Lattice gauge theory and physics beyond the standard model

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Outline

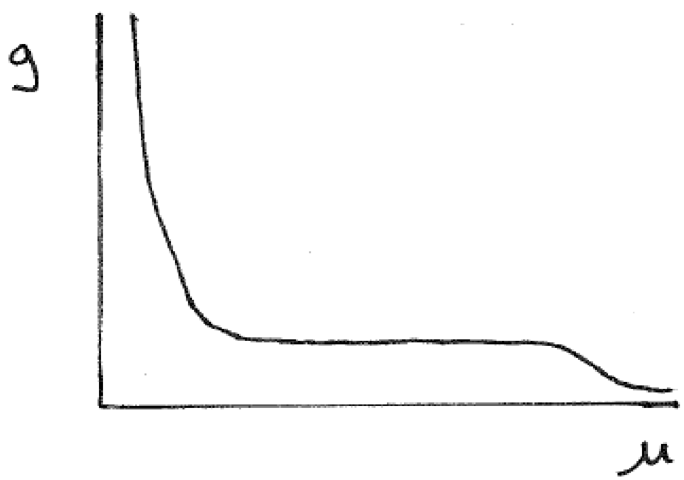
- Dynamical electroweak symmetry breaking/Technicolor
  - Conformal/confining
  - Measurements
- Not included (w/ apologies):
  - Supersymmetry
  - Condensed matter
  - Matrix models
  - Quantum gravity
  - Extra dimensions
  - Weak interaction
  - Higgs physics
  - Standard model tests
  - Generic quantum field theory

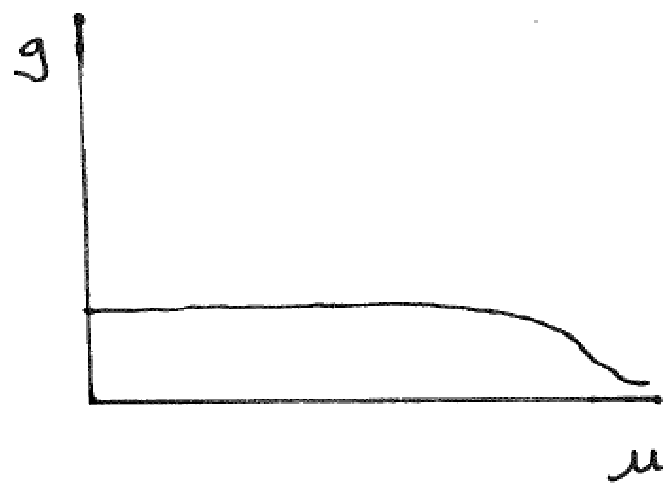
Technicolor

- Standard model Higgs suffers from hierarchy problem and triviality problem
- Replace Higgs vev with condensate of new fermions under new color group (technicolor)
- To avoid FCNCs, need to push up scale associated with exchange particle (ETC) that generates four-fermion terms from which quark/lepton masses are derived.
- Need large condensate at ETC scale to get viable quark masses: walking with $\gamma=1$

$$\langle\bar{\psi}\psi\rangle_{ETC} = \langle\bar{\psi}\psi\rangle_{TC} \left(\frac{\Lambda_{ETC}}{\Lambda_{TC}}\right)^{\gamma}$$

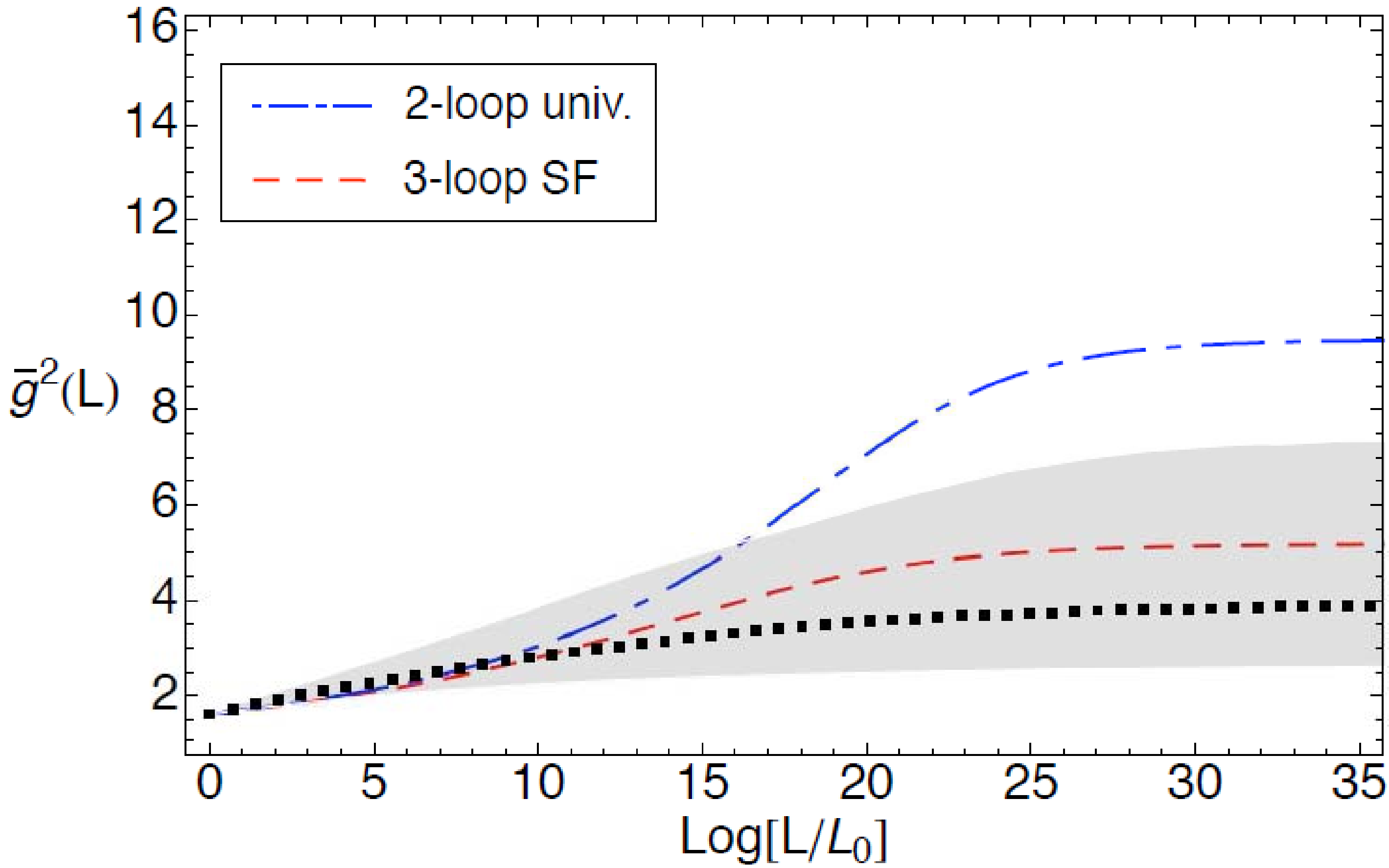
- Much effort has gone into attempting to distinguish between the two pictures below.
- It's not easy given that on a single lattice we only see a small range of μ and it's necessary to keep bare $\boldsymbol{g}$ small to have small $\boldsymbol{a}$.
- In addition, we often have a nonzero mass, which drives the theory away from the fixed point at long distances (RHS $\rightarrow$ LHS).



