# Infrared Fixed Point of SU(3) gauge theory with 12 flavor staggered fermions

Kenji Ogawa
National Chiao-Tung University
⇒ Chung Yuan Christian University

lattice 2012, Cairns

*In this talk, I show my trial to extract RG Beta function by simultaneous fitting w.r.t. bare coupling and lattice size. I show the application of this method to the data in arXiv: 0901.3766. (T.Appelquist et.al,) and arXiv: 1205.6076 (D.Lin et.al,)*

## Outline

Motivation

Details of Method

Application

Summary and Comments

# Motivation

In search of BSM (Walking Technicolor)

Step Scaling Function with fixed step size

=>

- The result might depend on step size s
- Beta interpolation + continuum extrapolation makes interpretation of chi-square complicate

Extract beta function from simultaneous fit in a limited region

Use more Information from data

(SSF with all available combination of step size)

One chi-square

# Review of Step Scaling Function

$$\sigma(u\,;s) = g^2(sL)\big|_{g^2(L)=u}$$

Relation between beta function

$$-\int_u^{\sigma(u;s)} \frac{1}{\beta(u)}\,du' = \log s$$

Continuum SSF satisfies

$$\sigma(u\,;s_1 s_2) = \sigma\left(\sigma(u\,;s_1)\,;s_2\right)$$

Lattice SSF

$$\begin{aligned}\Sigma(u\,;s,\hat{L}) &= g^2_{\mathrm{lattice}}\left(g_0^2, s\hat{L}\right)\Big|_{g^2_{\mathrm{lattice}}(g_0^2,\hat{L})=u} \\ &= \sigma(u\,;s) + f(u\,;s,\hat{L}) \\ &= \sigma(u\,;s) + f^{(1)}(u\,;s)\frac{1}{\hat{L}} + f(u\,;s)^{(2)}\frac{1}{\hat{L}^2} + \cdots\end{aligned}$$

which satisfies

$$\Sigma\left(u\,;s_1 s_2,\hat{L}\right) = \Sigma\left(\Sigma(u\,;s_1,\hat{L});s_2,\underline{s_1\hat{L}}\right)$$

