Nuclear Physics from Lattice Simulations

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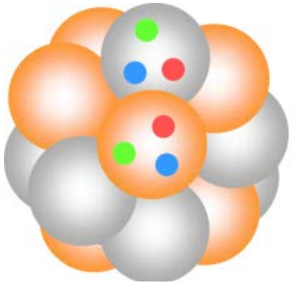
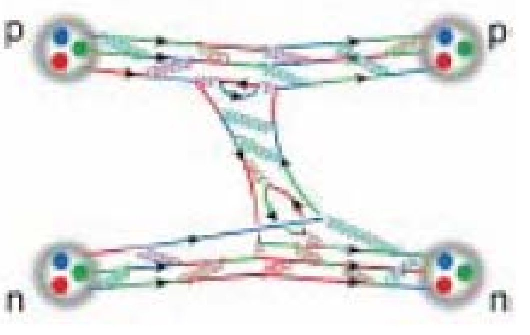
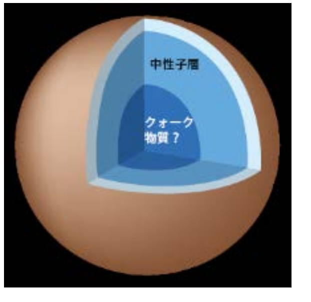




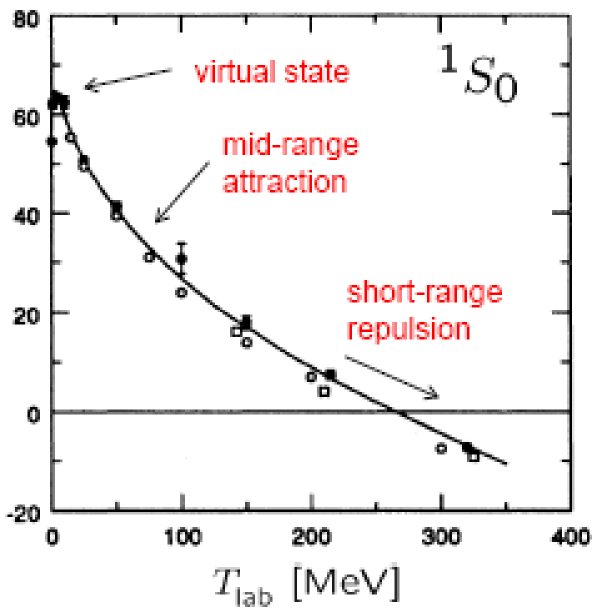


- Nuclear Physics is in the era of **Renaissance**

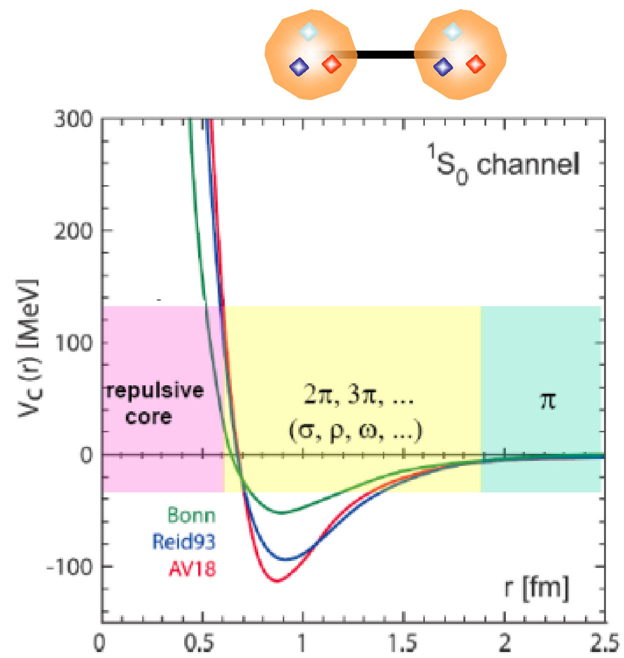
- Lattice QCD predictions play a crucial role

- <u>Outline</u>
  - Introduction
  - Nuclear physics on the lattice
  - NN interactions
  - Hyperon interactions
  - NNN interactions
  - Summary and Prospects

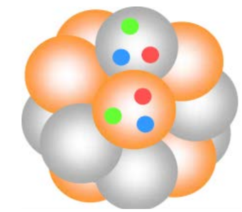
(1) Build a foundation from QCD



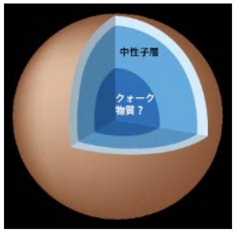
NN phase shifts from experiments



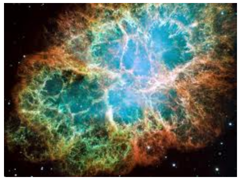
Phenomenological Nuclear Forces



Nuclei



Neutron Stars

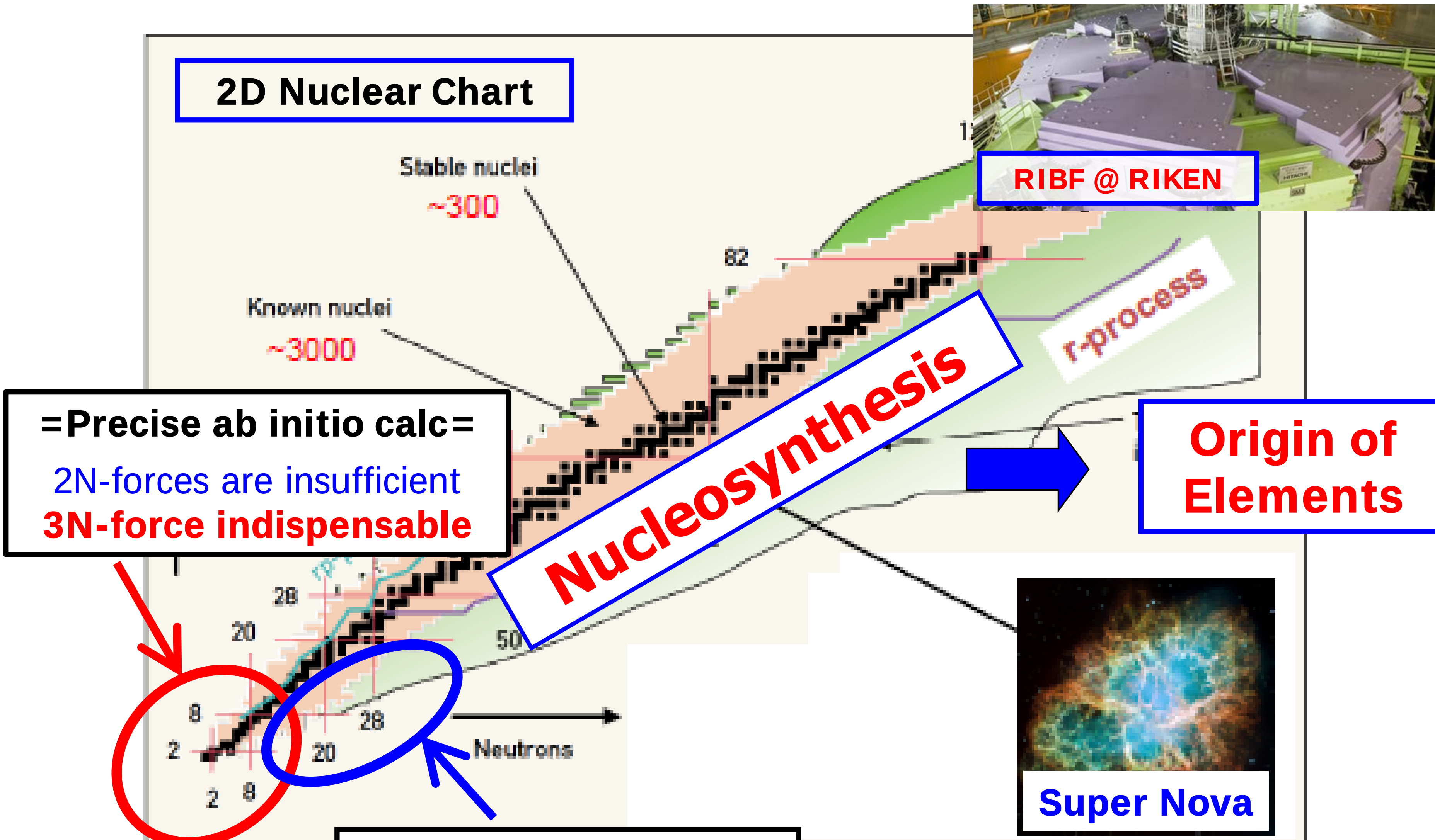


Super Novae

Various applications

- ***Nuclear Forces*** *play* ***crucial roles***
  - *Yet, no clear connection to QCD so far*

(2) Predict Unknown Interactions (NNN, YN, YY)

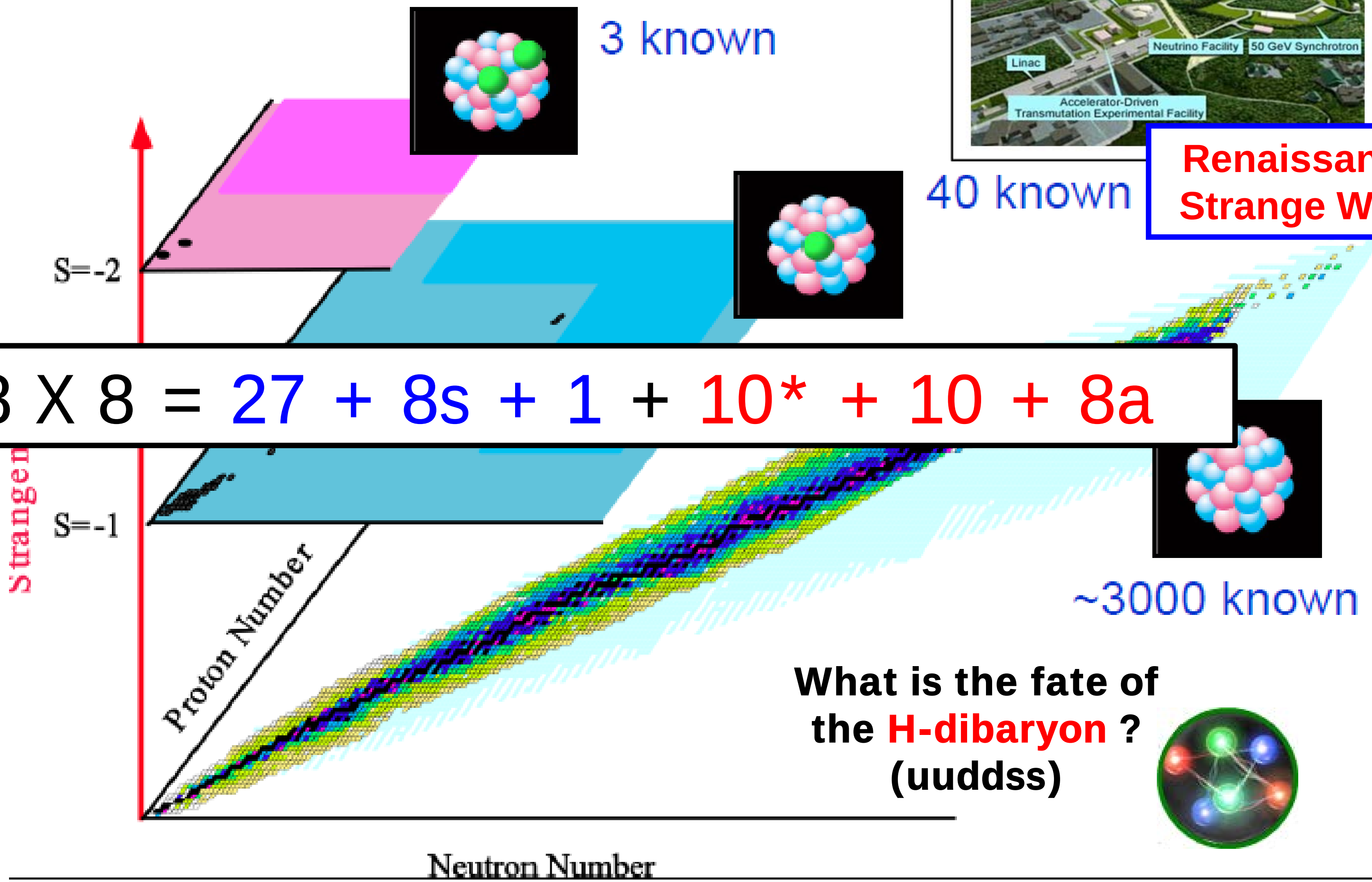


Paradigm Shift in
Unstable Nuclei
(New Magic Numbers !)

