Large-N reduction in lattice field theory with adjoint fermions

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In collaboration with Barak Bringoltz and Stephen R. Sharpe

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INNOVATIVE ECONOMY NATIONAL COHESION STRATEGY

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- Eguchi, Kawai, 82: Large volume lattice YM theory $\equiv$ single site YM when $N \to \infty$ *if center symmetry* $(\mathbb{Z}_N)^4$ *unbroken*
- Bhanot, Heller, Neuberger, 82: $\mathbb{Z}_N^4$ broken, as seen in PT
- Several fixes, none of them fully satisfactory

Volume reduction

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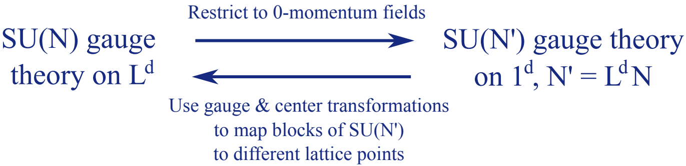
- Bhanot, Heller, Neuberger, 82: $\mathbb{Z}_N^4$ broken, as seen in PT
- Several fixes, none of them fully satisfactory
- Kovtun, Ünsal, Yaffe, 04: Modern view of reduction (large-N orbifold equivalence):



- KÜY, 07: Add massless adjoint fermions with PBC $\Rightarrow$ reduction holds in PT

Definition of AEK model

- Adjoint Eguchi-Kawai (EK + adj. Wilson fermions):

$$\mathcal{Z} = \int D[U, \psi, \bar{\psi}]\, e^{(S_{\text{gauge}} + \sum_{j=1}^{N_f} \bar{\psi}_j\, D_{\text{W}}\, \psi_j)}$$

$$S_{\text{gauge}} = 2Nb \sum_{\mu<\nu} \mathrm{ReTr}\, U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger, \quad b = \frac{1}{g^2 N}$$

$$D_W = 1 - \kappa \left[\sum_{\mu=1}^{4} (1-\gamma_\mu)\, U_\mu^{\text{adj}} + (1+\gamma_\mu)\, U_\mu^{\dagger\text{adj}} \right]$$

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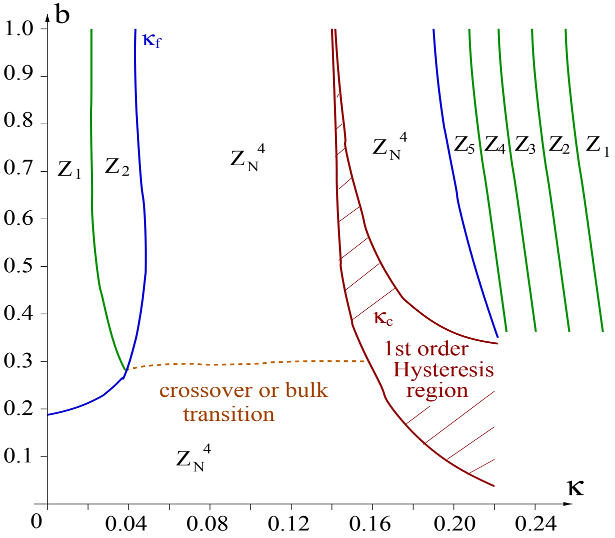
- Probes of $\mathbb{Z}_N^4$: general "open loops":

$$K_n \equiv \frac{1}{N} \mathrm{Tr}\, U_1^{n_1}\, U_2^{n_2}\, U_3^{n_3}\, U_4^{n_4}, \text{ with } n_\mu = 0, \pm 1, \pm 2, \ldots$$

- In particular, we find Polyakov loops $\mathrm{Tr}\, U_\mu$ and "corner variables" $\mathrm{Tr}\, U_\mu U_\nu^{\pm 1}$ very helpful.

