Lattice study on two-color QCD with six flavors of dynamical quarks

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Motivation

- The motivation of this work is to search a gauge system with nearly conformal dynamics, that serve masses of all known elementary particles, namely, $W^{\pm}$, Z^0, leptons and quarks.
- Thus far, many lattice groups have investigated dynamics of $\mathrm{SU}(3)_\mathrm{C}$ gauge theory with N_F flavors of "quarks". *i.e.*, N_F massless Dirac fermions in the fundamental representation.
- We focus on a series of $\mathrm{SU}(2)_\mathrm{C}$ with N_F flavors of quarks, and attempt to find out the system with N_F^{crt} in which $\overline{\psi}\psi$ has large anomalous dimension.

Motivation

- $\mathrm{SU}(2)_\mathrm{C}$ gauge theory is one of $\mathrm{SP}(N_C)_\mathrm{C}$ gauge theories, not $\mathrm{SU}(N_C)_\mathrm{C}$;
  - Fundamental representation is pseudo-real ($\Leftarrow$ symplectic form $\mathcal{J}_{jk}$).
  - Chiral symmetry is enhanced to $\mathrm{SU}(2N_F)$ ($\supset \mathrm{SU}(N_F)_\mathrm{L} \times \mathrm{SU}(N_F)_\mathrm{R} \times \mathrm{U}(1)_\mathrm{V}$).
  - Plausible breaking pattern: $\mathrm{SU}(2N_F) \to \mathrm{SP}(2N_F)$
  - The unbroken symmetry $\mathrm{SP}(2N_F)$ is exactly the one respected by degenerate Dirac mass terms, and lattice simulation can thus be applicable to study the system.
  - For simple charge assignments to the fermions, $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$-invariant chiral condensate can be formed, *i.e.*, $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y} \subset \mathrm{SP}(2N_F)$.
  - *Vacuum alignment* issue must be addressed. The interactions not involved in $\mathrm{SU}(2)_\mathrm{C}$ gauge theory (those originating from ETC gauge theory) could play essential roles for the vacuum with broken electroweak symmetry to be realized.

Motivation

- Confinement/deconfinement transition is (conjectured to be) second-order in pure $\mathrm{SU}(2)_{\mathrm{C}}$ Yang-Mills theory ($\equiv 3$-dim Ising model in the vicinity of T_c), while it is first-order in pure $\mathrm{SU}(3)_{\mathrm{C}}$ Yang-Mills theory.
- We anticipate that nonperturbative chiral dynamics ($\mathrm{SU}(2N_F) \to \mathrm{SP}(2N_F)$) of $\mathrm{SU}(2)_{\mathrm{C}}$ with $N_F^{\mathrm{crt}}[\mathrm{SU}(2)_{\mathrm{C}}]$ flavors could quantitatively differ from those of $\mathrm{SU}(3)_{\mathrm{C}}$ with $N_F^{\mathrm{crt}}[\mathrm{SU}(3)_{\mathrm{C}}]$, and they will thus deserve intensive study by lattice simulation.

Motivation

- Perturbatively $\beta(g^2)_{\mathrm{SU(2)_C}}$ hints us that $N_F^{\mathrm{crt}} = (6 \sim 9)$.
- Preceding lattice study of $\mathrm{SU(2)_C}$ gauge theory with N_F flavors;
  - phase structure of Wilson fermions (Y. Iwasaki,*et.al.*,[arXiv:hep-lat/0309159])
    $\Rightarrow$ $N_F = 3$ is conformal in the infrared (IR) limit.
  - running coupling constant g^2_{SF} for $N_F = 6$ defined under Schrödinger functional (SF) boundary condition (F. Bursa, *et.al.*, arXiv:1007.3067[hep-ph])
    $\Rightarrow$ $N_F = 6$ is conformal in IR limit.
  - g^2_{SF} using clover fermion with $c_{SW}|_{\text{1-loop}}$ (T. Karavirta, *et.al.*, arXiv:1111.4104 [hep-lat])
    $\Rightarrow$ IR fixed point is also allowed for $N_F = 6$.
  - coupling constant g^2_{twist} defined by twisted boundary condition for $N_F = 8$ (H. Ohki, *et.al.*,arXiv:1011.0373 [hep-lat])
    $\Rightarrow$ $N_F = 8$ is conformal in IR limit.
- We report our preliminary result of the lattice study on $\mathrm{SU(2)_C}$ gauge theory with six-flavors.