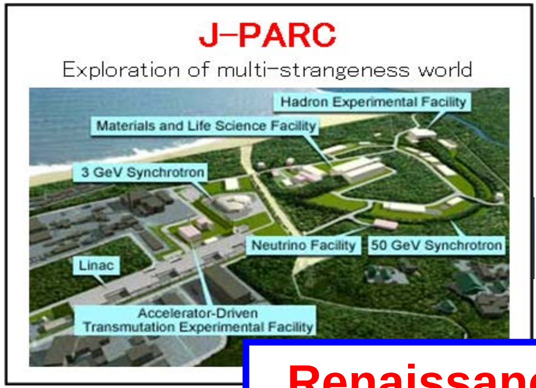
← **Important role of 3N-force**

(2) Predict Unknown Interactions (NNN, YN, YY)

3D Nuclear Chart



$$8 \times 8 = 27 + 8s + 1 + 10^* + 10 + 8a$$

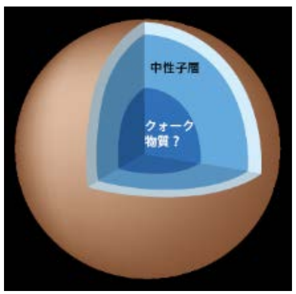


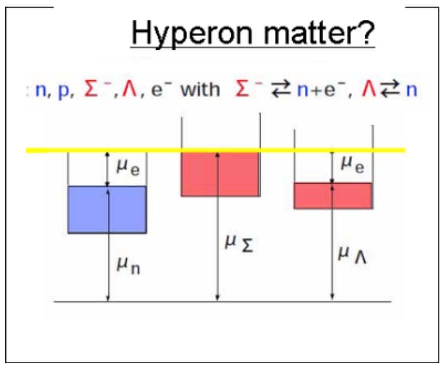
Renaissance in Strange World !

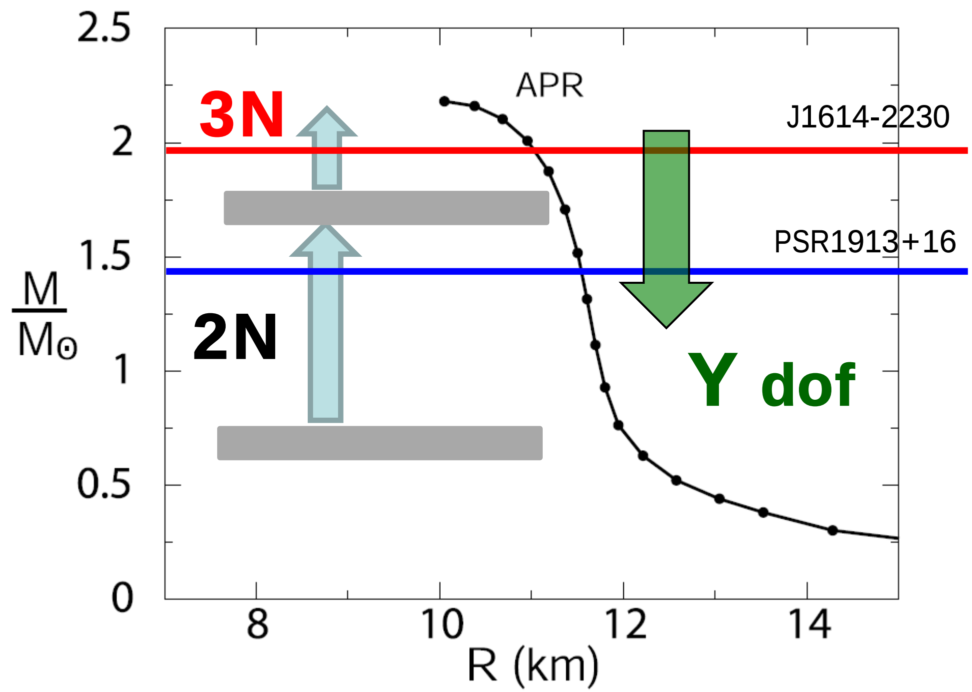
What is the fate of the H-dibaryon ? (uuddss)

Dense Matter ← Interactions of <u>YN, YY, NNN,… are crucial</u>

- Neutron Stars, Super Novae ↔ EoS

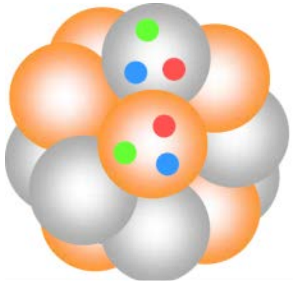


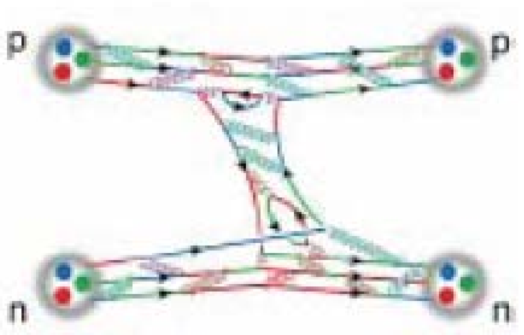


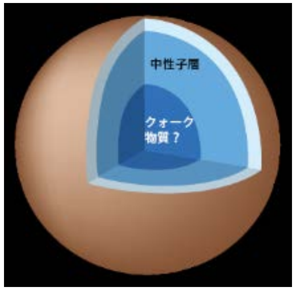


How to sustain a neutron star against gravitational collapse ?

Akmal et al.('98), Nishizaki et al.('02), Takatsuka et al.('08)







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Nuclear Physics on the Lattice

- NN phase shift (Luscher's formula)
  - Fukugita et al. PRL73(1994)2176
  - NPLQCD Coll, reviewed in Prog.Part.Nucl.Phys 66(2010)1 [K.Orginos @ Lat2011 plenary]
- NBS wave function ➔ NN potential
  - Ishii-Aoki-Hatsuda PRL99(2007)022001, PTP123(2010)89
  - HAL QCD Coll. (2009-), reviewed in arXiv:1206.5088 (PTEP in press)
- Light nuclei on the lattice [T.Yamazaki @ Lat2010 plenary]
  - Yamazaki-Kuramashi-Ukawa (PACS-CS Coll.) PRD81(2010)111504, PRD84(2011)054506

Other approaches, e.g., strong coupling limit

de Forcrand and Fromm, PRL104(2010)112005

Hadrons to **A**tomic nuclei from **L**attice QCD (**HAL** QCD Collaboration)

S. Aoki, N. Ishii, H. Nemura, K. Sasaki, M. Yamada (Univ. of Tsukuba)

B. Charron (Univ. of Tokyo)

T. Doi, T. Hatsuda , K. Murano (RIKEN)

Y. Ikeda (Tokyo Inst. Tech.)

T. Inoue (Nihon Univ.)

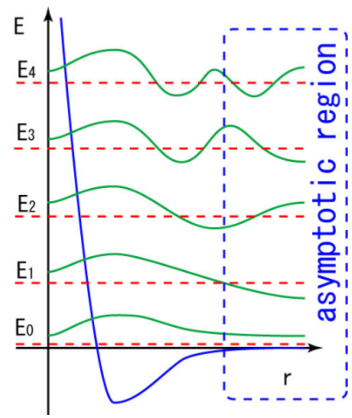
Phase shifts are encoded in wave functions

- Nambu-Bethe-Salpeter (NBS) wave function

$$\psi(\vec{r}) = \langle 0 | N(\vec{x}+\vec{r}) N(\vec{x}) | 2N \rangle$$

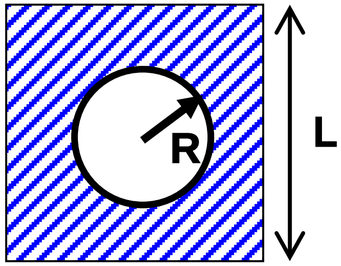
$$E = 2\sqrt{m^2 + k^2}$$

$$(\nabla^2 + k^2)\psi(\vec{r}) = 0, \qquad r > R$$



  – Wave function ←→ phase shifts

$$\psi(r) \simeq A \frac{\sin(kr - l\pi/2 + \delta(k))}{kr}$$



M.Luscher, NPB354(1991)531

CP-PACS Coll., PRD71(2005)094504

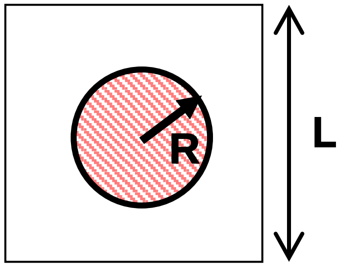
C.-J.Lin et al., NPB619(2001)467

Ishizuka, Pos LAT2009 (2009) 119

“Potential” as a representation of S-matrix (The HAL QCD method)

- Consider the wave function at “interacting region”

$$(\nabla^2 + k^2)\psi(\boldsymbol{r}) = m \int d\boldsymbol{r}' U(\boldsymbol{r}, \boldsymbol{r}')\psi(\boldsymbol{r}'), \quad r < R$$



  - U(r,r’): faithful to the phase shift by construction
    - U(r,r’) below inelastic threshold is

$$U(\boldsymbol{r}, \boldsymbol{r}') = \frac{1}{m} \sum_{n,n'}^{n_{\rm th}} (\nabla_{\boldsymbol{r}}^2 + k_n^2)\psi_n(\boldsymbol{r}) \mathcal{N}_{nn'}^{-1} \psi_{n'}^*(\boldsymbol{r}') \qquad \mathcal{N}_{nn'} = \int d\boldsymbol{r} \psi_n^*(\boldsymbol{r})\psi_{n'}(\boldsymbol{r})$$

    - U(r,r’): E-independent, while non-local in general
    - Non-locality ➔ derivative expansion   Okubo-Marshak(1958)

$$U(\vec{r}, \vec{r'}) = \underset{\textbf{LO}}{V_c(r)} + \underset{\textbf{LO}}{S_{12}V_T(r)} + \underset{\textbf{NLO}}{\vec{L}\cdot\vec{S}V_{LS}(r)} + \underset{\textbf{NNLO}}{\mathcal{O}(\nabla^2)}$$

Aoki-Hatsuda-Ishii PTP123(2010)89

Check on convergence: K.Murano et al., PTP125(2011)1225

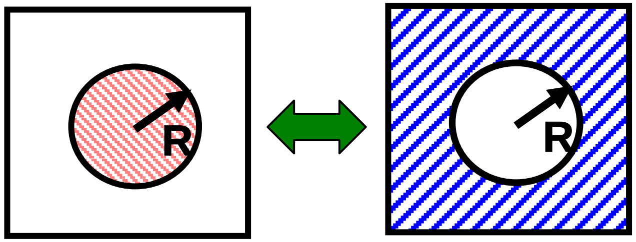
A few remarks on the Lattice Potential

- Potential is NOT an observable and is not unique:
  They are, however, phase-shift equivalent potentials.
  - Choosing the pot. (sink op.) ⇐⇒ choosing the "scheme"

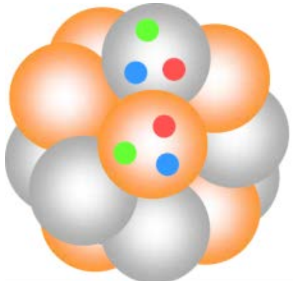
- We study potential (+ phase shifts), since:
  - Convenient to understand physics
  - Essential to study many-body

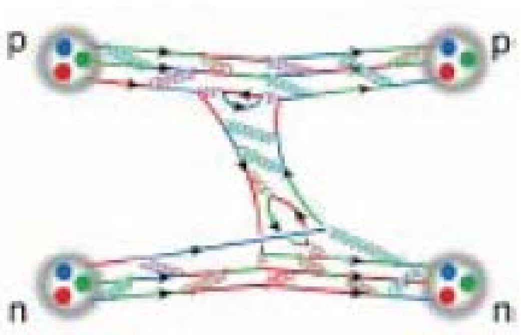
$$Lat \rightarrow \delta_E \rightarrow U(\boldsymbol{r}) \rightarrow \text{many-body}$$

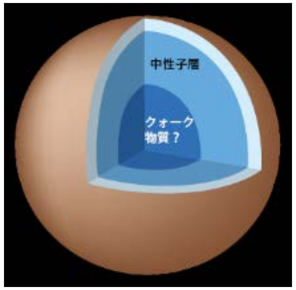
$$Lat \rightarrow \stackrel{\rightarrow}{\delta_E} \rightleftarrows U(\boldsymbol{r}) \rightarrow \text{many-body}$$



  - Finite V artifact better under control
  - Excited states better under control







- <u>Outline</u>
  - Introduction
  - Nuclear physics on the lattice
    - Major challenges: (1) S/N issue (2) computational cost
  - NN interactions
  - Hyperon interactions
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Challenges in multi-baryons on the lattice (1)

