Light scalar spectrum in extra-dimensional gauge theories

Enrico Rinaldi

SUPA, School of Physics and Astronomy
The University of Edinburgh

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[work in collaboration with Luigi Del Debbio]

Cutoff dependence of scalar masses

Problem:

the mass of a scalar field in any 4D QFT depends on the cutoff scale

$$\delta m^2 \sim \Lambda_{\mathrm{UV}}^2$$

How can we get a light scalar and cancel the cutoff dependence?

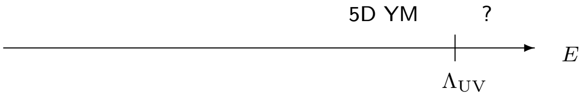
Higher dimensional Effective Field Theories

Start with a theory in five dimensions

- Consider a Yang–Mills theory in 5 dimensions

$$\mathcal{S} = \mathrm{Tr} \int \mathrm{d}^4x \int \mathrm{d}x_5 \; -\frac{1}{2} F_{MN} F^{MN}$$

- this 5D gauge theory is perturbatively non–renormalizable and is considered in the framework of Effective Field Theories
- an ultra–violet cutoff Λ_{UV} must be kept in place for the theory to be well defined: it determines the energy scale of our ignorance



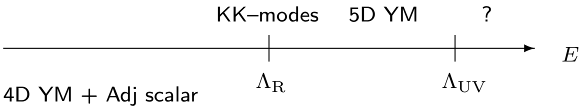
Dimensional reduction and scalar particles

Compactify one dimension on $\mathcal{S}_1$ with radius R

- a massless scalar field appears naturally in 4D from the compactified component of the higher–dimensional gauge vector field
- other massive particles (KK–modes) appear from this compactification at energies $E \sim \Lambda_{\mathrm{R}} \approx R^{-1}$
- by integrating out degrees of freedom heavier than Λ_{R}, the low energy effective action is

$$\mathcal{S}_{\mathrm{eff}} \;\sim\; 2\pi R \,\mathrm{Tr} \int \mathrm{d}^4x \; - \frac{1}{2} F^{(0)}_{\mu\nu} F^{(0)\mu\nu} + (D_\mu A^{(0)}_5)^2$$

- therefore we have an effective 4D YM + massless adjoint scalar at $E \ll \Lambda_{\mathrm{R}} \approx R^{-1}$



Light scalar from compactified extra dimensions

The resulting effective 4D action allows us to write a gauge–invariant mass term for the scalar

$$\mathcal{S}_{\text{eff}} \;\sim\; \int \mathsf{d}^4x \;-\frac{1}{2}\mathrm{Tr}\;[F_{\mu\nu}F^{\mu\nu}] + \mathrm{Tr}\;(D_\mu A_5)^2 + m_5^2 \mathrm{Tr}\;A_5^2$$

The 1–loop correction to the zero bare mass can be calculated in perturbation theory using different approaches:

- in the 4D effective field theory by accounting for all the KK–modes in the sum [Cheng]
- by writing an effective potential for a background field [Hosotani]
- using an explicit realization of a 5D theory regularized at $\Lambda_{\mathrm{UV}} \gg \Lambda_{\mathrm{R}}$ [Del Debbio]

The last calculation suggests that any regularization that preserves locality and gauge invariance will give the same result, independent of the cutoff scale, as long as $\Lambda_{\mathrm{UV}} \gg \Lambda_{\mathrm{R}}$

$$\delta m_5^2 \;=\; \frac{9 g_4^2 N_c}{16\pi^2 R^2}\zeta(3)$$

SU(2) Yang–Mills theory on the lattice

Motivations:

- perturbation theory is not the whole story: can we still say that the scalar mass is independent of the cutoff if the coupling constant is not small?
- 4D Yang–Mills theories develop a dynamical mass gap σ non–perturbatively
- if $m_5 \gg \sqrt{\sigma} \to$ decouples from the 4D physics
- the lattice provides a gauge–invariant regularization that allows us to study a non–renormalizable theory, by keeping the cutoff at all times
- the model in the lattice regularization can be studied non–perturbatively using Monte Carlo numerical simulations

[Ejiri, de Forcrand, Farakos, Knechtli]

SU(2) Yang–Mills theory on the lattice

Simulations are performed on asymmetric ${N_4}^4 \times N_5$ lattice with the Action

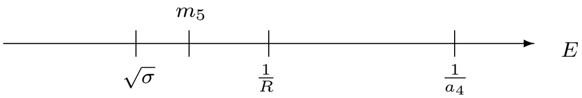
$$\mathcal{S}_W = \beta_4 \sum_{x;1\leq\mu\leq\nu\leq 4} \left[1 - \frac{1}{2}\mathrm{Re\,Tr}\ P_{\mu\nu}(x)\right] + \beta_5 \sum_{x;1\leq\mu\leq 4} \left[1 - \frac{1}{2}\mathrm{Re\,Tr}\ P_{\mu 5}(x)\right]$$

- The model has 4 tunable parameters:

$$(\beta_4, \beta_5, N_4, N_5) \quad \text{or} \quad (\beta, \gamma, N_4, N_5)$$

- The first 2 are the coupling constants and dynamically set the 2 lattice spacings: a_4 in the 4D subspace and a_5 in the extra direction
- γ is the bare anisotropy and, at tree level, corresponds to $\gamma \sim \xi = a_4/a_5$
- Restrict to $\gamma \geq 1$ gives $a_4 \geq a_5$ and $\Lambda_{\mathrm{UV}} \sim a_4^{-1}$
- The spatial volume is $V = (a_4 N_4)^3$
- The size of the extra dimension is $L_5 = 2\pi R = a_5 N_5$ and the compactification scale is $\Lambda_{\mathrm{R}} \sim 1/a_5 N_5$

Low energy regime and scale separation



- separate the compactification scale from the cutoff scale $\frac{\Lambda_{\mathrm{UV}}}{\Lambda_{\mathrm{R}}} \gg 1$

$$\frac{a_5 N_5}{a_4} = \frac{N_5}{\xi} \sim \frac{N_5}{\gamma} \gg 1$$

- separate the 4D physics from the cutoff scale $\Lambda_{\mathrm{UV}} \sim \frac{1}{a_4}$

$$a_4\sqrt{\sigma} \ll 1 \; ; \qquad a_4 m_5 \ll 1$$

- separate the 4D physics from the compactification scale $\Lambda_{\mathrm{R}} \sim \frac{1}{a_5 N_5}$

$$a_4\sqrt{\sigma}\frac{N_5}{\xi} \ll 1 \; ; \qquad a_4 m_5 \frac{N_5}{\xi} \ll 1$$

- find a scalar mass in 4D physical units which is independent of Λ_{UV}

$$\frac{m_5^2}{\sigma} \propto \frac{1}{R^2} \approx \Lambda_{\mathrm{R}}^2$$

Features of the lattice model

- Express the energy scales of the model in units of the low–energy 4D physics $\rightarrow \sqrt{\sigma}$ (Also assume this scale does not depend on the parameters)
- Assume no N_4 dependence of the 4D physics (introduce systematic errors)
- We have 3 distinct energy scales: m_5, Λ_{R} and Λ_{UV}
- We have 3 parameters that we can play with to change the 3 scales of the model
- How does m_5 depend on the other 2 scales?
- We can do non–perturbative numerical simulations and measure m_5 directly for different values of Λ_{R} and Λ_{UV}
- We can use one–loop relations between the lattice model and the continuum theory as a guide for numerical simulations

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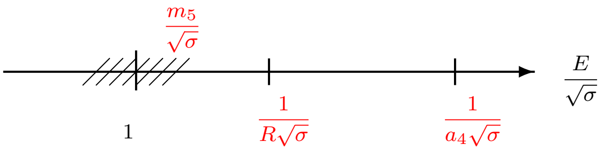

