Spontaneous supersymmetry breaking in the $2d$ $\mathcal{N} = 1$ Wess-Zumino model

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- Supersymmetric field theories interesting per se.
- Application of new approach to simulate (Majorana) fermions:

  $\Rightarrow$ fermion loop formulation.
- Non-perturbative description of a spontaneous supersymmetry breaking phase transition.

- Overview:
  - Description of the model
  - Fermion loop formulation
  - Vacuum structure and phase transition
  - Mass spectrum

$\mathcal{N} = 1$ Wess-Zumino model

- The $\mathcal{N} = 1$ Wess-Zumino model in 2D:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}P'(\phi)^2 + \frac{1}{2}\overline{\psi}\left(\partial\!\!\!/ + P''(\phi)\right)\psi$$

  - with ψ a two component Majorana field,
  - and ϕ real bosonic field,
  - superpotential, e.g. $P(\phi) = \frac{m^2}{4g}\phi + \frac{1}{3}g\phi^3$.

- Symmetries:
  - Supersymmetry

$$\begin{aligned} \delta\phi &= \bar{\epsilon}\psi \\ \delta\psi &= (\partial\!\!\!/\phi - P')\epsilon \\ \delta\overline{\psi} &= 0 \end{aligned}$$

  - $\mathbb{Z}(2)$ chiral symmetry

$$\begin{aligned} \phi &\rightarrow -\phi \\ \psi &\rightarrow \gamma_5\psi \\ \overline{\psi} &\rightarrow -\overline{\psi}\gamma_5 \end{aligned}$$

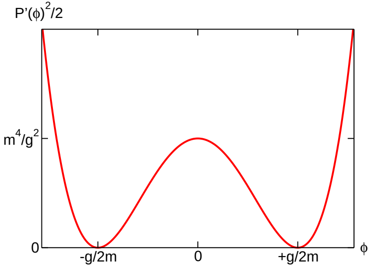
- Witten index $W = 0$:

  $\Rightarrow$ allows for spontaneous supersymmetry breaking

Classical potential

The scalar potential is a standard ϕ^4 theory:

$$\frac{1}{2}P'(\phi)^2 = \frac{1}{2}\frac{m^2}{2}\phi^2 + \frac{1}{2}g^2\phi^4 + \text{const}$$



- $\mathbb{Z}(2)$ symmetry may be broken.
- Witten index is vanishing ($W = 0$) with one bosonic and one fermionic ground state.
- Integrating out Majorana fermions yields indefinite Pfaffian:

$$\int \mathcal{D}\phi\, e^{-S_{\mathrm{b}}(\phi)}\, \mathrm{Pf}\, M_{pp}(\phi) \propto W = 0\,,$$

$\mathbb{Z}(2)$ and supersymmetry breaking

- For large $\frac{m}{g}$ the $\mathbb{Z}(2)$ symmetry is spontaneously broken.
- $\overline{\phi} = \pm m/2g$ selects a definite ground state:

$$\begin{array}{llll} \overline{\phi} = +m/2g & \Longrightarrow & \mathrm{Pf}\, M_{pp} = +\mathrm{Pf}\, M_{pa} & \text{bosonic} \\ \overline{\phi} = -m/2g & \Longrightarrow & \mathrm{Pf}\, M_{pp} = -\mathrm{Pf}\, M_{pa} & \text{fermionic} \end{array}$$

$\Rightarrow$ SUSY unbroken

- For small $\frac{m}{g}$ the $\mathbb{Z}(2)$ symmetry is unbroken.
- $\overline{\phi} = 0$ allows both bosonic and fermionic ground state:

$\Rightarrow$ SUSY broken

- Tunneling between the two ground states corresponds to the Goldstino mode.

$\mathcal{N} = 1$ Wess-Zumino model on the lattice

- Using Wilson lattice discretisation for the fermionic part:

$$\mathcal{L} = \frac{1}{2}\xi^T \mathcal{C}(\gamma_\mu \tilde{\partial}_\mu - \frac{1}{2}\partial^* \partial + P''(\phi))\xi \,,$$

  - ξ is a real, 2-component Grassmann field,
  - $\mathcal{C} = -\mathcal{C}^T$ is the charge conjugation matrix.

