to \pi\pi$ [Bernard et al. PRD 32 (1985)]
  $\Longrightarrow$ Compute $K \to \pi$ and $K \to vac$ matrix elements in LQCD
  $\Longrightarrow$ relate to physical transition amplitude via Chiral Perturbation Theory

$\Delta I = 1/2$ rule (II)

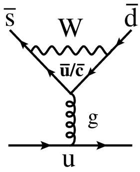
$T(K \to (\pi\pi)_I) = A_I e^{i\delta_I}, \qquad I = 0, 2, \qquad |A_0|/|A_2| \simeq 22.1$ "$\Delta I = 1/2$ rule"

Several possible origins/contributions of long-distance QCD

- final state interactions
- physics at $E_{QCD} \approx 250$ MeV
- physics at $\mu = m_{charm} \sim 1$ GeV (via penguins)

  Role of charm quark unclear. Classic arguments suggest: Large up-charm quark mass difference may be important
  [Shifman et al. NPB 120 (1977)]

- all of the above (no dominating mechanism)



Separate intrinsic QCD effects from physics at m_c
[Giusti, Hernández, Laine, Weisz, Wittig, JHEP11 (2004)]

Strategy (I) [Giusti, Hernández, Laine, Weisz, Wittig, JHEP11 (2004)]

Approach:

- CP-conserving $\Delta S = 1$ effective weak Hamiltonian $\mathcal{H}_{eff}^{\Delta S=1}$ with active charm quark
- Express ratio of kaon decay amplitudes A_I via LECs $[\hat{g}_1^{\pm}]$ in ChPT

$$\frac{|A_0|}{|A_2|} = \frac{1}{\sqrt{2}}\left(\frac{1}{2} + \frac{3}{2}\frac{\hat{g}_1^-}{\hat{g}_1^+}\right) \qquad \text{(at LO)}$$

- Determine $\hat{g}_1^- / \hat{g}_1^+$ in lattice QCD

Step 1:	$m_c = m_u = m_d = m_s$	$SU(4)$-LQCD matched to $SU(4)$-ChPT
Step 2:	$m_c \gg m_u = m_d = m_s$	$SU(4)$-LQCD matched to $SU(3)$-ChPT

$\Longrightarrow$ monitor A_0, A_2 as a function of m_c

Strategy (II) [Giusti, Hernández, Laine, Weisz, Wittig, JHEP11 (2004)]

Ingredient:

Overlap fermions: $D_N = \frac{1+s}{a}\left\{1 - \frac{A}{(A^\dagger A)^{1/2}}\right\}, \qquad A = 1 - aD_W,$

- Renormalization & mixing patterns like in the continuum, provided

$$\psi \rightarrow \tilde{\psi} = \left(1 - \frac{1+s}{2a}D\right)\psi, \qquad \bar{\psi} \rightarrow \bar{\psi}$$

  No mixing with lower dimensional operators [Capitani, Giusti, PRD (2001) 014506]

- Allows simulating quark masses near chiral limit $\longrightarrow$ ChPT most reliable
- Numerical treatment expensive $\longrightarrow$ first results quenched
  Quantitative attempt to understand large $\Delta I = 1/2$ enhancement!

$\Delta S = 1$ weak Hamiltonian with an active charm quark

OPE: separates long-distance and short-distance effects

$$\mathcal{H}_w^{eff} = \frac{g_w^2}{2M_W^2} V_{ud} V_{us}^* \sum_{\sigma=\pm} \{ k_1^\sigma \mathcal{Q}_1^\sigma + k_2^\sigma \mathcal{Q}_2^\sigma \}$$

Short distance QCD $\longrightarrow$ moderate enhancement $\quad \frac{k_1^-}{k_1^+} \approx 2.8$ (2 loop PT)

Enhancement dominated by matrix elements of operators

$$\mathcal{Q}_1^\pm = \{(\bar{s}\gamma_\mu P_- u)(\bar{u}\gamma_\mu P_- d) \pm (\bar{s}\gamma_\mu P_- d)(\bar{u}\gamma_\mu P_- u)\} - (u \to c)$$
$$\mathcal{Q}_2^\pm = (m_u^2 - m_c^2)(m_d \bar{s} P_+ d + m_s \bar{s} P_+ d)$$

- $SU(4)_L \times SU(4)_R$ chiral group: 4-quark operator $\mathcal{Q}_1^+$: $(84, 1)$, $\mathcal{Q}_1^-$: $(20, 1)$
  $\longrightarrow$ no mixing under renormalization for GW fermions
- $\mathcal{Q}_2^\pm$ don't contribute to phys. $K \longrightarrow \pi\pi$ decay, mix with $\mathcal{Q}_1^\pm$ for $m_c \neq m_u$
- Nice: Active charm quark + GW-fermions $\longrightarrow$ only logarithmic divergences !

