WALKING NEAR A CONFORMAL FIXED POINT

MICHELE PEPE

INFN Sez. Milano – Bicocca

Milan (Italy)

IN COLLABORATION WITH

PHILIPPE DE FORCRAND

ETH – Zurich (Switzerland)

Cern – Geneve (Switzerland)

UWE-JENS WIESE

University of Bern

Bern (Switzerland)

PLAN OF THE TALK

- Introduction
- The 2-d O(3) model
  - main facts
  - θ-vacuum effects
- A toy model for walking technicolor (numerical results)
- Conclusions

Introduction

Electroweak symmetry breaking: Higgs mechanism by elementary scalar field

Higgs → mass to $W^{\pm}$, Z^0, fermions

no dynamical explanation, unnatural (fine-tuning), hierarchy problem

Technicolor: $\overline{Q}Q$

Weinberg, Susskind, Farhi
Late `70s

→ χPT: $\bar{q}q$ condensate

$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_{L=R}$

→ Superconductivity: Cooper pairs

(Abelian Higgs model)

IDEA: replace the Higgs sector of the SM with a high energy copy of QCD with techni-gluons and techni-quarks undergoing spontaneous χSB

$\overline{\psi}\psi\,\overline{Q}Q$ — heavy fermion masses

t e n s i o n

$\overline{\psi}\psi\,\overline{\psi}\psi$ — suppressed FCNC

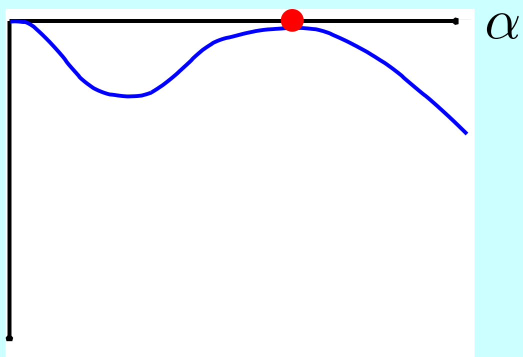
input from phenomenology

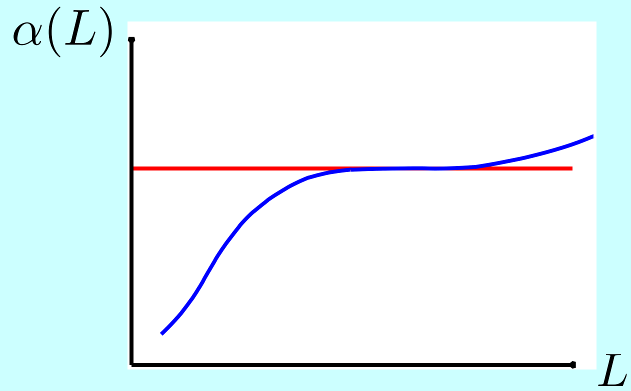
electroweak precision measurements

The dynamics of the TC extended Standard Model must be different than QCD-like

Holdom, Appelquist et al. `80s

$$\beta(\alpha) = -\frac{d\alpha(L)}{d\log L}$$





walking coupling

Slowly walking Technicolor models are possible candidates for EW symmetry breaking

Non-perturbative investigations from first principles

Lattice formulation of the QFT

Very challenging from the numerical viewpoint:

available results are often questionable

2-d O(3) model with a θ-term is an ideal theoretical laboratory to investigate slowly walking technicolor models with high accuracy

Same suggestion Nogradi, 2012

Main facts about the 2-d O(3) model

$$\mathcal{S}_0 = \frac{1}{2g^2} \int d^2x \;\; \partial_\mu \vec{e} \cdot \partial_\mu \vec{e}$$

g^2 is the bare coupling and $\vec{e} = (e_1, e_2, e_3)$ unit-vector

- It is asymptotically free like Yang-Mills theory
- It has a non-perturbatively generated massgap M
- There are instantons and topological sectors $\Pi_2(S^2) = \mathbb{Z}$
- There are non-trivial θ-vacuum effects

$$Q[\vec{e}] = \frac{1}{8\pi} \int d^2x \; \epsilon_{\mu\nu} \, \vec{e} \cdot (\partial_\mu \vec{e} \times \partial_\nu \vec{e}) \quad \longrightarrow \quad \mathcal{S}_\theta = \mathcal{S}_0 + i\theta Q[\vec{e}]$$

- The topological susceptibility is logarithmically divergent

$$\langle Q^2 \rangle (L) \sim \log(\Lambda L)$$

Schwab (1982),
Berg and Lüscher (1981)
Balog and Niedermayer (1997)
Vicari (2008)
Bietenholz, Gerber, Pepe, Wiese (2010)

However the topological density 2-point function $\langle q(x) q(0) \rangle$ is well-defined for x>0

Is the concept of different topological sectors meaningful?

