Mass anomalous dimension from Dirac eigenmode scaling in conformal and confining systems

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In collaboration with A. Cheng, D. Schaich and G. Petropoulos
(Talks on Monday, 4:40 , Thursday 2:30)

Colorado nHYP – BSM project

Investigate SU(3) gauge with N_f=8 and 12 fundamental flavors

- nHYP smeared fermions, fundamental-adjoint plaquette gauge action
- 12^3x24 → 32^3x64 volumes, am=0.0025 – 0.025 , wide gauge coupling range form weak to strong coupling

Use a variety of techniques & operators to cover many aspects of these systems

- Phase structure, D. Schaich, Monday 4:40
- Finite temperature transitions, D. Schaich, Monday 4:40
- (Spectrum – in progress)
- Dirac eigenmode scaling
- MCRG & step scaling function, G. Petropoulos, Thursday 2:30

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- Goal : compare and contrast N_f=8 and 12

Dirac eigenvalue spectrum

Both the eigenvalue density $\rho(\lambda)$

and the mode number $\nu(\lambda) = V\int_0^{\lambda} \rho(\lambda')\, d\lambda'$ are RG invariant

In the infinite volume, m=0 chiral limit they scale

$$\rho(\lambda) \sim \lambda^{\alpha}$$

$$\nu(\lambda) \sim V\lambda^{\alpha+1} \sim (L\lambda^{(\alpha+1)/4})^{4}$$

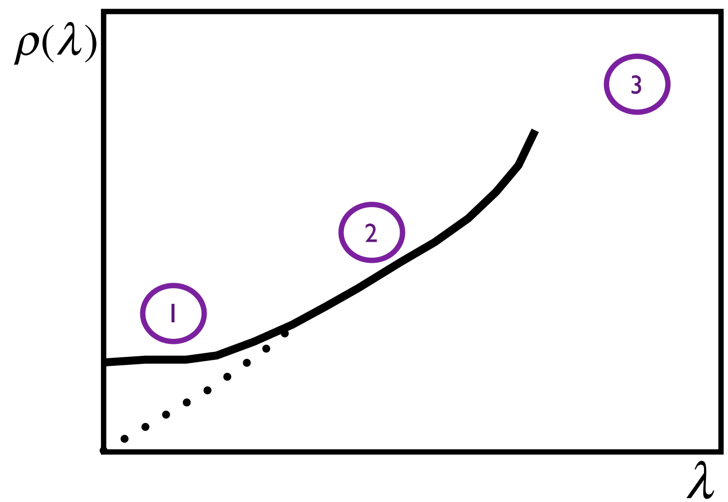
The RG invariance of $\nu(\lambda)$ implies

$$y_m = 1 + \gamma_m = \frac{4}{\alpha + 1}$$

Dirac eigenvalue spectrum

There are 3 distinct regions:

1. Confining regime, $\lambda < F_\pi$ or Λ_{QCD} ,
   → predicts Σ:
   - individual eigenmode distributions compared to RMT
   - global fit to $\nu(\lambda)$ (Giusti, Luescher)



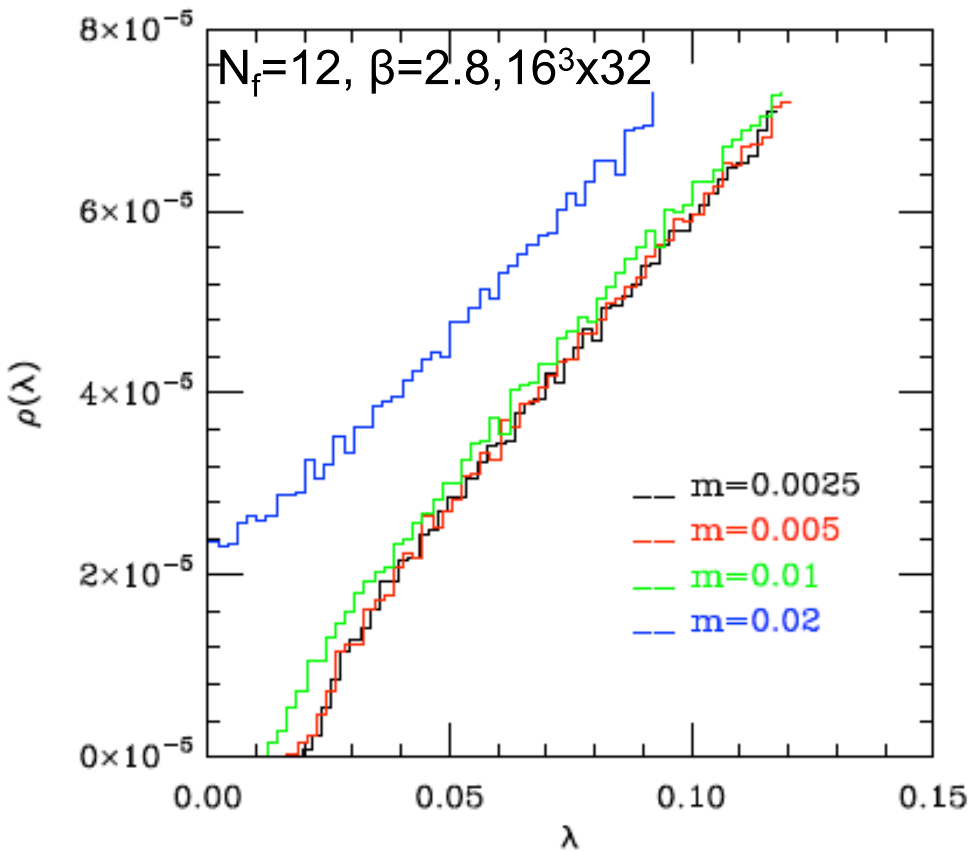
2. Perturbative regime, $F_\pi < \lambda < \Lambda_{cutoff}$,
   → predicts γ_m
   - global fit to many volumes, eigenvalues (Cheng, AH,Schaich)
   - global fit to $\nu(\lambda)$ in fixed, large volume (Patella)

3. UV regime, $\Lambda_{cutoff} < \lambda$,
   → non-universal

In conformal systems only regions 2-3 exist

Fermion mass dependence

We calculate the eigenvalues of the massless Dirac operator, but the sea fermion mass is finite