# Extracting Beta function by Simultaneous Fitting

I. Parametrize beta function
and derive the relation between lattice SSF Σ

II. Fix the parameters by simultaneous fitting with,

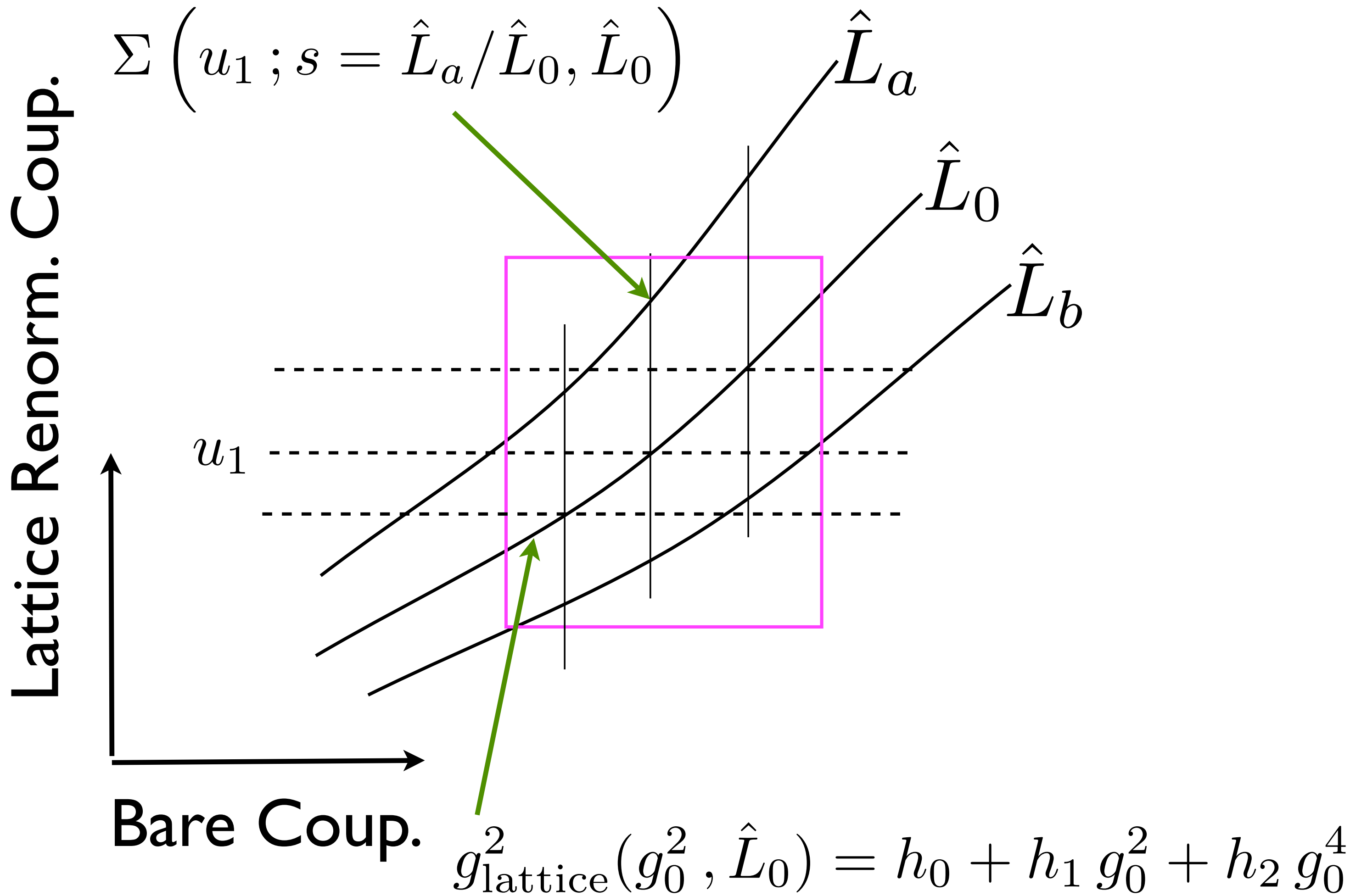
$$g^2_{\rm lattice}(g_0^2, \hat{L}) = \Sigma\left( g^2_{\rm lattice}(g_0^2, \hat{L}_0)\,;\, s = \hat{L}/\hat{L}_0, \hat{L}_0 \right)$$

$\hat{L}_0$ : Base Lattice Size
Can be any number between minimum and maximum lattice size in data.

One have to make up a fit function for $g^2_{\rm lattice}(g_0^2\,, \hat{L}_0)$

e.g. $g^2_{\rm lattice}(g_0^2\,, \hat{L}_0) = h_0 + h_1\, g_0^2 + h_2\, g_0^4$

We do simultaneous fit, but don't do global fit



$\Sigma\left(u_1\,; s = \hat{L}_a/\hat{L}_0, \hat{L}_0\right)$
$\hat{L}_a$
$\hat{L}_0$
$\hat{L}_b$
$u_1$
Lattice Renorm. Coup.
Bare Coup.
$g^2_{\mathrm{lattice}}(g_0^2\,, \hat{L}_0) = h_0 + h_1\, g_0^2 + h_2\, g_0^4$

# General Procedure to Constrain Lattice SSF

Parametrize Beta Function

$$\beta(u) = \beta(u; \{c_i\})$$

Get the expression of sigma by Integration

$$-\int_{u}^{\sigma(u;s)} \frac{1}{\beta(u)}\, du' = \log s \quad \Rightarrow \quad \sigma(u\,;s) = \sigma(u\,;s,\{c_i\})$$

Lattice SSF is given as,

$$\Sigma(u\,;s,\hat{L}) = \sigma(u\,;s) + f(u\,;s,\hat{L})$$

I assume that f is analytic function of u, s, ^L .