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Low-energy counterpart of weak effective Hamiltonian at LO

$$\mathcal{H}_w^{ChPT} = \frac{g_w^2}{2M_W^2} V_{ud} V_{us}^* \sum_{\sigma=\pm} \hat{g}_1^\sigma \left\{ [\hat{\mathcal{O}}_1^\sigma]_{suud} - [\hat{\mathcal{O}}_1^\sigma]_{sccd} \right\}$$

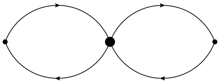
4-quark operator $\leftrightarrow$ $[\hat{\mathcal{O}}_1]_{\alpha\beta\gamma\delta} = \frac{1}{4} F^4 (U \partial_\mu U^\dagger)_{\gamma\alpha} (U \partial_\mu U^\dagger)_{\delta\beta}$

$U \in SU(4)$: Goldstone boson field, F: pion decay constant

Step 1: Degenerate charm [Giusti,Hernández,Laine,Pena,Wennekers,Wittig '07]

Study $K \to \pi$ transitions in the SU(4)-symmetric theory: $m_u = m_d = m_s = m_c$

$\Longrightarrow$ only "Figure-8" graphs contribute



$$C_1^{\pm}(x,y) = \left\langle Tr\left[\gamma_0 P_- S(x,0)^\dagger \gamma_0 P_- S(x,0)\right] Tr\left[\gamma_0 P_- S(y,0)^\dagger \gamma_0 P_- S(y,0)\right]\right\rangle$$
$$\mp \left\langle Tr\left[\gamma_0 P_- S(x,0)^\dagger \gamma_0 P_- S(y,0) \gamma_0 P_- S(y,0)^\dagger \gamma_0 P_- S(x,0)\right]\right\rangle$$

	$\hat{g}_1^+$	$\hat{g}_1^-$
Lattice	0.51(3)(5)(6)	2.6(1)(3)(3)
"Exp"	~ 0.5	~ 10.4
large N_c	1	1

errors
-1st: statistical
-2nd: matching to ChPT
-3rd: renormalization

- $\Delta I = 3/2$ amplitude A_2: close to "experiment"
- $\Delta I = 1/2$ amplitude A_0: factor ~ 4 too small
- Observe significant enhancement in $SU(4)$-symmetric limit: $\frac{A_0}{A_2} \sim 6$

Low-mode averaging (LMA)

$$S(x,y) = \frac{1}{V}\sum_i \frac{v_i(x)\otimes v_i^\dagger(y)}{\lambda_i + m}$$

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$$S(x,y) = \frac{1}{V}\sum_{i}^{n_{low}} \frac{v_i(x) \otimes v_i^\dagger(y)}{\lambda_i + m} + S^h(x,y) \quad (\text{“low part”} + \text{“high part”})$$

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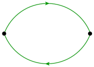
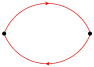
2pt function: $\sum_{\vec{x}} \langle J_0(x) J_0(0)\rangle = C^{ll}(t) + C^{lh}(t) + C^{hh}(t)$

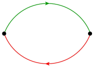
$$C^{hh}(t) = -\sum_{\vec{x}} \langle Tr\{\gamma_0 P_- S^h(x,0)^\dagger \gamma_0 P_- S^h(x,0)\}\rangle$$



$$C^{ll}(t) \propto -\frac{1}{V}\sum_{k,l=1}^{n_{low}} \sum_{x,y} \delta_{t,t_x-t_y} \left\langle [v_k^\dagger \gamma_0 P_- v_l](x)[v_l^\dagger \gamma_0 P_- v_k](y)\right\rangle$$



$$C^{hl}(t) \propto -\frac{1}{L^3}\sum_{k=1}^{n_{low}} \sum_{x,\vec{y}} \delta_{t,t_x-t_y} \left\langle v_k^\dagger(x)\gamma_0 P_- S^h(x,y)\gamma_0 P_- v_k(y)\right\rangle + (x \leftrightarrow y)$$

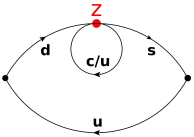


3pt function: $\longrightarrow$ 5 contributions

$$C^{3pt} = C^{llll} + C^{hlll} + C^{hhll} + C^{hhhl} + C^{hhhh}$$

	C^{llll}	C^{hlll}	C^{hhll}	C^{hhhl}	C^{hhhh}	
	$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$	
# of diagrams	1	4	6	4	1	$=$ **16**

Step 2: Decouple charm quark $m_c > m_u$



$$S(z,z) = \frac{1}{V}\sum_{i}^{n_{low}} \frac{v_i(z) \otimes v_i^\dagger(z)}{\lambda_i + m} + S^h(z,z)$$