Phase diagram of $N_f = 2$ AEK



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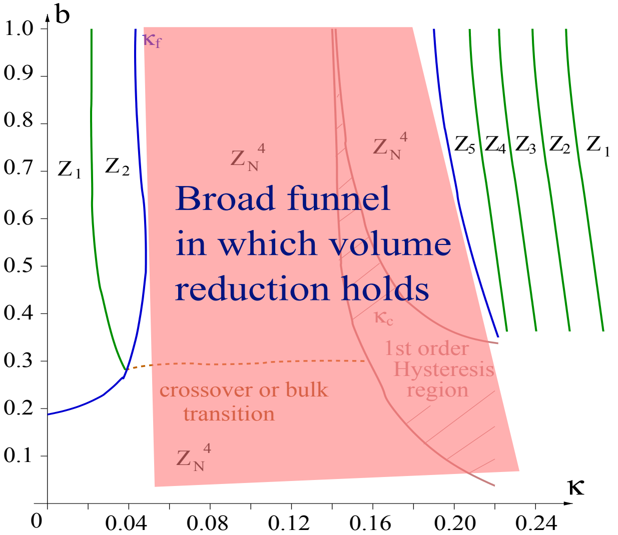

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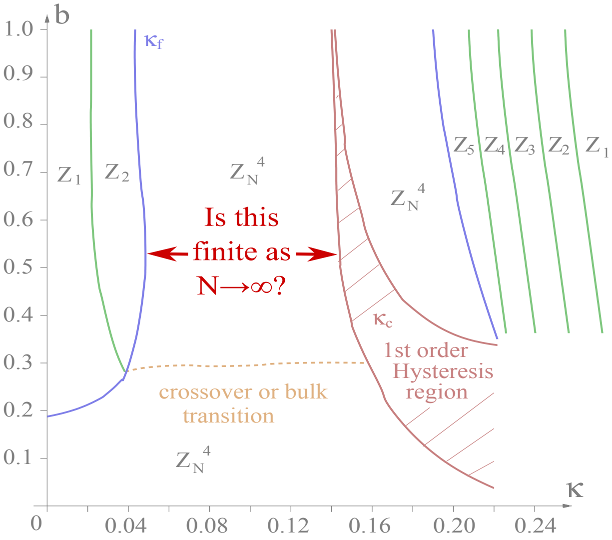
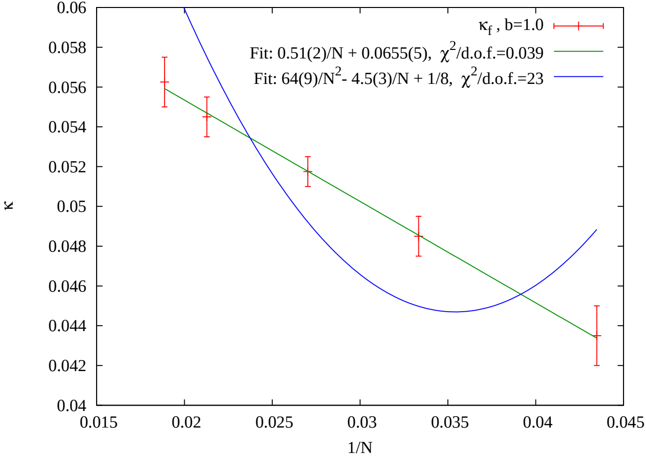


Phase diagram cntd. Funnel edge (κ_f) vs N

- Evidence of finite width of the funnel at $b = 1$ as $N \to \infty$



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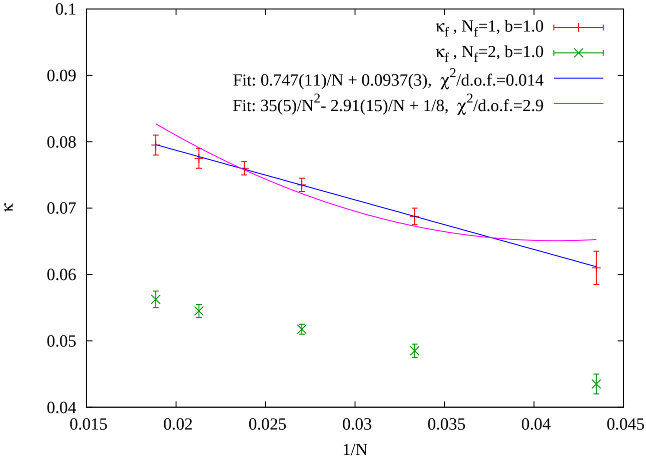
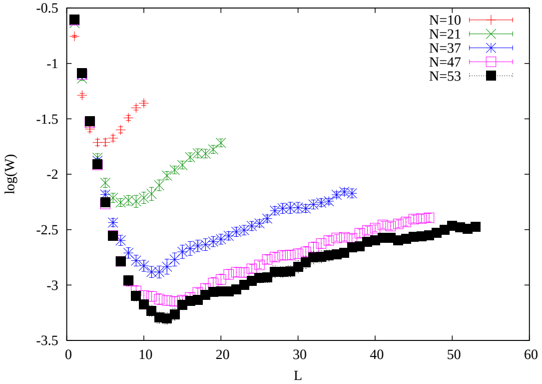


Wilson loops

- For given μ, ν: $W(M,L) = \langle \frac{1}{N} \mathrm{Tr}\, U_\mu^M U_\nu^L U_\mu^{\dagger M} U_\nu^{\dagger L} \rangle \sim e^{-V(M)L}$



Log of $1 \times L$ Wilson loops vs L for various N

Wilson loops cntd.