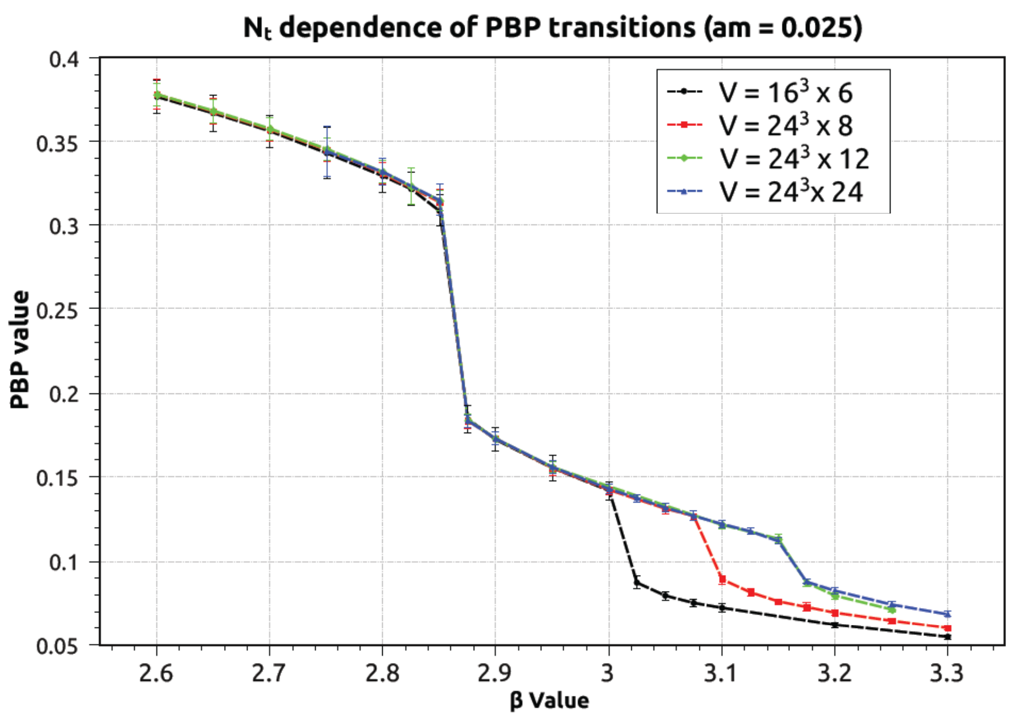


12 flavors of fundamentals in SU(3)

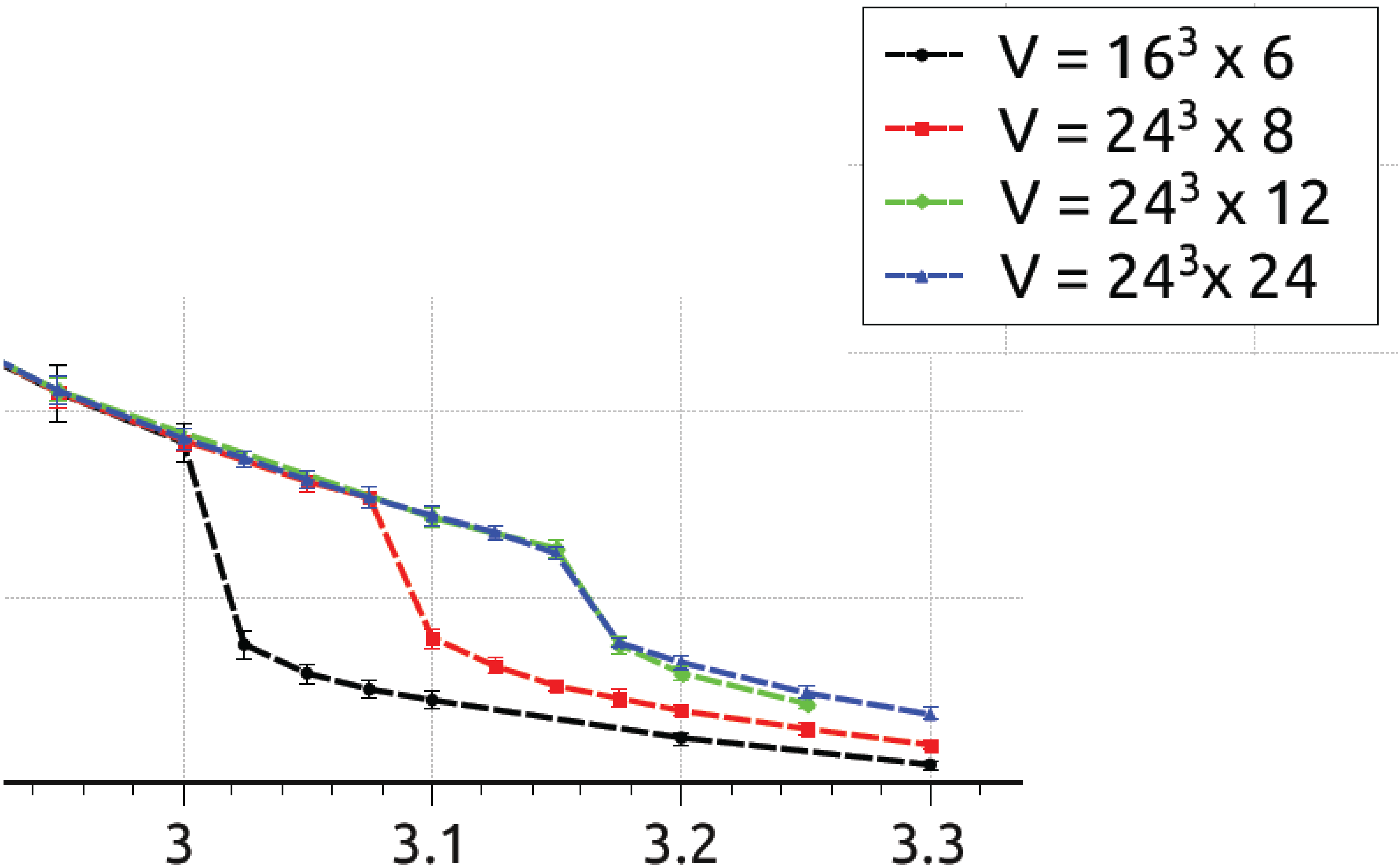
- Of course you have the Appelquist, Fleming, Neil result…



- More recently, Deuzeman, Lombardo, Nunez da Silva & Pallante observe two transitions, one of which is clearly bulk and the other is argued to be bulk:

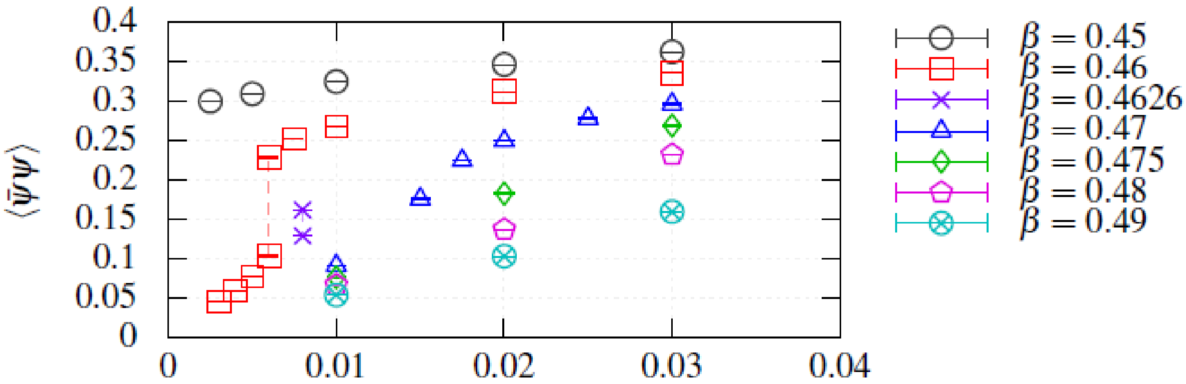


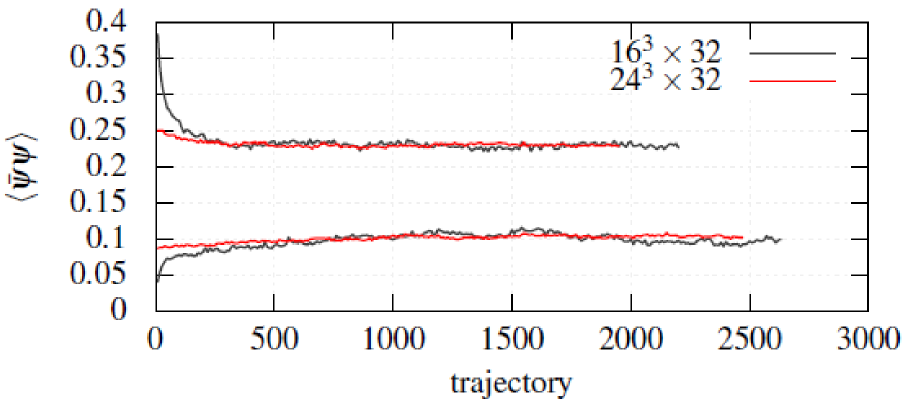
Lattice 2011 proceedings, 1111.2590



V = 16³ x 6
V = 24³ x 8
V = 24³ x 12
V = 24³x 24
3
3.1
3.2
3.3

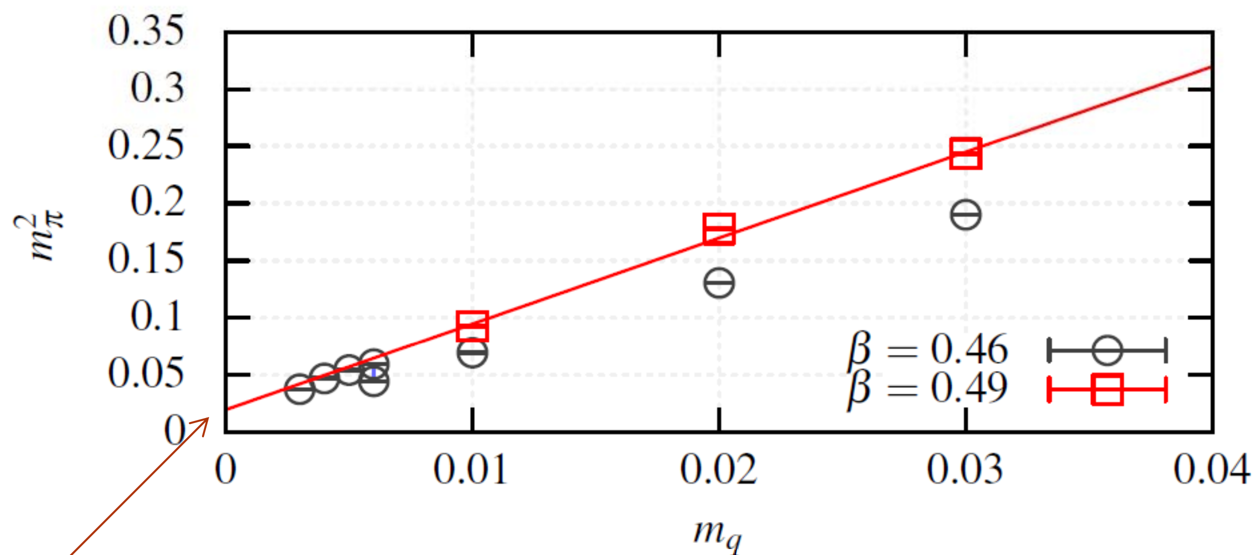
- Jin & Mawhinney observe a bulk transition with two different actions