Is θ just an irrelevant parameter that renormalizes to 0 non-perturbatively?

θ is a relevant parameter and characterizes a different QFT

Bögli, Niedermayer, Pepe, and Wiese (2012)

The 2-d O(3) model is integrable at θ=0; massgap $M = \frac{8}{e}\Lambda_{\overline{MS}}$

Hasenfratz, Maggiore, Niedermayer (1990)

Perturbation theory is blind to the vacuum angle θ: the same mass scale $\Lambda_{\overline{MS}}$ is present in the spectrum for every value of θ. This is true also non-perturbatively.

2-d O(3) model at θ=π $\xrightarrow{\text{low energy}}$ k=1 WZNW model

$$S[U] = \frac{1}{4\pi}\int d^2x \ \mathrm{Tr}[\partial_\mu U^\dagger \partial_\mu U] - 2\pi i k S_{WZNW}[U]$$

Zamolodchikov and Fateev (1986), Haldane (1983), Affleck and Haldane (1987), Affleck et al. (1989)

where $S_{WZNW}[U] = \frac{1}{24\pi^2}\int_{H^3} d^2x \ dx_3 \ \varepsilon_{\mu\nu\rho}\mathrm{Tr}[U^\dagger\partial_\mu U U^\dagger \partial_\nu U U^\dagger \partial_\rho U]$ and $U(x) \in SU(2)$

the symmetry is $SU(2)_L \times SU(2)_R \sim O(4):$ $\quad U'(x) = LU(x)R^\dagger$

Wess and Zumino (1971), Novikov (1981), Witten (1984)

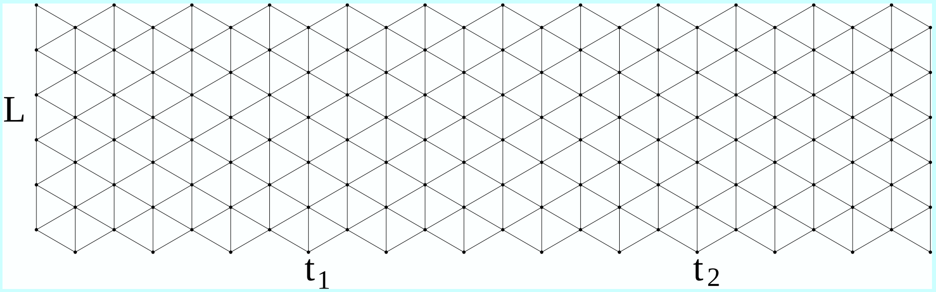
at very low energy (E≪M) the O(3) symmetry is enhanced to O(4)

Controzzi and Mussardo (2004)

The k=1WZNW model is a conformal field theory: the 2-d O(3) model at θ≈π is an ideal theoretical laboratory to study a slowly walking asymptotically free theory near a conformal fixed point.

The numerical study

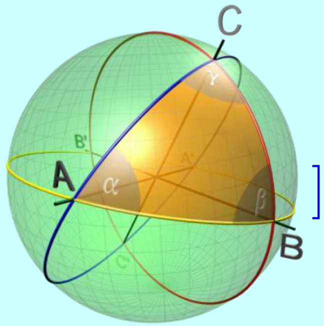
The 2-d O(3) model with the θ-term is regularized on the lattice



$$s(\vec{e}_x, \vec{e}_y) = \begin{cases} \frac{1}{g^2}(1 - \vec{e}_x \cdot \vec{e}_y) & \text{if} \quad \vec{e}_x \cdot \vec{e}_y > -\frac{1}{2} \\ \infty & \text{otherwise} \end{cases}$$

$$S[\vec{e}] = \sum_{\langle xy \rangle} s(\vec{e}_x, \vec{e}_y)$$

We then add the θ-term: $Q[\vec{e}] = \sum_{\triangle} q(\triangle)$ $\quad q(\triangle_{ABC}) = \frac{1}{4\pi}\mathrm{Area}[\ \ ]$



Berg and Lüscher (1981)