- Signal / Noise estimate Lepage(1989)
  - pion

$$\frac{\text{Signal}}{\text{Noise}} \sim \frac{\exp(-m_\pi t)}{\sqrt{\exp(-2m_\pi t)}} \sim \boxed{\text{const.}}$$

  - nucleon

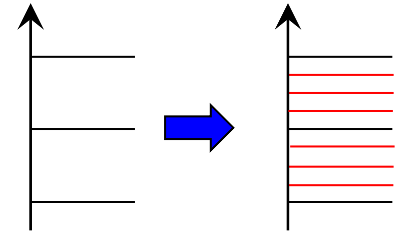
$$\frac{\text{Signal}}{\text{Noise}} \sim \frac{\exp(-m_N t)}{\sqrt{\exp(-3m_\pi t)}} \sim \boxed{\exp[-(m_N - 3/2 m_\pi)t]}$$

$$\frac{\text{Signal}}{\text{Noise}} \sim \boxed{\exp[-\mathbf{A}(m_N - 3/2 m_\pi)t]}$$ (for mass number = A)

➔ Variational method ? e.g., Luscher-Wolff (1990)

  - Large spectral density by scatt.

$$\Delta E \simeq \frac{\vec{p}^2}{m_N} \simeq 15\text{MeV} \quad \text{for } L = 10\text{fm}$$



Solution: Extract the signal from excited states

N.Ishii et al. (HAL QCD Coll.) PLB712(2012)437

E-indep of potential U(r,r') ➔ *(excited) scatt states share the same U(r,r')*

*They are **not contaminations**, **but signals***

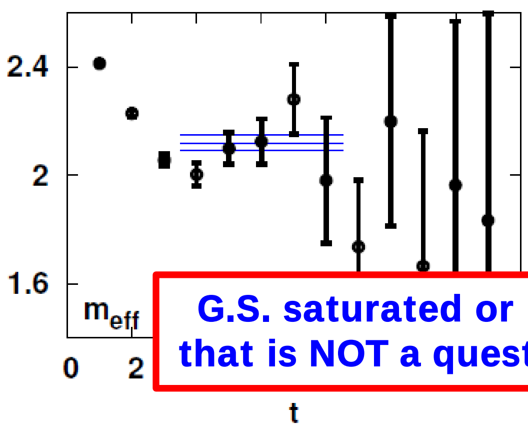
➔ **Time-dependent Schrodinger Eq.**

$$2\sqrt{m^2 + k_n^2} = E_n = -\frac{\partial}{\partial_t}$$

$$\left(-\frac{\partial}{\partial t} + \frac{1}{4m}\frac{\partial^2}{\partial t^2} - H_0\right) R(\boldsymbol{r}, t) = \int d\boldsymbol{r}' U(\boldsymbol{r}, \boldsymbol{r}') R(\boldsymbol{r}', t)$$

$$R(\boldsymbol{r}, t) \equiv C_{NN}(\boldsymbol{r}, t)/C_N(t)^2$$

Grand State (G.S.) saturation is NOT necessary !



G.S. saturated or not, that is NOT a question !

Explicit Lat calc for I=2 pipi phase shift

Beautiful agreement between

(1) Luscher's formula w/ g.s. saturation

(2) the HAL QCD method w/ & w/o g.s. saturation

➔ Talk by T.Kurth (Thu.)

Challenges in multi-baryons on the lattice (2)

- Enormous computational cost for correlators
  - # of Wick contraction (permutation)

$$N_{\rm perm} = N_u! \times N_d! \sim [\,(\tfrac{3}{2}A)!\,]^2$$ for mass number A

(← can be reduced by 2^A by inner-baryon exchange)

  - # of color / spinor contractions

$$N_{\rm loop} = 6^A \cdot 4^A \quad \text{or} \quad 6^A \cdot 2^A \;(\text{"half-spin"})$$

(color) (spinor)

$$N = \epsilon_{abc}(q^T C \gamma_5 q) q$$

  - Total cost: $N_{\rm perm} \times N_{\rm loop}$
    - ^{2}H : 9 x 144 = 1 x 10^3
    - ^{3}H : 360 x 1728 = 6 x 10^5
    - ^{4}He : 32400 x 20736 = 7 x 10^8

c.f. T.Yamazaki et al., PRD81(2010)111504

$N_{\rm perm} = 1107$ for ^{4}He in the isospin limit

c.f. recursive method for multi-meson
Detmold et al., PRD82(2010)014511

Solution: Unified contraction algorithm

TD, M.Endres, arXiv:1205.0585

- Traditional algorithm

color/spinor contractions (ξ')

$$\Pi^{2N} \simeq \langle qqqqqq(t)\bar{q}(\xi'_1)\bar{q}(\xi'_2)\bar{q}(\xi'_3)\bar{q}(\xi'_4)\bar{q}(\xi'_5)\bar{q}(\xi'_6)(t_0)\rangle \times \mathrm{Coeff}^{2N}(\xi'_1,\cdots,\xi'_6)$$

Permutations

color ϵ_{abc}, spinor $(C\gamma_5)$, etc.

- New algorithm

[impose the same spacial label at source]

  – Permutation applies to color/spinor indices at “Coeff”

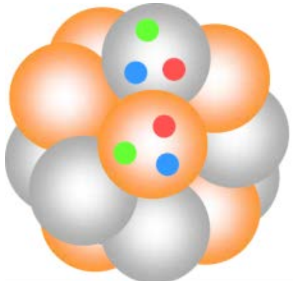
Permuted Sum

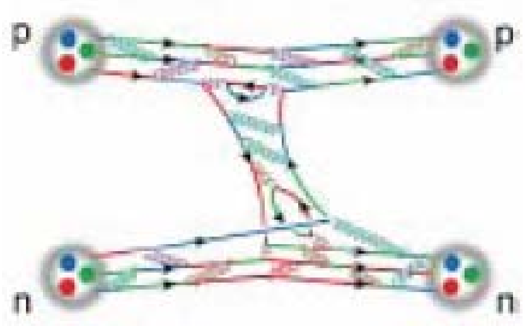
$$\Pi^{2N} \simeq \langle qqqqqq(t)\bar{q}(\xi'_1)\bar{q}(\xi'_2)\bar{q}(\xi'_3)\bar{q}(\xi'_4)\bar{q}(\xi'_5)\bar{q}(\xi'_6)(t_0)\rangle \times \mathrm{Coeff}^{2N}(\xi'_1,\cdots,\xi'_6)$$

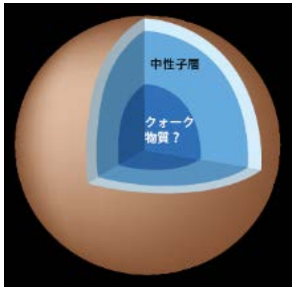
Sum over color/spinor unified list

  – **Permutation DONE beforehand**
    - (Wick contraction and color/spinor contractions are unified)
  – Significant improvement

$\times 192$ for ${}^3\mathrm{H}/{}^3\mathrm{He}$, $\times 20736$ for ${}^4\mathrm{He}$, $\times 10^{11}$ for ${}^8\mathrm{Be}$ (x add'l. speedup)







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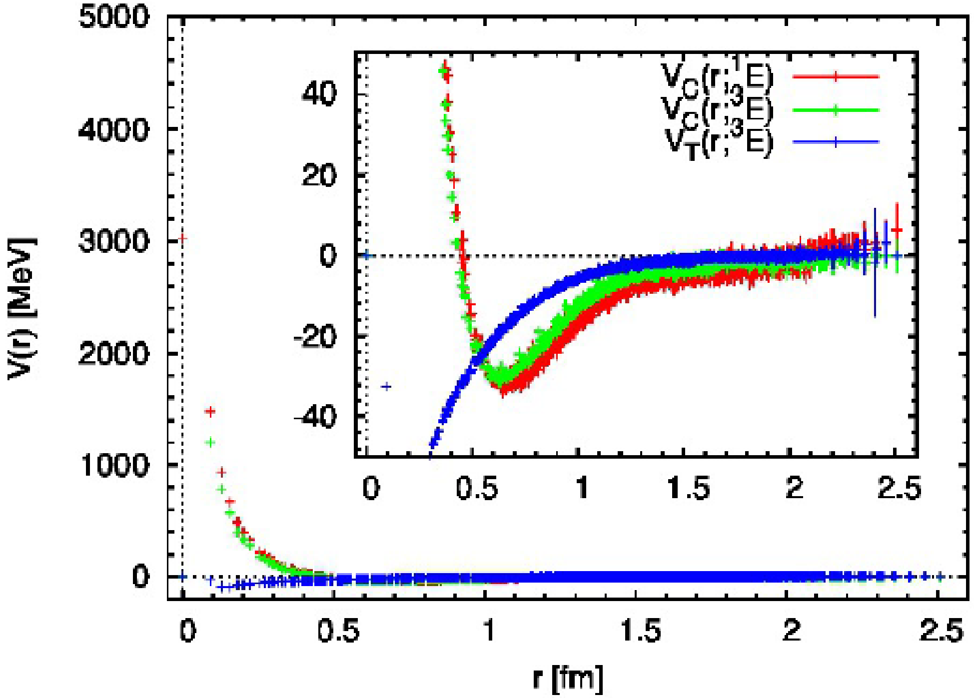
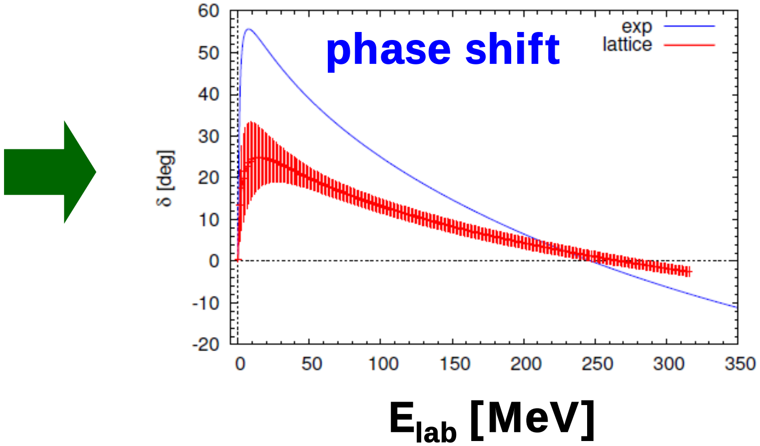
NN potential on the lattice (positive parity)

$^{2S+1}L_J$

- "di-neutron" channel $^1S_0$ ➔ central force
- "deuteron" channel $^3S_1-{}^3D_1$ ➔ central & tensor force





Not Bound $a({}^1S_0) = 1.6(1.1)$ fm

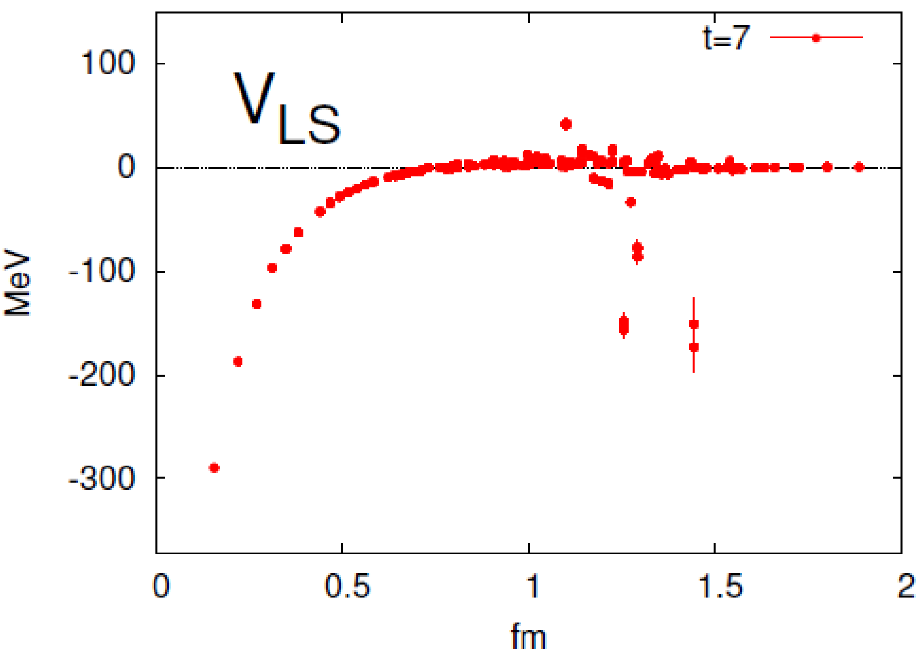
Nf=2+1 clover (PACS-CS), 1/a=2.2GeV,
L=2.9fm, m_π=0.7GeV, m_N=1.6GeV

[N.Ishii (Wed.)]