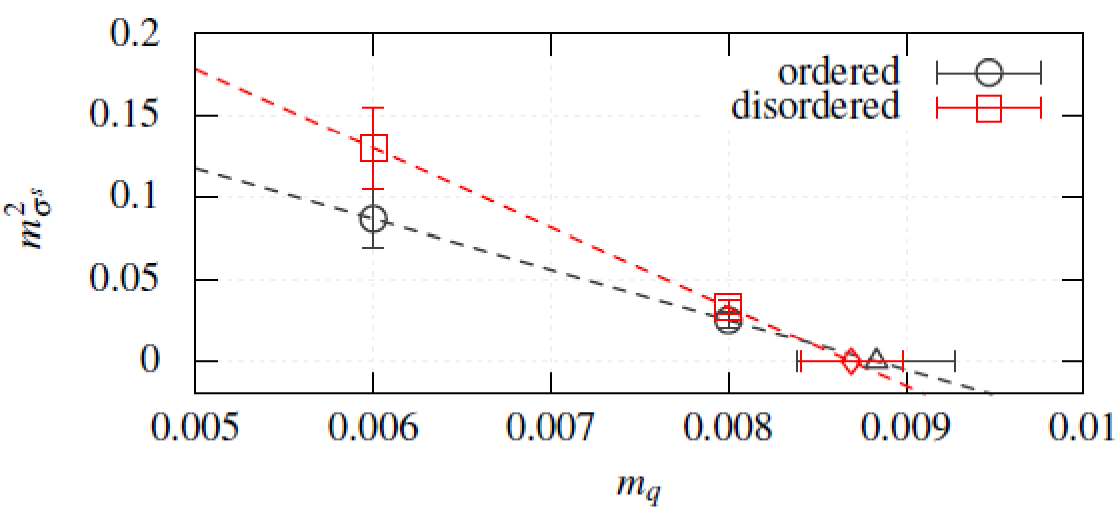
Lattice 2011 proc.,
1203.5855

- On the weak side they see behavior consistent (?) with χSB



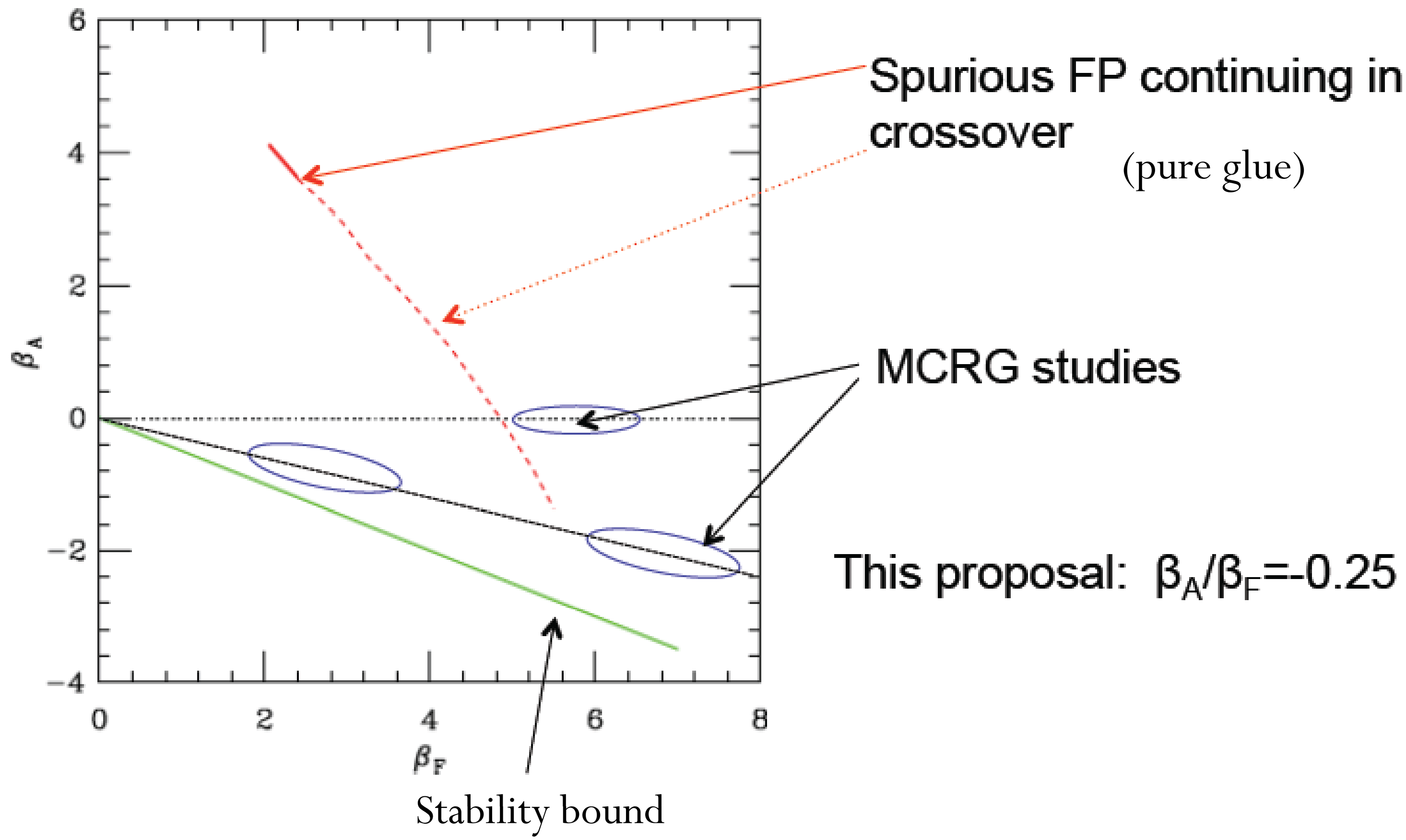
Nonzero intercept?

- They also find a massless scalar at the end of the first order line and interpret it as a UVFP



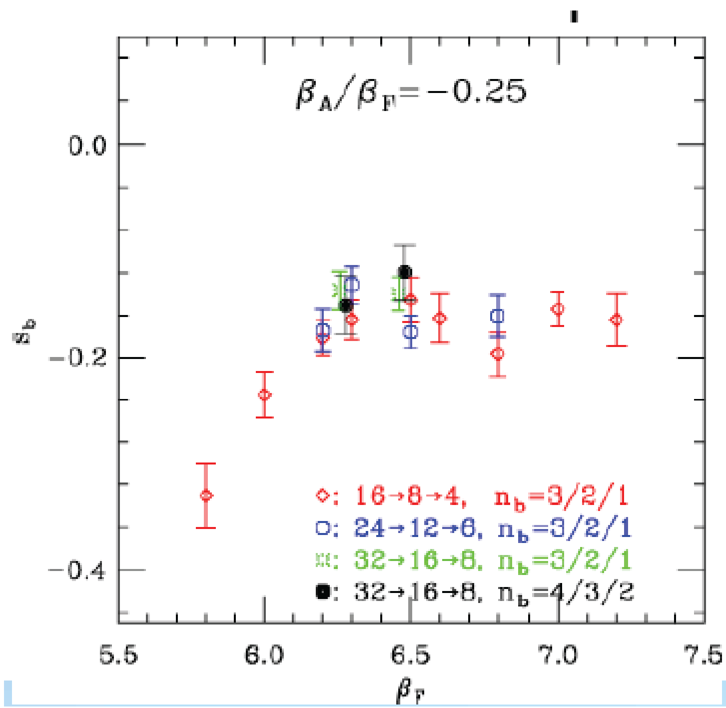
The negative adjoint explorations

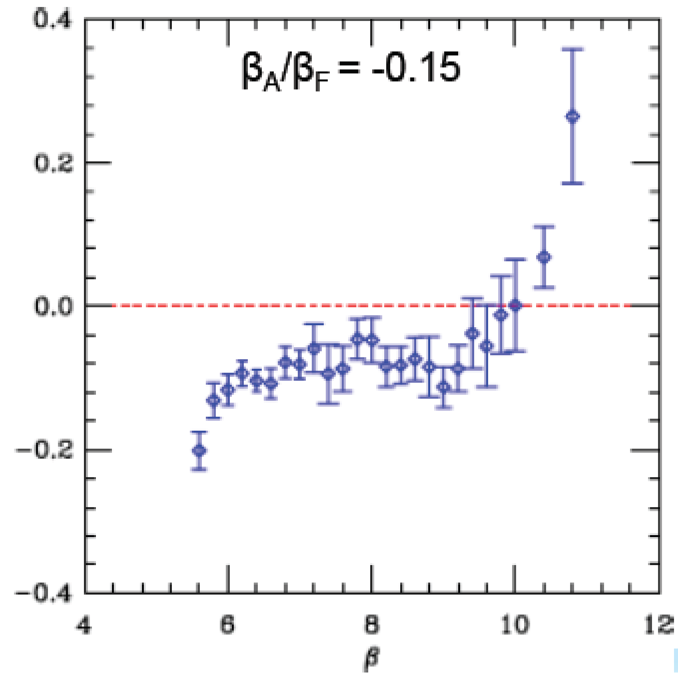
A. Hasenfratz



12 flavor results

MCRG (See talk Thurs. by Petropoulos)