The partition function: $Z(\theta, L) = \int \mathcal{D}\vec{e}\ \mathrm{e}^{-S[\vec{e}]+i\theta Q[\vec{e}]}$

aim: scale dependence of a renormalized coupling; from running to walking

massgap scheme: $\alpha(\theta, L) = g^2(\theta, L) = m(\theta, L)L$

Lüscher, Weisz and Wolff (1991)

$$\langle \vec{E}(t_1) \cdot \vec{E}(t_2) \rangle = \frac{1}{Z(\theta, L)} \int \mathcal{D}\vec{e}\ \vec{E}(t_1) \cdot \vec{E}(t_2)\ \mathrm{e}^{-S[\vec{e}]+i\theta Q[\vec{e}]} = A\,\mathrm{e}^{-m(\theta,L)\,(t_2 - t_1)}$$

- We have analytical results for m(θ,L) at θ=0 and π

Balog and Hegedus (2004,2005,2010), Balog (private communication)

- We know the InfraRed Fixed Point: $\alpha(\pi, L) = \pi - \frac{\#}{\log(L)} + \ldots$

Affleck and Haldane (1987), Affleck et al. (1989)

- We know the scaling law near the IRFP: $m(\theta, L \to \infty) \sim |\theta - \pi|^{2/3} |\log(|\theta - \pi|)|^{-1/2}$

Important technical issues - 1

- Sign problem: $\mathrm{e}^{i\theta Q[\vec{e}]}$ => use of the meron-cluster algorithm **Bietenholz, Pochinsky and Wiese (1995)**

Choose a Wolff direction $\vec{r}$ and split the spin configuration into clusters $\mathcal{C}$ using the Wolff construction and respecting the angle constraint $\vec{e}_x \cdot \vec{e}_y > -\frac{1}{2}$ between n.n. spins.

$$Q[\vec{e}] = \sum_{\mathcal{C}} Q[\mathcal{C}] \quad \text{where} \quad Q[\mathcal{C}] = \frac{Q[\vec{e}] - Q[\vec{e}]_{\mathcal{C}\ \text{flipped}}}{2}$$

independent of the orientation of the other clusters
exponential gain!

- Lattice with cylinder geometry L × *time* to extract $m(\theta, L)$ and $\alpha(\theta, L) = m(\theta, L)L$

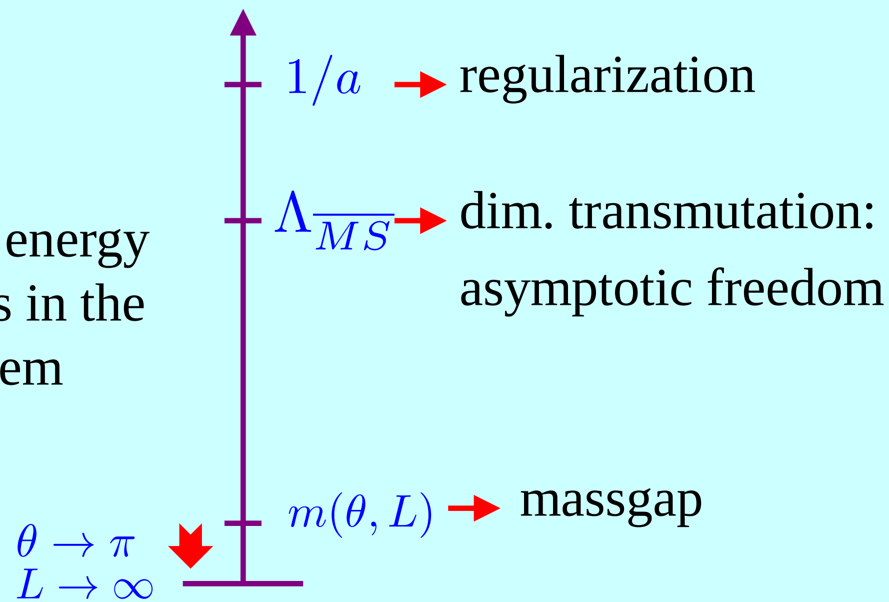
$\mathrm{e}^{i\theta Q[\vec{e}]}$ is put into the observable => improved estimator for the 2-point function