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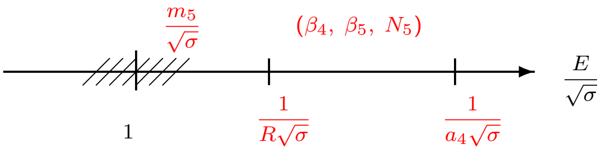

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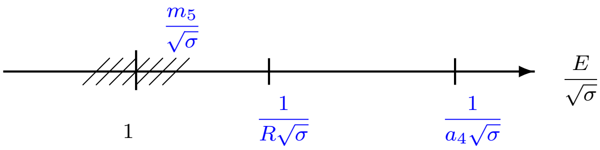

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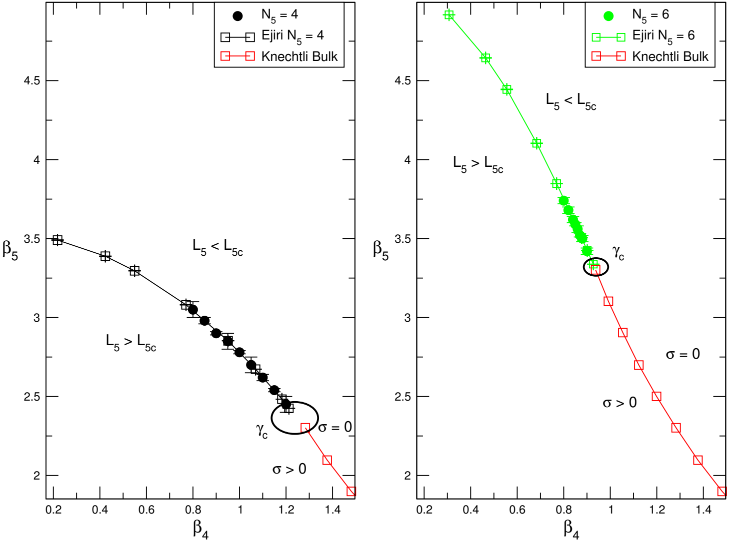
Sketch of the phase diagram



[Del Debbio, Hart, ER, arXiv:1203.2116]

Measuring masses

- We use standard lattice spectroscopic techniques and we extract masses from Euclidean 2–point functions
- We use gauge–invariant, zero–momentum lattice operators $\mathcal{O}(t)$ coupling to the states of interest, that is with the same quantum numbers and symmetries of the states whose mass we are interested in
- We correlate the operators in the time direction (which is assumed to be one of the 4 directions with N_4 lattice sites) and we average over the N_5 slices in the extra dimension
- We find the best linear combination of operators within a basis of operators with the same quantum numbers, and extract the mass from fitting its correlator at large temporal distances

$$\Phi(t) = \sum_{\alpha} v_\alpha \mathcal{O}_\alpha(t) \; ; \qquad \left\langle \Phi^\dagger(t)\Phi(0) \right\rangle = |c_0|^2 \cosh\left(m_0 t - N_t/2\right)$$

- We define the relative projection of the extracted state onto each of the basis operators $\mathcal{O}_\alpha$

$$\mathrm{proj}_\alpha = \frac{|v_\alpha|^2}{\sum_i |v_i|^2}$$

Measuring masses

1. String tension from spatial Polyakov loops

$$\mathcal{O}(t) \;=\; \sum_{x,i} l_i(x,t) \;; \qquad l_i(x,t) = \prod_{j=1}^{N_4} \mathcal{U}_i(x + j a_4 \hat{i}, t)$$

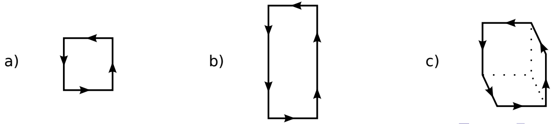
2. Scalar mass from compact Polyakov loops

$$\mathcal{O}_1(t) \;=\; \sum_{x} \mathrm{Tr}\,\left[l_5(x,t)\right] \;; \qquad l_5(x,t) = \prod_{j=1}^{N_5} \mathcal{U}_5(x + j a_5 \hat{5}, t)$$

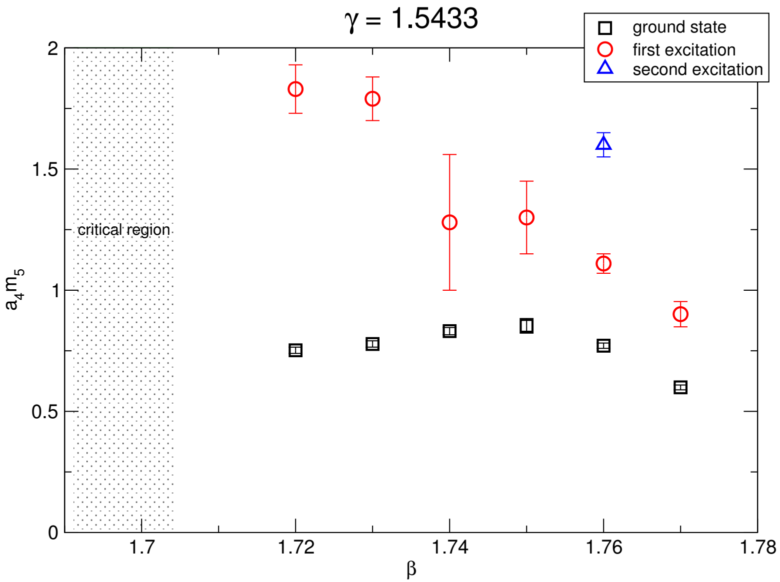
$$\mathcal{O}_2(t) \;=\; \sum_{x} \mathrm{Tr}\,\left[\phi(x,t)\phi^\dagger(x,t)\right] \;; \qquad \phi(x,t) = \frac{l_5 - l_5^\dagger}{2}$$

3. Glueball mass from spatial Wilson loops:

$$\mathcal{O}_G(t) \;=\; \sum_{x} \mathrm{Tr} \prod_{l \in \mathcal{C}(\vec{x})} U_l$$