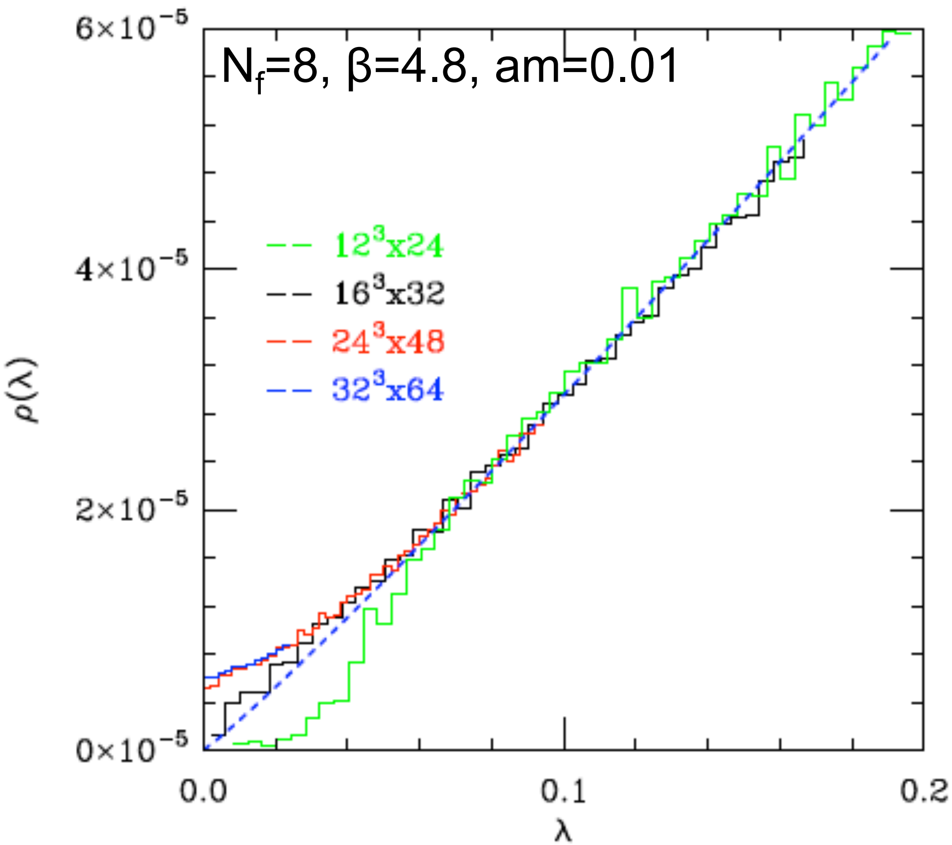


Eigenvalue density is sensitive to the mass but stable for $m \leq 0.005$

(we use $m \leq 0.0025$)

Volume dependence

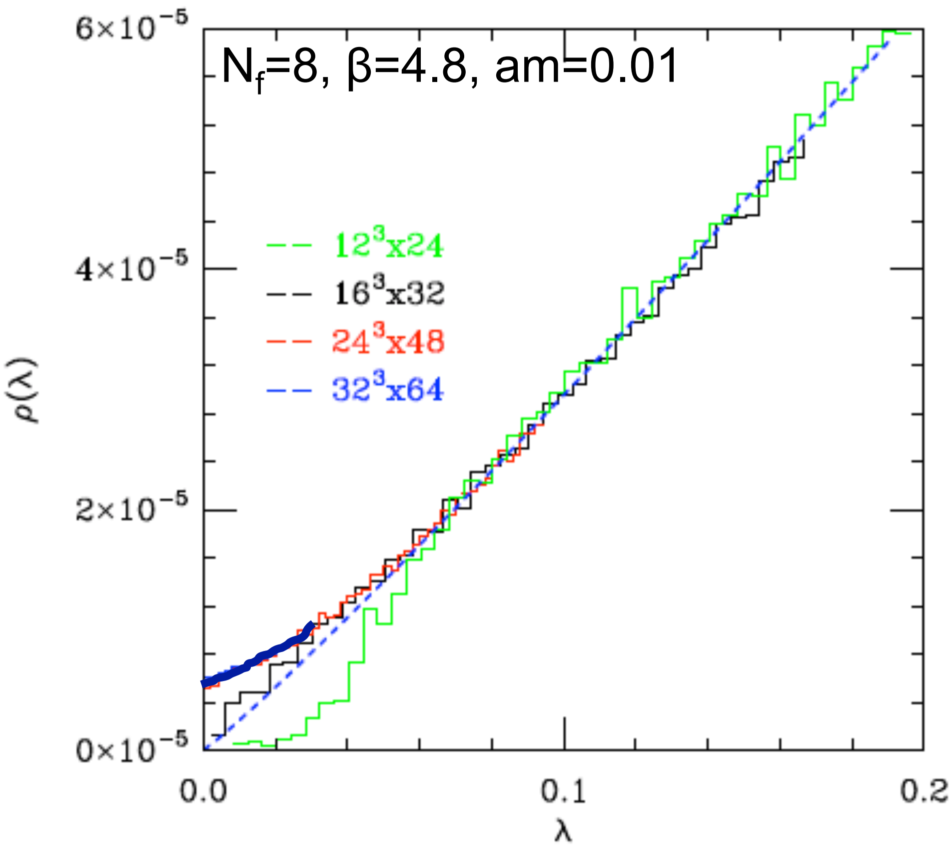
Scaling formulae are valid in infinite volume



Finite volume affects mainly the low eigenmodes

Volume and quark mass dependence

Scaling formulae are valid in infinite volume.