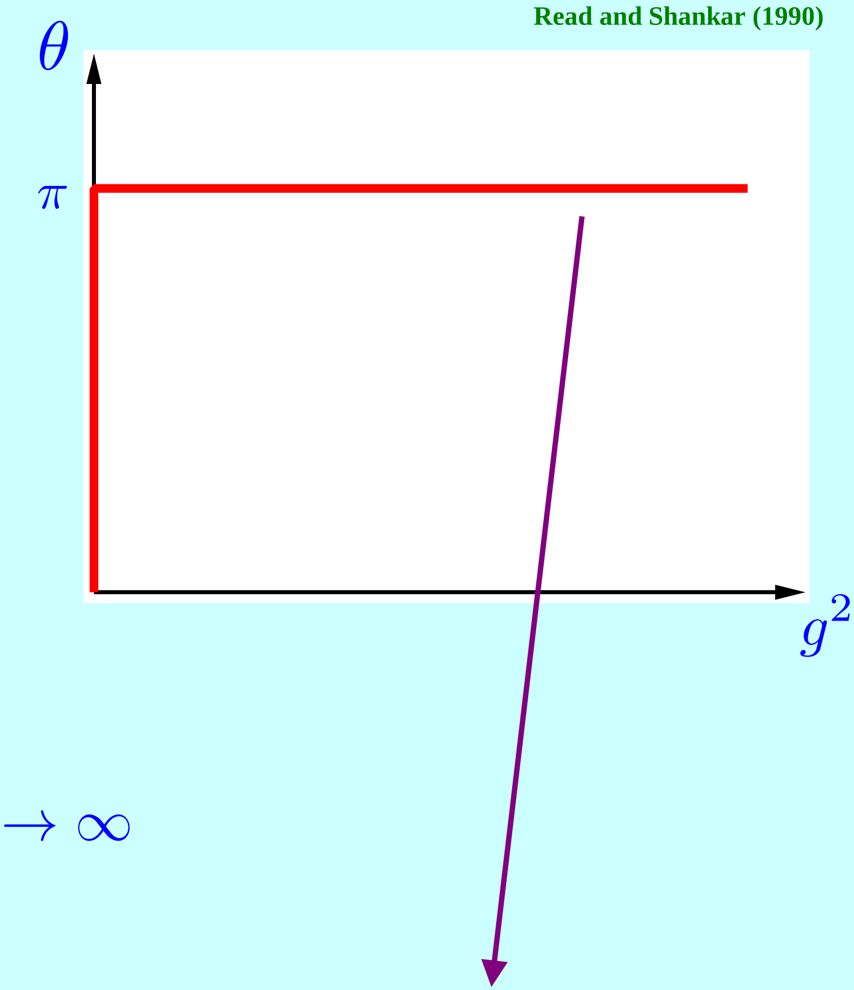
$$\langle \vec{e}_x \cdot \vec{e}_y\ \mathrm{e}^{i\theta Q} \rangle = \langle (\vec{e}_x \cdot \vec{r})(\vec{e}_y \cdot \vec{r}) \prod_{\mathcal{C}} \cos(\theta Q_{\mathcal{C}})[\delta_{\mathcal{C}_x,\mathcal{C}_y} + (1 - \delta_{\mathcal{C}_x,\mathcal{C}_y}) \tan(\theta Q_{\mathcal{C}_x}) \tan(\theta Q_{\mathcal{C}_y})] \rangle$$

$\mathcal{C}_x$ and $\mathcal{C}_y$ are the clusters that contain $\vec{e}_x$ and $\vec{e}_y$.

- The continuum limit extrapolation: $a \to 0$

three energy scales in the problem





Continuum Limit : first $g^2 \to 0$ and then $L \to \infty$

BUT

information valid only for the low-energy part of the spectrum:

OK if interested only in the IRFP

The numerical results

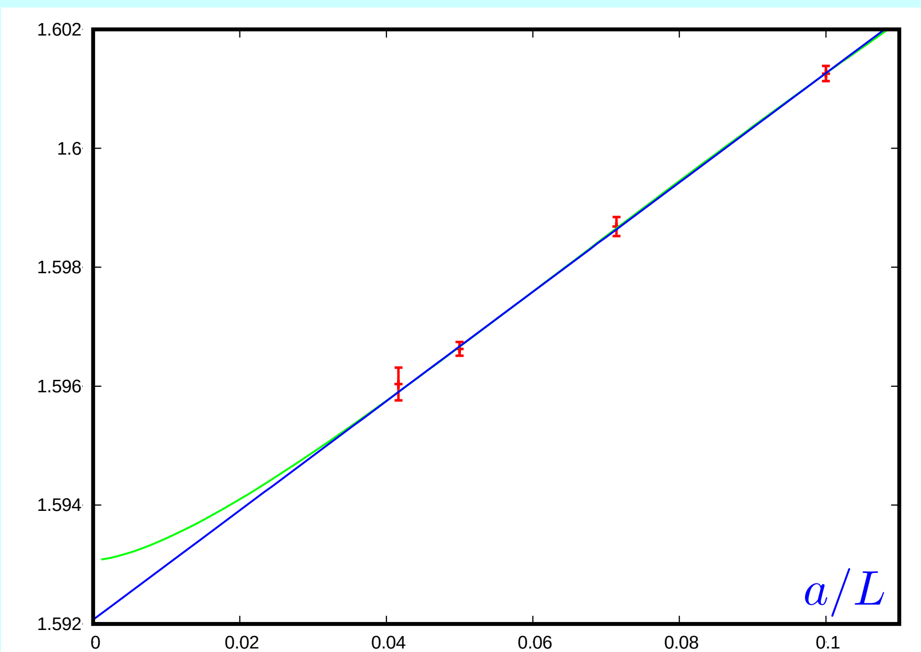
Lüscher, Weisz and Wolff (1991)