$g^2_{\mathrm{SF}}(L)$

- We compute SF gauge coupling constant $g^2_{\mathrm{SF}}(L)$ by unimproved Wilson fermions;
  - Data with larger lattices ($L/a = 18$, $L/a = 24$) and more number of $\beta \equiv 4/g_0^2$.
  - Finer tuning of $\kappa_c(L/a,\, \beta)$ for every $(L/a,\, \beta)$ by demanding that

$$\left| am_{\mathrm{PCAC}}(L/a,\, \beta,\, \kappa)|_{\kappa=\kappa_c(L/a,\, \beta)} \right| \leq 0.0007 \,.$$

  - We employ one-loop improvement. That is, we fit the data to the presumed functional form

$$\frac{g_0^2}{g^2_{\mathrm{fit}}(L/a,\, g_0^2)} = \frac{1 - a^{(1)}_{L/a}\, g_0^4}{1 + p_{L/a,\,1} g_0^2 + \sum_{k=2}^{N} a^{(k)}_{L/a}\, g_0^{2k}} = \frac{1}{1 + p_{L/a,\,1} g_0^2 + O(g_0^4)} \,,$$

with the coefficient $p_{1,\, L/a}$ obtained from one-loop calculation. (See also S. Sint and P. Vilaseca, arXiv:1111.2227 [hep-lat].)

$g^2_{\rm SF}(L)$

- We show the result for the continuum limit of the discrete beta function

$$B^{(s)}(u;\, l_1 \to l_2 = s l_1) \equiv \left[\frac{1}{g^2_{\rm fit}(l_2 = s l_1,\, g_0^2)} - \frac{1}{g^2_{\rm fit}(l_1,\, g_0^2)}\right]_{u = g^2_{\rm fit}(l_1,\, g_0^2)}$$

$$= \left.\frac{1}{g^2_{\rm fit}(l_2 = s l_1,\, g_0^2)}\right|_{u = g^2_{\rm fit}(l_1,\, g_0^2)} - \frac{1}{u}\,,$$

which is negative in the asymptotically free region.

$g^2_{\mathrm{SF}}(L)$

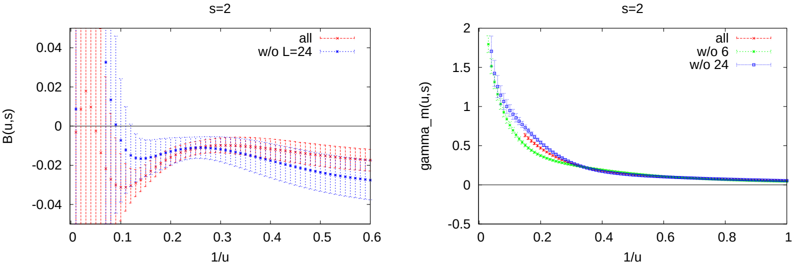

Figure: Discrete beta function (left panel) and mass anomalous dimension (right panel) versus $1/u$ with $u \equiv g^2_{\mathrm{SF}}$ for the size of step scaling $s = 2$; without $L/a = 24$ (Blue), with $L/a = 24$ (Red), and without $L/a = 6$ (Green).

Spectroscopy

- We also study dependence of mesonic spectrum on quark masses in $\mathrm{SU}(2)_\mathrm{C}$ with 6 flavors using unimproved Wilson fermion.
- Computation of lattices of $L/a = 24$ and $L/a = 32$ has been carried out using the computer "φ" at Nagoya university, which also has the servers equipped with 3 GPGPUs per node.
- HMC simulation can be done efficiently on a single node by flavor-parallelized multi-GPUs, *i.e.*, mixed precision solver with 3 single-precision pseudo-fermions distributed to the individuals of 3 GPUs (K. -I. Ishikawa, M. Hayakawa, Y. Osaki, S. Takeda, S. Uno and N. Yamada, *Improving many flavor QCD simulations using multiple GPUs*, PoS LATTICE **2010**, 325 (2010)).