Finite volume affects mainly the low eigenmodes

200 eigenmodes on 32^3x64

Volume and quark mass dependence

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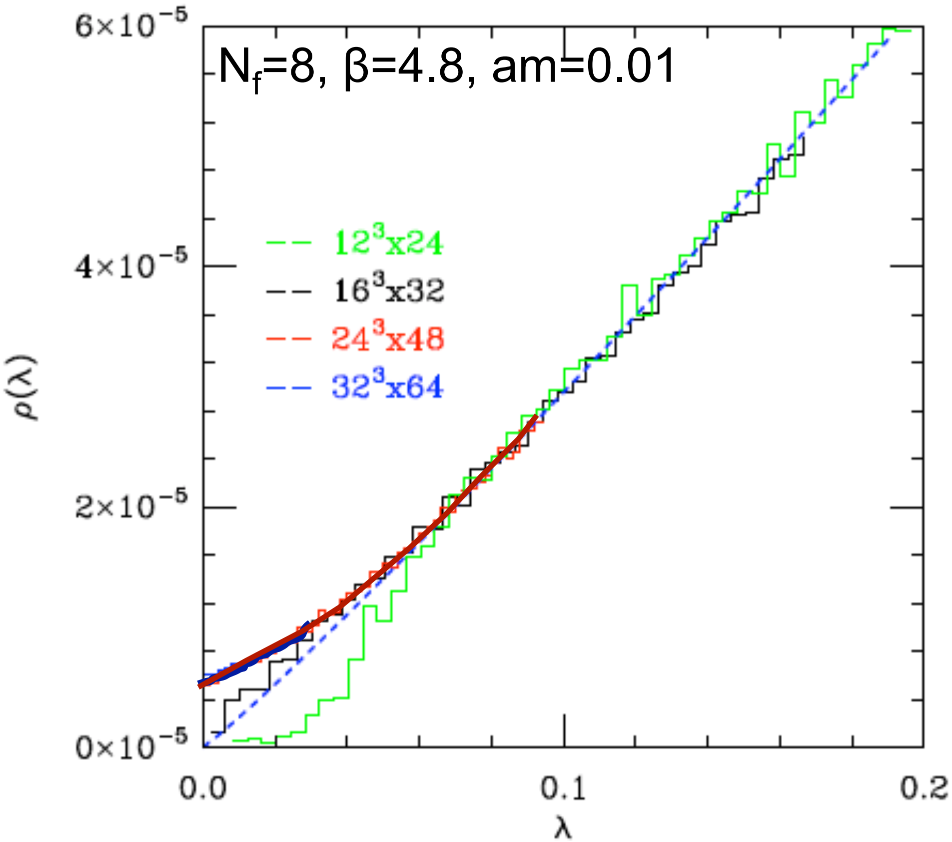
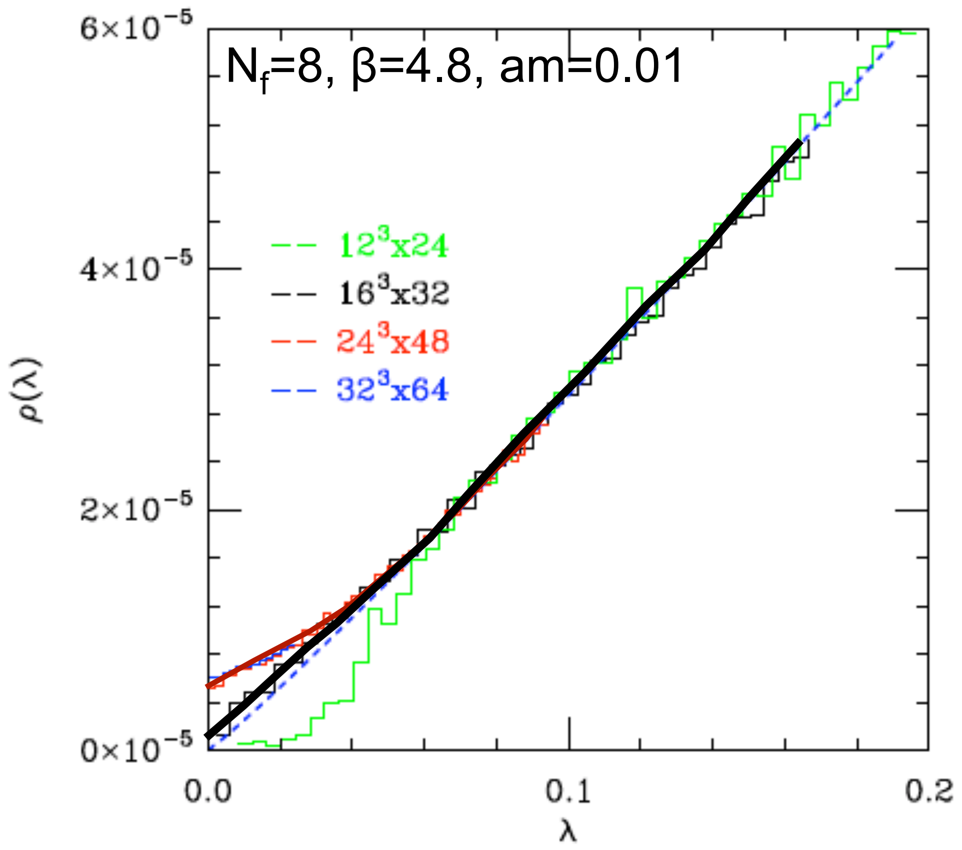


Finite volume affects mainly the low eigenmodes;
200 eigenmodes on 32^3x64
500 eigenmodes on 24^3x48

Volume and quark mass dependence

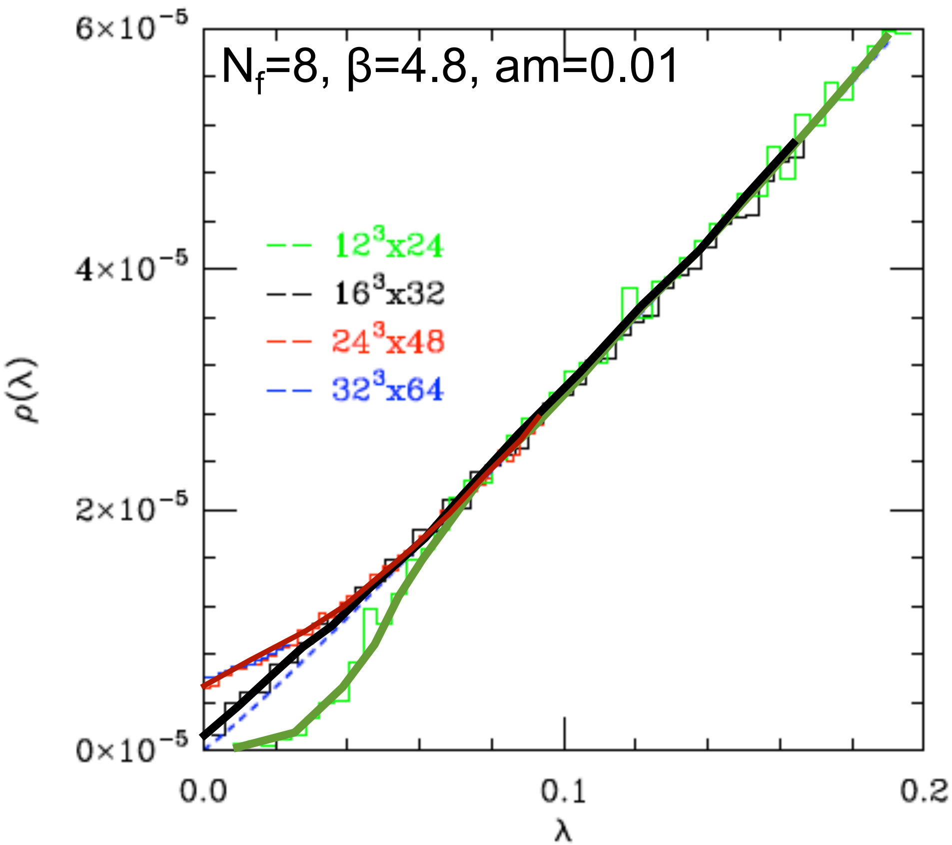
Scaling formulae are valid in infinite volume.



Finite volume affects mainly the low eigenmodes;
200 eigenmodes on 32^3x64
500 eigenmodes on 24^3x48
300 eigenmodes on 16^3x32

Volume and quark mass dependence

Scaling formulae are valid in infinite volume.