NN potential on the lattice (negative parity)

$^{2S+1}L_J$

- S=1 channel: $^3P_0,\ ^3P_1,\ ^3P_2 - ^3F_2$
  - Central & tensor forces in LO
  - Spin-orbit force in NLO
    - Inject a momentum ➔ $J^P = A_1^-, T_1^-, T_2^-$



Nf=2 clover (CP-PACS), L=2.5fm, mπ=1.1GeV

$\vec{L}\cdot\vec{S} = +1 \text{ for } ^3P_2$

Superfluidity $^3P_2$ in neutron star ←→ neutrino cooling

[K.Murano (Thu.)]

NN spectra on the lattice

- **PACS-CS Coll.** T. Yamazaki et al. PRD84(2011)054506
  - quenched, m_π= 0.8GeV, L= 3, 6, 12fm, (variational study)
    - di-neutron ($^1S_0$) : B.E. = 5.5(1.1)(1.0)MeV
    - deuteron ($^3S_1$ –$^3D_1$) : B.E. = 9.1(1.1)(0.5)MeV

Bound

  - Nf=2+1, m_π= 0.5GeV, L= 3 – 6 fm
    - Both channels still bound w/ similar B.E.
    - di-neutron ($^1S_0$) : B.E. = 7.4(1.3)(0.6)MeV
    - deuteron ($^3S_1$ –$^3D_1$) : B.E. = 11.5(1.1)(0.6)MeV

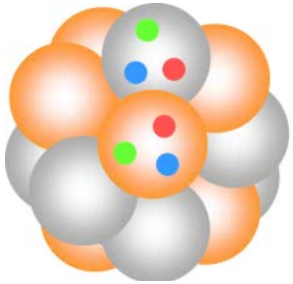
[T.Yamazaki (Tue.)]

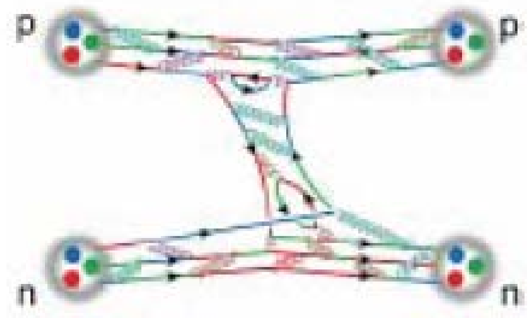
(preliminary)

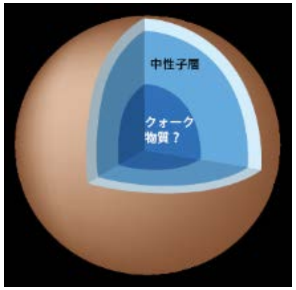
- **NPLQCD Coll.** S.Beane et al. PRD85(2012)054511
  - Nf=2+1, m_π= 0.39GeV, L= (2, 2.5), 3, 4 fm
    - di-neutron ($^1S_0$) : B.E. = 7.1(5.2)(7.3)MeV
    - deuteron ($^3S_1$ –$^3D_1$) : B.E. = 11 (05)(12)MeV

Suggestion of bound states

[K.Orginos (Wed.)]







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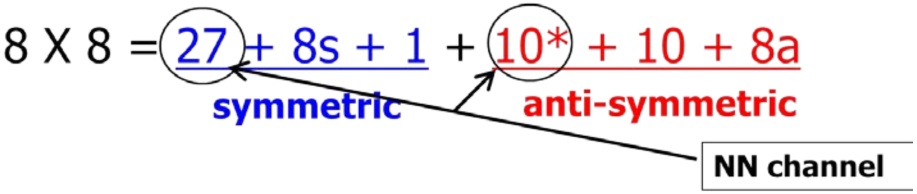
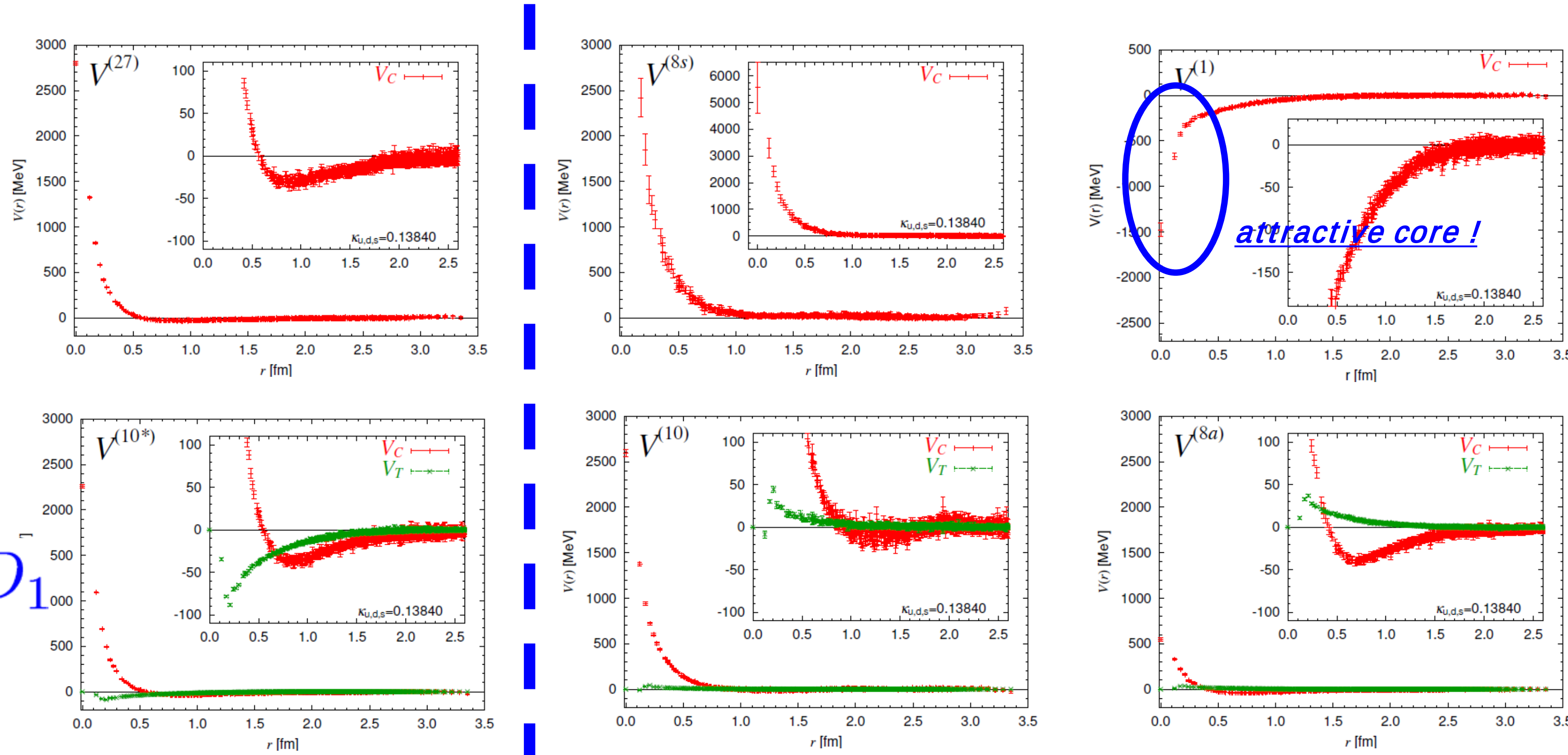


BB potentials

a=0.12fm, L=3.9fm, m(PS)= 0.47-1.2GeV

$^1S_0$

$^3S_1-^3D_1$



27,10*:
Same as NN

8s,10:
strong repulsive core

1s: deep attractive pocket
8a: weak repulsive core

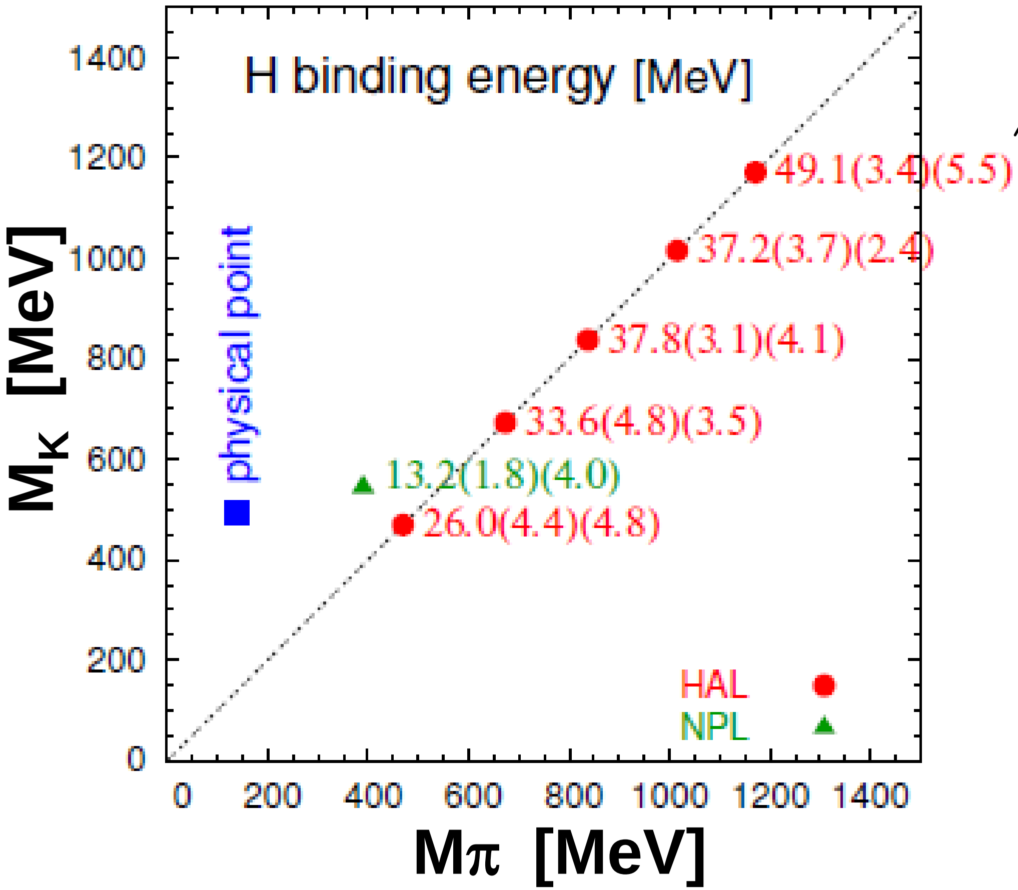
T.Inoue et al. (HAL QCD Coll.), NPA881(2012)28

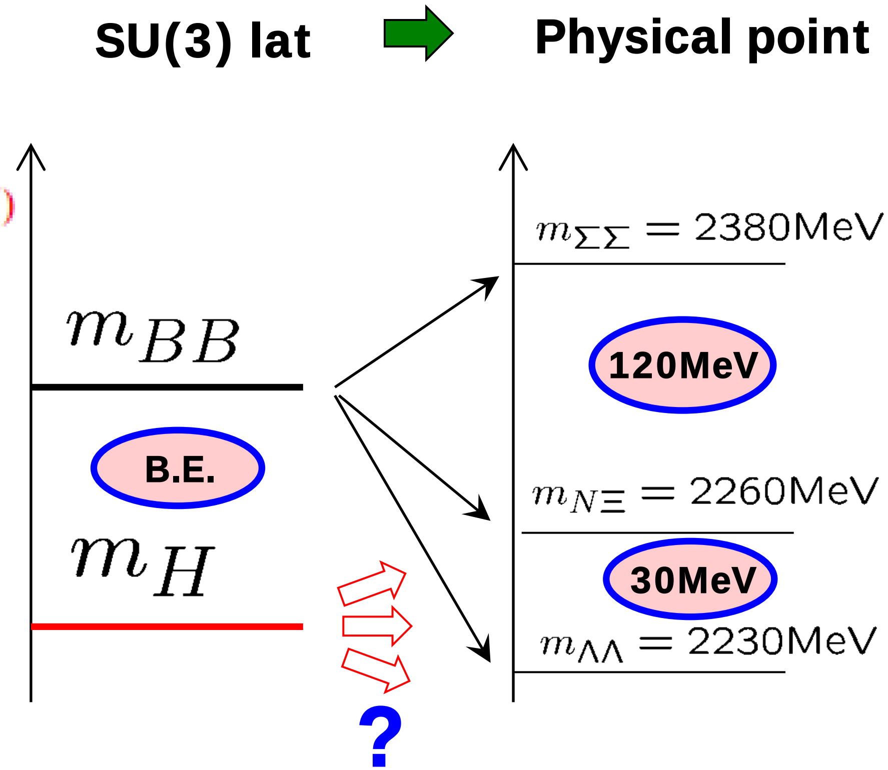
Repulsive core
← Pauli principle !