Scalar spectrum at $\gamma \sim 1.54$



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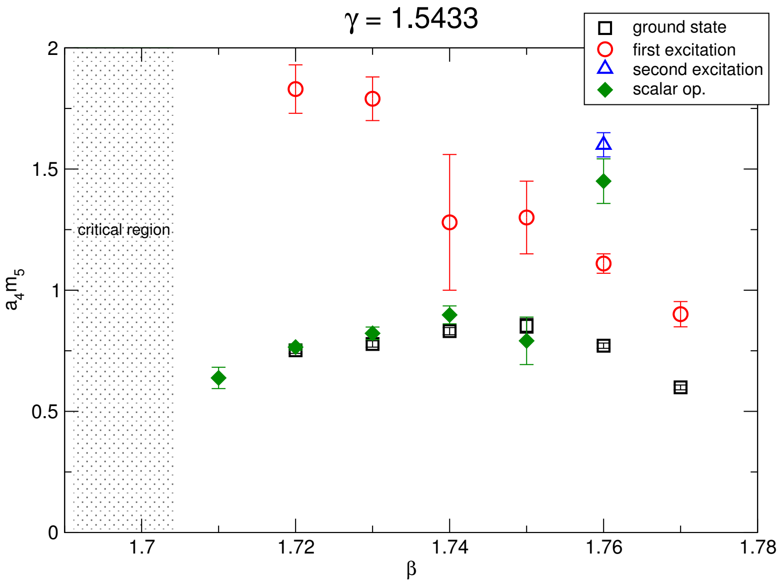
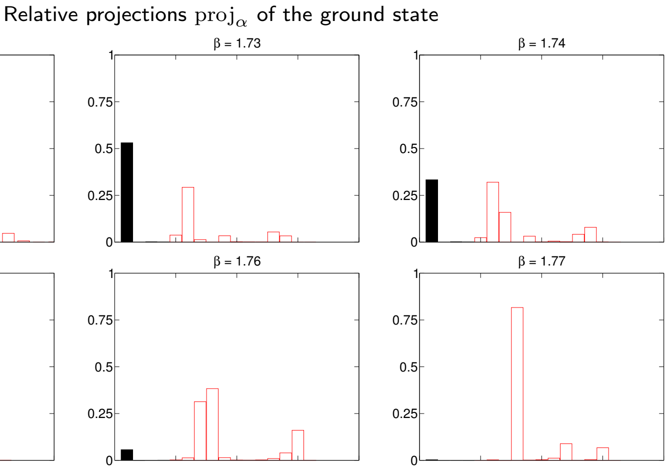
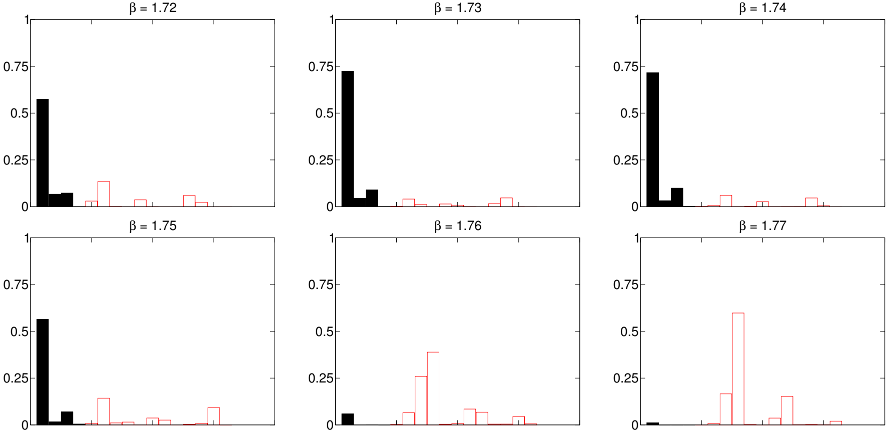


Operator content of the scalar spectrum



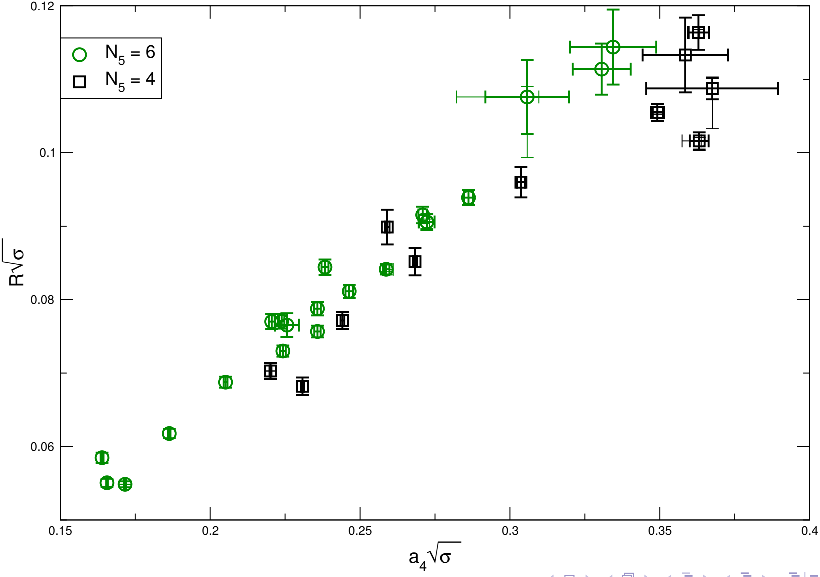
Operator content of the scalar spectrum

Relative projections proj_{α} of the 1^{st} excitation



Scalar mass dependence at $N_5 = 4$ and $N_5 = 6$

The simulated points fall in the following region of a_4 and R



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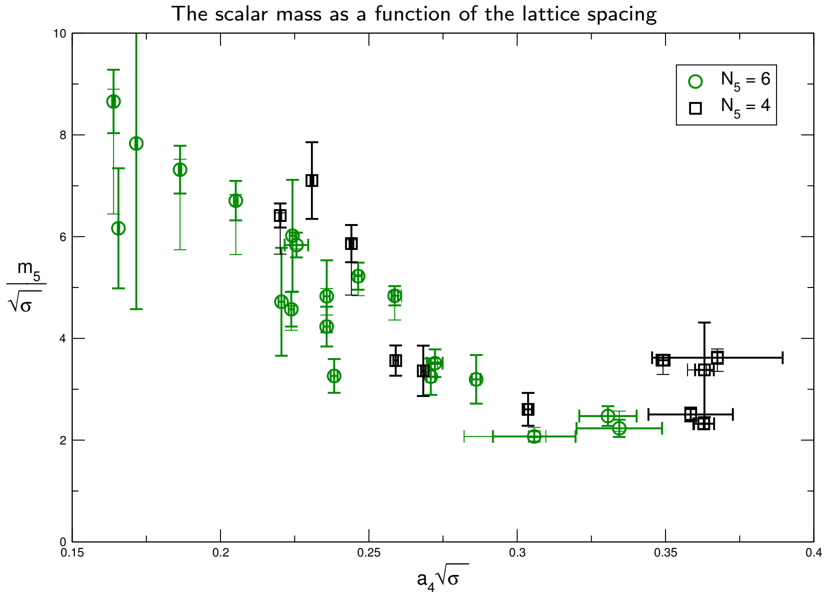

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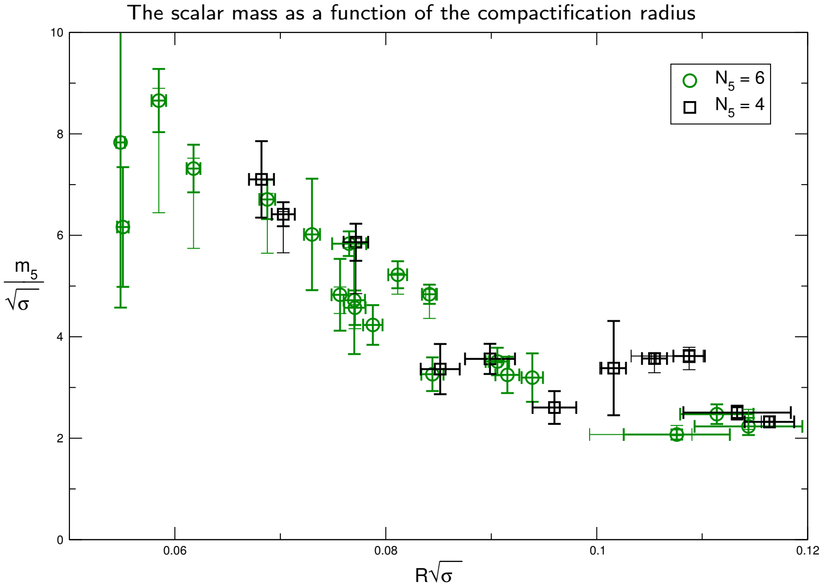

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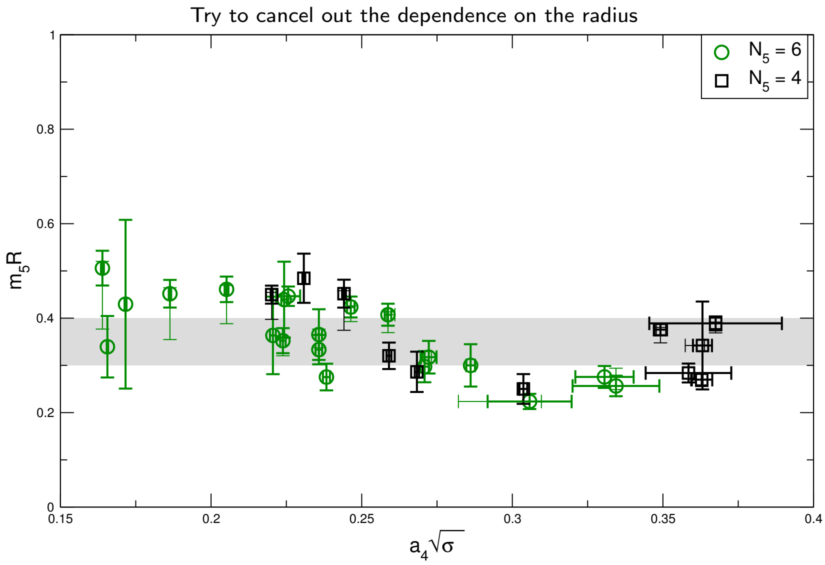

