Mapping the Conformal Window: SU2 with 4, 6 and 10 flavors of fermions

Jarno Rantaharju, Kari Rummukainen
Department of Physics, University of Helsinki and
Helsinki Institute of Physics,
Tuomas Karavirta and Kimmo Tuominen
Department of Physics, University of Jyväskylä and
Helsinki Institute of Physics

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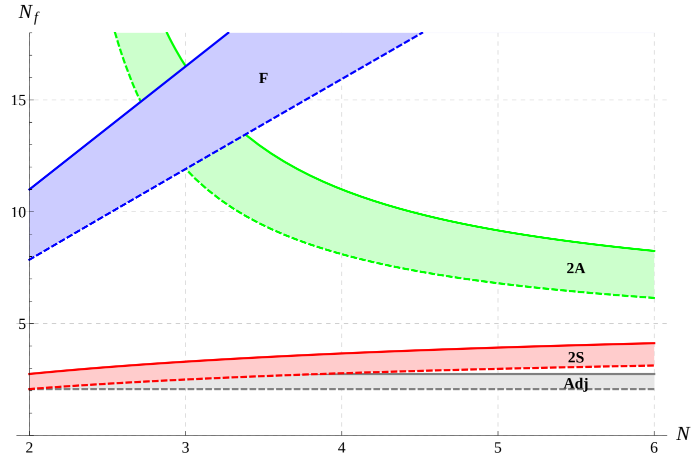
Introduction

- The conformal window
- The model
- The coupling
- Continuum limit
- Conclusions

The conformal window



The conformal window

- Well behaved upper limit when asymptotic freedom lost
- Lower limit when chiral symmetry broken before the fixed point

SU(2), $N_f = 10$ Inside the conformal window

SU(2), $N_f = 6$ Close to the lower limit

SU(2), $N_f = 4$ Below the conformal window

The conformal window

- Previous studies show large discretization errors
- Slow running requires accuracy,
  Correlation times long
- Increasing lattice size very challenging

The Model

- A perturbatively order a improved Wilson action

$$
\begin{aligned}
S =& S_G + S_F, \\
S_F =& a^4 \sum_{\alpha=1}^{N_f} \sum_x \Big[\bar{\psi}_\alpha(x)(iD_W + m_0)\psi_\alpha(x) \\
&+ a c_{\mathrm{sw}} \bar{\psi}_\alpha(x) \frac{i}{4} \sigma_{\mu\nu} F_{\mu\nu}(x) \psi_\alpha(x) \Big]
\end{aligned}
$$

The Model

- The Schrödinger Functional
  with the boundary conditions

$$U_\mu(\bar{x}, t = 0) = e^{-i\eta\sigma_3 a/L}$$
$$U_\mu(\bar{x}, t = L) = e^{-i(\pi-\eta)\sigma_3 a/L}$$

- The coupling

$$\langle \frac{\partial S}{\partial \eta} \rangle = \frac{k}{g^2}.$$

M. Luscher et al. Nucl. Phys. Proc. Suppl. **30** (1993) 139-148.

The Model

- Boundary improvement terms

$$\delta S_{c_t} = \frac{\beta_L}{4} \sum_{p_T} (c_t - 1) \mathrm{tr}(1 - U(p))$$

$$\delta S_{\tilde{c}_t} = a^4 \sum_{x} (\tilde{c}_t - 1) \frac{1}{a} \bar{\psi}(x) \psi(x) (\delta(x_0 - a) + \delta(x_0 - (L - a)))$$

The Model

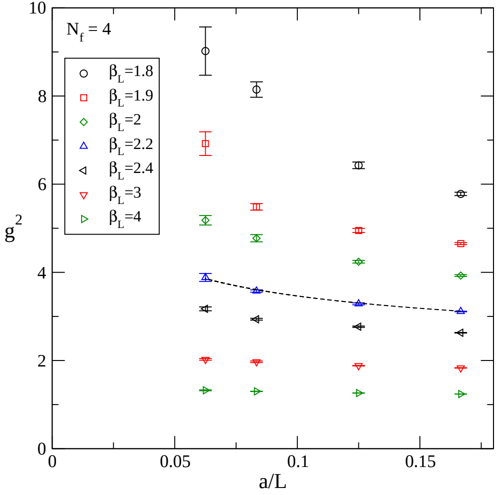
- With perturbative improvement

$$
\begin{aligned}
c_{\mathrm{sw}} =& 1 + 0.1551(1) g_0^2 + \mathcal{O}(g_0^4), \\
c_t =& [-0.0543(3) + 0.0192(2) * N_f]\, g_0^2 + \mathcal{O}(g_0^4), \\
\tilde{c}_t =& -0.0101(3) g_0^2 + \mathcal{O}(g_0^4), \\
c_A =& -0.004252(8) g_0^2 + \mathcal{O}(g_0^4)
\end{aligned}
$$

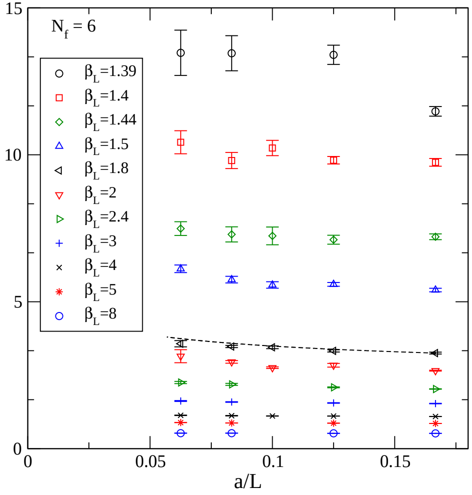
M. Luscher and P. Weisz, Nucl. Phys. B **479** (1996) 429.