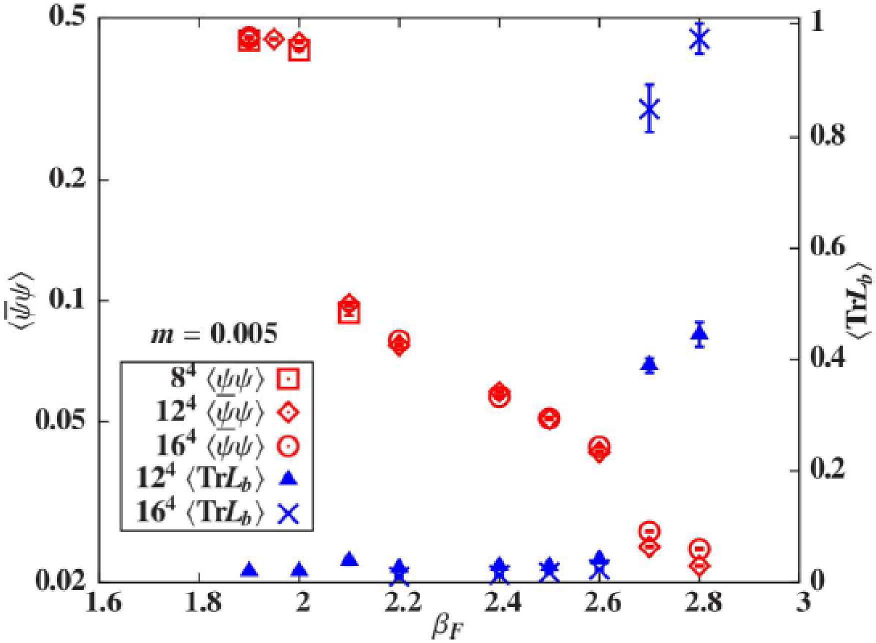


Backward flow

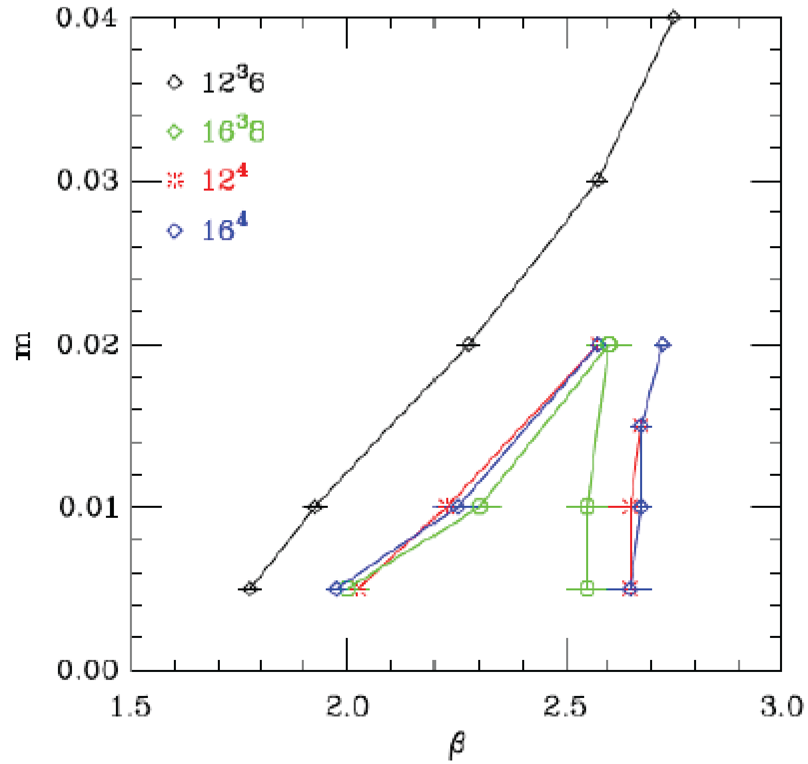
IRFP

Cheng, Hasenfratz & Schaich

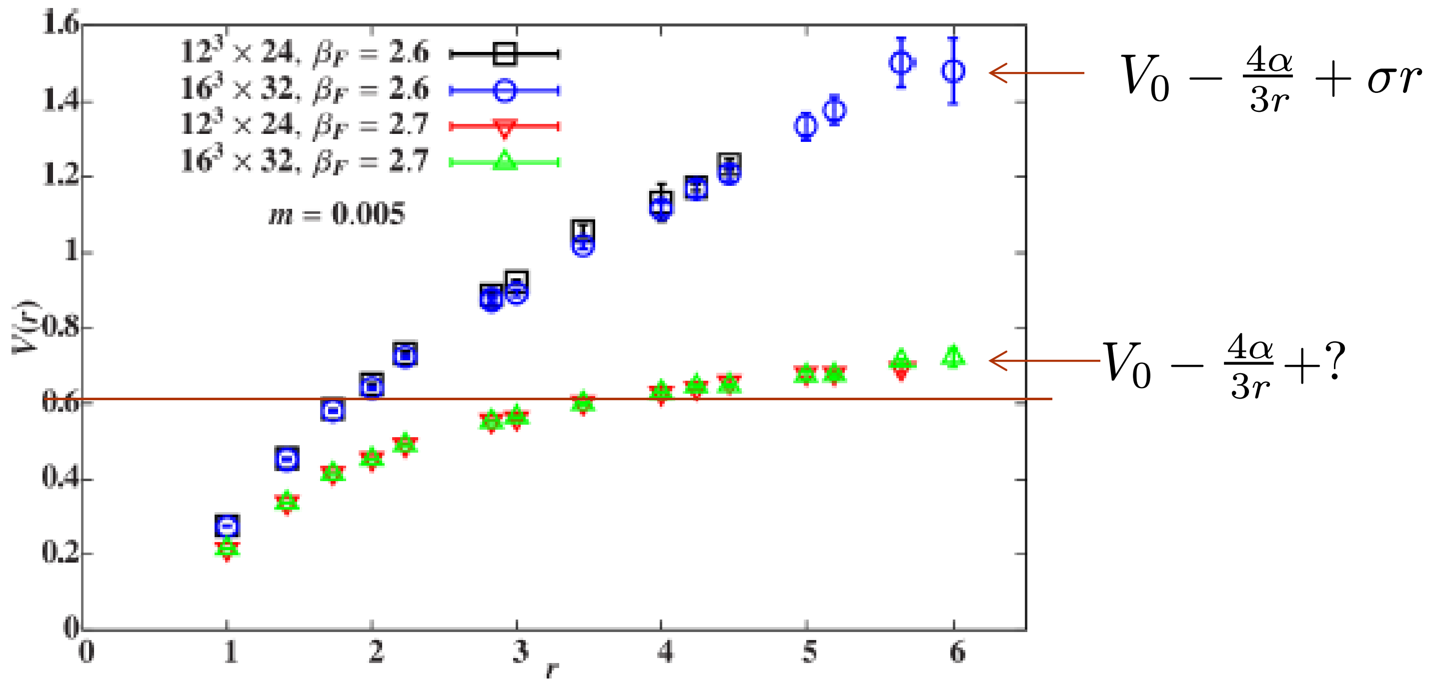
See talk by Schaich from Mon.



ma = 0.005

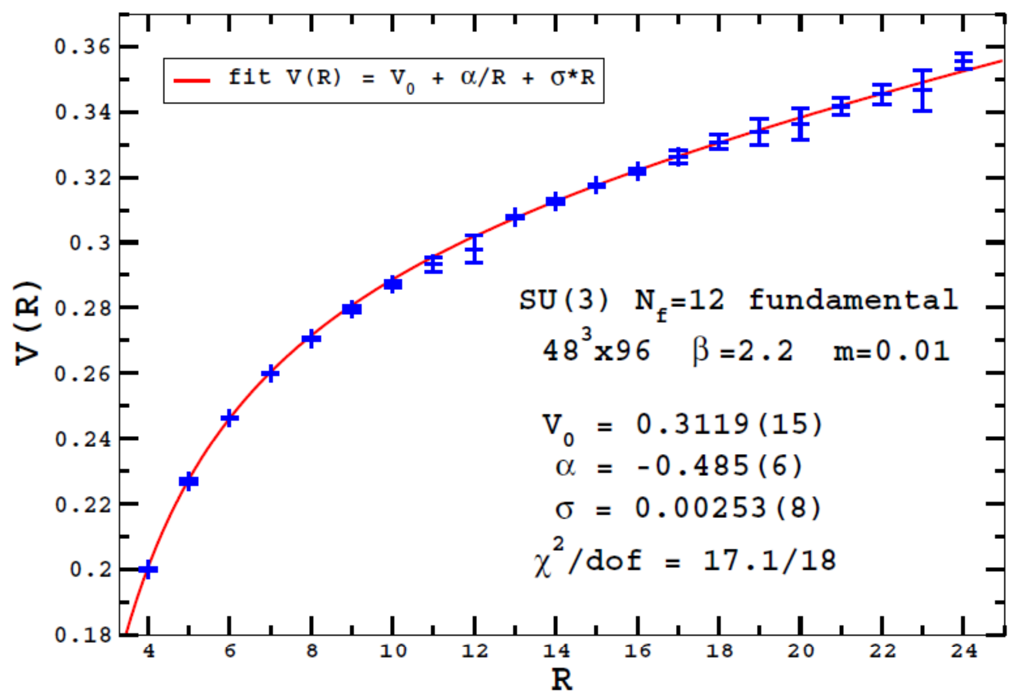


- One thing that differs significantly between the two phases is the static potential…