Conclusions

Summary:

- Non–perturbative study of scalar mass corrections using an explicit regularization of a non–renormalizable gauge theory
- The parameter space of the model has a very rich structure and we found a region where the desired separation of scales takes place
- We are able to follow lines of constant physics and to study the dependence of the scalar mass on the 2 energy scales of the system
- The measured scalar mass is independent of the cutoff when the separation of scales takes place and the data confirm the perturbative prediction
- Mixing with scalar glueball states becomes non negligible as the theory approaches the weak–coupling limit

Conclusions

Still a work in progress:

- The current understanding is a good starting point
- Increase N_4 to explore the region with smaller $a_4\sqrt{\sigma}$ (reduce finite size errors)
- Increase N_5 to explore the region with smaller $R\sqrt{\sigma}$ (reduce finite size errors)
- Find operators with better overlap on the adjoint scalar particle
- Match the spectrum of 5D lattice gauge theory wiht the corresponding dimensionally reduced 4D lattice theory coupled to a scalar field

References

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Hiding extra dimensions

- Models with extra dimensions have been used many times in elementary particle physics and cosmology
- Results rely on perturbation theory or string theory
- Phenomenologically interesting for Gauge-Higgs unification, hierarchy problem, dynamical EW symmetry breaking, etc...

Problem:

why don't we see extra dimensions?

Dimensional reduction to 4D can happen through different mechanisms:

- compactification (*Kaluza-Klein*)
- localisation (*brane scenario*) [ADD,Randall-Sundrum,Dvali-Shifman,Fu-Nielsen,D-theory]

Kaluza-Klein reduction

- Hide the extra dimension at low energies by making it small and compact
$$x_5 \to R\theta \qquad \theta \in [-\pi, \pi]$$
- The field–strength tensor can be written as
$$\mathcal{S} \;=\; \mathrm{Tr} \int \mathrm{d}^4 x \int \mathrm{d}x_5 \; -\frac{1}{2} F_{\mu\nu} F^{\mu\nu} - F_{\mu 5} F^{\mu 5}$$
- The gauge field can be expanded in Fourier modes and the component $A_5(x, x_5)$ can be gauge–fixed to be θ–independent (*almost* axial gauge)
$$\begin{aligned} A_\mu(x,\theta) &= A_\mu^{(0)}(x) + \sum_{n=1}^{\infty} \left[A_\mu^{(n)}(x) e^{in\theta} + A_\mu^{(n)\star}(x) e^{-in\theta} \right] \\ A_5(x,\theta) &= A_5^{(0)}(x) \end{aligned}$$
- Expanding the field–strength tensors keeping only quadratic terms gives
$$\begin{aligned} \mathcal{S} \;=& 2\pi R \,\mathrm{Tr} \int \mathrm{d}^4 x \left\{ -\frac{1}{2} (\partial_\mu A_\nu^{(0)} - \partial_\nu A_\mu^{(0)})^2 + \frac{1}{2} (\partial_\mu A_5^{(0)})^2 \right. \\ & \left. + \sum_{n=1}^{\infty} \left[-\frac{1}{2} |\partial_\mu A_\nu^{(n)} - \partial_\nu A_\mu^{(n)}|^2 + \frac{n^2}{R^2} |A_\mu^{(n)}|^2 \right] \right\} \end{aligned}$$
- Below the mass scale $m_{\mathrm{KK}} = n/R$ the quadratic action is
$$\mathcal{S}_{\mathrm{eff}} \;\sim\; 2\pi R \,\mathrm{Tr} \int \mathrm{d}^4 x \; -\frac{1}{2} F_{\mu\nu}^{(0)} F^{(0)\mu\nu} + (D_\mu A_5^{(0)})^2$$

Scale separation and the scalar mass

- The effective coupling constant $g_4 \equiv \frac{g_5}{\sqrt{2\pi R}}$ is dimensionless
- Determined by the naive running of the dimensionless $\hat{g}_5^2(E) = g_5^2(E)E$
- At the compactification scale $\Lambda_{\mathrm{R}} = R^{-1}$ we have

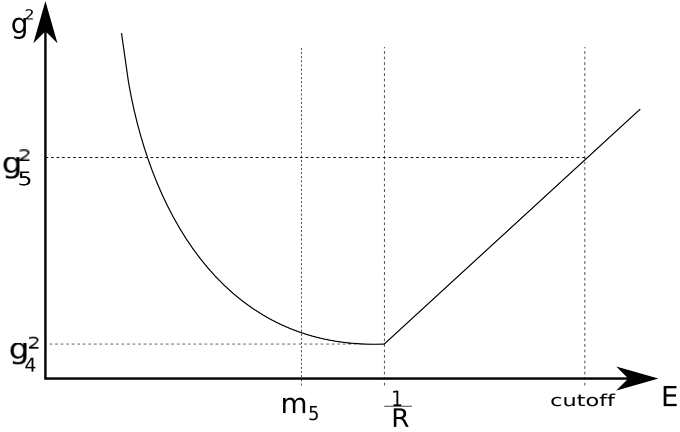
$$\hat{g}_4^2 \equiv g_4^2 = g_5^2(\Lambda_{\mathrm{R}})\Lambda_{\mathrm{R}}$$

$$\hat{g}_4^2 = \hat{g}_5^2(\Lambda_{\mathrm{UV}}) \left(\frac{\Lambda_{\mathrm{R}}}{\Lambda_{\mathrm{UV}}} \right)$$

- The scalar mass at 1–loop becomes

$$m_5^2 = \frac{9\hat{g}_5^2 N_c}{16\pi^2 R^2} \left(\frac{\Lambda_{\mathrm{R}}}{\Lambda_{\mathrm{UV}}} \right) \zeta(3)$$

Running of the coupling constant



SU(2) Yang–Mills theory on the lattice

Toy Model:

Start from the continuum 5D $\mathrm{SU}(2)$ Yang–Mills Euclidean Action

$$\mathcal{S} = \int \mathrm{d}^4x \int_0^{2\pi R} \mathrm{d}x_5 \, \frac{1}{2g_5^2} \mathrm{Tr}\, F_{MN}^2$$

and discretize it using an anisotropic Wilson Action

$$\mathcal{S}_W = \beta_4 \sum_{x;1\leq\mu\leq\nu\leq 4} \left[1 - \frac{1}{2}\mathrm{Re}\,\mathrm{Tr}\, P_{\mu\nu}(x)\right] + \beta_5 \sum_{x;1\leq\mu\leq 4} \left[1 - \frac{1}{2}\mathrm{Re}\,\mathrm{Tr}\, P_{\mu 5}(x)\right]$$

- asymmetric lattice with dimensions ${N_4}^4 \times N_5$
- periodic boundary conditions in all the 5 directions
- two equivalent parametrization can be used

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$$\mathcal{S}_W = \frac{\beta}{\gamma} \sum_{x;1\le\mu\le\nu\le 4} \left[1 - \frac{1}{2}\mathrm{Re}\,\mathrm{Tr}\; P_{\mu\nu}(x)\right] + \beta\gamma \sum_{x;1\le\mu\le 4} \left[1 - \frac{1}{2}\mathrm{Re}\,\mathrm{Tr}\; P_{\mu 5}(x)\right]$$