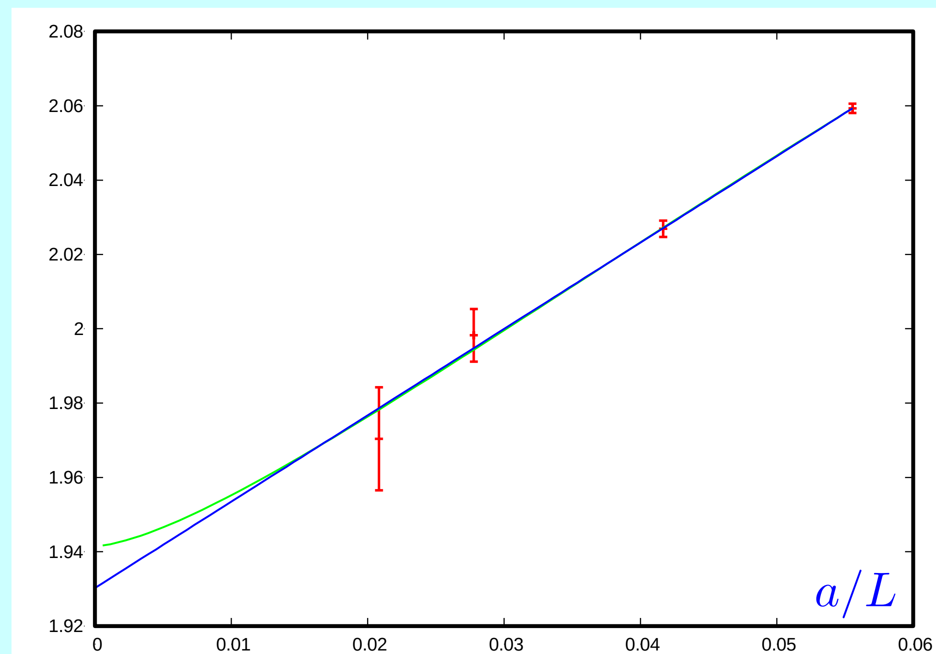
Renormalization point: $\alpha(\theta = 0, ML_0 = 0.2671536) = m(\theta = 0, ML_0 = 0.2671536)\, L_0 = 1.0595$

Other scales: $ML = s\, ML_0$

- infrared: $s = 2, \ldots, 32$
- ultraviolet: $s = 0.15, \ldots, 1$ + $s = 10^{-16}, \ldots, 0.1$ (3-loops perturbative)

For each ML and θ we extrapolate to the continuum limit:



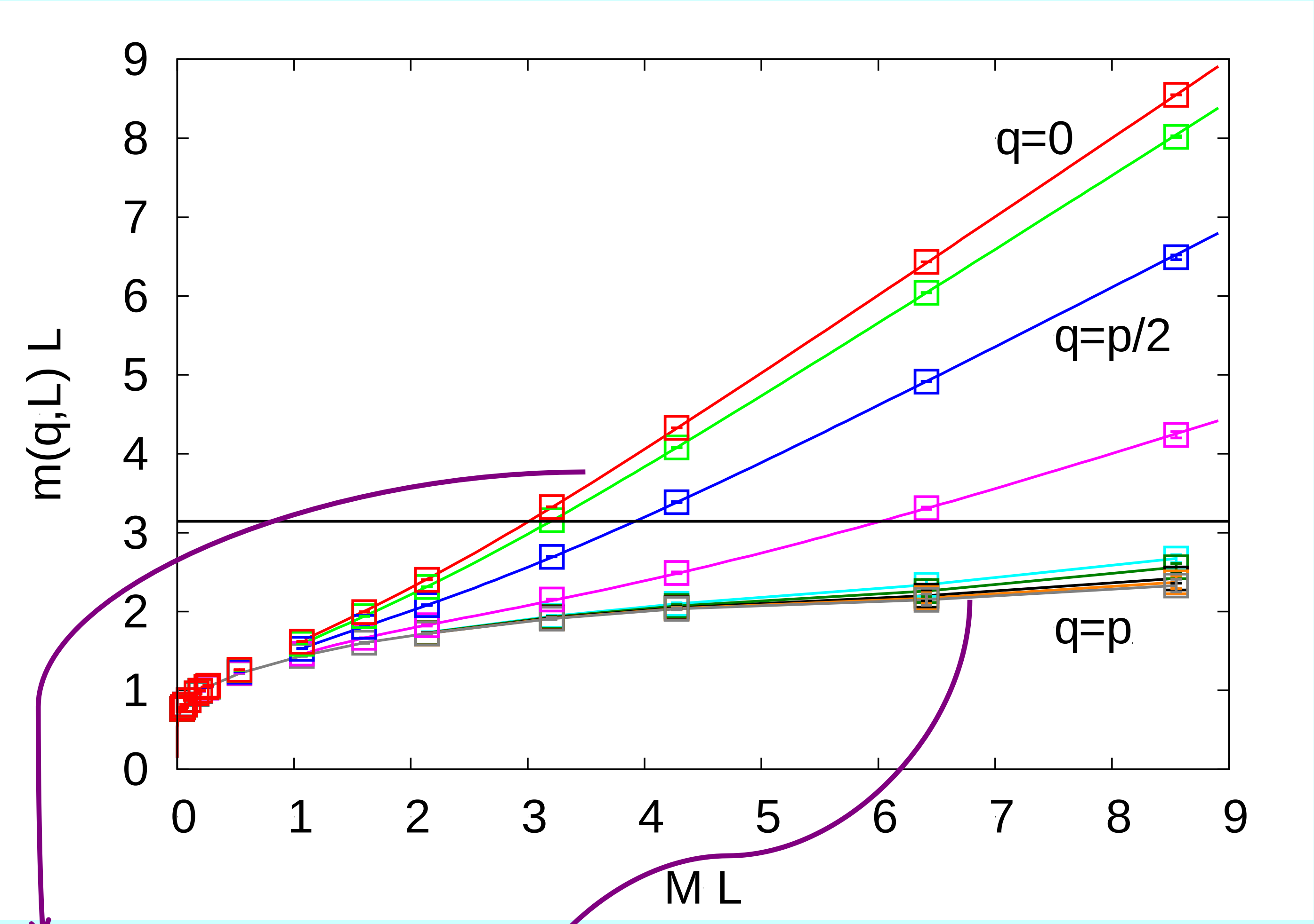


$s = 4; \theta = \pi/4$

$A + Bx^2(\log^3(x) + C\log^2(x))$

$A + Bx$

$s = 12; \theta = 0.92\,\pi$



very well described by: $$m(\theta,L)L \sim m(\theta)L\left(1 + A\frac{e^{-m(\theta)L}}{\sqrt{m(\theta)L}}\right)$$