M.Oka et al., NPA464(1987)700

Also seen in SU(2)c , Takahashi et al.,, PRD82(2010)094506
Charmonium-N, Kawanai-Sasaki, PRD82(2010)091501
Meson-baryon, **Y.Ikeda** et al., arXiv:1111.2663, **Talk (Tue.)**

H-dibaryon (uuddss, I=0, $^1S_0$)





➔ [HAL] T.Inoue (Wed.)

➔ [NPL] K.Orginos (Wed.)

Coupled channel study is essential

$\Lambda\Lambda$–$N\Xi$–$\Sigma\Sigma$

➔ Talk by K.Sasaki (Thu.)

- **NPLQCD: New SU(3) study** Beane et al., arXiv:1206.5219
  - m_{PS} = 0.81GeV, L=(3.4, 4.5), 6.7fm, a=0.145fm
- **➔ Can be compared to one of the HAL QCD setup:**
  - m_{PS} = 0.84GeV, L=(2, 3), 3.9fm, a=0.121fm

NPLQCD

At least 4 channels bound

➔ Two bound H-dibaryons (71MeV, 19MeV)

G.S. saturation is necessary !

State	I	J^π	SU(3) irrep	B[illegible]ng Energy (MeV)
d	0	1^+	$\overline{10}$	25(3)(2)
nn	1	0^+	[illegible]	19(3)(1)
$n\Sigma$	$\frac{3}{2}$	[illegible]	[illegible]	3(3)(1)
H	[illegible]	[illegible]	[illegible]	71(3)(1)
[illegible]	[illegible]	[illegible]	[illegible]	36(3)(1)

27[illegible] on")

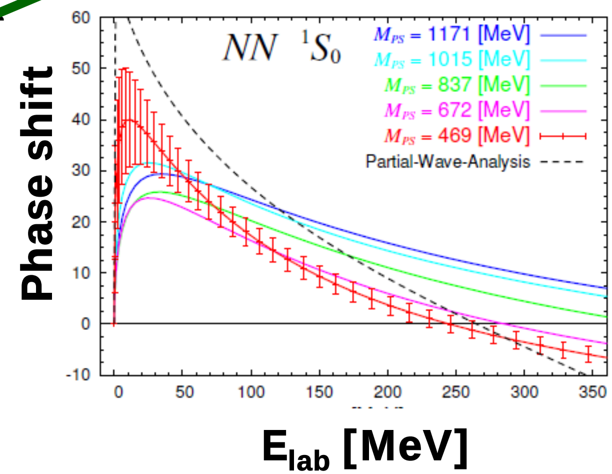
The results are "scattering"

HALQCD

Bound state [illegible]

➔ [illegible]aryon (38MeV)

G.S. saturated or not, that is NOT a question !



PACS-CS

Quenched, but m_π= 0.80GeV

27plet and 10bar bound:
27plet: B.E. = 5.5(1.1)(1.0)MeV

- **NPLQCD: New SU(3) study** Beane et al., arXiv:1206.5219
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State	I	J^π	SU(3) irrep	Binding Energy (MeV)
d	0	1^+	$\overline{10}$	25(3)(2)
nn	1	0^+	**27**	19(3)(1)
$n\Sigma$	$\frac{3}{2}$	1^+	**10**	3(3)(1)
H	0	0^+	**1**	71(3)(1)
$n\Xi$	0	1^+	$\mathbf{8}_A$	36(3)(1)

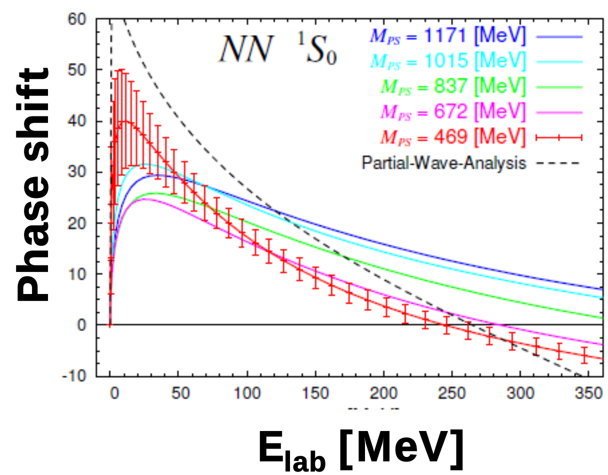
27plet : ("di-neutron")

HALQCD

Bound state only in singlet

➔ One bound H-dibaryon (38MeV)

G.S. saturated or not, that is NOT a question !



PACS-CS

Quenched, but m_π= 0.80GeV

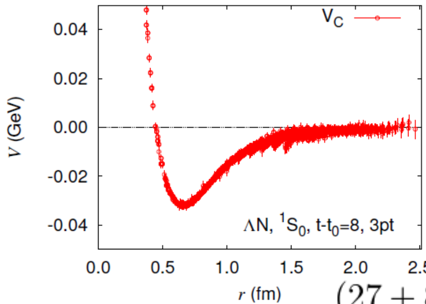
27plet and 10bar bound:
27plet: B.E. = 5.5(1.1)(1.0)MeV

Hyperon Interactions in S= -1

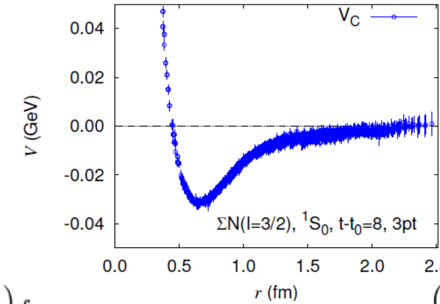
– HALQCD: ΛN $\Sigma N(I=3/2)$

Nemura et al.
Nf=2+1, L=2.9fm,
mπ= 0.70GeV
arXiv:1203.3320

$^1S_0$

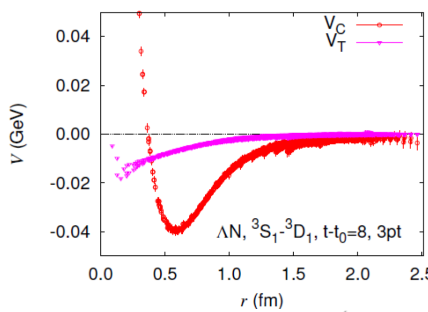


$(27+8_s)_f$

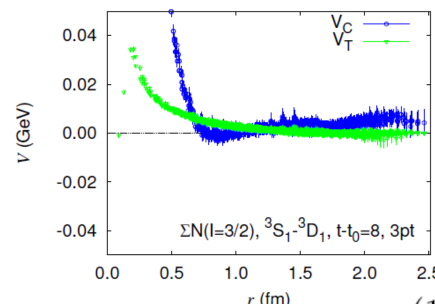


(27_f)

$^3S_1{-}^3D_1$



$(10^*+8_a)_f$



(10_f)

$\Lambda N(^1S_0, ^3S_1)$
$\Sigma N(I=3/2, ^1S_0)$
➔ Attractive
Not bound

Crucial input for the core of neutron star

– NPLQCD: Nf=2+1, mπ= 0.39GeV

Beane et al.
PRD81(2010)054505,

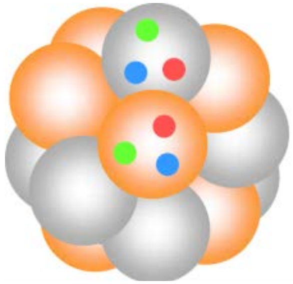
- $\Lambda N(^1S_0, ^3S_1)$ ➔ repulsive (L=2.5fm)
- $\Sigma N(I=3/2, ^1S_0)$ ➔ bound, BE=25(9.3)(11)MeV (L=3,4fm)
- $\Sigma N(I=3/2, ^3S_1)$ ➔ strong repulsive

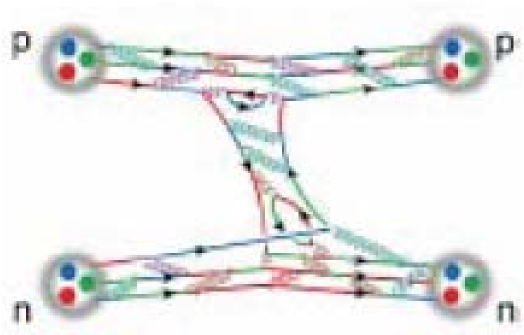
Beane et al. arXiv:1204.3606 ➔ M.Savage (Thu.)

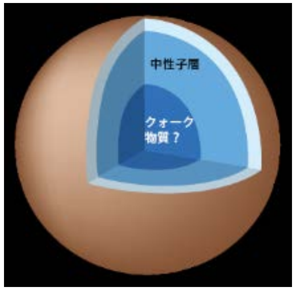
Other Hyperon Interactions

- S= -2 $\Lambda\Lambda$–$N\Xi$–$\Sigma\Sigma$ $(I=0, {}^1S_0)$ (H-dibaryon channel)
- S= -2, $N\Xi$–$\Sigma\Sigma$–$\Lambda\Sigma$
- S= -3 $\Lambda\Xi$–$\Sigma\Xi$ ➔ Systematic Study by K.Sasaki (Thur.)
- S= -4 $\Xi\Xi$
  - NPLQCD : L=(2, 2.5), 3, 4fm S.Beane et al. PRD85(2012)054511
    - $\Xi^-\Xi^-$ $(I=1, {}^1S_0)$ bound, B.E. = 14.0(1.4)(6.7)MeV
- S= -6: $\Omega\Omega$ ➔ Talk by K.Orginos (Wed.)
  - Buchoff et al. : same as NPL, but L=2.5, 3fm arXiv:1201.3596
    - J=0 : weak repulsive $a = -0.16(22)\text{fm}$
    - J=2: highly repulsive

➔ Talk by J.Wasem (Mon.)



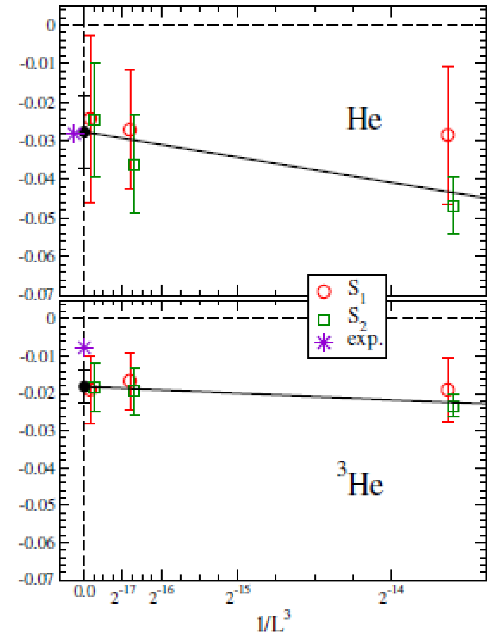




- <u>Outline</u>
  - Introduction
  - Nuclear physics on the lattice
  - NN interactions
  - Hyperon interactions
  - NNN interactions
  - Summary and Prospects

Spectroscopy on the lattice

- **PACS-CS Coll.** Yamazaki et al. PRD81(2010)111504
  - quenched, mπ= 0.8GeV, L= 3, 6, 12fm
    - ^{3}He=(^{3}H) : B.E. = 18.2 (3.5)(2.9) MeV
    - ^{4}He : B.E. = 27.7 (7.8)(5.5) MeV
  - Nf=2+1, mπ= 0.5GeV, L= 3 – 6 fm
    - Both nuclei are still bound w/ similar B.E.
    - ^{3}He=(^{3}H) : B.E. = 20.3 (4.0)(2.0) MeV (preliminary)
    - ^{4}He : B.E. = 43 (12) (8) MeV



[T.Yamazaki (Tue.)]