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Naive continuum limit

By matching the naive continuum limit $(a_4, a_5 \to 0)$ of the lattice action with the continuum action we obtain:

1. $$(\beta_4 \ , \ \beta_5) \quad \to \begin{cases} \beta_4 \simeq \dfrac{4a_5}{g_5^2} \\ \beta_5 \simeq \dfrac{4a_4^2}{g_5^2 a_5} \end{cases}$$

2. $$(\beta \ , \ \gamma) \quad \to \begin{cases} \beta = \sqrt{\beta_4 \beta_5} \simeq \dfrac{4a_4}{g_5^2} \\ \gamma = \sqrt{\dfrac{\beta_5}{\beta_4}} \simeq \dfrac{a_4}{a_5} \end{cases}$$

3. $$\tilde{N}_5 \quad \to \quad \frac{N_5}{\gamma} \simeq \frac{2\pi R}{a_4}$$

One–loop expressions for lattice observables

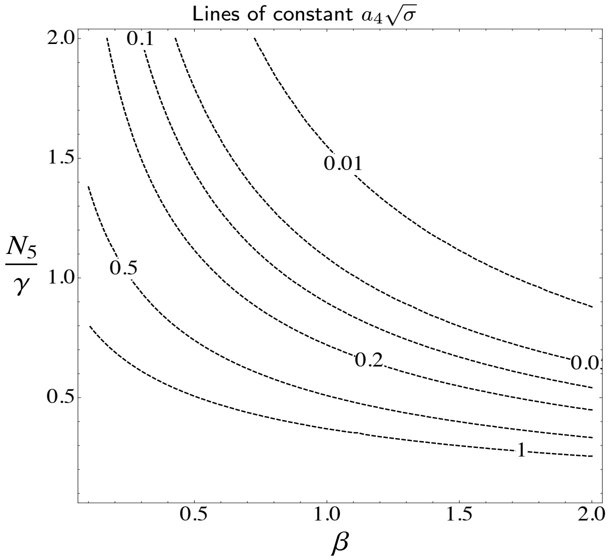
We can express lattice obervables like $a_4^2\sigma$ or $\frac{m_5}{\sqrt{\sigma}}$ as functions of the lattice model's parameters β, γ and N_5. This is only a rough guide to understand the behaviour of observables as the parameters are changed.

A simple–minded approach consists in using the classical relation between the lattice and the continuum, together with one–loop formulae for the string tension and for the scalar mass. The results can be written as

$$a_4^2\sigma \sim \frac{\gamma^2}{N_5^2} \exp\left\{-\frac{\beta N_5}{2N_c b_0 \gamma}\right\}$$

$$\frac{m_5}{\sqrt{\sigma}} \sim \sqrt{\frac{2N_c\gamma}{\beta N_5}} \exp\left\{\frac{\beta N_5}{4N_c b_0 \gamma}\right\}$$

String tension and scalar mass in parameter space



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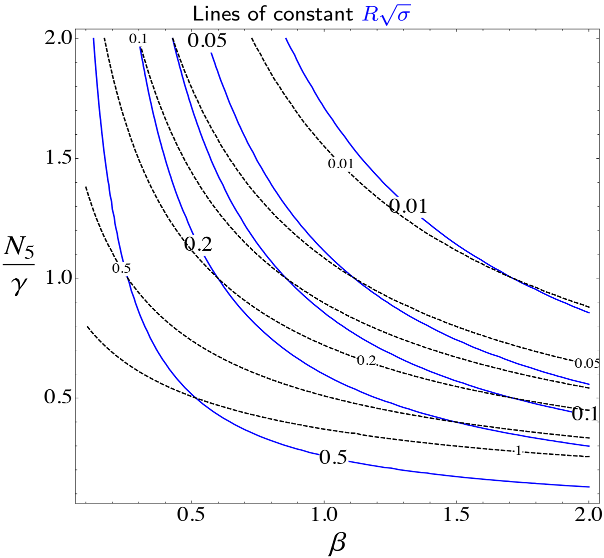

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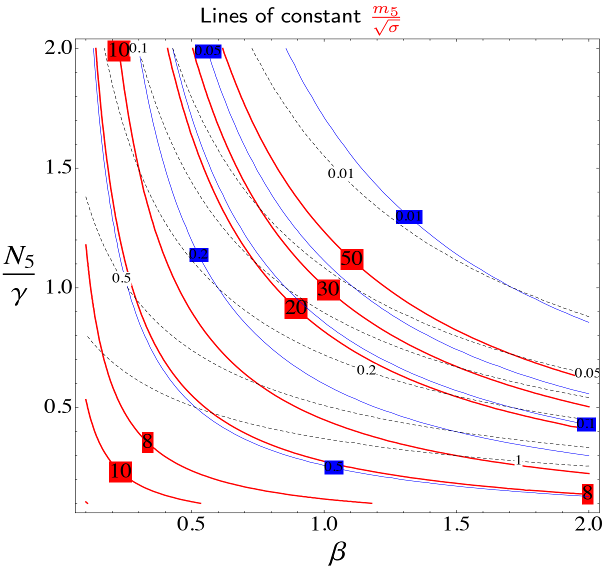


Monitoring phase transitions

The investigation of the phase structure in the lattice model requires monitoring the behaviour of order parameters, such as the following gauge–invariant observables:

1. 4D Plaquette

$$p_4 = \frac{\sum_{1\leq\mu<\nu\leq 4}\sum_x \mathrm{Re}\,\mathrm{Tr}\;P_{\mu\nu}(x)}{6N_c N_4^4 N_5}$$

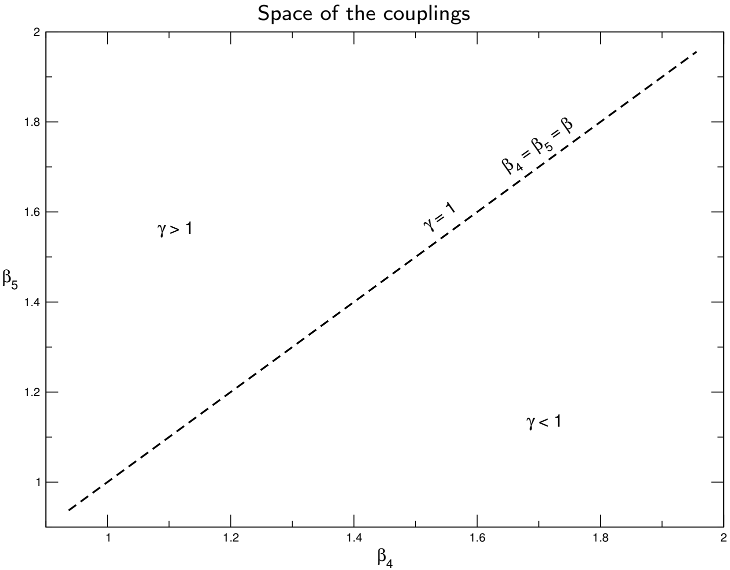
2. transverse Plaquette

$$p_5 = \frac{\sum_{1\leq\mu\leq 4}\sum_x \mathrm{Re}\,\mathrm{Tr}\;P_{\mu 5}(x)}{4N_c N_4^4 N_5}$$

3. compact Polyakov loop

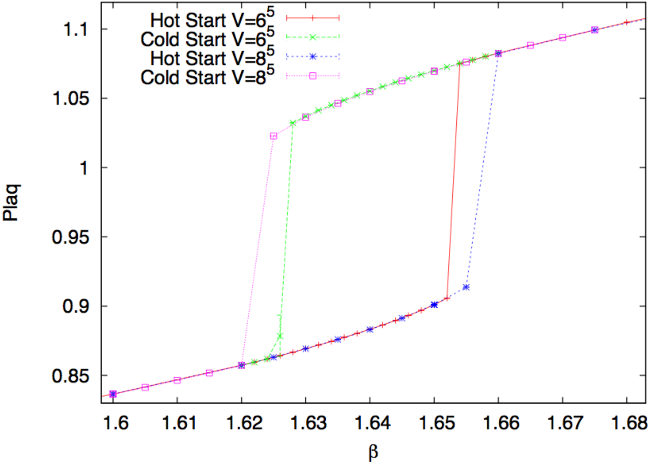
$$l_5 = \frac{\sum_{x=1}^{N_4^4} \mathrm{Tr}\;\prod_{i=1}^{N_5} \mathcal{U}_5(x+\hat{5}ia)}{N_c N_4^4}$$