- Need the same derivative for the bosons [Golterman, Petcher '88]:
  $\Rightarrow$ guarantees supersymmetric continuum limit

- Wilson derivative spoils both $\mathbb{Z}(2)$ and chiral symmetry!

- We use exact reformulation in terms of fermion loops. Together with fluctuating fermionic boundary conditions:

$\Rightarrow$ sign of Pfaffian under perfect control

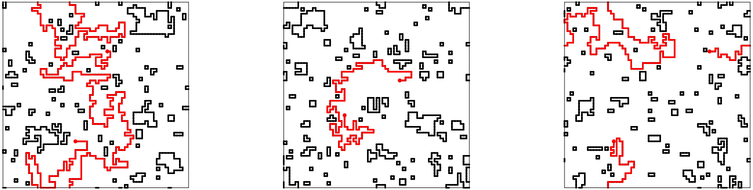
- Expand fermion action using the nilpotency of Grassmann elements.
- Only closed, non-intersecting paths survive the integration.
- Partition function becomes a sum over all non-oriented, self-avoiding fermion loops ℓ

$$Z_{\mathcal{L}} = \int d\phi \sum_{\{\ell\}\in\mathcal{L}} \omega[\ell,\phi], \quad \mathcal{L} \in \mathcal{L}_{00} \cup \mathcal{L}_{10} \cup \mathcal{L}_{01} \cup \mathcal{L}_{11}$$

Represents system with unspecified fermionic b.c.

- Simulate fermions by enlarging the configuration space by one open fermionic string [Wenger '08].

- The open fermionic string corresponds to the insertion of a Majorana fermion pair $\{\xi^T(x)\mathcal{C}, \xi(y)\}$ at position x and y:



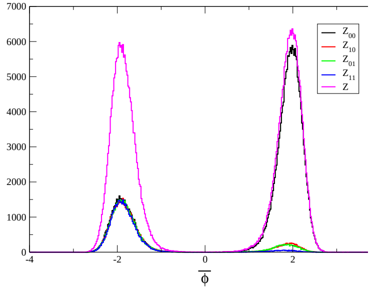
- It samples the relative weights between $Z_{\mathcal{L}_{00}}, Z_{\mathcal{L}_{10}}, Z_{\mathcal{L}_{01}}, Z_{\mathcal{L}_{11}}$.
- Reconstruct the Witten index a posteriori

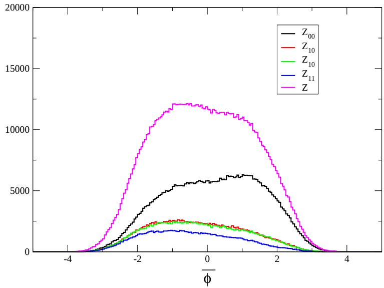
$$W \equiv Z_{pp} = \underbrace{Z_{\mathcal{L}_{00}}}_{\text{bosonic vacuum}} - \underbrace{(Z_{\mathcal{L}_{10}} + Z_{\mathcal{L}_{01}} + Z_{\mathcal{L}_{11}})}_{\text{fermionic vacuum}},$$

or a system at finite temperature

$$Z^{pa} = Z_{\mathcal{L}_{00}} - Z_{\mathcal{L}_{10}} + Z_{\mathcal{L}_{01}} + Z_{\mathcal{L}_{11}}.$$

Symmetry breaking pattern and vacuum structure





- $\mathbb{Z}(2)$ broken phase:

  $\langle\overline{\phi}\rangle \simeq -m/2g : Z_{pp} \simeq -Z_{pa}$

  $\Rightarrow$ fermionic vacuum

  $\langle\overline{\phi}\rangle \simeq +m/2g : Z_{pp} \simeq Z_{pa}$

  $\Rightarrow$ bosonic vacuum

- supersymmetry unbroken

- $\mathbb{Z}(2)$ symmetric phase:

  $\langle\overline{\phi}\rangle \simeq 0 : Z_{pp} \simeq 0$

  $\Rightarrow$ vacuum tunneling

- supersymmetry broken

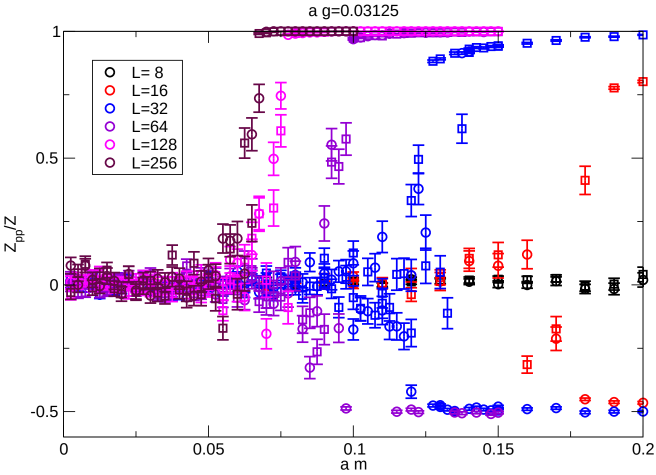
- Setting up model with Wilson derivative for bosons and fermions yields supersymmetric continuum limit [Golterman, Petcher '88].
- The model is superrenormalisable:
  $\Rightarrow$ need to tune only mass parameter m to obtain renormalised theory [Golterman, Petcher '88]

$$m^2 = m_R^2 + 2g^2/\pi \ln m_R^2$$

  $\Rightarrow$ use coupling g to set the scale: $\hat{g} = ag$
- Continuum limit of supersymmetry breaking transition:

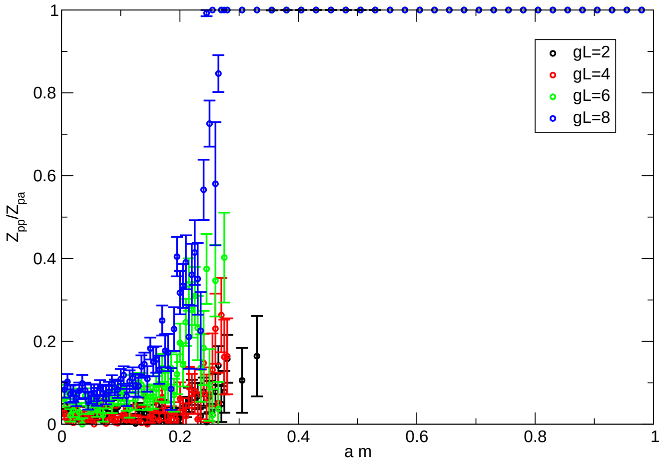
$$f_c = \lim_{\hat{g}\to 0} \frac{g}{m_R}\bigg|_c$$

Supersymmetry breaking phase transition



- Witten index $W \propto Z_{pp}/Z_{pa}$ as an order parameter:
  - bosonic vacuum preferred ($\mathbb{Z}(2)_\chi$ broken at $a \neq 0$)
  - $am_c \simeq 0.05$ for $L \to \infty$