Lattice SSF satisfy the relation,

$$\Sigma\left(u\,;s_1 s_2,\hat{L}\right) = \Sigma\left(\Sigma(u\,;s_1,\hat{L});s_2,s_1\hat{L}\right)$$

The above relation constrains the lattice artifact $f(u\,;s,\hat{L})$

## Simplest Case: Beta Function = Constant

$$-\int_{u}^{\sigma(u;s)} \frac{1}{c}\, du' = \log s \;\Rightarrow\; \sigma(u\,;s) = u - c\,\log s$$

$$\Sigma(u\,;s,\hat{L}) = \sigma(u\,;s) + f(u\,;s,\hat{L})$$

When lattice artifact is $\mathcal{O}(a)$, $f(u\,;s,\hat{L}) = f(u\,;s)\frac{1}{\hat{L}}$

Condition for Lattice SSF

$$\Sigma\left(u\,;s_1 s_2,\hat{L}\right) = \Sigma\left(\Sigma(u\,;s_1,\hat{L});s_2,s_1\hat{L}\right)$$

$$\Rightarrow\; f(s_1 s_2) = f(s_1) + f(s_2)\frac{1}{s_1}$$

u dependence are omitted

This is satisfied with $f(s) = k\left(\frac{1}{s} - 1\right)$

$$\boxed{\Sigma(u\,;s,\hat{L}) = u + c\,\log s + \frac{k}{\hat{L}}\left(\frac{1}{s} - 1\right)}$$

## Linear

u dependence are omitted

$$\beta(u) = c\,(u - u_0)$$

$$\Sigma(u\,;s,\hat{L}) = u_0 + |u - u_0|\, s^c + \frac{k_1}{\hat{L}}\left(\frac{1}{s} - s^c\right)$$

higher order of lattice artifact term is given as $\frac{k_i}{\hat{L}^i}\left(\frac{1}{s^i} - s^c\right)$

## Quadratic with/without x-intercepts

$$\beta(u) = \underset{(-)}{-}c_0^2 + c_1^2\,(u - c_2)^2$$

$$\sigma(u) = c_2 + \frac{c_0}{c_1}\underset{(\tan)}{\tanh}\left(c_0\,c_1 \log s - \underset{(\arctan)}{\operatorname{arctanh}}\left(c_1\frac{c_2 - u}{c_0}\right)\right)$$

lattice artifact term are not simple

## Perturbation 1-Loop

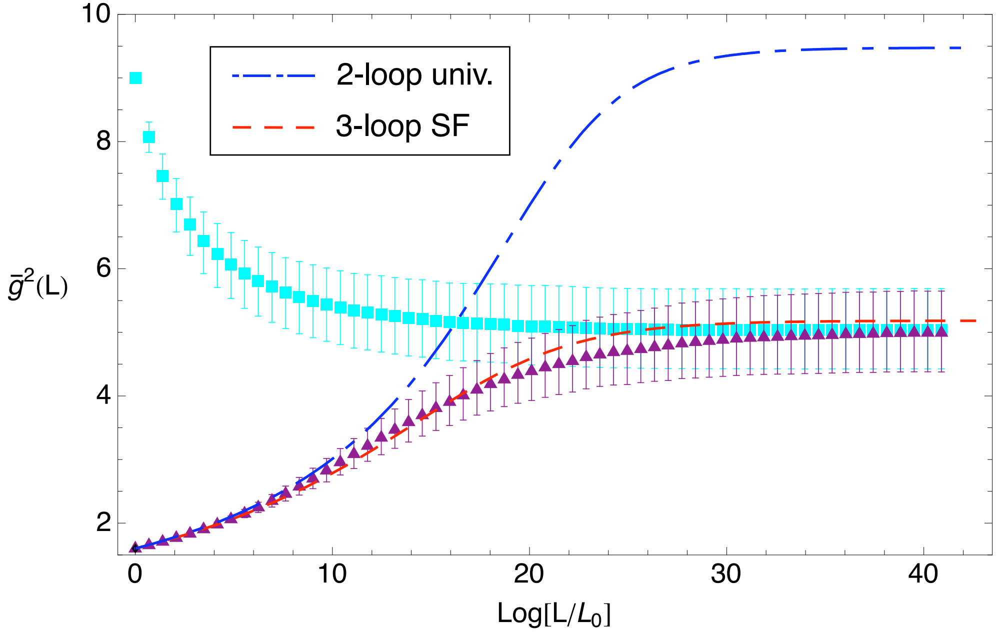
$$\beta(u) = -c_0 u^2 \qquad \sigma(u) = \frac{u}{1 - c_0\, u \log s}$$