- Rise of $W(M, L)$ at large L – finite N corrections
- It can be seen in strong coupling expansion – contrary to large volume one never encounters integrals $\int dUU_{ij} = 0$ – zero-th order does not disappear.
- In pure gauge (for $0 < L, M \leq N$):

$$W(L, M) \xrightarrow{b,\kappa\to 0} \frac{1}{N^2 - 1} \left(L + M - 1 - LM/N^2\right)$$

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- Idea: use 2^4 lattices with only odd values of L, M.

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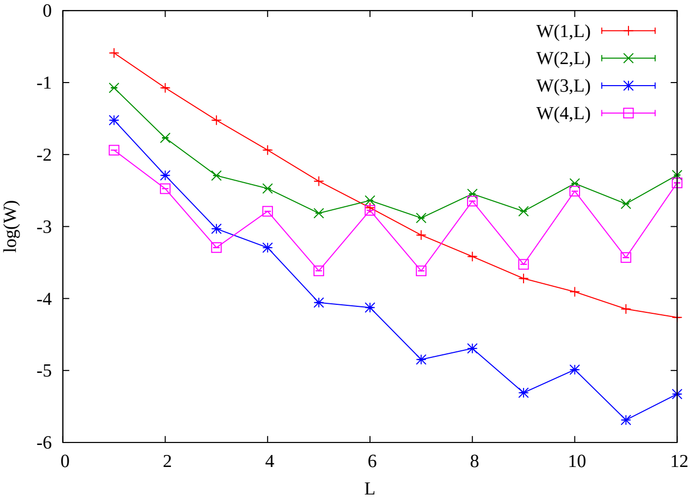

Wilson loops on 2^4 lattice, $N = 10$, $b = 0.35$, $\kappa = 0.1$

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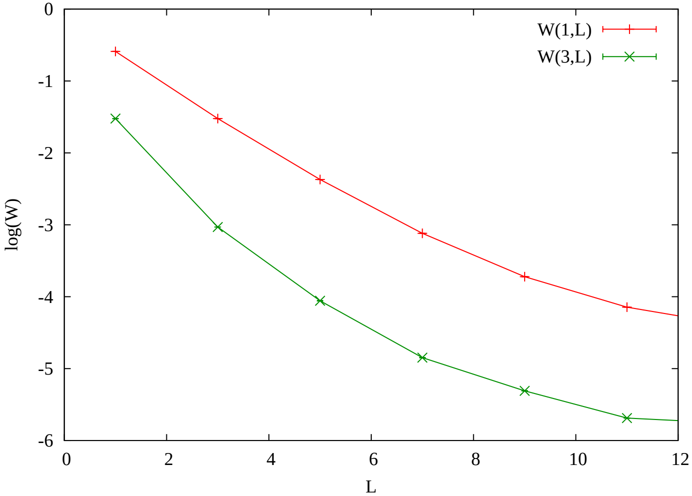

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Meson masses

- Narayanan, Neuberger, 05: "quenched momentum prescription"
- Multiply temporal links by additional $U(1)$ factors interpreted as "quenched momentum": $U_4 \to U_4\, e^{ip}$
- $\mathcal{M}_\pi(p,m) = \mathrm{Tr}\,\left\{\gamma_5\, D^{-1}(U_4\, e^{ip/2}, m)\, \gamma_5\, D^{-1}(U_4\, e^{-ip/2}, m)\right\},$

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- First results: hard to obtain light meson masses. Need larger N / further improvements.
- Caveat: While legitimate for fundamental mesons, can this be extended to adjoint mesons?
- Future plan: Compare with one extended dimension.

Summary & outlook

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- Center symmetry is unbroken in AEK in a wide range of physically interesting parameters ⇒ volume reduction works!
- "No free lunch" – calculation of physical quantities requires large values of N.

TODO list:

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- Perform simulations with one "long" dimension – meson & glueball masses, thermodynamics...

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Thank you for your attention!