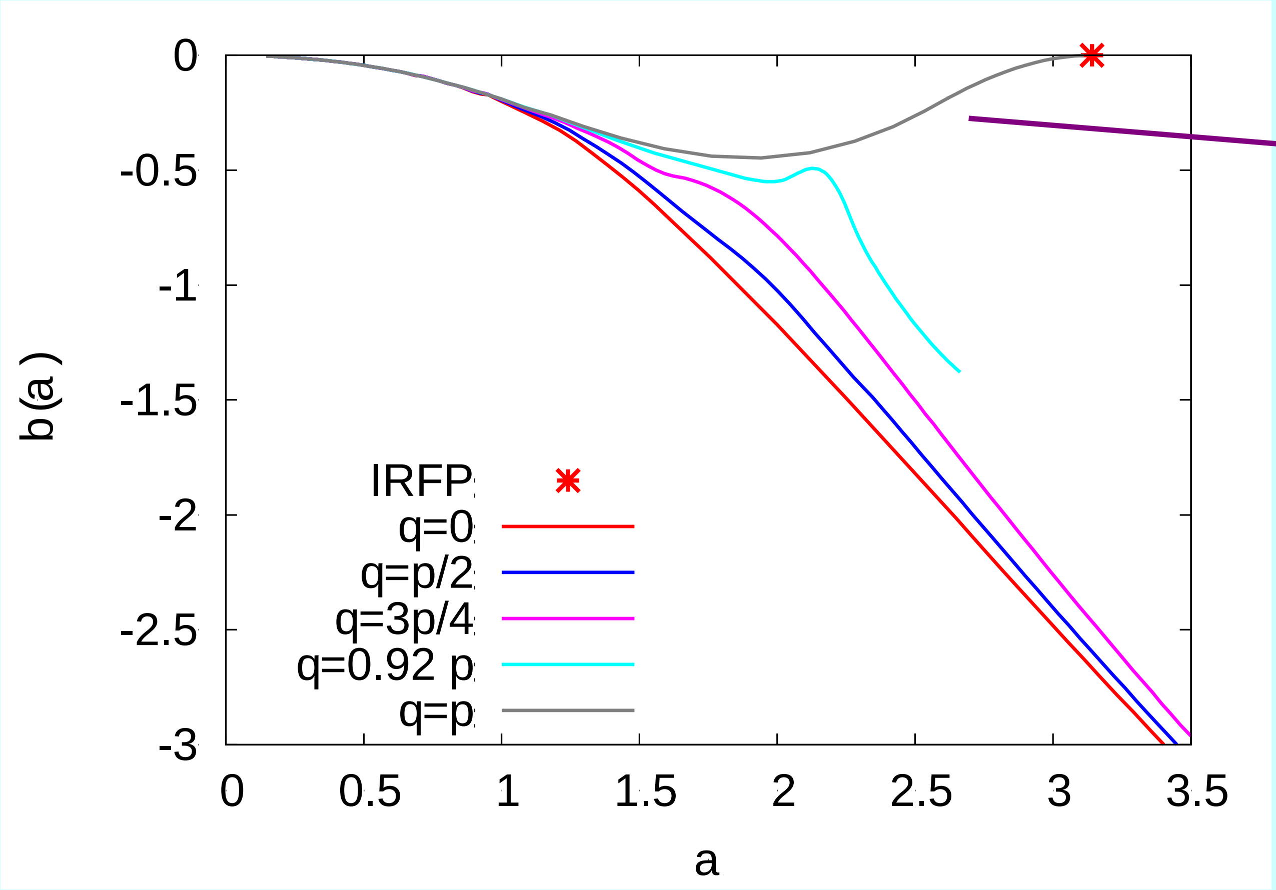
very slow approach: $$m(\theta,L)L = \pi - \frac{A}{\log(ML)} + \ldots$$

Cardy (1986),
Affleck and Haldane (1987),
Affleck et al. (1989)

marginally irrelevant operator $$SU(2)_L \times SU(2)_R \sim O(4) \longrightarrow O(3)$$

The β-function

$$\beta(\theta,\alpha) = -L\,\frac{\partial\alpha(\theta,L)}{\partial L}$$



Balog: private communication and confirmed by numerical simulations

$$\beta(\alpha) \simeq -C(\alpha-\alpha^*)^2 \quad \Rightarrow \quad \alpha(L) \simeq \alpha^* - \frac{1}{C\log(L/L_0)}$$

The slow, logarithmic approach to the IRFP is a general feature of a walking theory.

Is the enhanced symmetry at the IRFP also a general feature of a walking theory?