Supersymmetry breaking phase transition



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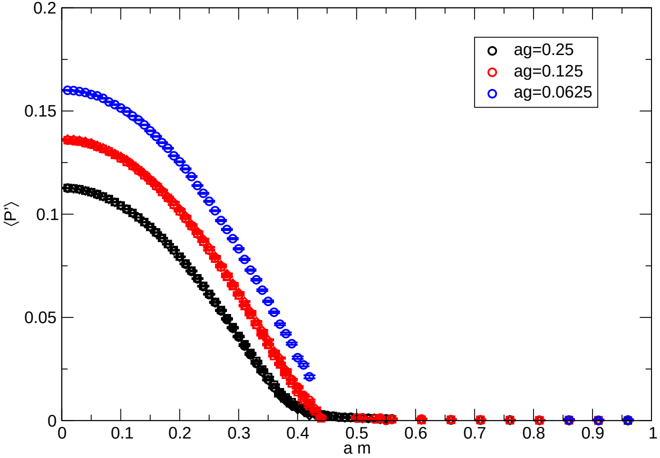
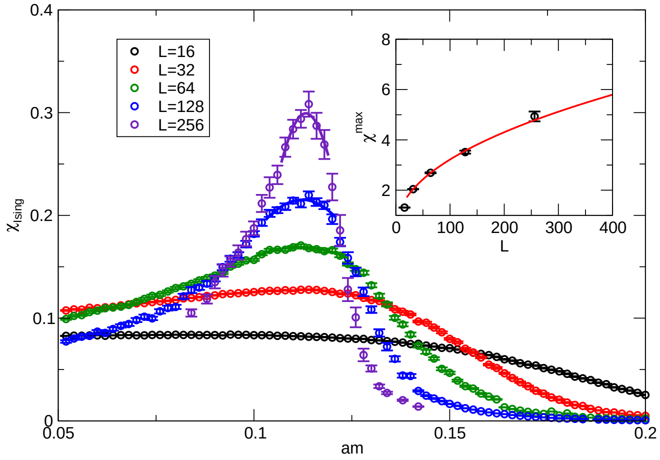
Supersymmetry breaking phase transition



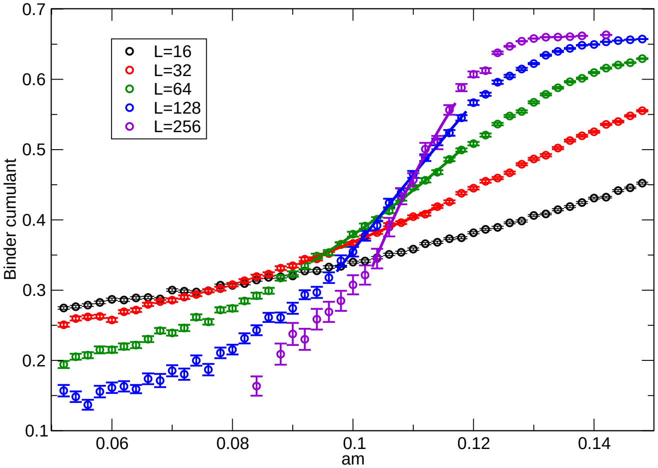
- Ward identity $\langle P' \rangle$ as an order parameter, volumes $gL = 2, 4, 6, 8$

$\mathbb{Z}(2)$ breaking phase transition



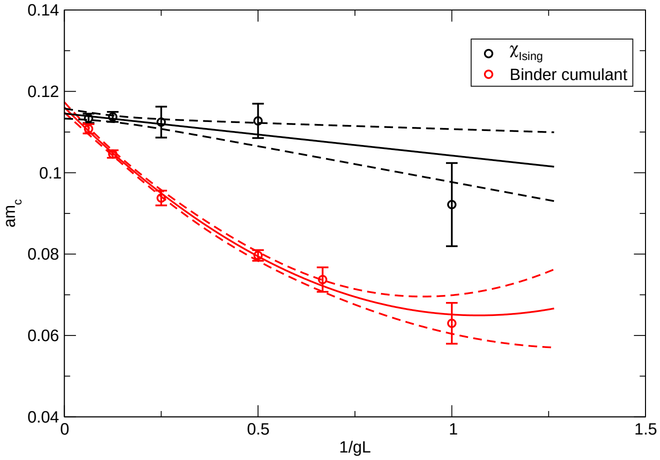
- 'Ising susceptibility' χ_m with $m = 1/V \sum_x \text{sign}[\phi_x]$