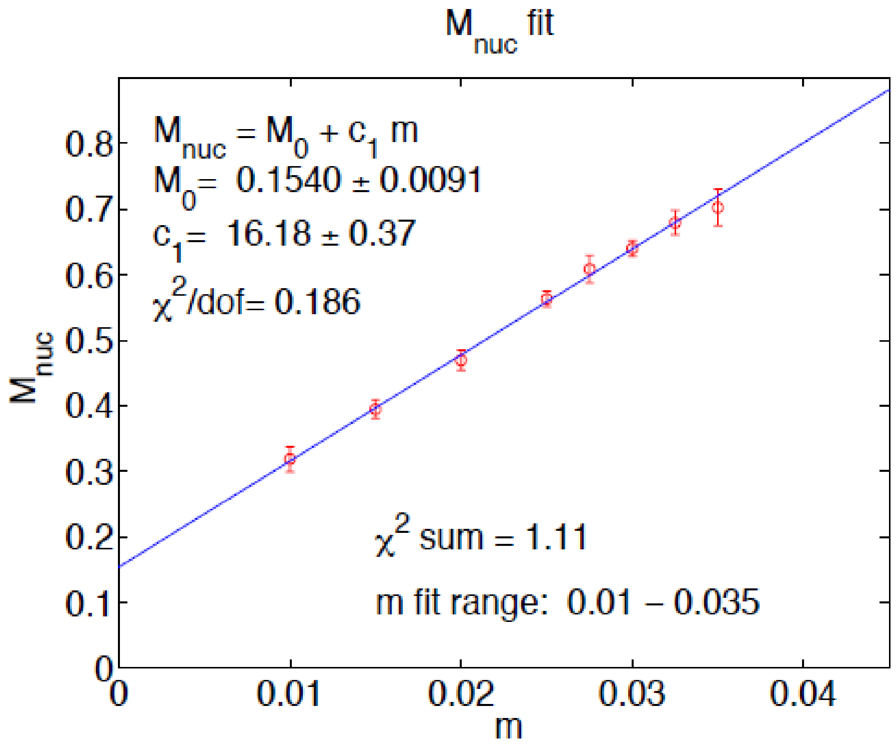


- This is to be compared with the results of LHC (Fodor, Holland, Kuti, Nogradi & Schroeder 2010) :

See talk by Holland from Mon.

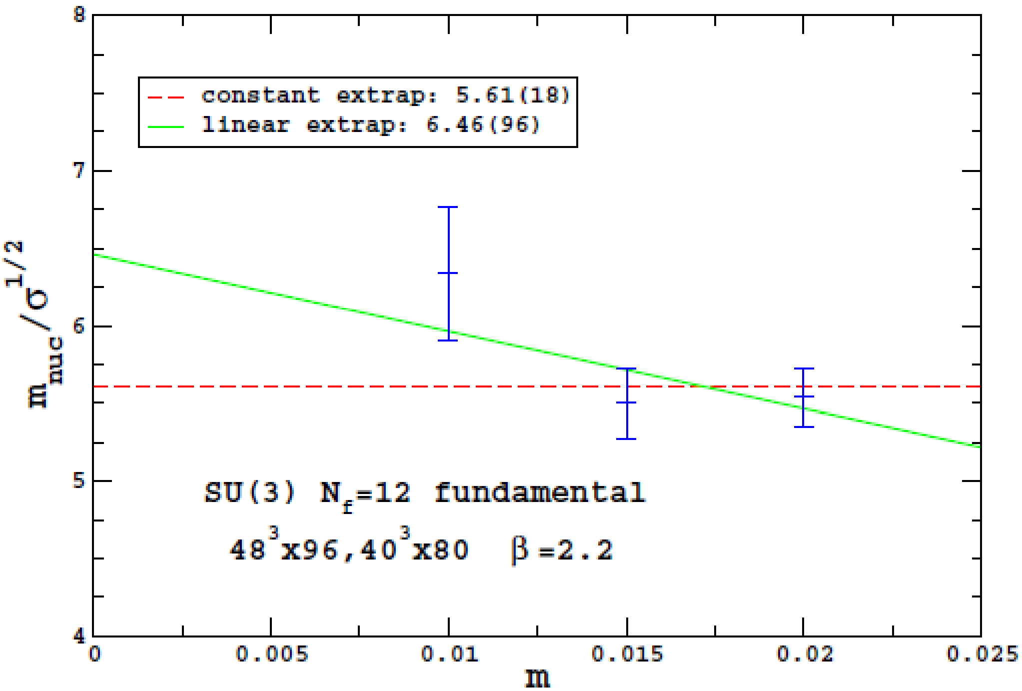


- LHC find a nonzero nucleon gap in the chiral limit:



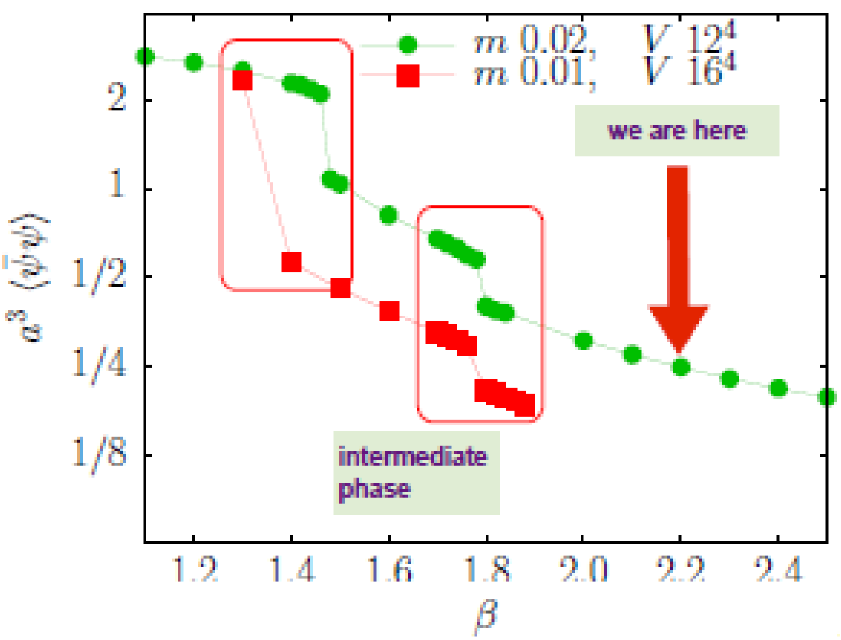
$$\frac{M_0}{\sigma_{M_0}} = 17$$

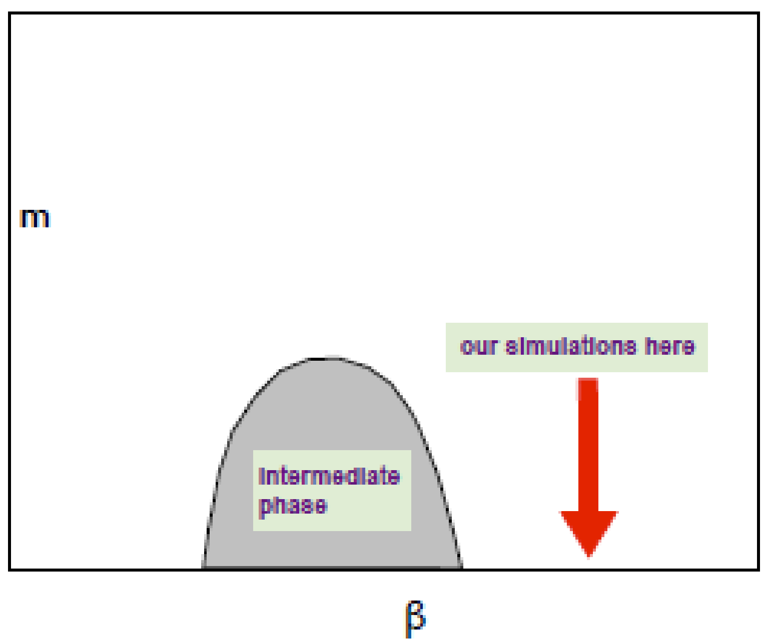
- When they measure the string tension in these units, extrapolating to the chiral limit, they have a nonzero result:



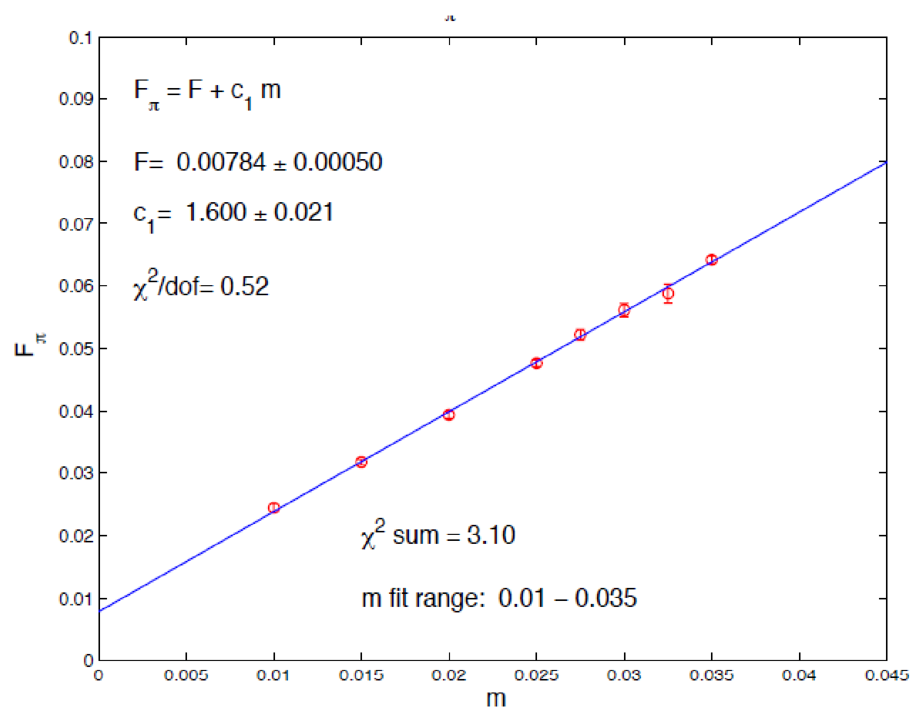
This ought to address the issue that the massive theory is always confining at long enough distance scale.

- LHC has identified the two phase transitions relative to their principal simulation point:



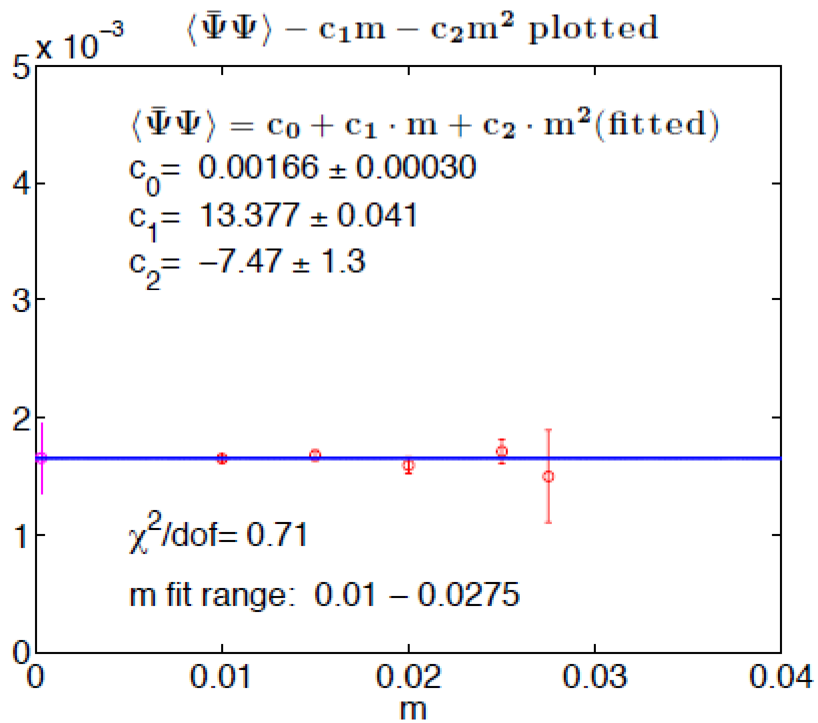


- F extrapolates to a nonzero value, indicating spontaneous chiral symmetry breaking.



L=32,40,48

- The chiral condensate comes out nonzero



- Similar results are found for the nucleon, f_0, rho and the a_1.
- It looks like the 12-flavor theory has a hadronic scale given by Ma=0.1 for this lattice spacing.
- The existence of a dynamical scale is forbidden by the IRCFT hypothesis.

Conformal fits

- The Lattice 2011 proceedings generalizes the function and includes finite volume (FSS):