Finite-size scaling near the IRFP

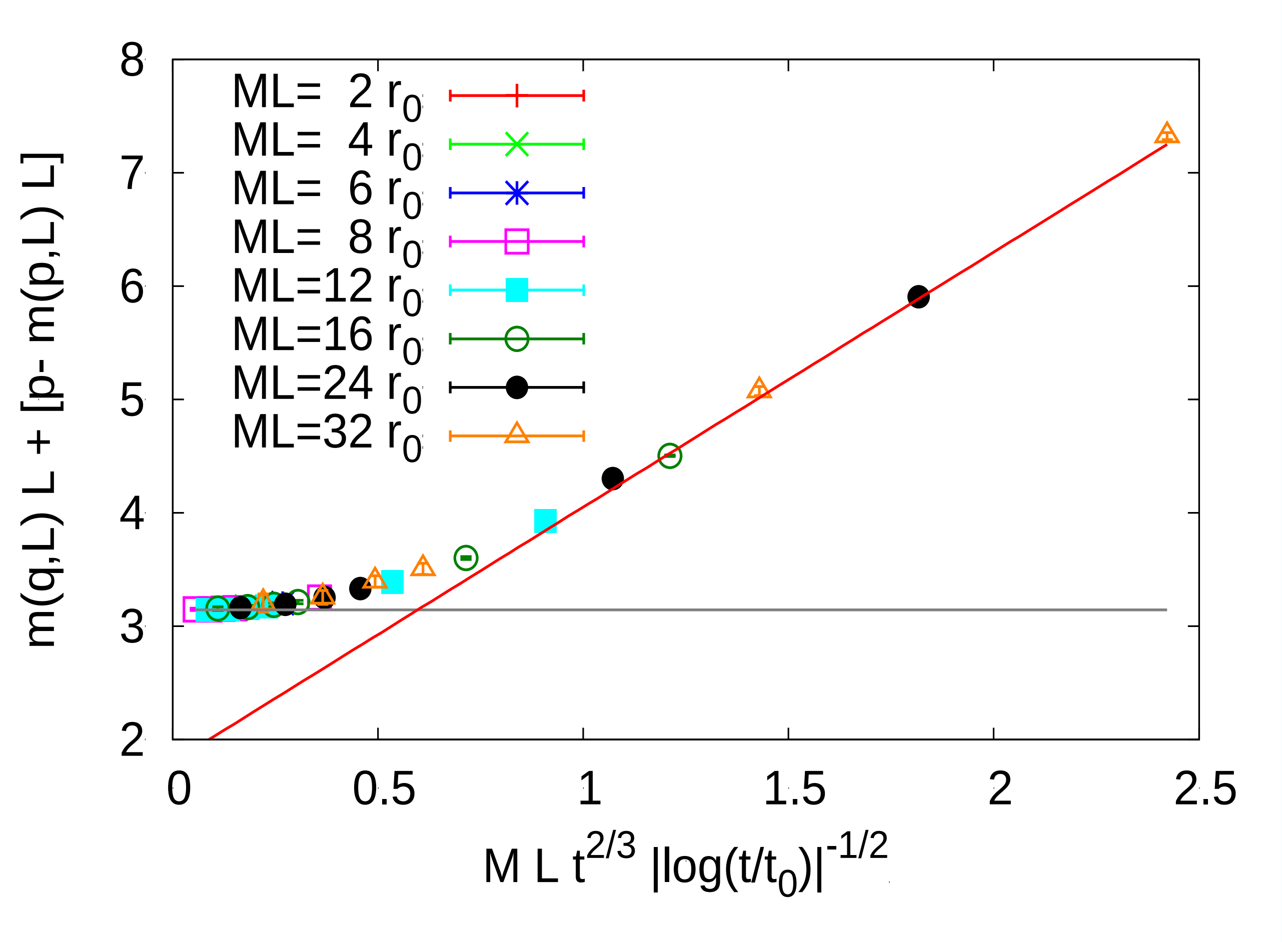
$$m(\theta, L \to \infty) \sim |\theta - \pi|^{2/3} |\log(|\theta - \pi|)|^{-1/2}$$

$$x = ML\, t^{2/3} \sqrt{|\log t/t_0|} \quad \text{where} \quad t = 1 - \frac{\theta}{\pi}$$

direct observation of FSS: $m(\theta, L)L = f(x)$ with $f(x) \xrightarrow[x \to 0]{} \pi$ and $f(x) \xrightarrow[x \to \infty]{} A\, x$

practically impossible due to the slow logarithmic approach

$$m(\theta, L)L - m(\pi, L)L + \pi$$



Conclusions

- The 2-d O(3) model with the θ-term is an ideal theoretical laboratory to investigate with high accuracy the dynamics and the main features of walking technicolor models.

- The model is affected by a hard sign problem that can be strongly reduced by the efficient meron-cluster algorithm.

- We studied the scale dependence of the renormalized coupling $\alpha(\theta, L) = m(\theta, L)L$ showing the slow, log approach to the IRFP. General feature of a walking theory.

- We measured the β function and showed evidence for the transition from a running to a walking behavior as the IRFP is approached.

- We performed a FSS study of the massgap near the IRFP and showed the collapse of the numerical data on a single universal curve.

2-d O(3) model	TC gauge models
• There is a continuous parameter, θ.	• There are 3 parameters, N_c, N_f, R.
• Logarithmic approach to the IRFP: marginally irrelevant operator $O(4) \to O(3)$	• Where does it come from? Symmetry enhancement, conformal symmetry,…?
• O(3) singlet that becomes light and degenerate with O(3) triplet as $\theta \to \pi$	• Technidilatons? Pseudo-Goldstone bosons of spontaneous breaking of conformal invariance (explicitly broken by the scale anomaly $\Lambda_{\overline{MS}}$) Yamawaki, Bando, Matumoto (1986), Hashimoto and Yamawaki (2011)
• Large lattice artifacts: $a^2 \log^3 a \sim a$	• Large lattice artifacts: fermions

Outlook: consider a different scheme for the renormalized coupling;

4-d SU(2) Yang-Mills theory with the θ term.