T. Karavirta et al. JHEP **1106** (2011) 061.

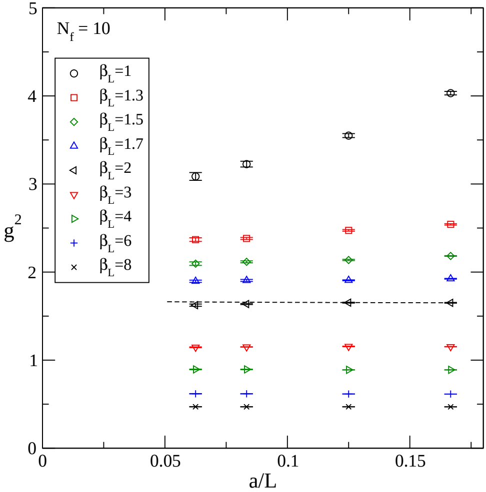
The Coupling, $N_f = 4$



The Coupling, $N_f = 6$



The Coupling, $N_f = 10$



The Continuum limit

- Continuum limit of the scaling
- Step scaling function

$$\Sigma(u, L/a) = g^2(g_0, 2L/a)|_{g^2(g_0, L/a)=u}$$
$$\Sigma(u, 2, L/a) = \sigma(u, 2) + c(u)\,(L/a)^{-2}$$

- Keeping g^2 constant

M. Luscher et al. Nucl. Phys. B **384**, 168 (1992).

The Continuum limit

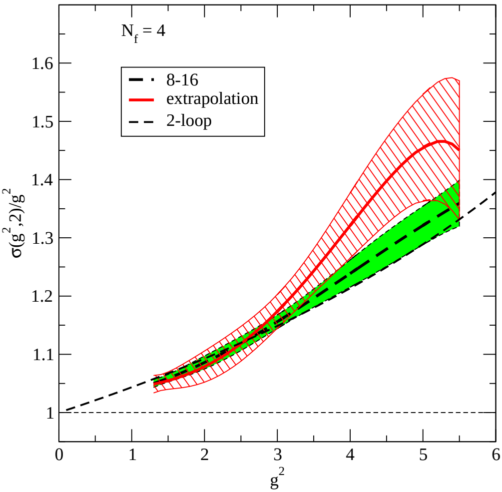
- To keep the coupling constant, interpolate between the measurements

$$\frac{1}{g^2(g_0^2, L/a)} = \frac{1}{g_0^2}\left[\frac{1+\sum_{i=1}^{n} a_i g_0^{2i}}{1+\sum_{i=1}^{m} b_i g_0^{2i}}\right]$$

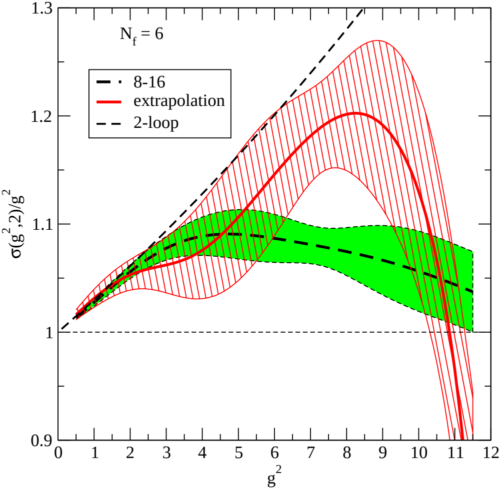
- $N_f = 4, 6$: $m = 2, n = 2$
- $N_f = 10$: $m = 1, n = 2$

T. W. Appelquist et al. Phys. Rev. Lett. **57**, 957 (1986).

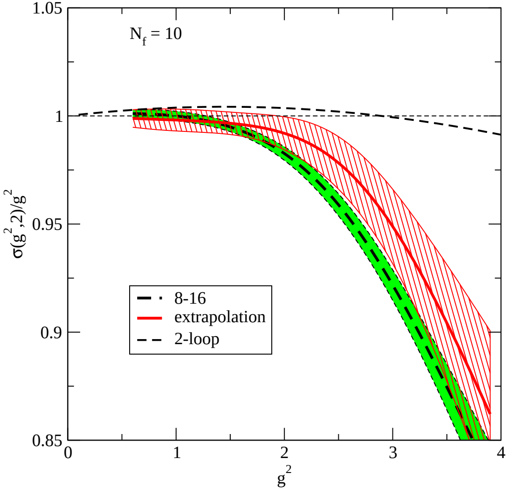
The Continuum limit, $N_f = 4$



The Continuum limit, $N_f = 6$



The Continuum limit, $N_f = 10$



Conclusions

- Determining the lower limit of the conformal window still poses problems
- Discretization effects seem large
- Clover improvement alone not enough

- Next: Find a model with reduced errors
- Hypercubic stout smearing
- Smearing in the gauge action