The phase diagram



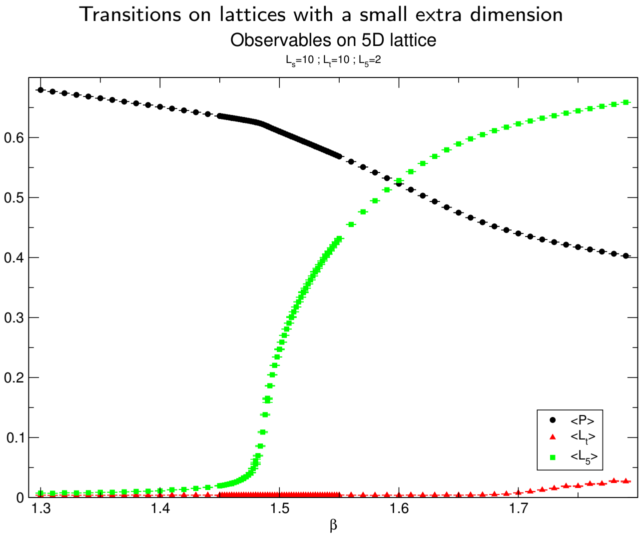
Isotropic model: $\gamma = 1$

Bulk transition on large symmetric lattices



[Knechtli, arxiv:1110.4210]

Isotropic model: $\gamma = 1$

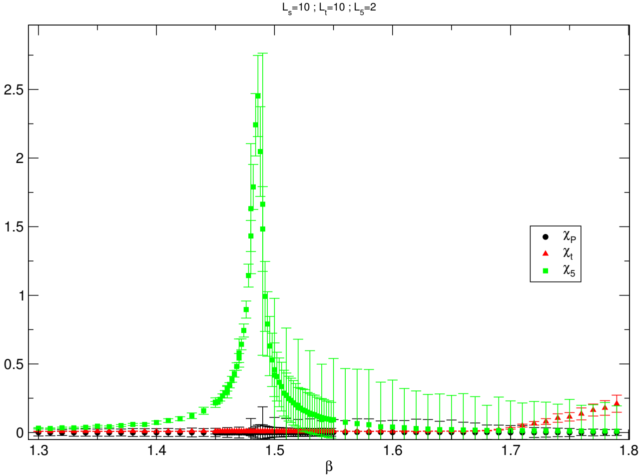


[Del Debbio, Hart, ER, arXiv:1203.2116]

Isotropic model: $\gamma = 1$

Transitions on lattices with a small extra dimension

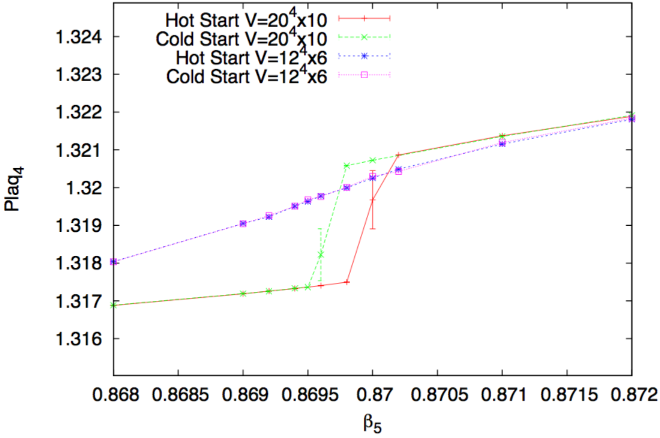
Susceptibility on 5D lattice



[Del Debbio, Hart, ER, arXiv:1203.2116]

Anisotropic model:$\gamma < 1$

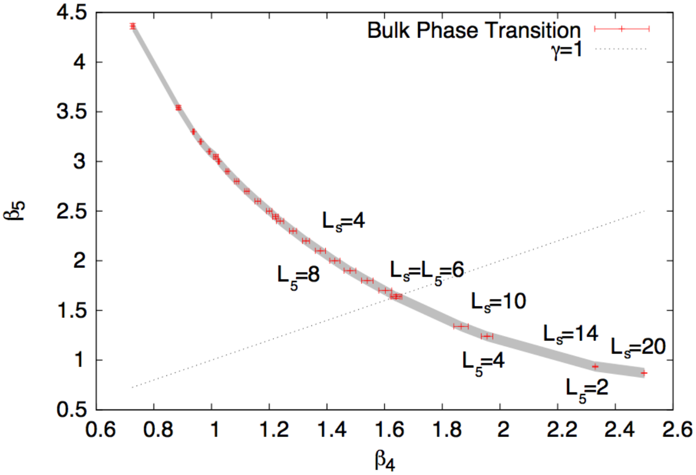
Bulk transition at small anisotropy ($\beta_4 = 2.5$)



[Knechtli, arxiv:1110.4210]

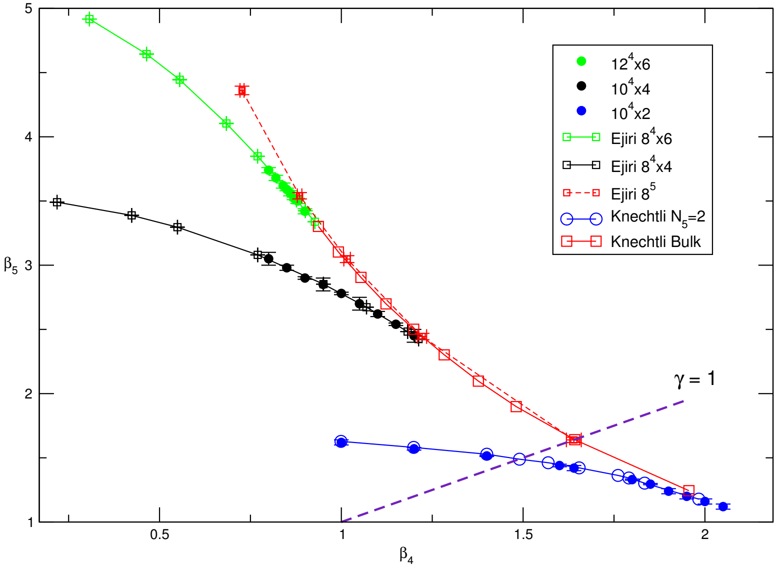
Anisotropic model: $\gamma < 1$

Bulk transition line and minimal lengths



[Knechtli, arxiv:1110.4210]

Phase diagram: $\gamma > 1$



Strategy for lattice simulations

- Fix a point in parameter space

$$(\beta_4, \beta_5, N_5)$$

- Now 2 scales are fixed

$$\Lambda_{\mathrm{UV}} \quad \text{and} \quad \Lambda_{\mathrm{R}}$$

- Measure 2 observables in units of the lattice spacing

$$a_4\sqrt{\sigma} \quad \text{and} \quad a_4 m_5$$

- These give us the actual values for Λ_{UV} and m_5 in units of the string tension
- We are not able to extract Λ_{R} from a measurement but