Spectroscopy

We start with surveying the phase structure of Wilson fermion using small lattice, $8^3 \times 24$ or $8^3 \times 32$.

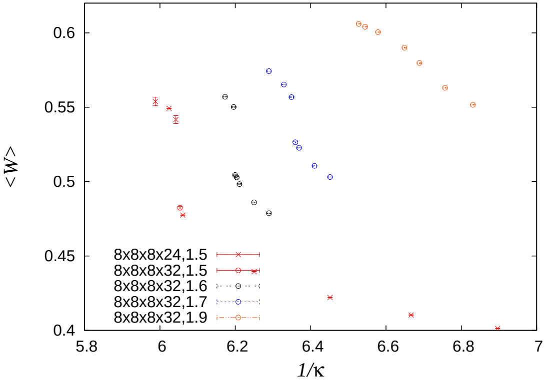

Figure: $\langle W(\Box) \rangle$ as a function of $1/\kappa$. The lattice size is $8^3 \times 32$ for $\beta = 1.6$, 1.7 and 1.9, and that of $\beta = 1.5$ is $8^3 \times 24$, except for one point.

Spectroscopy

We thus choose $\beta = 2.0$.

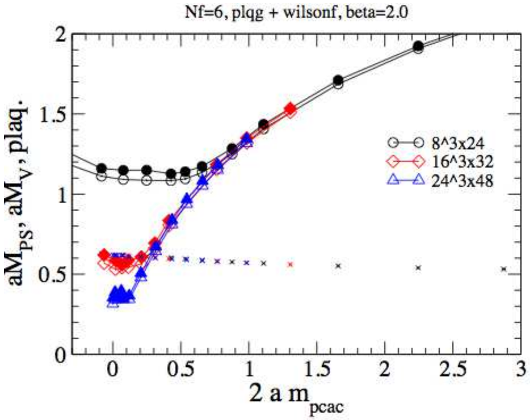

Figure: The lightest (non-singlet) pseudoscalar meson mass (empty dot) and the lightest (non-singlet) vector meson mass (filled dot) versus $2 \times m_{\mathrm{PCAC}}$ in unit of a^{-1} at $\beta = 2.0$.

Spectroscopy

- Meson masses are bounded from below at small m_{PCAC}.
- Hierarchical branching structure indicates that the saturation is attributed to finite size effect.
- High degeneracy between the lightest pseudo-scalar (PS) and vector (V) meson masses.
- Such a degeneracy may occur if size is so small that quark and anti-quark are bounded only by Coulombic force. Heavy quark potential will help to see whether this is actually the case or not.
  That is, if heavy quark potential turns out to exhibit appreciable range of linear term within our lattices,
  - (valence) quarks feel sufficient confining force,
  - high degeneracy between PS and V results from such dynamics.

Heavy quark potential and string tension

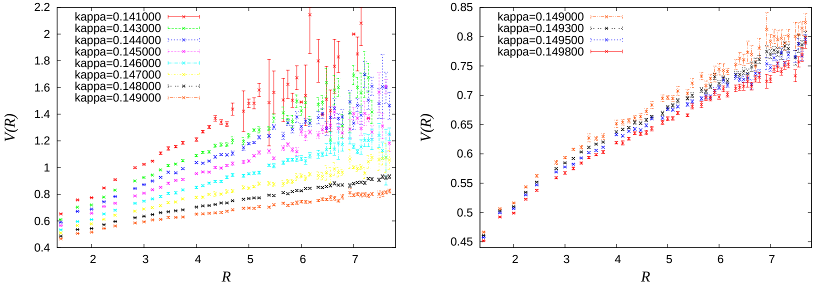

Figure: Heavy quark potential for lattice with $24^3 \times 48$ and $\beta = 2.0$. The horizontal axis is the separation between infinitely heavy quark and anti-quark in unit of lattice spacing, a. Left panel plots $a^3V(R)$ for $0.1410 \leq \kappa \leq 0.1490$. Right panel plots $a^3V(R)$ for $0.1490 \leq \kappa \leq 0.1498$ with vertical axis rescaled.