# Application 1

*arXiv: 0901.3766. (T.Appelquist et.al,)*

Schrodinger Functional

s=2, L=6,8,10 3 point constant fit




renorm. coup
More data points in
smaller lattice

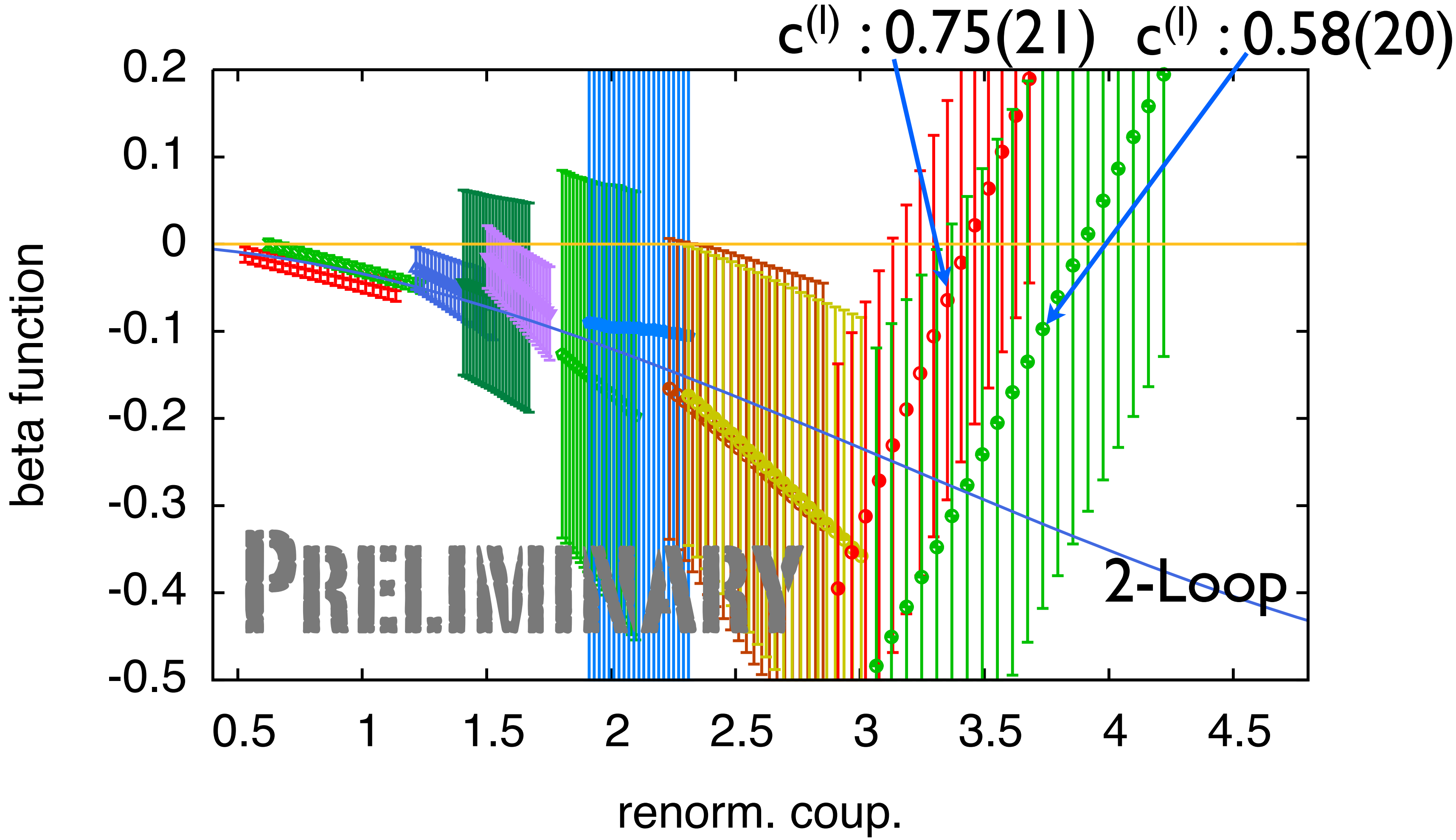
Our Fit

Linear Beta Function + O(a) lattice artifact

15 data points are used in each region after sorting

# Beta Function by simultaneous fitting

$$\beta^{(l)}(u) = c^{(l)}\,(u - u_0^{(l)})$$




Chisq/dof 0.6 ~2.2

# Application II

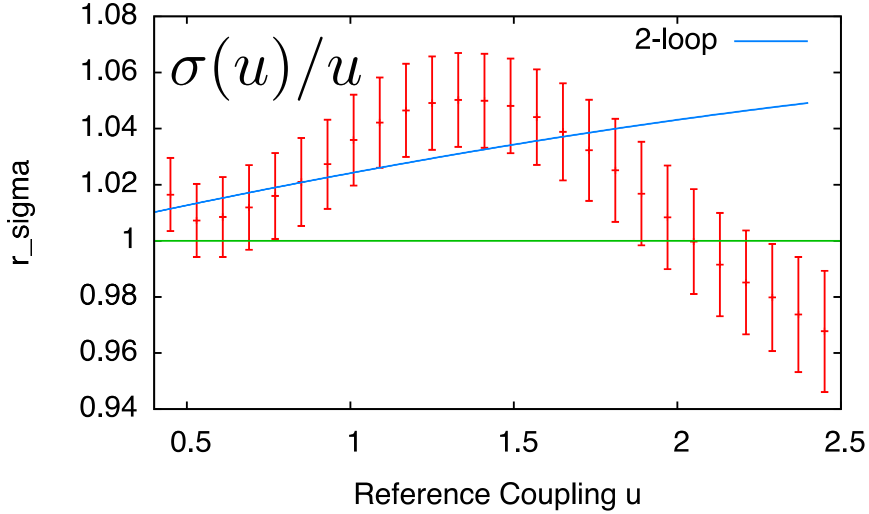
*arXiv: 1205.6076 (D.Lin et.al,)*

Polyakov Loop Scheme
step size s = 2
3 point linear fit

| $L/a$ | $\beta$ | $\bar{g}^2_{\rm latt}$ | # of traj. |
|---|---|---|---|
| 16 | 5.30 | 3.065 ( 71 ) | 321200 |
| 16 | 5.36 | 3.06 ( 11 ) | 187232 |
| 16 | 5.50 | 2.950 ( 67 ) | 256050 |
| 16 | 5.53 | 2.953 ( 83 ) | 191286 |
| 16 | 5.70 | 2.851 ( 63 ) | 235080 |
| 16 | 5.81 | 2.728 ( 70 ) | 186009 |
| 16 | 6.12 | 2.490 ( 65 ) | 183776 |
| 16 | 6.47 | 2.387 ( 44 ) | 273140 |
| 16 | 6.50 | 2.259 ( 57 ) | 286230 |
| 16 | 6.76 | 2.165 ( 66 ) | 136446 |
| 16 | 7.11 | 1.997 ( 40 ) | 244791 |
| 16 | 7.82 | 1.697 ( 47 ) | 136365 |
| 16 | 8.00 | 1.725 ( 50 ) | 141570 |