- "EYE"-diagram: Signal still lost despite LMA
- Problem: Use of point-to-all propagators $S^h(z,0)$ does not allow for averaging over $\vec{z}$, the position of 4-quark operator insertion
- Approach: Estimate entire propagator $S^h(z,z)$ ("high part") stochastically; not only single column! ["Hybrid-approach", Peardon et al. CPC 172 (2005)]
- Not only loop requires stochastic all-to-all propagator

Reminder: Random noise sources

[Bernardson et al. CPC 78 (1993) '93, Dong, Liu PLB 328 (1994)]

- Source method: Quark propagator by solving the linear system

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  $\langle \ldots \rangle_{src}$: expectation value over the distribution of noise vectors

- Taking the solution vector $\Phi_{[r]} = D^{-1}\eta_{[r]} \longrightarrow$ estimate of entire propagator matrix

$$\left\langle \Phi^a_\alpha(x)_{[r]} \right\rangle_{\mathrm{src}} = \left\langle (D^{-1})^{ac}_{\alpha\gamma}(x,z)\eta^c_\gamma(z)_{[r]} \right\rangle_{\mathrm{src}}$$

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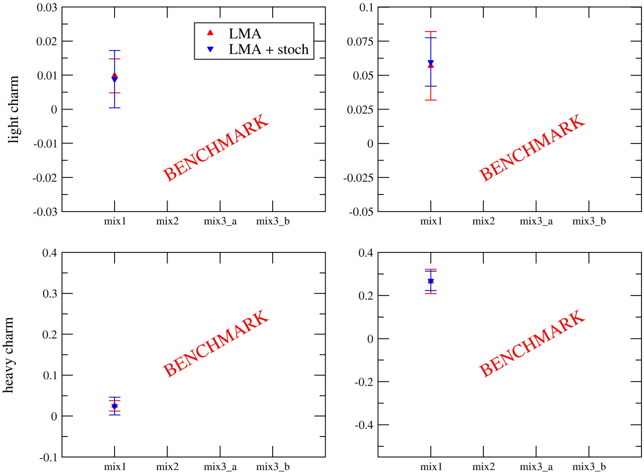
$$= S^{ab}_{\alpha\beta}(x,y)$$

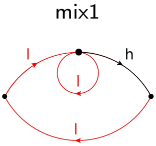
Preliminary results: Eye-diagram

$R_{-} = C^{-}_{EYE} / C^{2}_{2pt}$ $\qquad$ $R_{+} = C^{+}_{EYE} / C^{2}_{2pt}$



$m_c = 2 \times m_u$

$m_c = 10 \times m_u$



- 82 cnfgs, $32x16^3$
- Quenched
- $\beta = 5.8485$
- $n_{low} = 20$
- Spin-dilution
- $m_{\pi} = 322$ MeV

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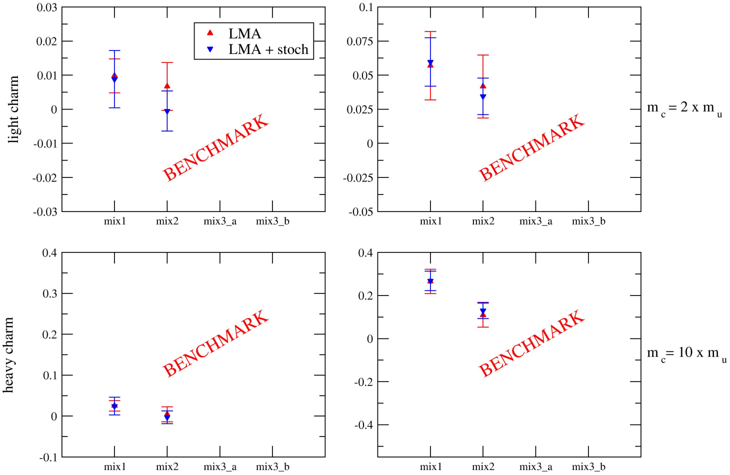

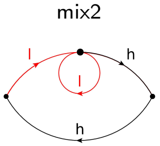

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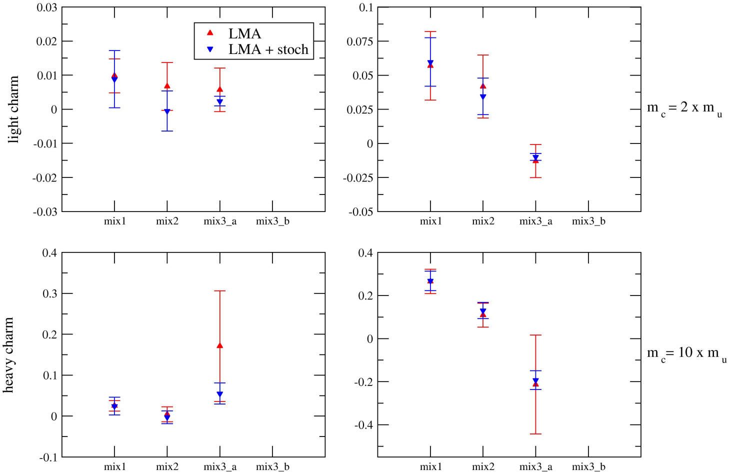