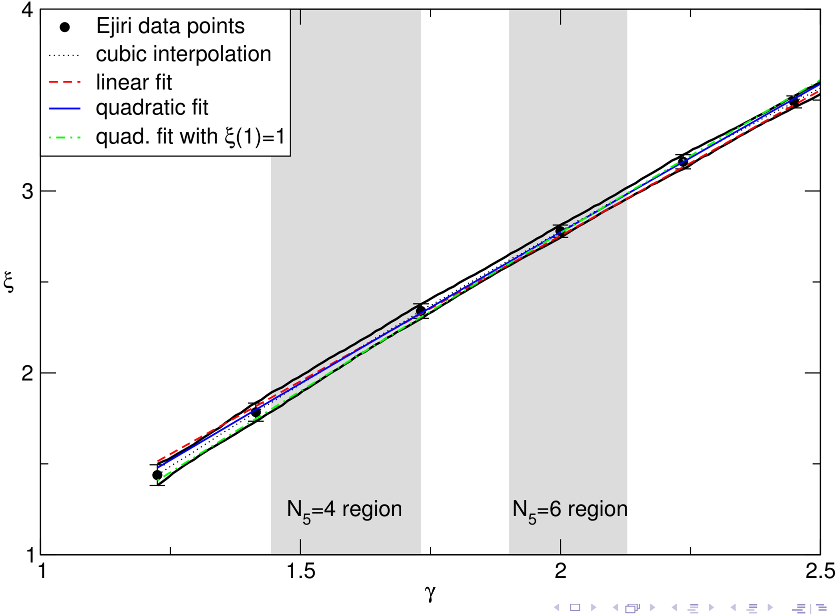
$$\xi = \frac{a_4}{a_5} \quad \rightarrow \quad \Lambda_{\mathrm{R}} = \frac{\xi}{N_5}\Lambda_{\mathrm{UV}}$$

  (the relation $\xi = f(\gamma, \beta)$ had already been mapped [Ejiri, hep-ph/0006217])
- For each set of bare parameters we obtain a set of scales

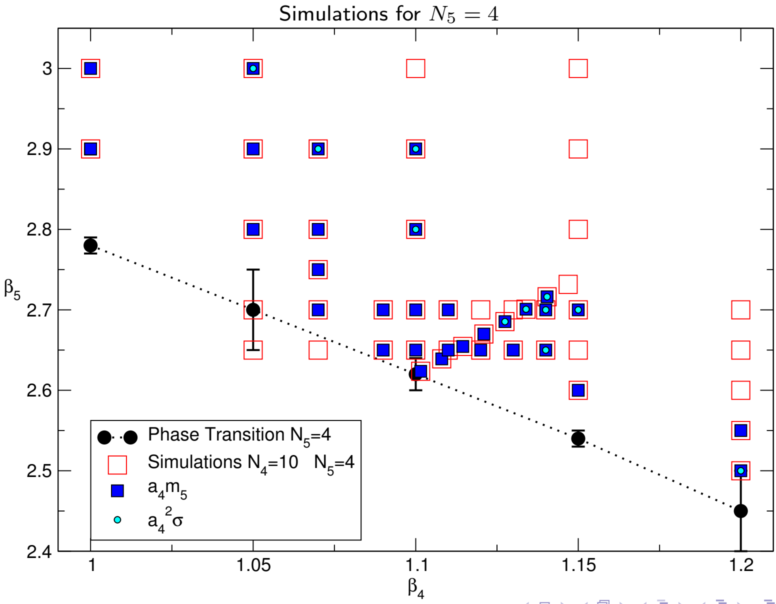
$$(\beta_4, \beta_5, N_5) \quad \rightarrow \quad (\Lambda_{\mathrm{UV}}, \Lambda_{\mathrm{R}}, m_5)$$