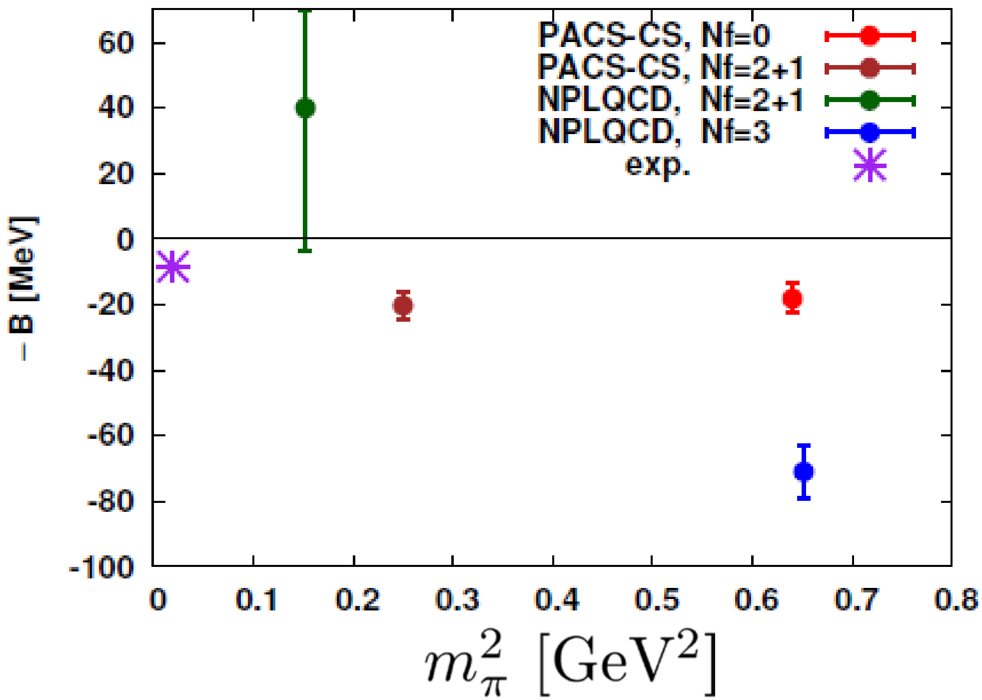
- **NPLQCD Coll.**
  - Nf=2+1, mπ= 0.39GeV, L= 2.5 fm only — S.Beane et al. PRD80(2009)074501, Prog.Part.Nucl.Phys 66(2010)1
    - Study $\Xi^0\Xi^0 n$ and pnn $\quad E_{pnn} - 3m_N = +40(21)(38)\mathrm{MeV}$
  - Nf=3, mπ= 0.81GeV, L= (3.4, 4.5), 6.7fm — Beane et al., arXiv:1206.5219
    - many (hyper) nuclei bound, e.g.,
    - ^{3}He=(^{3}H) : B.E. = 71 (6) (5) MeV
    - ^{4}He : B.E. = 110 (20)(15) MeV

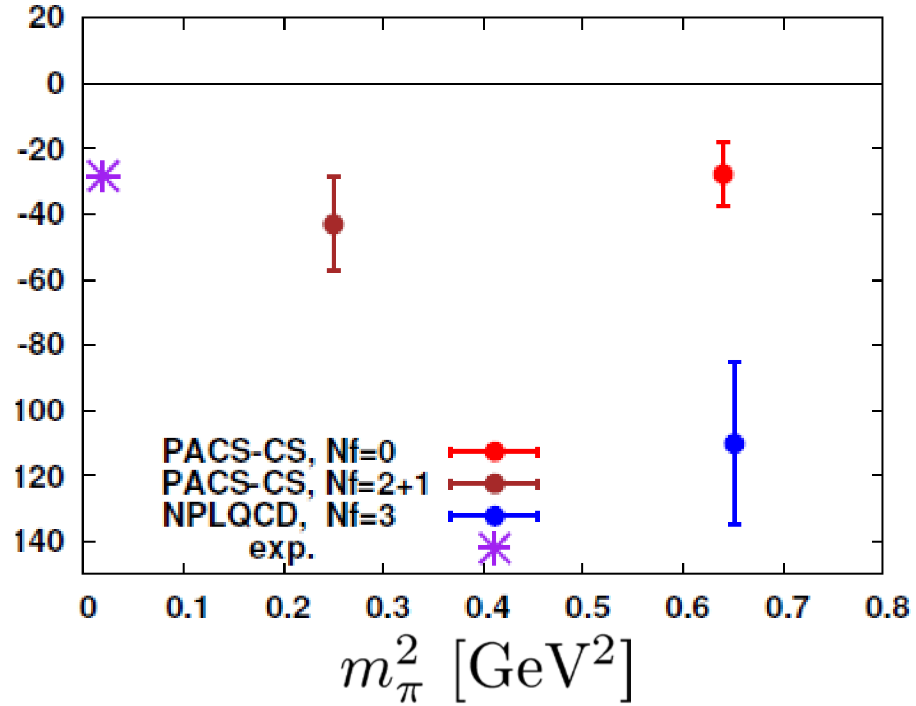
[K.Orginos (Wed.)]

Spectroscopy on the lattice

$^3\mathrm{H}\ (={}^3\mathrm{He})$

$^4\mathrm{He}$





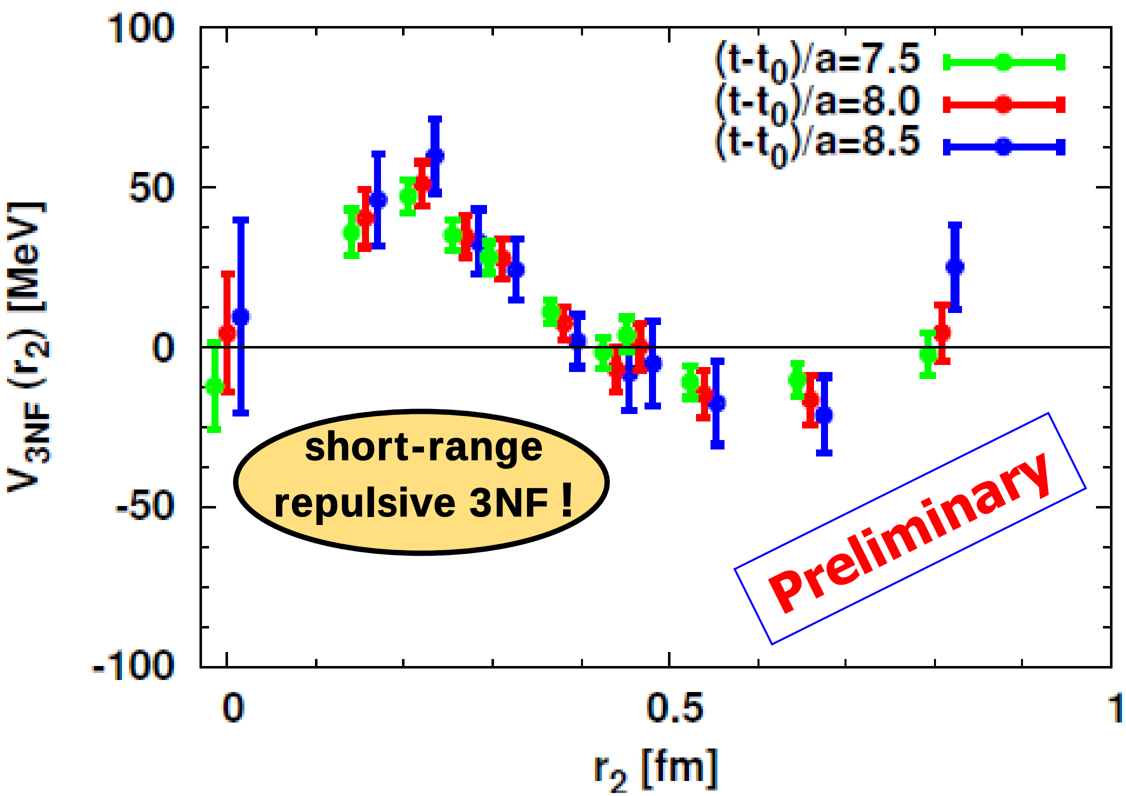
[T.Yamazaki (Tue.)]

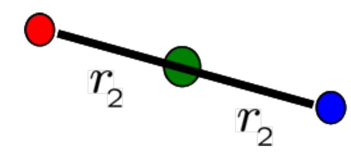
[K.Orginos (Wed.)]

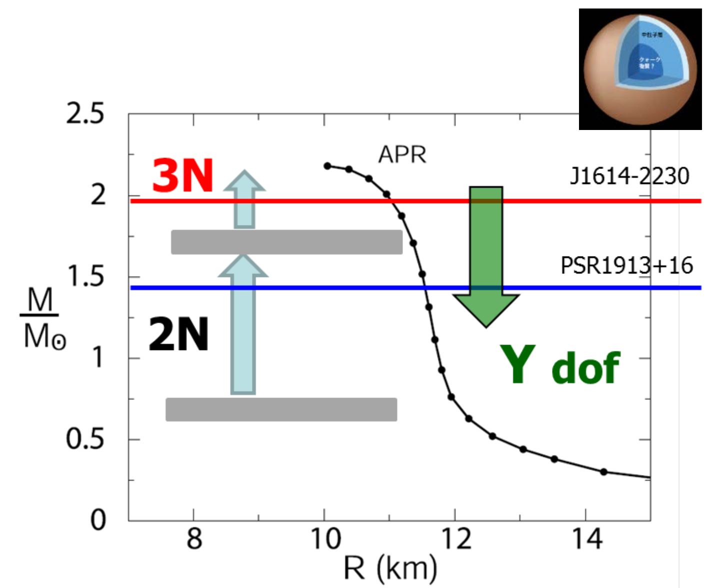
(NB: PACS-CS Nf=2+1: preliminary)

3N-forces (3NF) on the lattice

T.D. et al. (HAL QCD Coll.) PTP127(2012)723





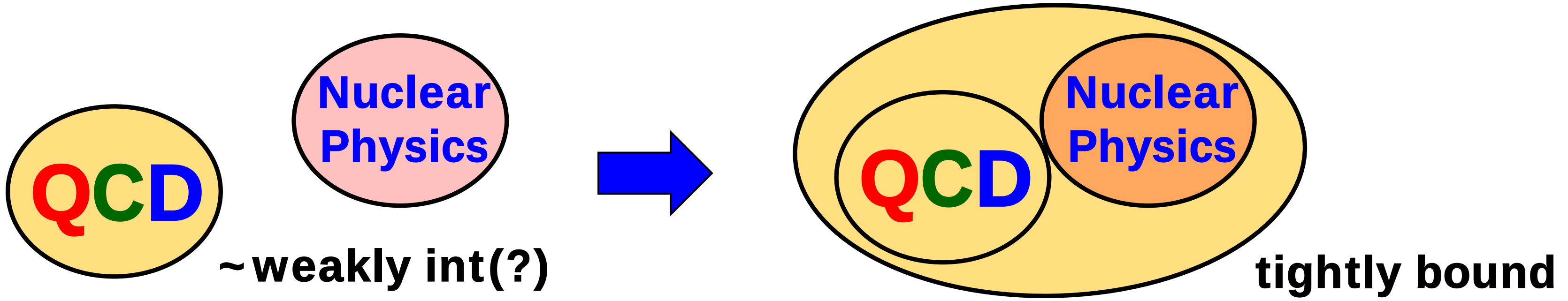


Nf=2 clover (CP-PACS), 1/a=1.27GeV,
L=2.5fm, m_π=1.1GeV, m_N=2.1GeV

How about other geometries ?
How about YNN, YYN, YYY ?

Summary and Prospects

- Nuclear Physics is in the era of **Renaissance**
- Lattice QCD predictions play a crucial role
- Vast amount of new results in
  - Lattice Potentials
  - Phase shifts and Bound states on the lattice
  - Breakthroughs in S/N issue & Comput. cost issue
- Bright future with the physical point simulation



Backup Slides

Scatterings on the lattice

- Luscher's formula

M.Luscher, CMP105(1986)156
NPB354(1991)531

  – Extract the phase shifts from spectrum in finite V

$$E = 2\sqrt{m^2 + k^2}$$

$$k \cot \delta(k) = \frac{2}{\sqrt{\pi}L} Z_{00}(1; q^2), \quad q = \frac{kL}{2\pi}$$

$$Z_{00}(s; q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\boldsymbol{n} \in \boldsymbol{Z}^3} \frac{1}{(\boldsymbol{n}^2 - q^2)^s}$$

$$\text{low energy:} \quad k \cot \delta(k) = \frac{1}{\mathbf{a}} + \frac{1}{2} \mathbf{r}\, k^2 + \cdots$$

Large V expansion

$$\Delta E = E - 2m = -\frac{4\pi \mathbf{a}}{mL^3} \left[1 + c_1 \frac{a}{L} + c_2 \left(\frac{a}{L} \right)^2 + \mathcal{O}\left(\frac{1}{L^3}\right) \right]$$

c_1, c_2: geometric constants

Bound state

Infinite V extrapolation has to be examined

Lattice QCD Setup

- **HAL QCD Coll.**
  - Nf=3 clover, 1/a=1.6GeV, L= (2,3),4 fm,
    m_{PS} = 0.47,.0.67, 0.84, 1.0, 1.2GeV, m_B=1.2, 1.5, 1.7, 2.0, 2.3GeV
  - Nf=2+1 clover, 1/a=2.2GeV, L=2.9fm, $m\pi$=0.7GeV, m_N=1.6GeV (PACS-CS)
  - Nf=2, clover, 1/a=1.3GeV, L=2.5fm, $m\pi$=1.1GeV, m_N=2.1GeV (CP-PACS)