Finite volume affects mainly the low eigenmodes;
200 eigenmodes on 32^3x64
500 eigenmodes on 24^3x48
300 eigenmodes on 16^3x32
300 eigenmodes on 12^3x24

The perturbative regime,
$F_\pi < \lambda < \Lambda_{cutoff}$
is volume independent.

Strategy

1. Work with m=0.0025 (and 0.005 to check for mass distortion)

2. Use 12^3x24, 16^3x32, 24^3x48 volumes

3. Evaluate 300-500 eigenmodes, but use only $\lambda < \lambda_{max} \sim 0.4$
   (we do not have a stochastic mode number estimator yet)

-- Fit the mode number $\nu(\lambda)$ between λ_0 and λ_{max} on each volume as

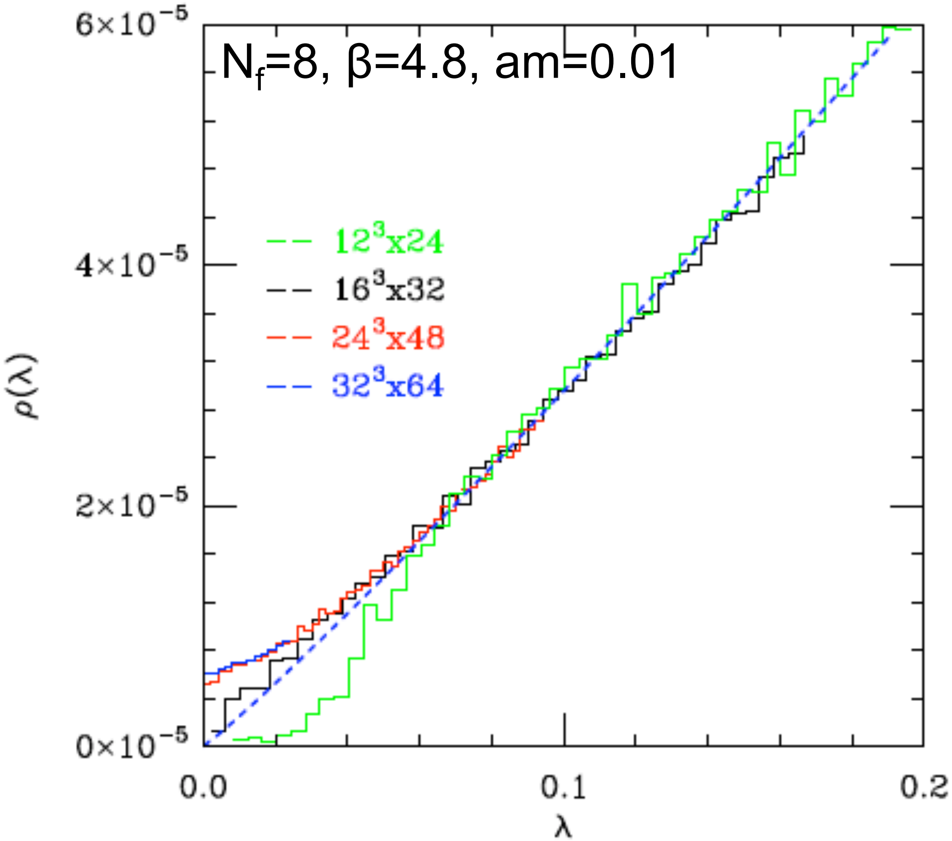
$$\nu(\lambda) - \nu(\lambda_0) = cV(\lambda^{4/y_m} - \lambda_0^{4/y_m})$$

tuning λ_0 until the fit stabilizes. (Only 2 parameters to fit: c & y_m !)

-- Combine different volumes for a global fit to predict γ_m . (We allow the constant c to depend on the volume.)

Volume dependence

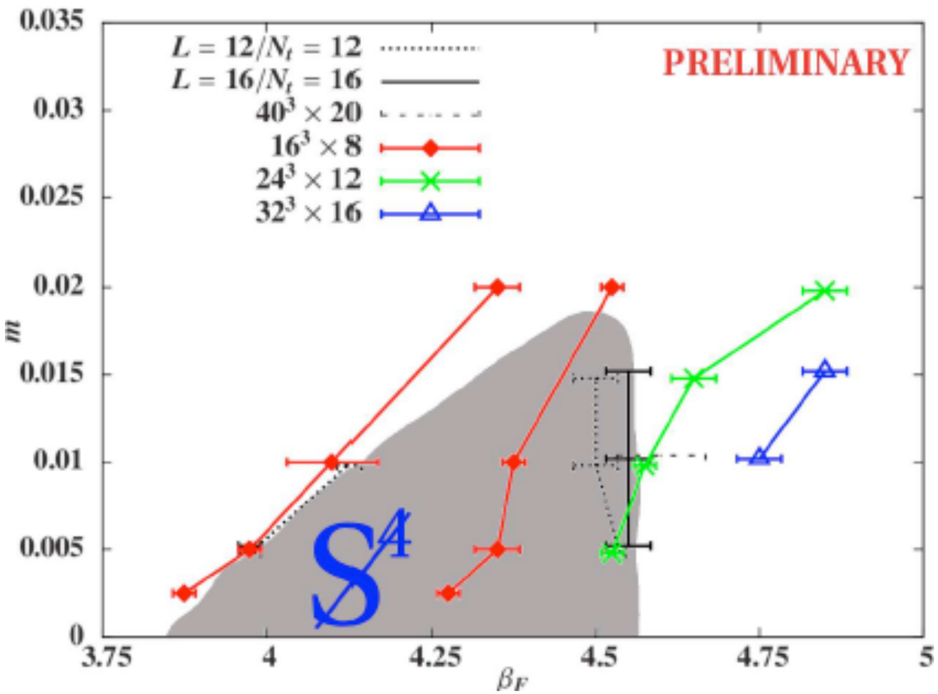
Scaling formulae are valid in infinite volume