| 16 | 8.45 | 1.520 ( 24 ) | 368201 |
| 16 | 9.00 | 1.379 ( 41 ) | 114100 |
| 16 | 9.42 | 1.229 ( 28 ) | 147603 |
| 16 | 11.15 | 0.964 ( 26 ) | 72562 |
| 16 | 12.00 | 0.836 ( 17 ) | 118000 |
| 16 | 13.85 | 0.700 ( 19 ) | 70801 |
| 16 | 15.23 | 0.566 ( 13 ) | 80752 |
| 16 | 16.00 | 0.5431 ( 89 ) | 116000 |
| 16 | 17.55 | 0.4785 ( 100 ) | 83657 |
| 16 | 18.00 | 0.469 ( 13 ) | 40000 |
| 16 | 20.00 | 0.3902 ( 86 ) | 44700 |
| 16 | 20.13 | 0.4135 ( 77 ) | 79816 |
| 16 | 50.00 | 0.1327 ( 16 ) | 60900 |
| 16 | 99.00 | 0.06326 ( 68 ) | 28050 |

| $L/a$ | $\beta$ | $\bar{g}^2_{\rm latt}$ | # of traj. |
|---|---|---|---|
| 20 | 5.70 | 2.940(58) | 1892896 |
| 20 | 6.00 | 2.663(67) | 443775 |
| 20 | 6.50 | 2.401(54) | 301480 |
| 20 | 7.00 | 2.108(45) | 430782 |
| 20 | 8.00 | 1.725(38) | 295316 |
| 20 | 9.00 | 1.450(33) | 322420 |
| 20 | 10.00 | 1.187(24) | 263795 |
| 20 | 12.00 | 0.8437(17) | 258279 |
| 20 | 14.00 | 0.6450(14) | 125942 |
| 20 | 16.00 | 0.5545(11) | 155575 |
| 20 | 18.00 | 0.4565(80) | 148488 |
| 20 | 20.00 | 0.4064(76) | 123948 |
| 20 | 50.00 | 0.1352(12) | 147168 |

TABLE IV: Raw data for the renormalised coupling in the TPL scheme.