Y.Ikeda (Tue.), N.Ishii (Wed.), T.Inoue (Wed.),
K.Sasaki (Thu.), K.Murano (Thu.), B.Charron (Poster)

- **PACS-CS Coll.**
  - quenched clover, 1/a=1.5GeV, L=3,6,12 fm, $m\pi$=0.8GeV, m_N=1.6GeV
  - Nf=2+1 clover, 1/a=2.2GeV, L= 3 – 6 fm, $m\pi$=0.5GeV, m_N=1.3GeV

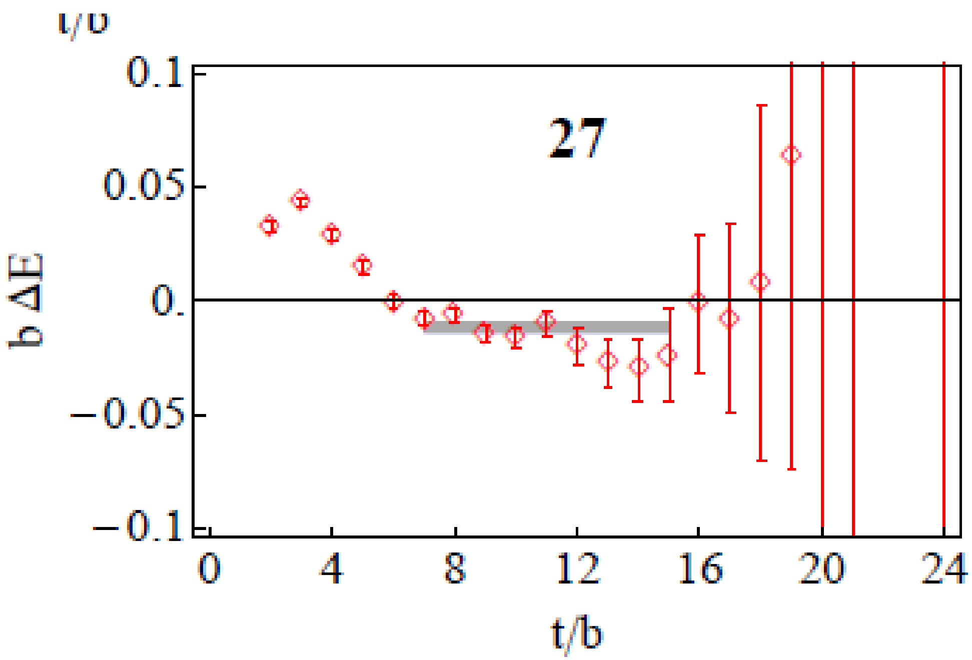
T.Yamazaki (Tue.)

- **NPLQCD Coll.** (a_s/a_t=3.5)
  - Nf=2+1 clover, 1/a_s=1.6GeV, L=(2, 2.5), 3, 4fm, $m\pi$=0.39GeV, m_N=1.2GeV
  - Nf=3 clover, 1/a=1.4GeV, L=(3.4,4.5), 6.7fm, m_{PS}=0.81GeV,m_B=1.6GeV

K.Orginos (Wed.), M.Savage(Thu.)

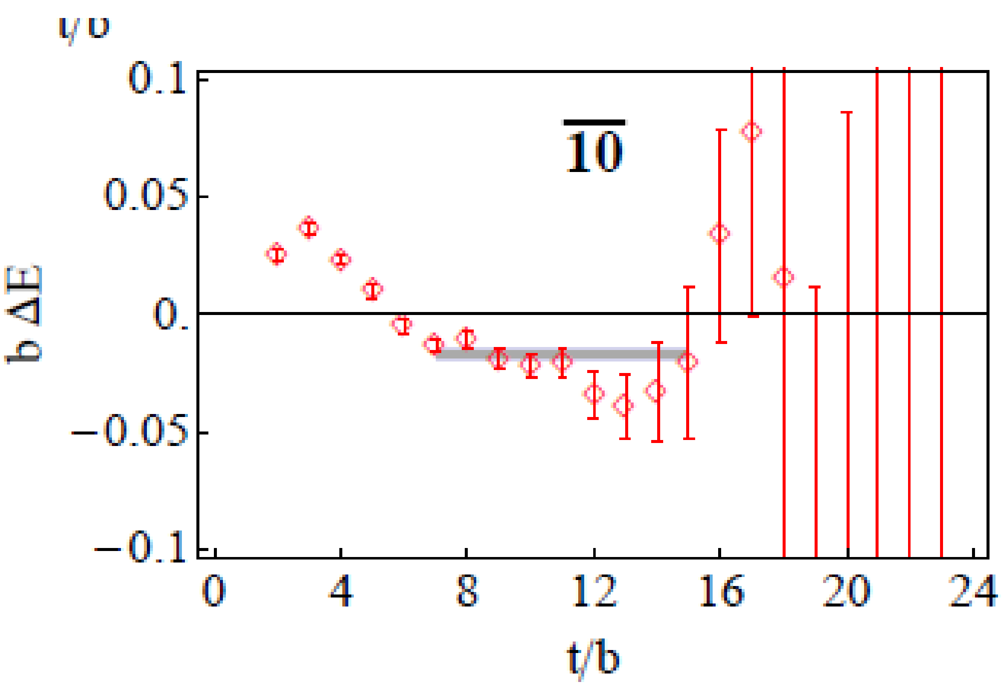
NPLQCD: SU(3) study

Beane et al., arXiv:1206.5219



“di-neutron” channel

G.S. saturated or not, that is the question



“deuteron” channel

^{3}H $(={}^3$He$)$

NPLQCD: SU(3) study

Beane et al., arXiv:1206.5219

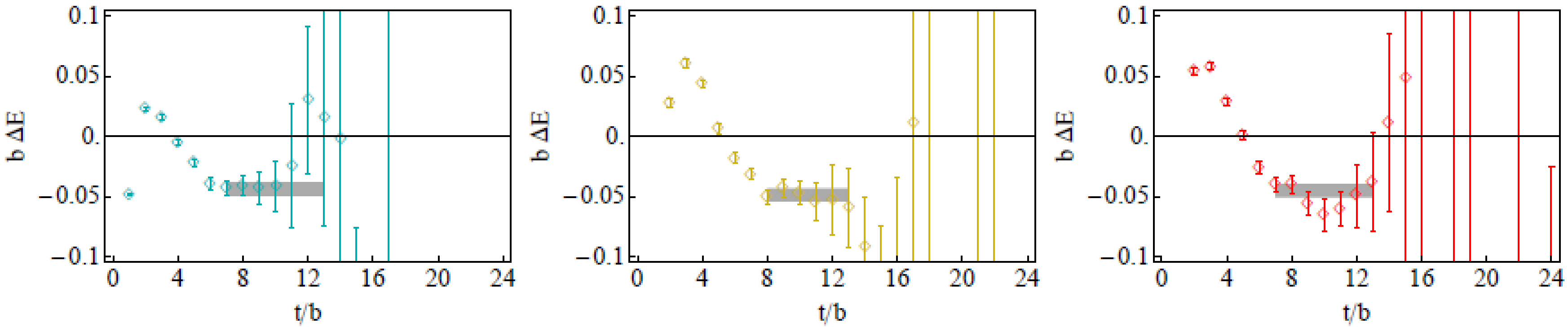


FIG. 6: EMP's associated with $J^\pi = \frac{1}{2}^+$ ^{3}He (^{3}H) $|\mathbf{P}| = 0$ correlation functions computed with the $24^3 \times 48$ (left), $32^3 \times 48$ (center) and $48^3 \times 64$ (right) e
to the statistical uncertainty associated with the shown f

G.S. saturated or not, that is the question

^{4}He

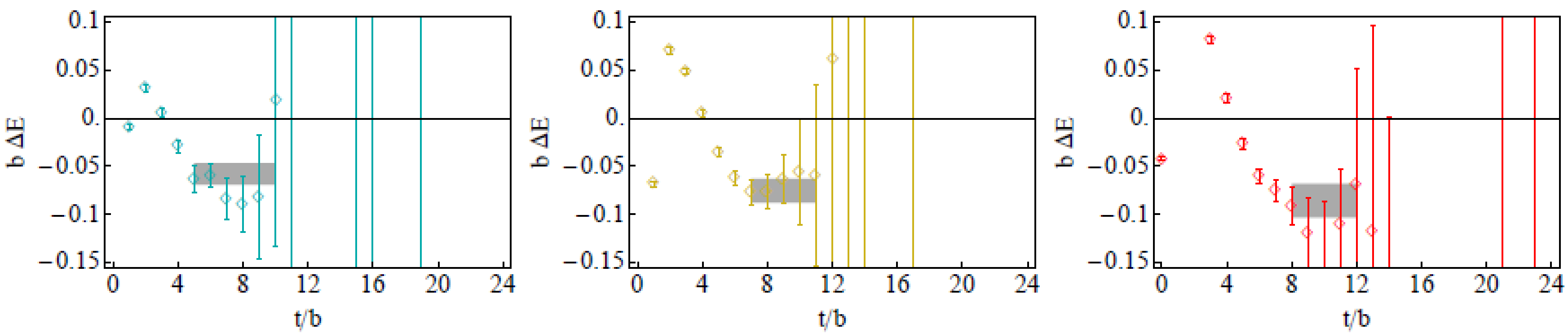
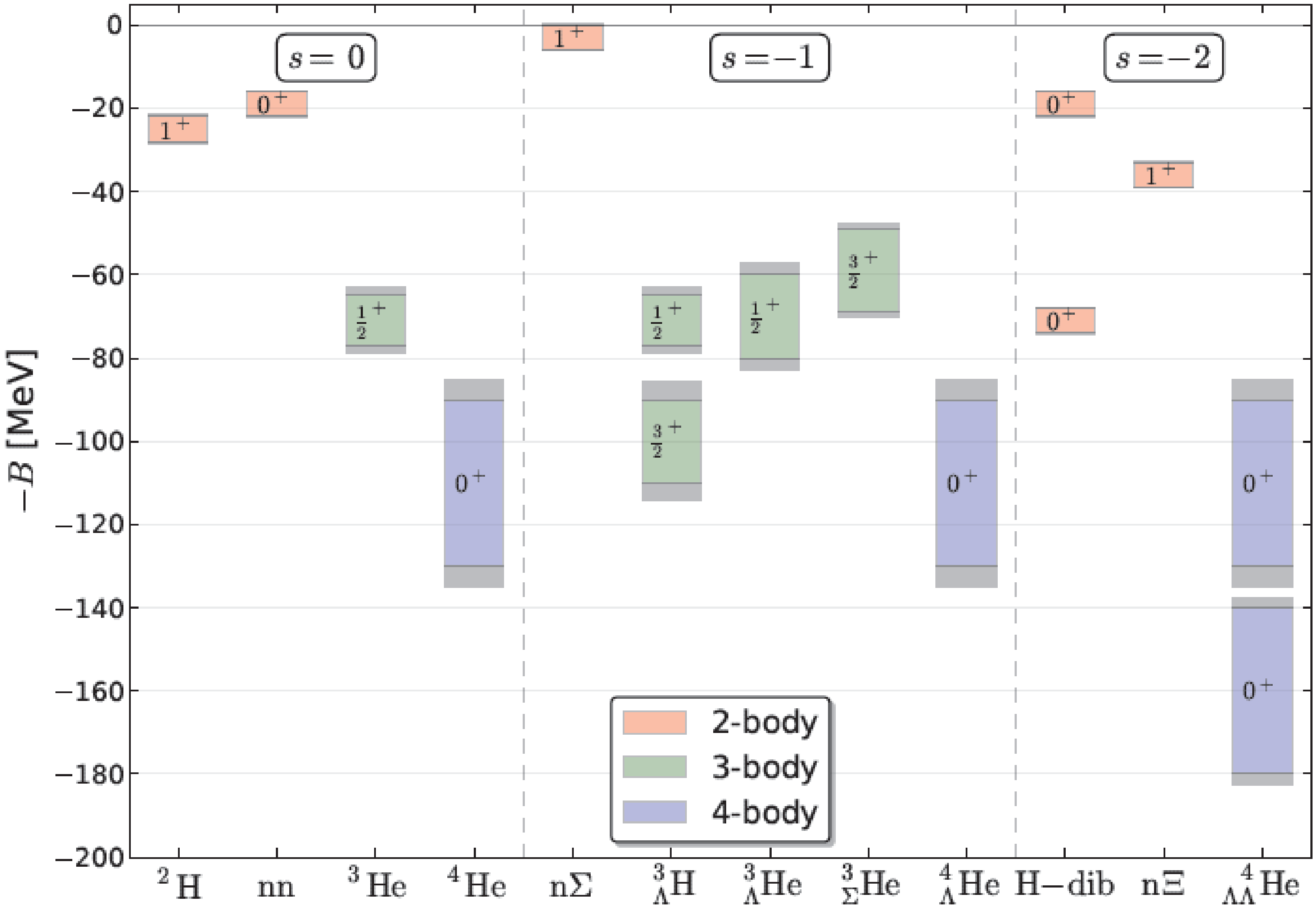


FIG. 14: EMP's associated with a $|\mathbf{P}| = 0$ $J^\pi = 0^+$ ^{4}He correlation function computed with the $24^3 \times 48$ (left), $32^3 \times 48$ (center) and $48^3 \times 64$ (right) ensembles. The shaded regions corresponds to the statistical uncertainty associated with the shown fitting interval.