Finite volume affects mainly the low eigenmodes

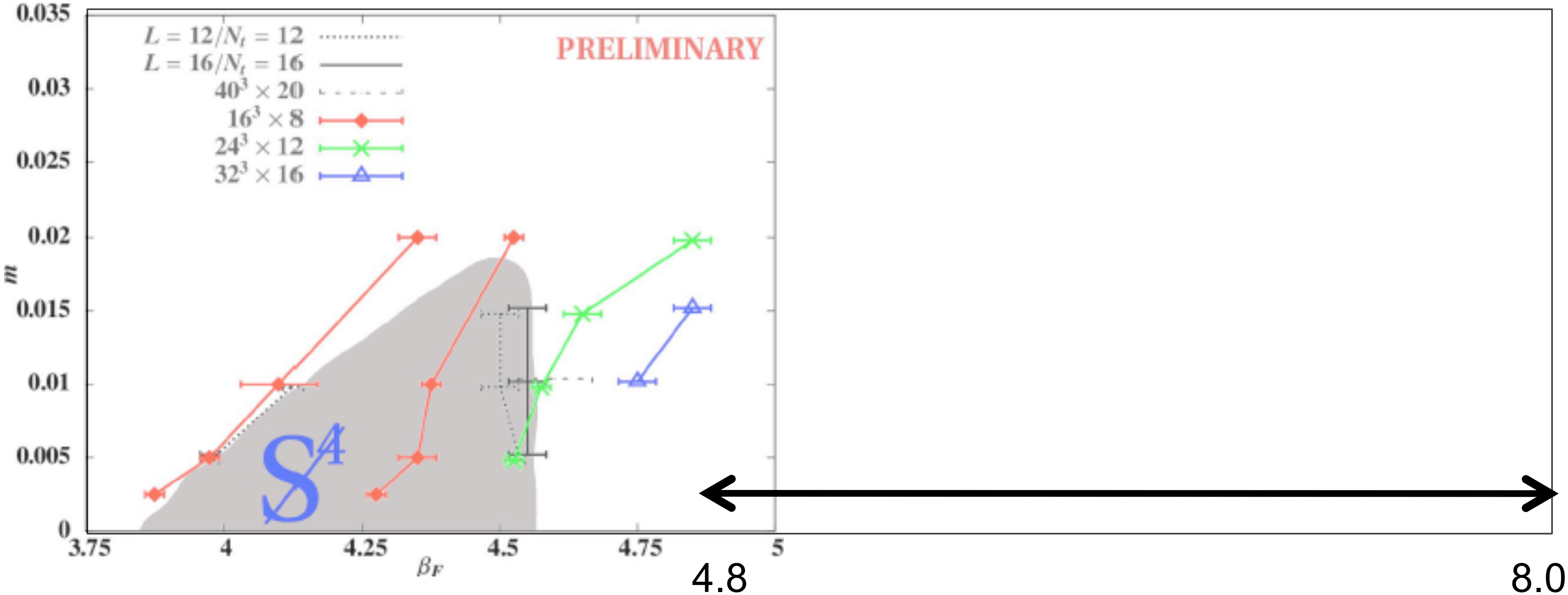
Parameter range, N_f=8

Phase structure (See D. Schaich's talk)



Gray area is the S^4b phase,
That has to be avoided

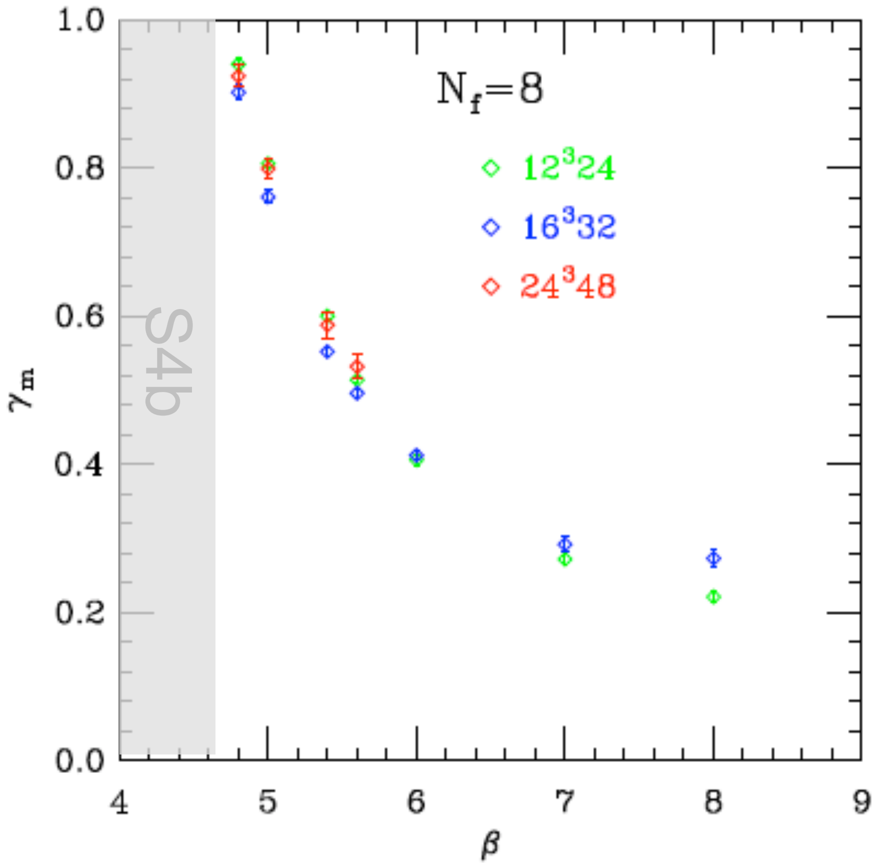
Parameter range, N_f=8



Cover range β= 4.8 – 8.0, m=0.0025
volumes 12^3x24, 16^3x32, 24^3x48 (32^3x64 in progress)