Heavy quark potential and string tension

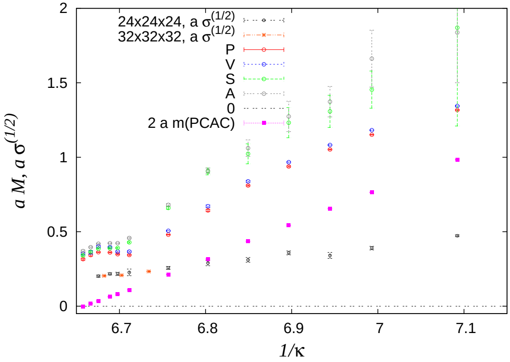

Figure: Meson masses, PCAC quark mass and the square root $\sigma^{\frac{1}{2}}$ of string tension σ in unit of a^{-1}, versus $1/\kappa$ for lattice with $24^3 \times 48$ and $\beta = 2.0$.

- The result for $a\sqrt{\sigma}$ obtained from the lattice with $32^3 \times 64$, $\beta = 2.0$ is also plotted (orange dot), indicating negligible finite size effect.
- Non-vanishing σ at $am_{\mathrm{PACA}} \ll 1$ indicates confinement in massless theory.

Polyakov loop

- Anti-periodic boundary condition is equivalent to periodic boundary condition in $\mathrm{SU}(2)_{\mathrm{C}}$ with N_F Dirac fermions in the fundamental representation (Z_2-odd conjugacy class);
    - $-\mathbb{I}_2 \in \mathrm{SU}(2)_{\mathrm{C}}$.
    - The minus sign realizing anti-periodic boundary condition in the hopping term in the t-direction, say,

$$
\begin{aligned}
&+\kappa \sum_{t=1}^{N_T-1} \sum_{\mathbf{x}} \overline{\psi}(t,\,\mathbf{x})\;(1-\gamma_0)\,U((t,\,\mathbf{x}),\,0)\psi(t+1,\,\mathbf{x}) \\
&+\kappa \sum_{\mathbf{x}} (-1)\overline{\psi}(N_T,\,\mathbf{x})\;(1-\gamma_0)\,U((N_T,\,\mathbf{x}),\,0)\psi(1,\,\mathbf{x})\,,
\end{aligned}
$$

    can be absorbed into link fields, $(-1)U((N_T,\,\mathbf{x}),\,0) \equiv U^{\diamondsuit}((N_T,\,\mathbf{x}),\,0)$, without changing gauge action, in which every $U((N_T,\mathbf{x}),\,0)$ always appears together with $U((N_T,\mathbf{x}'),\,0)^{-1}$ in all terms.
    - Hence, for instance, the results for Z_2-even quantities in $32^3 \times 64$ can be regarded as those in $32^2 \times 64 \times 32_A$.

Polyakov loop

- Anticipating that finite size effect does not modify qualitative features even though the aspect ratio $N_Z/(L/a) \sim 1$, the average $\langle P \rangle_{\rm P.B.C.} = - \langle P \rangle_{\rm A.P.B.C}$ of Polyakov loop P wrapped around the direction of $N_Z = 32_P$ was measured for $\beta = 2.0,\ \kappa = 0.14965$.

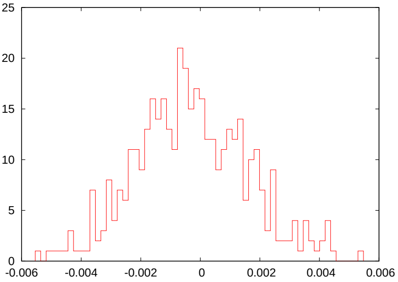

Figure: Distribution of Polyakov loops along z-direction ($N_Z = 32$, periodic) for the lattice with $32^3 \times 64$, $\beta = 2.0$, $\kappa = 0.14965$. $\langle P \rangle = -0.00028\ (10)$, tiny asymmetry due to dynamical quarks.