NPLQCD: SU(3) study

Beane et al., arXiv:1206.5219



Solution: Extract the signal from excited states

N.Ishii et al. (HAL QCD Coll.) PLB712(2012)437

E-indep of potential U(r,r') ➔ *(excited) scatt states share the same U(r,r')*

They are ***not contaminations, but signals***

$$\left(k_n^2/m - H_0\right)\psi_n(\boldsymbol{r}) = \int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')\psi_n(\boldsymbol{r}')$$

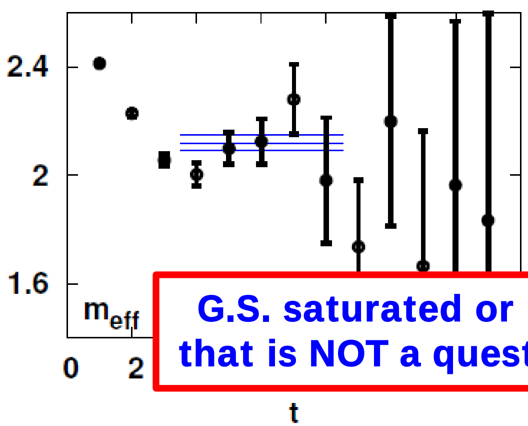
⇩

$$\left(-\frac{\partial}{\partial t} + \frac{1}{4m}\frac{\partial^2}{\partial t^2} - H_0\right) R(\boldsymbol{r},t) = \int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}') R(\boldsymbol{r}',t)$$

$$2\sqrt{m^2 + k_n^2} = E_n = -\frac{\partial}{\partial t}$$

$$R(\boldsymbol{r},t) \equiv C_{NN}(\boldsymbol{r},t)/C_N(t)^2$$

Grand State (G.S.) saturation is NOT necessary !



G.S. saturated or not, that is NOT a question !

Explicit Lat calc for I=2 pipi phase shift

Beautiful agreement between

(1) Luscher's formula w/ g.s. saturation

(2) the HAL QCD method w/ & w/o g.s. saturation

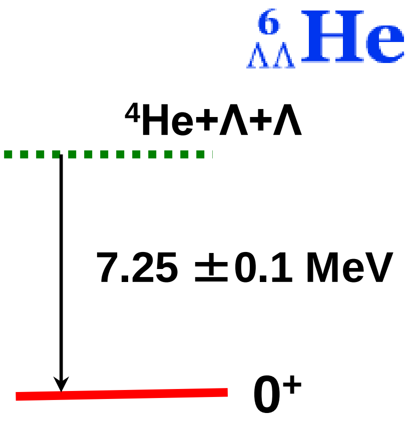
➔ Talk by T.Kurth (Thu.)

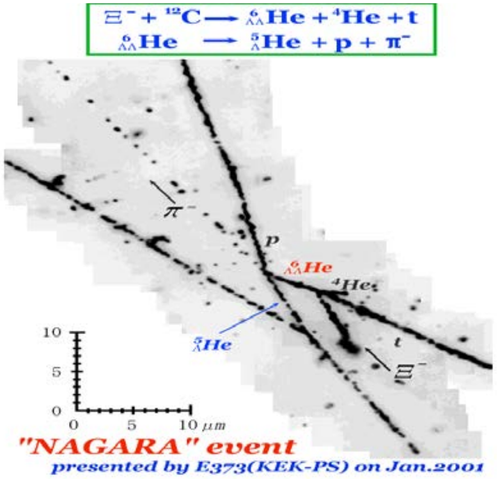
H-dibaryon

- Predicted by R.Jaffe ('77) [uuddss], I=0,$^1S_0$
  - MIT bag model / diquark picture…
- Lattice studies
  - Mackenzie et al., ('85), Iwasaki et al. ('88), Negele et al. ('99), Wetzorke et al. ('00, '03)
  - HAL QCD Coll. PRL106(2011)162002
  - NPLQCD Coll, PRL106(2011)162001
  - Luo et al., arXiv:1106.1945

Bound H found !

- Experiments:

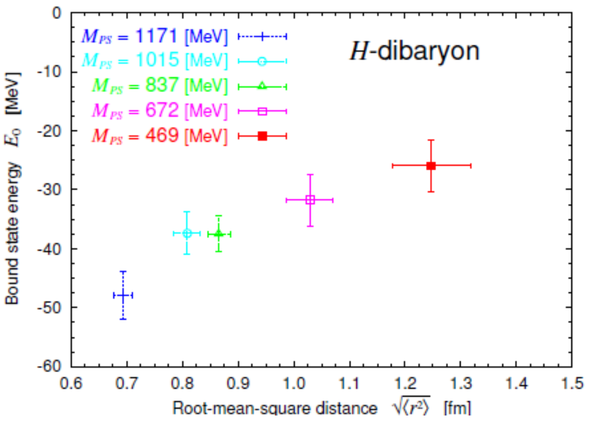




- ΛN attraction
- ΛΛ weak attraction
- No deeply bound H-dibaryon

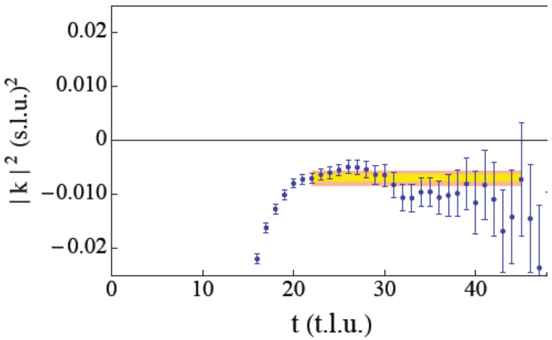
H-dibaryon (uuddss, I=0, $^1S_0$)

- HAL QCD Coll. T.Inoue et al. NPA881(2012)28
  - Nf=3 clover, L=2, 3, 4fm, a=0.12fm
  - m_{PS} = 0.47-1.2GeV, m_B=1.2-2.3GeV
  - ➔ **bound**, B.E. = 26-49MeV



➔ Talk by T.Inoue (Wed.)

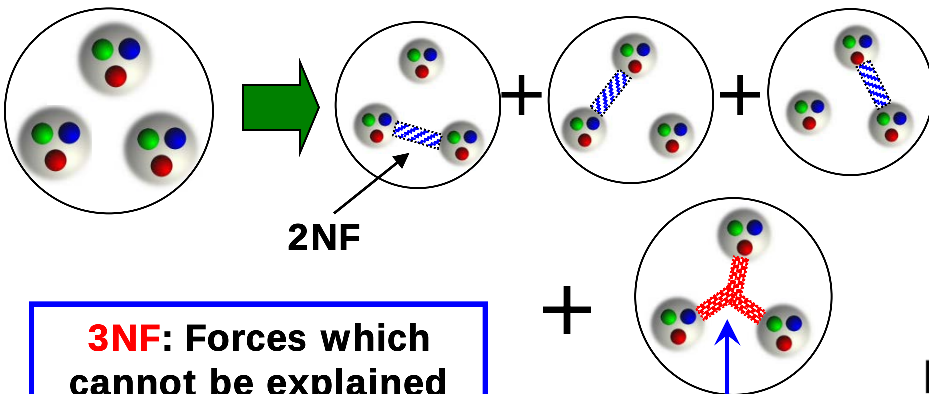
- NPLQCD Coll. S.Beane et al. PRD85(2012)054511
  - Nf=2+1 clover, L=(2, 2.5), 3, 4fm, a_s=0.12fm (a_s/a_t=3.5)
  - $m\pi$= 0.39GeV, m_N=1.2GeV
  - ➔ **bound**, B.E. = 13.2(1.8)(4.0)MeV



➔ Talk by K.Orginos (Wed.)

Frontier in Hadron-Hadron Interactions
⇒Three-Nucleon Forces (3NF)

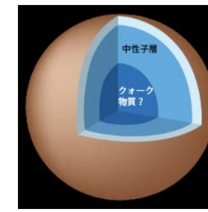
What is 3NF ?

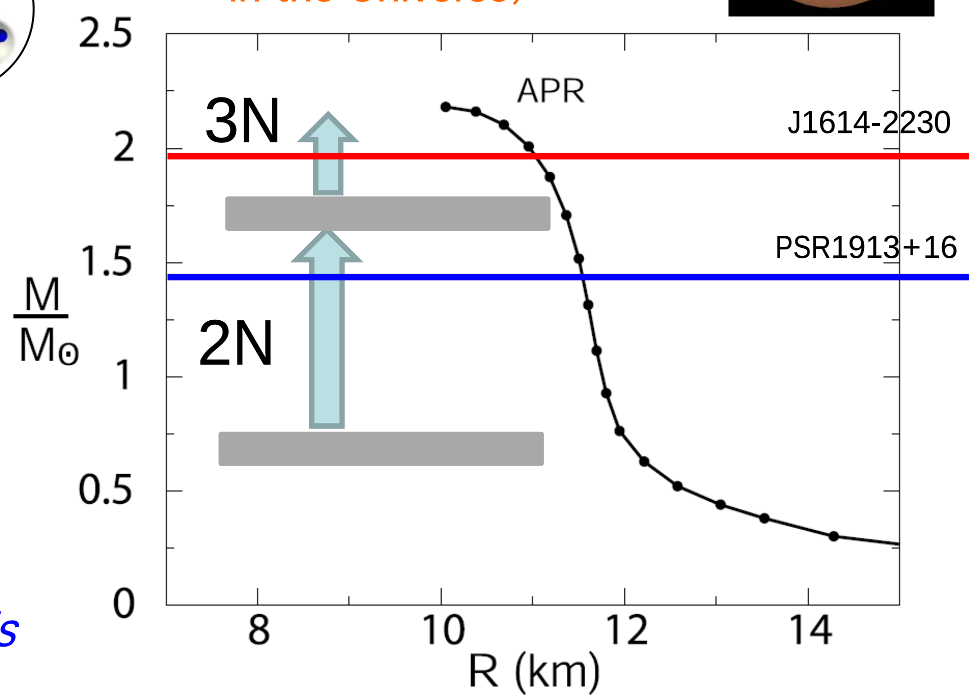


3NF: Forces which cannot be explained by pair-wise 2NF

Neutron Star

（Densest system in the Universe）





- *EoS of high density matter*
- *Neutron rich nuclei / Nucleosynthesis*
- *B.E. of light nuclei*

Short-range repulsive 3NF is required
Can we understand it from QCD ?

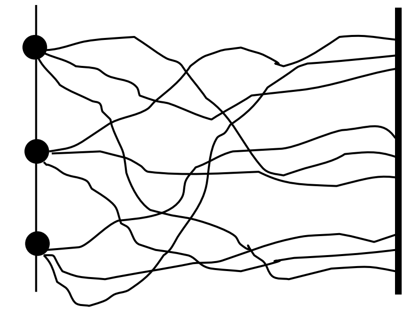
Lattice QCD setup for 3NF

- Nf=2 clover fermion + RG improved gauge action (CP-PACS)
  - 598 configs x 32 measurements
  - beta=1.95, (a^{-1}=1.27GeV, a=0.156fm)
  - 16^3 x 32 lattice, L=2.5fm
    - M(π) = 1.13GeV
    - M(N) = 2.15GeV
    - M(Δ) = 2.31GeV

(κ(ud) =0.13750)

(Mπ L=14)

CP-PACS Coll. S. Aoki et al., Phys. Rev. D65 (2002) 054505



- Correlators
  - **Standard nucleon op** to define the wave function / potential **at sink**

$$N = \epsilon_{abc}(q_a^T C\gamma_5 q_b)q_c$$

  - **Non-rela limit op** is used to create 3N state **at source**

$$G(\vec{r}_2, t-t_0) = \sum_{\vec{x}} \langle 0 | \underbrace{N(\vec{x}+\vec{r}_2, t)N(\vec{x}-\vec{r}_2, t)N(\vec{x}, t)}_{\text{sink}} \underbrace{\overline{NNN}(t_0)}_{\text{source}} | 0 \rangle$$

See also T.Yamazaki et al., PRD81(2010)111504