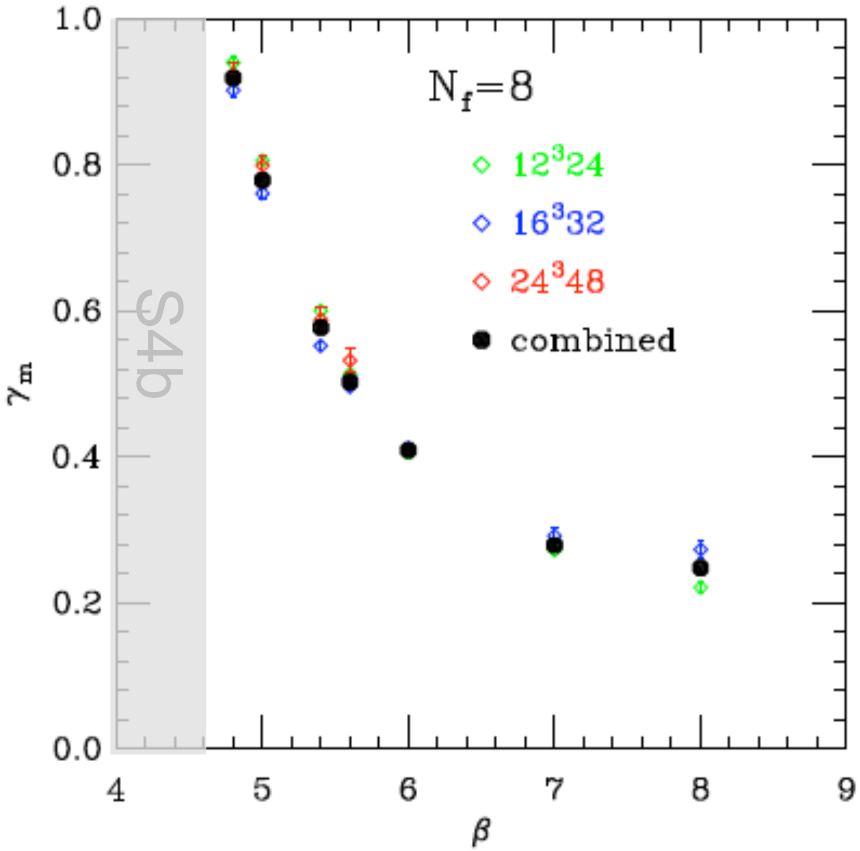
Results: N_f=8

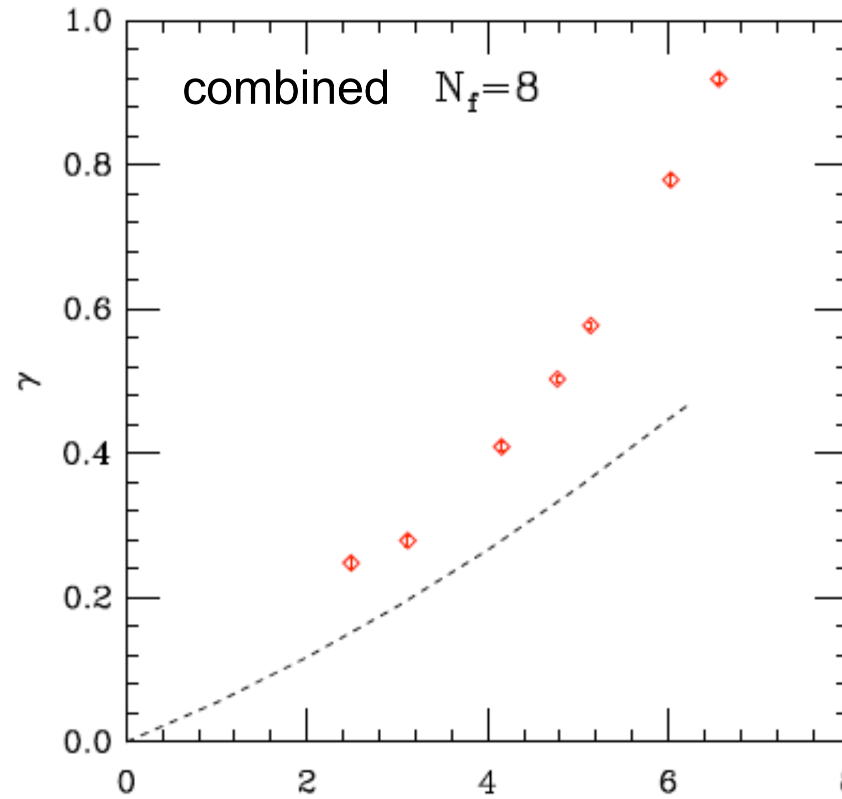
Fit results to individual volumes are consistent