Polyakov loop

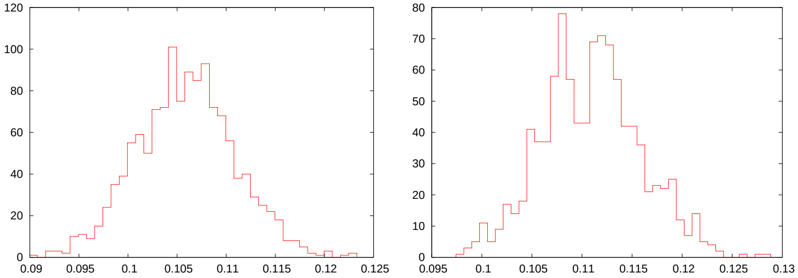

Figure: Distribution of Polyakov loops wrapped around anti-periodic temporal direction (8_A) for the lattice with $18^2 \times 48 \times 8_A$, $\beta = 2.0$. Left panel is for $\kappa = 0.1492$ and right for $\kappa = 0.1498$. All data have the same sign, signaling deconfinement.

- Gauge configurations for $18^2 \times 48 \times 8_A$ were obtained with anti-periodic boundary condition imposed explicitly on fermion fields using a single core with newest *iMac*.

Insight from finite size scaling

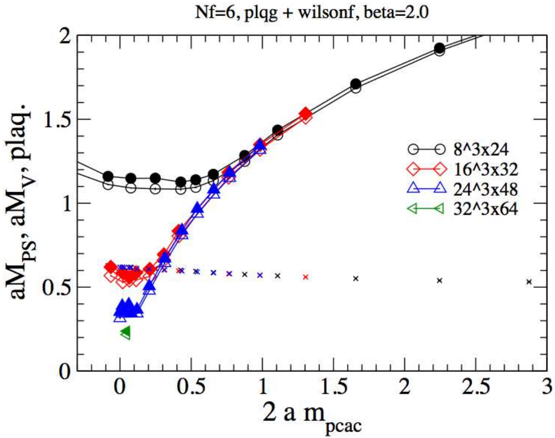

Figure: The lightest (non-singlet) pseudoscalar meson mass (empty dot) and the lightest (non-singlet) vector meson mass (filled dot) versus $2 \times m_{\rm PCAC}$ in unit of a^{-1} at $\beta = 2.0$. Here, the result for $L/a = 32$ is included.

Insight from finite size scaling

- Finite size effect increases meson masses, as in ordinal QCD ($\mathrm{SU}(3)_\mathrm{C}$ gauge theory with light 2-flavors).
- Such a tendency is in contrast to that of $\mathrm{SU}(2)_\mathrm{C}$ gauge theory with 2 adjoint fermions (L. Del Debbio, *et.al.*, arXiv:1111.4672 [hep-lat].) , where finite size effect tends to decrease meson masses.
- $\mathrm{SU}(2)_\mathrm{C}$ gauge theory with 2 adjoint fermions is shown to have scalar singlet (glueball) 0_{++} lighter than any mesons for small quark masses (L. Del Debbio, *et.al.*, arXiv:1004.3206 [hep-lat]);

$$\frac{M_{0++}}{M_P} \sim 0.73\,, \quad \frac{\sqrt{\sigma}}{M_P} \sim 0.14 \quad \text{in } \mathrm{SU}(2)_\mathrm{C} + 2 \times \text{adjoint}\,.$$

- Such a tendency seems to be compatible with Lüscher's formula for the finite size effect on the meson mass;

$$\begin{aligned} M_P(L) - M_P = &- \frac{3}{16\pi M_P^2 L} \lambda^2 (\cdots < 0) \\ &- \frac{3}{16\pi L} \int_{-\infty}^{\infty} dy\, e^{-M_P\sqrt{1+y^2}} F(iM_P y)(\cdots > 0)\,. \end{aligned}$$

  where λ is dimensionful $0_{++}PP$ coupling constant. *If* 0_{++} *survives as an active D.O.F in the low energy effective theory*, it is possible that the first term can dominate over the second term.
- Knowledge on the mass of 0_{++} is important to understand dynamics of the target system, $\mathrm{SU}(2)_\mathrm{C}$ + 6-flavors.