$\mathbb{Z}(2)$ breaking phase transition



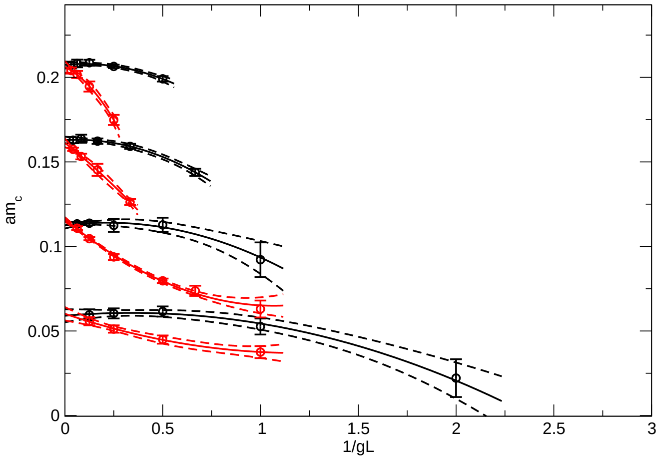
- Binder cumulant

$\mathbb{Z}(2)$ breaking phase transition



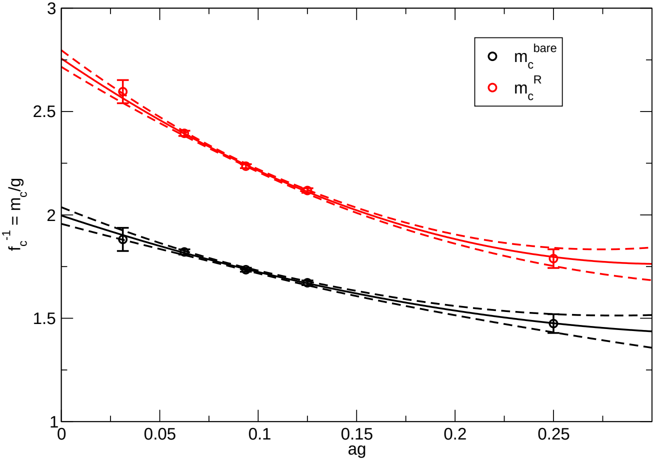
- Thermodynamic limit

$\mathbb{Z}(2)$/SUSY breaking phase transition



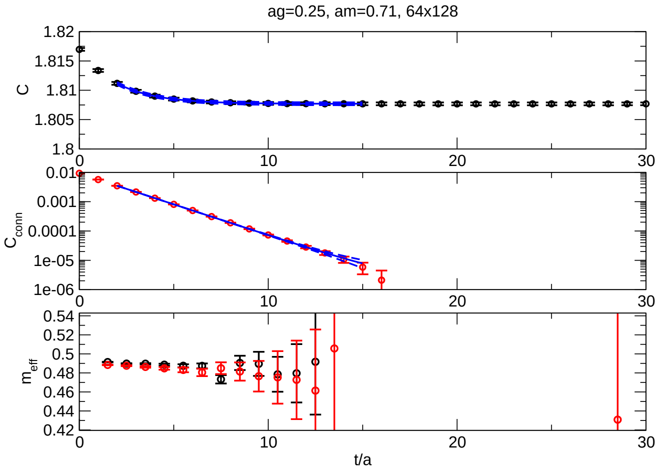
- Thermodynamic limit

$\mathbb{Z}(2)$/SUSY breaking phase transition



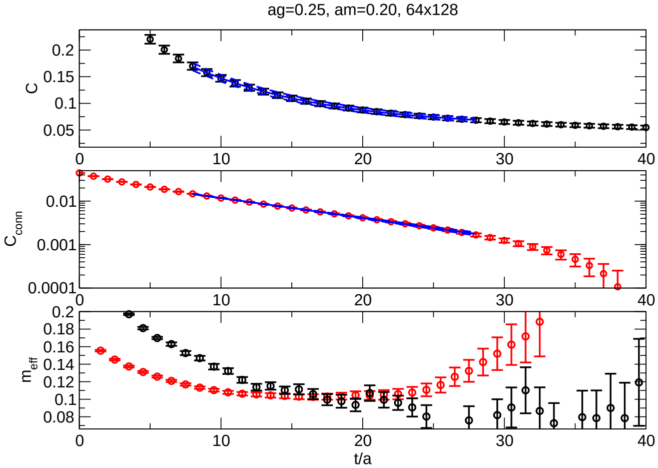
- Continuum limit: 1-loop renormalised critical coupling

Boson field correlator in the SUSY symmetric/$\mathbb{Z}(2)$ broken phase:



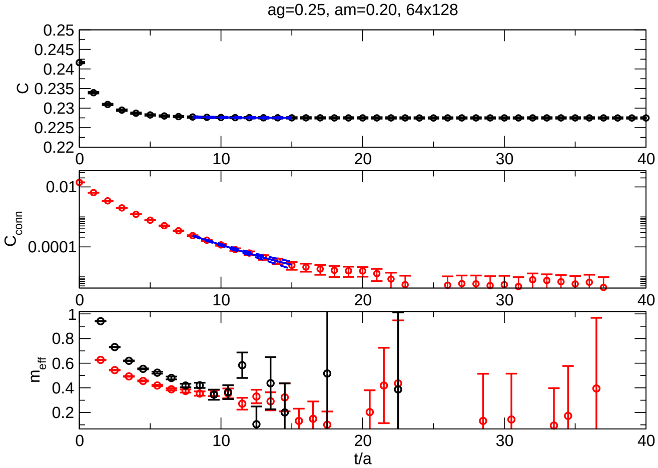
perfect fit with $e^{-M_B t}$ plus constant shift

Boson field correlator in the SUSY broken/$\mathbb{Z}(2)$ symmetric phase, $\mathbb{Z}(2)$ odd state ϕ:



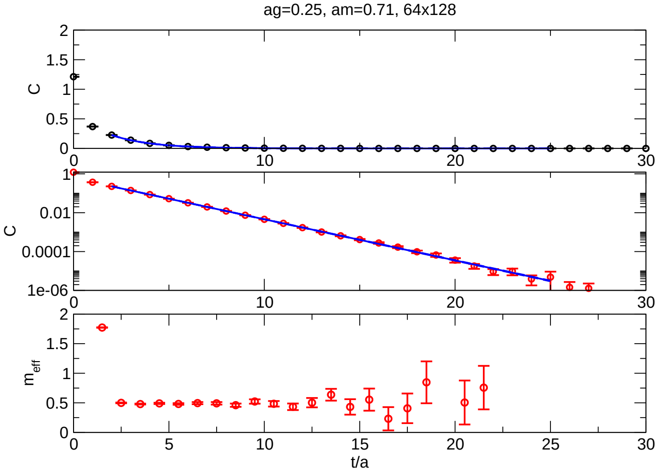
perfect fit with $A_0 e^{-M_B^{(0)} t} + A_1 e^{-M_B^{(1)} t}$

Boson field correlator in the SUSY broken/$\mathbb{Z}(2)$ symmetric phase, $\mathbb{Z}(2)$ even state ϕ:



perfect fit with $e^{-M_B^{(2)}t}$ plus constant shift

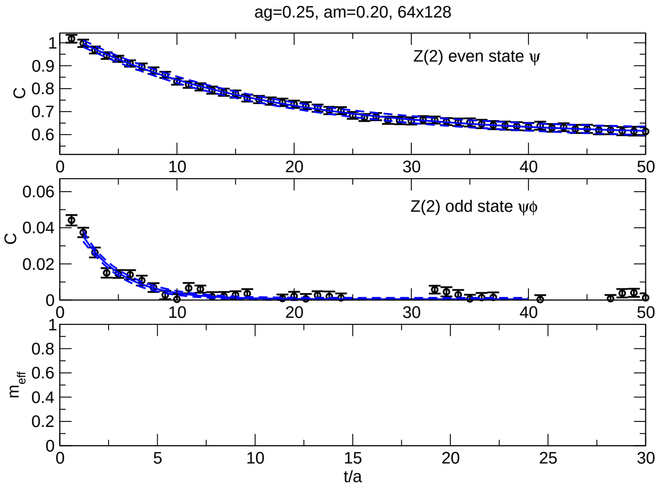
Fermion field correlator in the SUSY symmetric/$\mathbb{Z}(2)$ broken phase (spans $\mathcal{O}(10^7)$ due to fermion loop algorithm):