Results: N_f=8

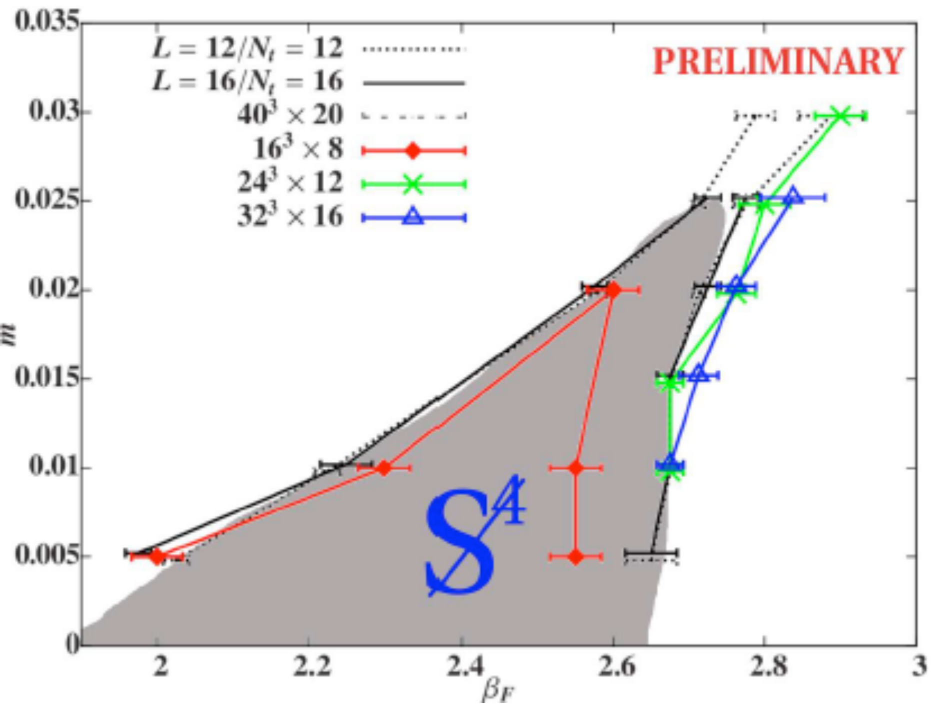
Global fit to all volumes





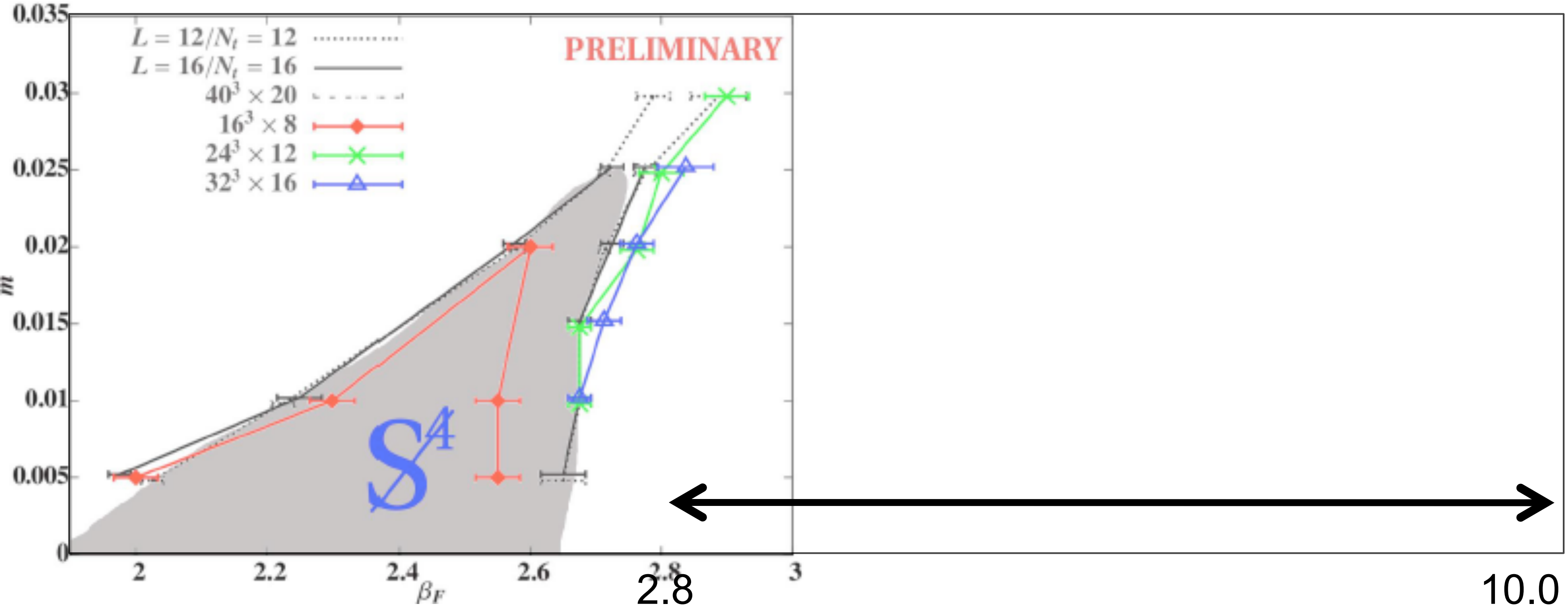
as function of tadpole improved g^2

Parameter space, N_f=12



Gray area is the S^4b phase

Dirac spectrum study, N_f=12

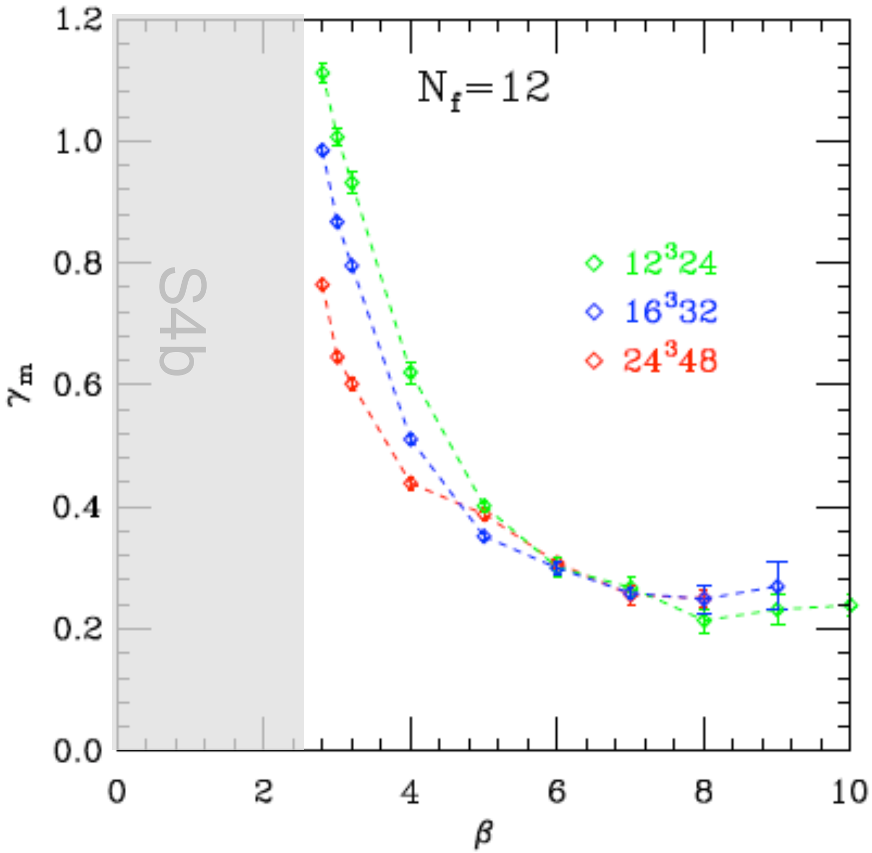


Cover range β=2.8 – 10, m=0.0025

volumes 12^3x24, 16^3x32, 24^3x48 (32^3x64 in progress)