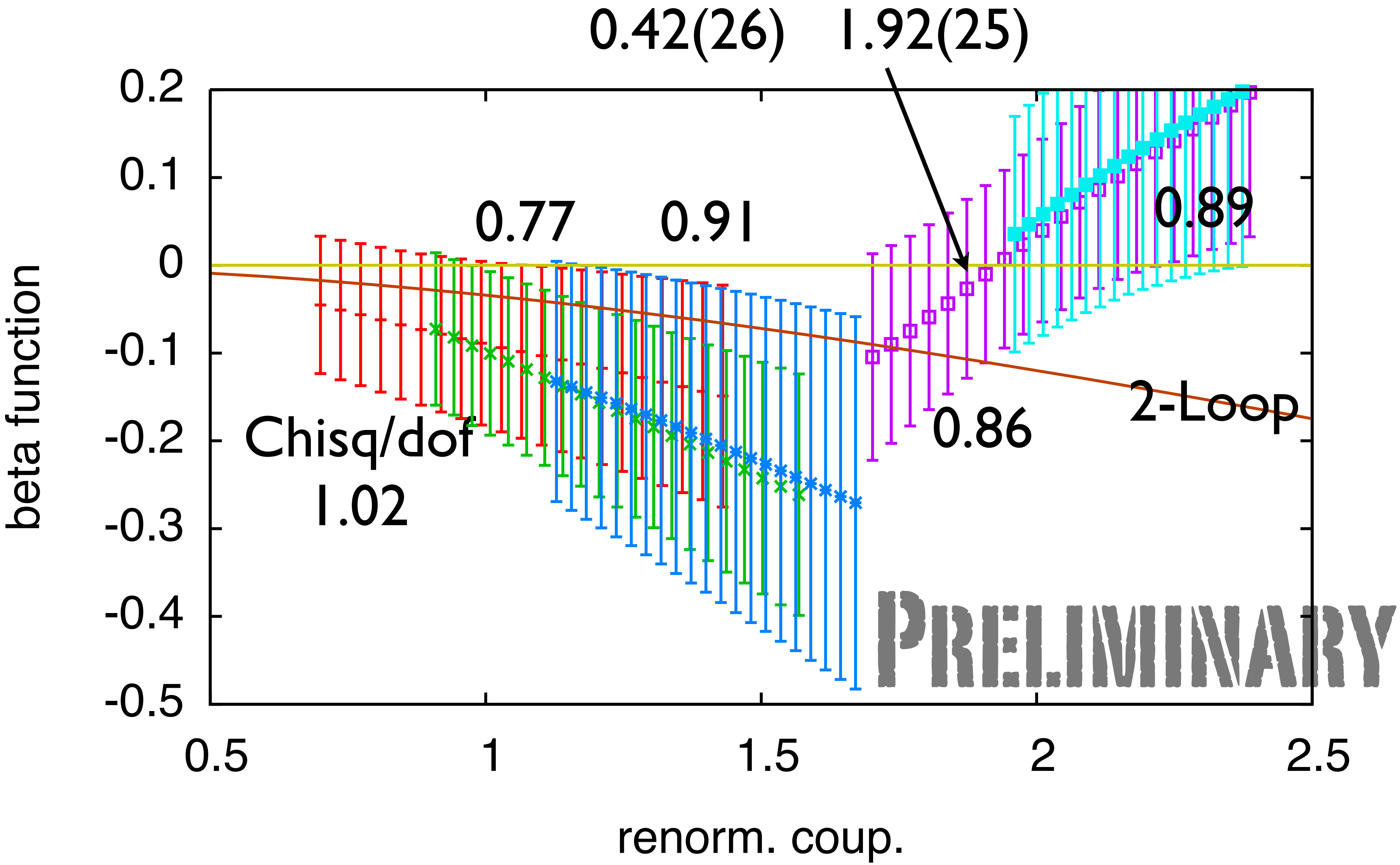

Fit assuming beta function is linear in a limited region,
with Lattice artifact $1/\hat{L}^2$

20 data points are used in each region after sorting

Small Size lattice L=6,8 data excluded

# Beta Function by simultaneous fitting

$$\beta^{(l)}(u) = c^{(l)}\,(u - u_0^{(l)})$$



# Summary and Comments

Key-equations

$$-\int_u^{\sigma(u;s)} \frac{1}{\beta(u)}\, du' = \log s \qquad \Sigma(u\,;s,\hat{L}) = \sigma(u\,;s) + f(u\,;s,\hat{L})$$

$$\Sigma\left(u\,;s_1 s_2,\hat{L}\right) = \Sigma\left(\Sigma(u\,;s_1,\hat{L});s_2,s_1\hat{L}\right) \qquad g^2_{\mathrm{lattice}}(g_0^2,s\hat{L}) = \Sigma\left(\,g^2_{\mathrm{lattice}}(g_0^2,\hat{L})\,;s,\hat{L}\right)$$

Lattice SSF is written with a simple function
when beta function is considered to be linear and
dependence of lattice artifact on u are omitted

Preferable if there are a lot of data points in a small region

Similar Result with SSF with fixed s

We treated a system in which running is slow

To be improved