perfect fit with $e^{-M_F t}$

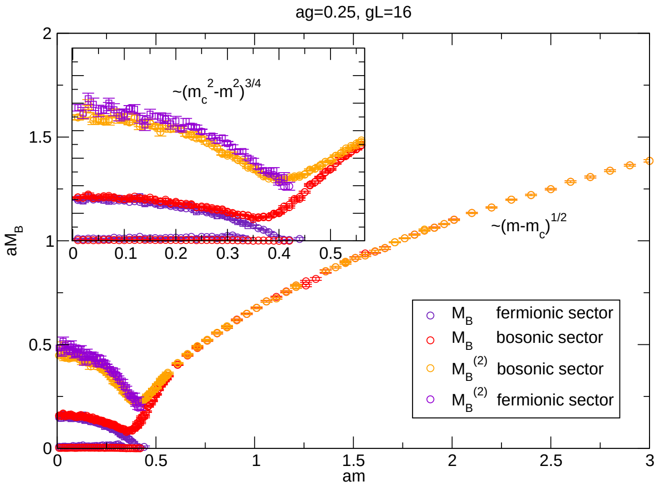
Fermion mass spectrum

Fermion field correlator in the SUSY broken/$\mathbb{Z}(2)$ symmetric phase, $\mathbb{Z}(2)$ even/odd state ψ and $\psi\phi$:

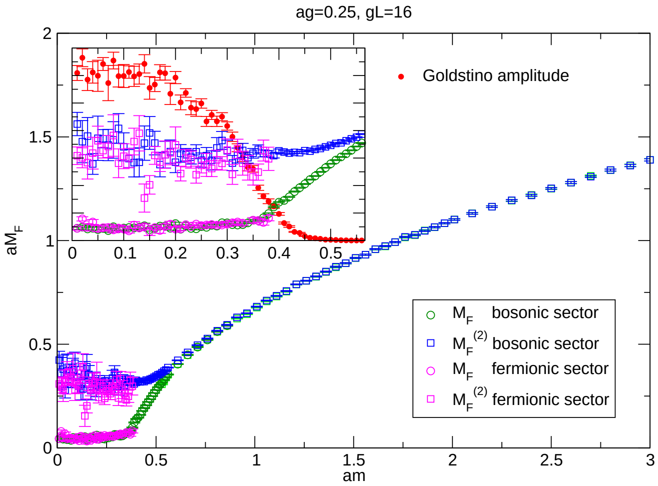


good fits with $A_0 e^{-M_F^{(0)}t} + A_1 e^{-M_F^{(1)}t}$ and $e^{-M_F^{(2)}t}$

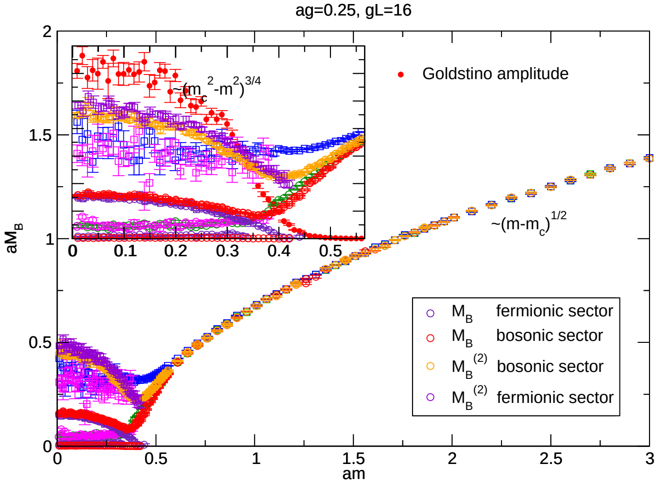
Boson mass spectrum



Fermion mass spectrum



Combined mass spectrum



Conclusions

- Reformulation of Majorana fermions in terms of fermion loops:
  - separation of partition function into bosonic and fermionic sector,
  - avoids the fermion sign problem,
  - allows simulations without critical slowing down.

- Determination of the critical coupling for the $\mathbb{Z}(2)$/SUSY breaking phase transition.

- Determination of the particle mass spectrum above and below the transition.

- First complete non-perturbative description of a spontaneous supersymmetry breaking phase transition.