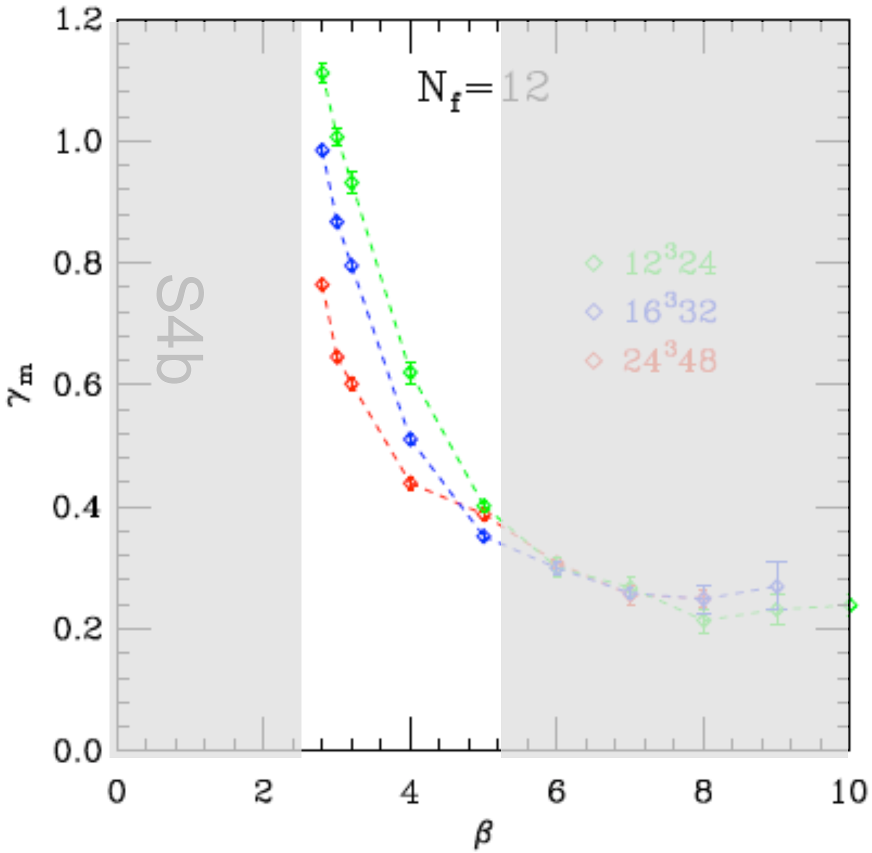
Results: N_f=12

Fit results to individual volumes are **not** consistent for β < 6.0



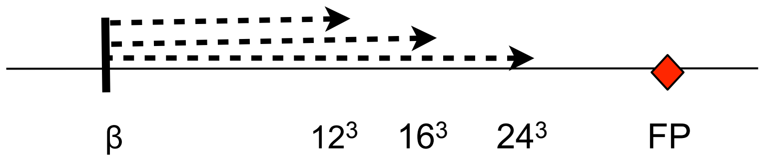
Results: N_f=12

Fit results to individual volumes are **not** consistent for β < 5.0



The strong coupling region does not follow expected scaling form

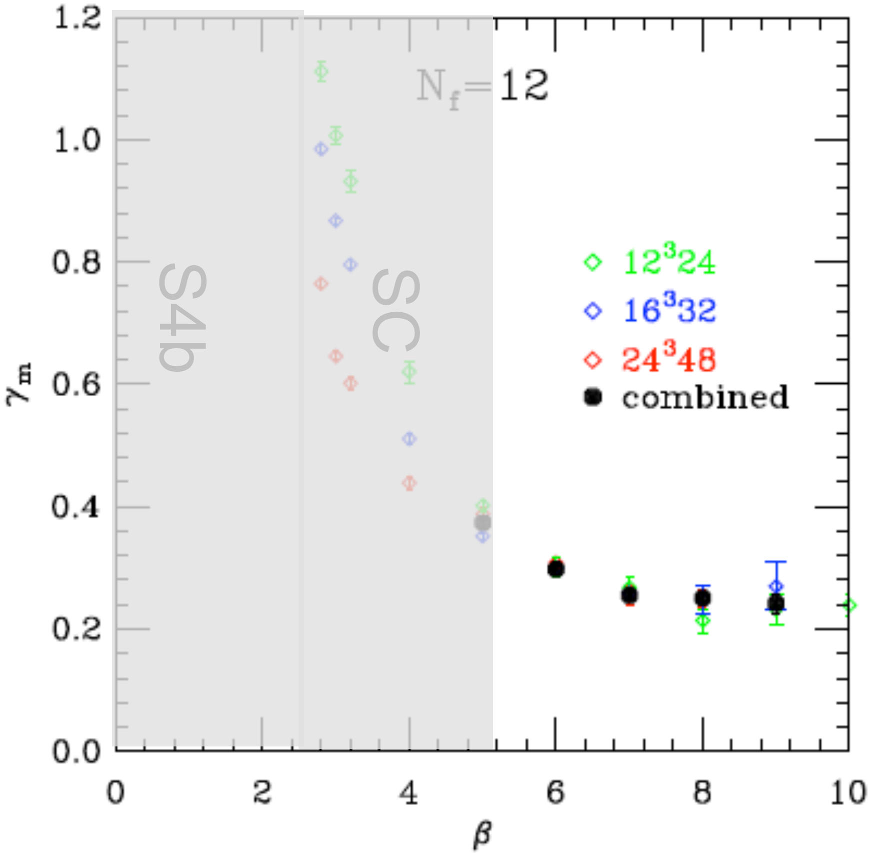
→Large lattice artifacts: not in the basin of attraction of the FP



→Could introduce scaling violations in finite size scaling (in progress)

Results: N_f=12

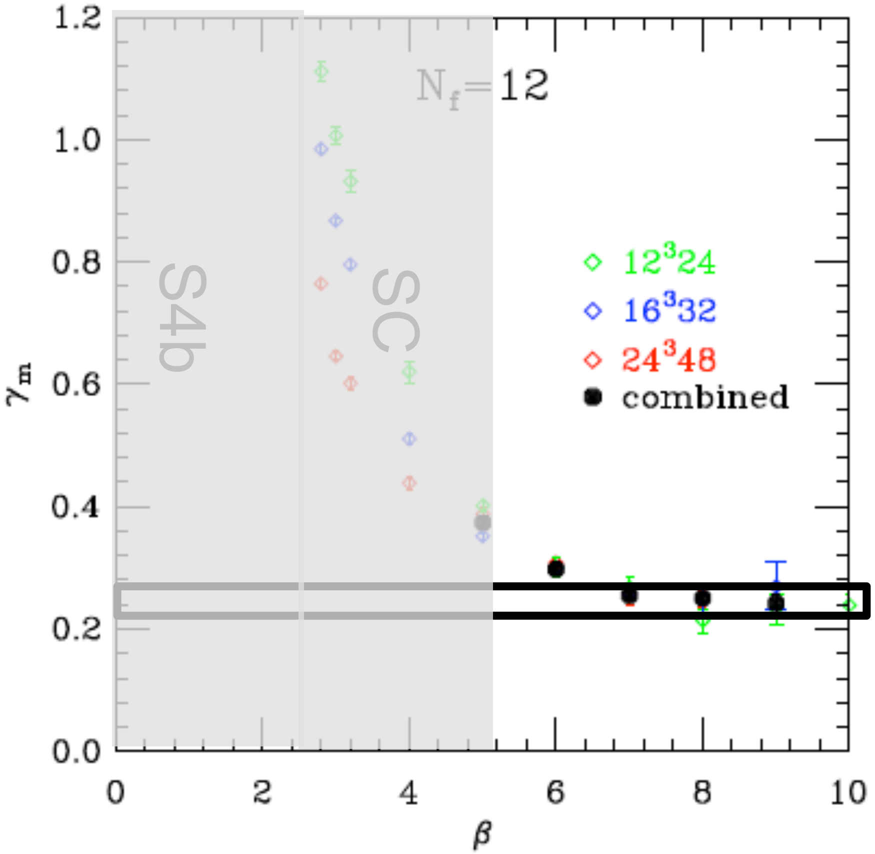
Global fit to all volumes β > 5.0



This data does not exclude chirally broken scenario, though the near constant γ_m does not look perturbative

Results: N_f=12

Global fit to all volumes β > 5.0



For β>5 the value

γ_m = 0.25(1)

is either

- the anomalous dimension at the IRFP

or

- the walking anomalous dimension
(but it is too small for that)

At smaller β we are too far form the IRFP

Conclusion

Dirac eigenmode/mode number scaling is a very effective approach to predict γ_m at given lattice scale or at IRFP

- Finite mass effects can be controlled
- Volume dependence can be used to constrain the fits

Nf=8 results

- γ_m vs β can be combined with step scaling function to predict running γ_m
- 2-loop perturbative form is approached around $g^2=1$

Nf=12 results

- show large lattice violation effects at strong(er) couplings
- predict surprisingly small $\gamma_m=0.25(1)$

Large volume data would benefit from more eigenmodes

→ stochastic approach in progress