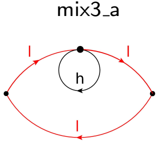

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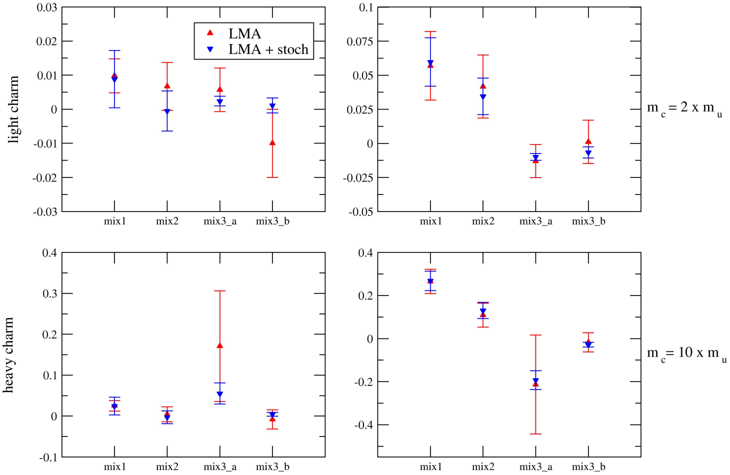

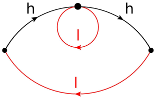

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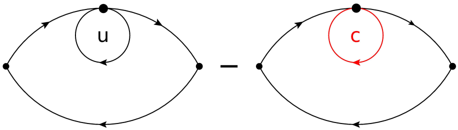
Nested loop propagator (I) [Giusti, Wittig (2005)]



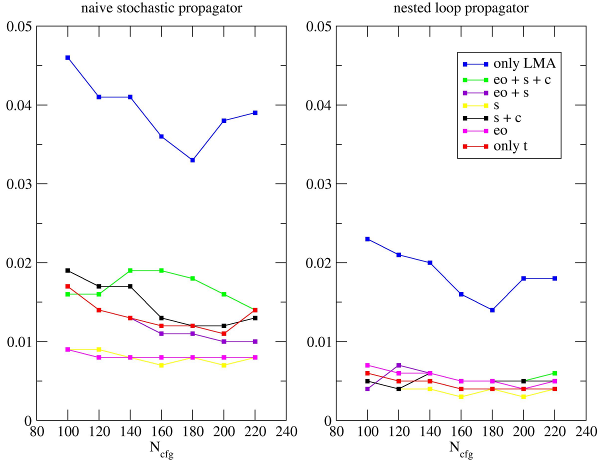
New approach:

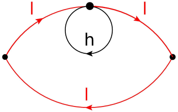
Use result of charm quark inversion as source for light quark inversion

$$P_-(S_{\mathbf{u}} - S_{\mathbf{c}})P_+ =$$

$$(m_{\mathbf{c}}^2 - m_{\mathbf{u}}^2)P_- \left(D_{m_{\mathbf{u}}}^\dagger D_{m_{\mathbf{u}}}\right)^{-1} \overbrace{P_- \left(D_{m_{\mathbf{c}}}^\dagger D_{m_{\mathbf{c}}}\right)^{-1} \underbrace{P_- (\gamma_5 D) P_+ \left(1 - \frac{\bar{a}}{2}\gamma_5 D\right) P_+}_{\text{charm quark source}}}^{\text{light quark source}}$$

abs. error $\times \sqrt{N_{cfg}}$





Dilution scheme

- eo: even-odd
- c : color
- s : spin
- t : time

Simulation details

- $16x8^3$
- $\beta = 5.8485$
- $n_{low} = 20$

Summary and Outlook

- Computational framework to quantify different sources of $\Delta I = 1/2$ enhancement: $\Delta S = 1$ weak effective Hamiltonian with active charm and Ginsparg-Wilson fermions
- Current stage: Decouple charm quark mass, i.e. $m_c > m_u$, to monitor dependence of amplitudes on m_c
- Problem: Signal lost due to closed quark loops in Eye-diagram
- Proposed solution: Combining LMA with stochastic and/or nested all-to-all propagators
- Status: Noise can be reduced significantly for some contributions, however, more statistics is needed. No physical statements so far...