- Study m_5 as a function of Λ_{UV} and Λ_{R}

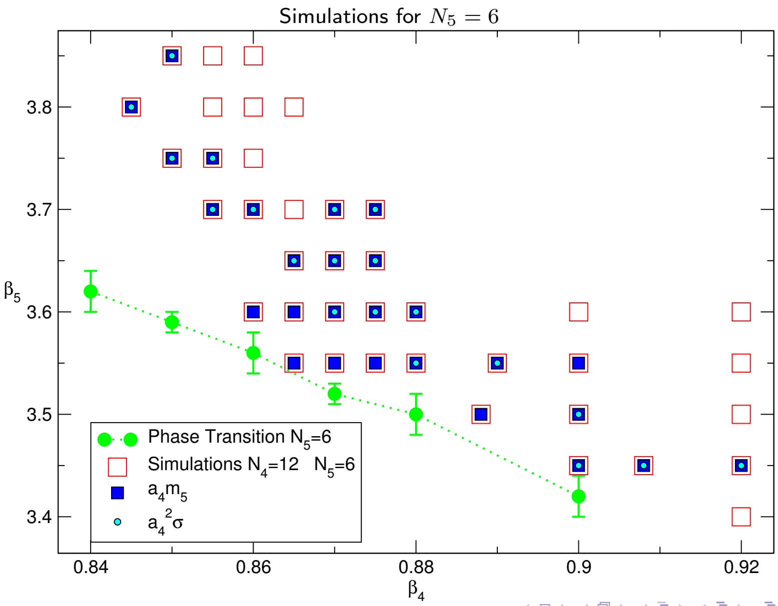
Renormalized anisotropy



Simulations points

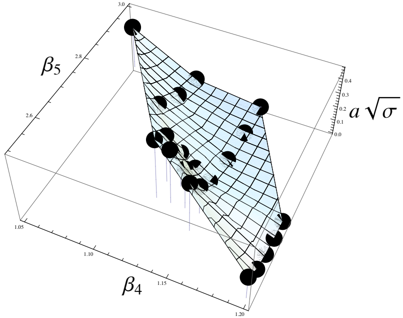


Simulations points

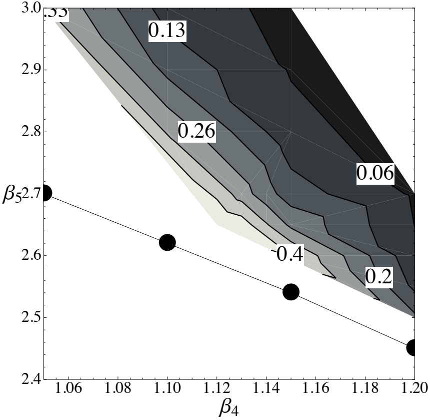


Explore parameter space at $N_5 = 4$

Data for $a_4\sqrt{\sigma}$

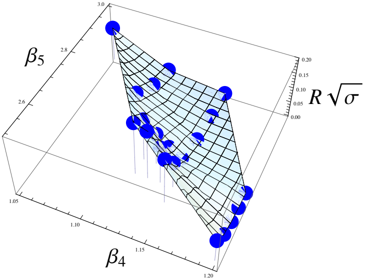


Contours $a_4\sqrt{\sigma}$ fixed

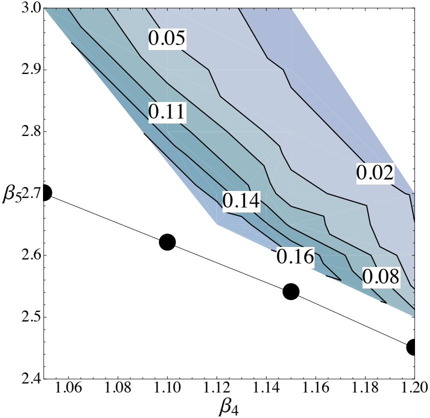


Explore parameter space at $N_5 = 4$

Data for $R\sqrt{\sigma}$

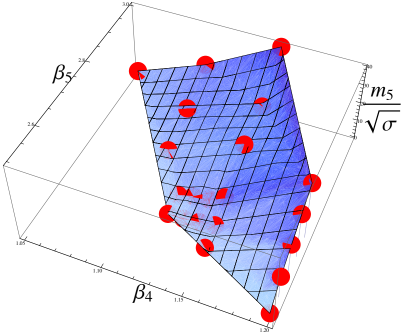


Contours $R\sqrt{\sigma}$ fixed

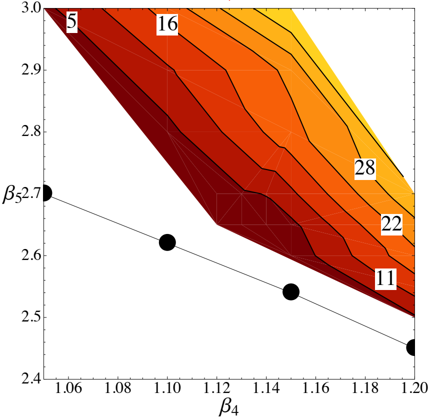


Explore parameter space at $N_5 = 4$

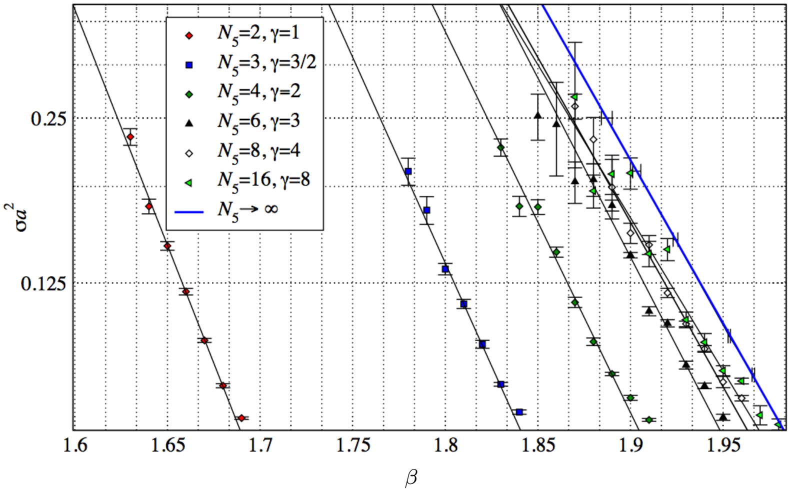
Data for $\frac{m_5}{\sqrt{\sigma}}$



Contours $\frac{m_5}{\sqrt{\sigma}}$ fixed

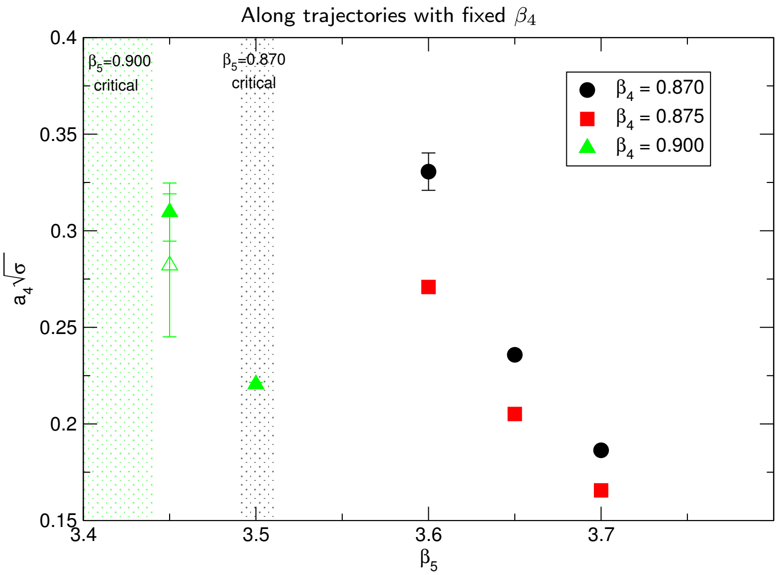


String tension at weak-coupling

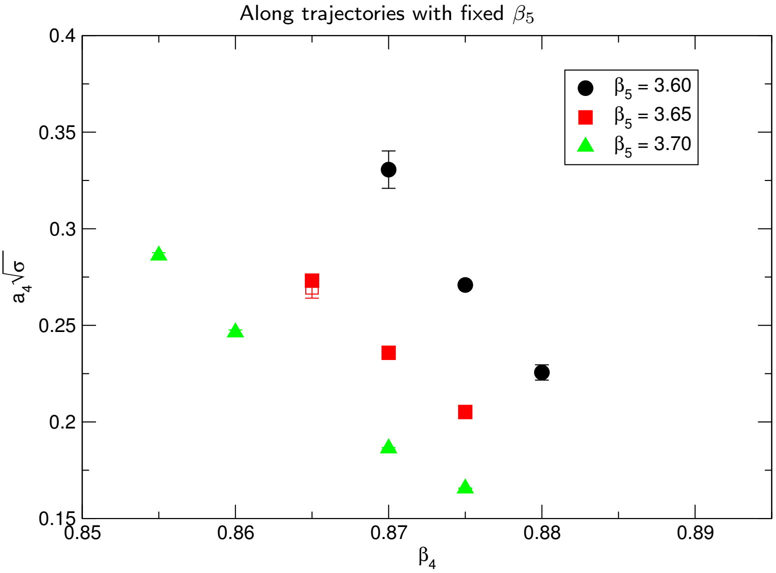


[de Forcrand, arxiv:1003.4643]

String tension at $N_5 = 6$

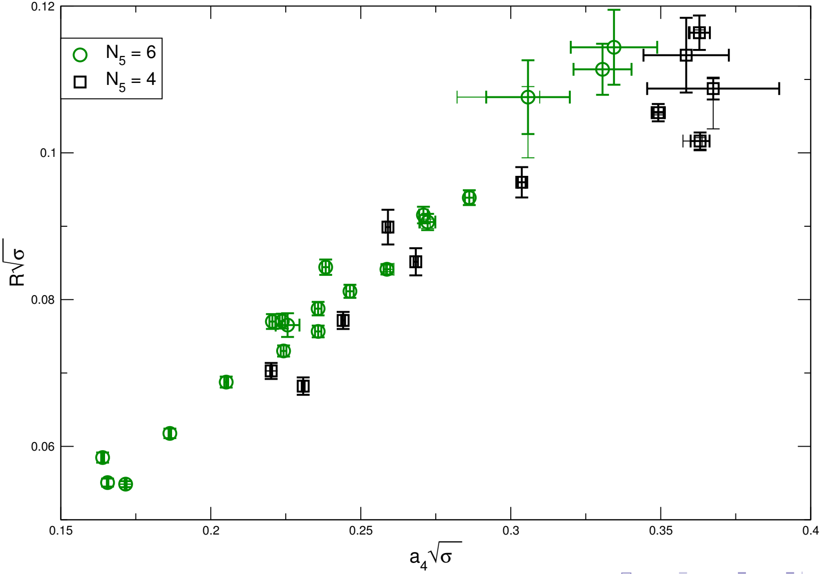


String tension at $N_5 = 6$

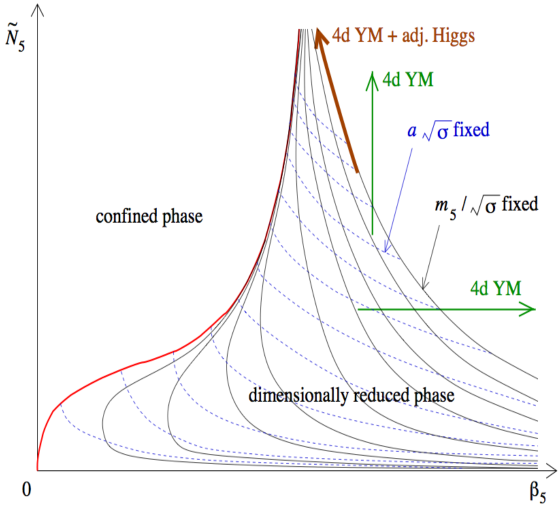


Extra dimension size and lattice spacing

The simulated points fall in the following region of a_4 and R



Perturbative *continuum limit*



de Forcrand, arxiv:1003.4643