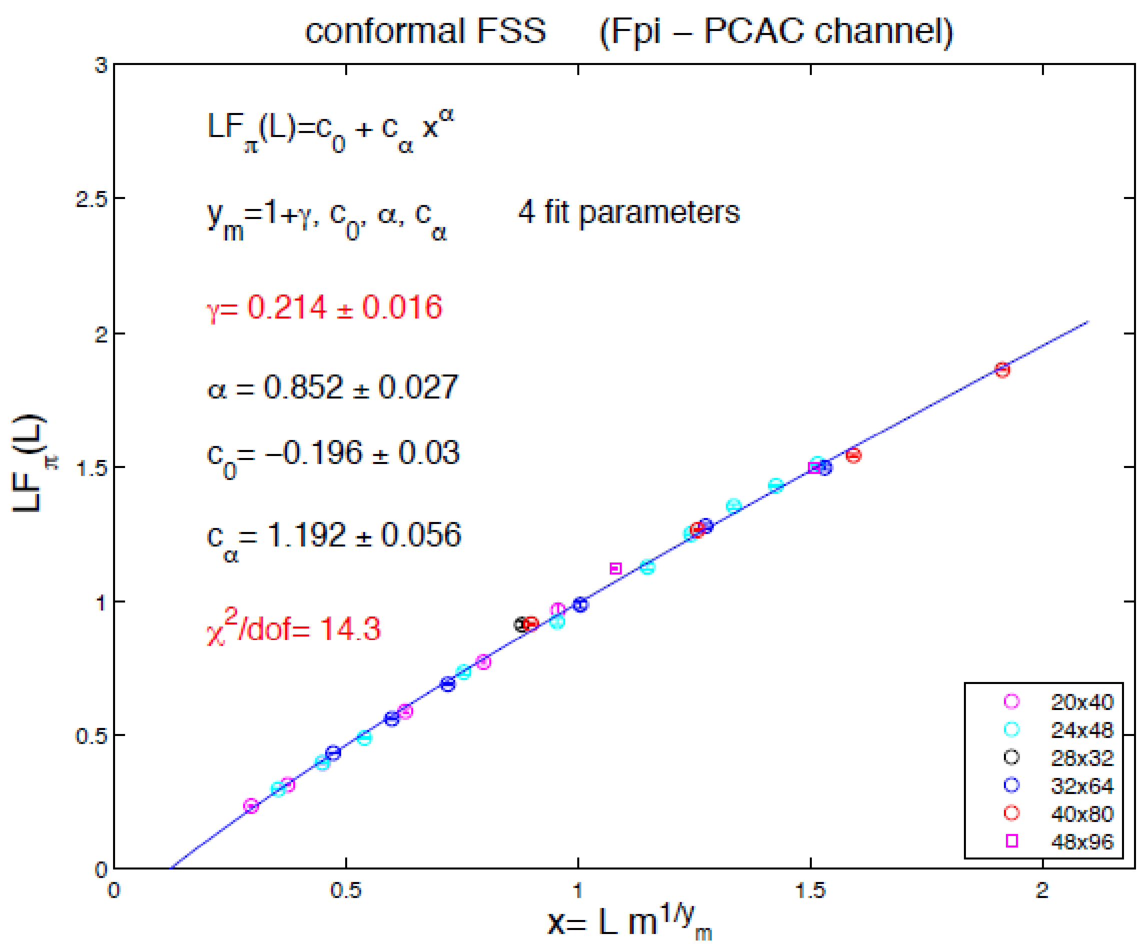
$$ML = f(x), \quad x = m^{1/y_m} L$$

$$f(x) = c_1 x + c_{\mathrm{exp}} (c_1 x)^{-1/2} \exp(-c_1 x), \quad x > x_{\mathrm{cut}}$$

$$f(x) = c_0 + c_\alpha x^\alpha, \quad x < x_{\mathrm{cut}}$$

  with qualifications.
- See talk by Wong from Mon.

channel	γ	χ^2/dof
pion	0.393(8)	2.83
F	0.214(16)	14.3
rho	0.300(17)	1.51
nucleon	0.288(27)	1.45



conformal FSS (Fpi – PCAC channel)
LF_π(L)=c_0 + c_α x^α
y_m=1+γ, c_0, α, c_α
4 fit parameters
γ= 0.214 ± 0.016
α = 0.852 ± 0.027
c_0= −0.196 ± 0.03
c_α= 1.192 ± 0.056
χ^2/dof= 14.3
LF_π(L)
x= L m^{1/y_m}
20x40
24x48
28x32
32x64
40x80
48x96
0
0.5
1
1.5
2
2.5
3

- The γ values don't agree within the errors derived from the fit.
- But how large is the systematic error associated with choice of fitting function?
- For example, could I bring the 0.39 value from the pion down to the 0.30 value from the rho by choosing a different function?

- DeGrand approached this problem by extracting γ from the FHKNS results using an approach that doesn't assume a specific form for the finite size scaling function f(x).

$$P(y_m) = \frac{1}{N_{over}} \sum_p \sum_{j \neq p} \sum_{i,over} \left(\frac{\xi_L(m_{i,j})}{L_j} - f_p(L_j^{y_m} m_{i,j}) \right)^2$$

TABLE I: Exponent y_m from various hadronic channels. Errors are from a single-elimination jackknife.

channel	y_m
pseudoscalar	1.35(23)
nucleon	1.43(26)
f_π	1.23(31)
vector	1.33(22)
axial vector	1.32(12)

- The LHC collaboration has responded with spline based general B-form fits

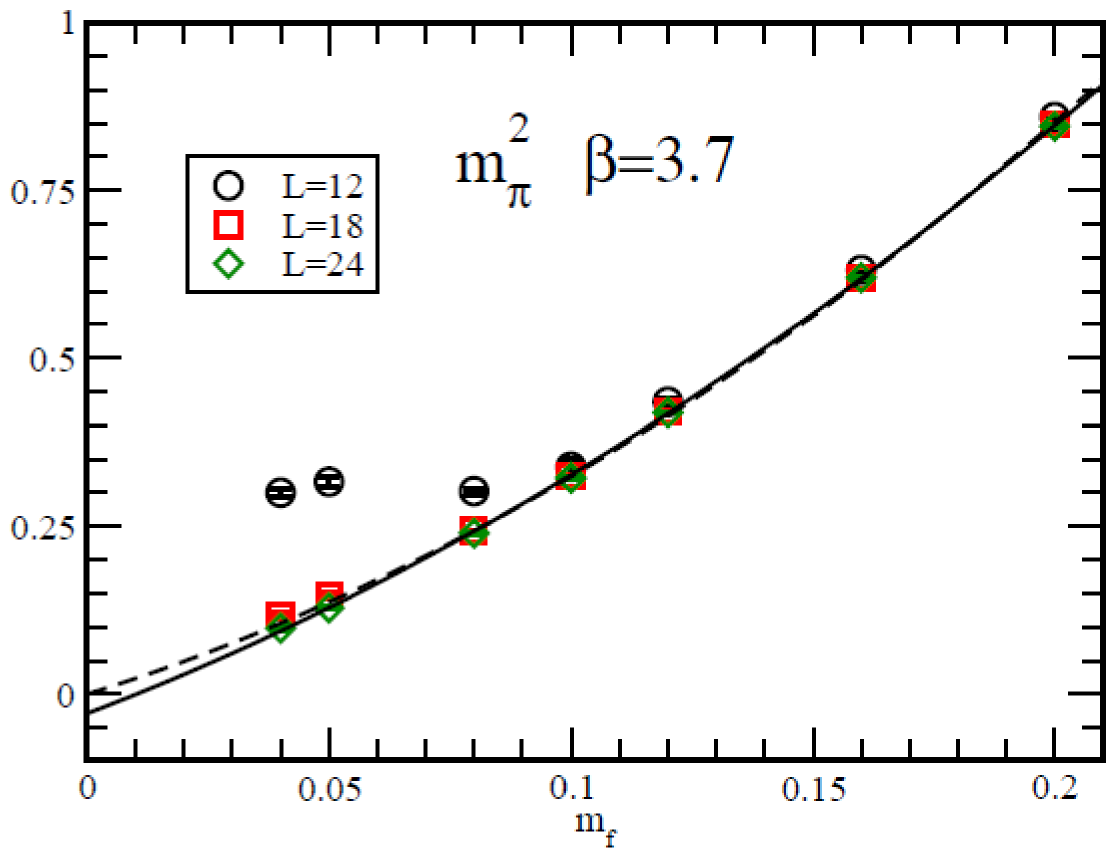
channel	γ	χ^2/dof
pion	0.405(21)	1.47
rho	0.315(75)	1.02
F	0.23(2)	8.05

- Cheng, Hasenfratz and Schaich have been looking at the mode number of the Dirac operator (following A. Patella) to determine the anomalous mass dimension.

$$\nu(\lambda) - \nu(\lambda_0) = cV(\lambda^{4/y_m} - \lambda_0^{4/y_m})$$

- They have added many more eigenvalues (x10) and now perform fits on separate volumes (Talk: Hasenfratz, Tues. afternoon).
- They find that it is necessary to go to fairly large β (well past the bulk phase transition) to see y_m volume independence.

- The LatKMI collaboration (Aoki et al.) have studied spectra.



$$m_\pi^2 = c_0 + c_1 m_f + c_2 m_f^2$$

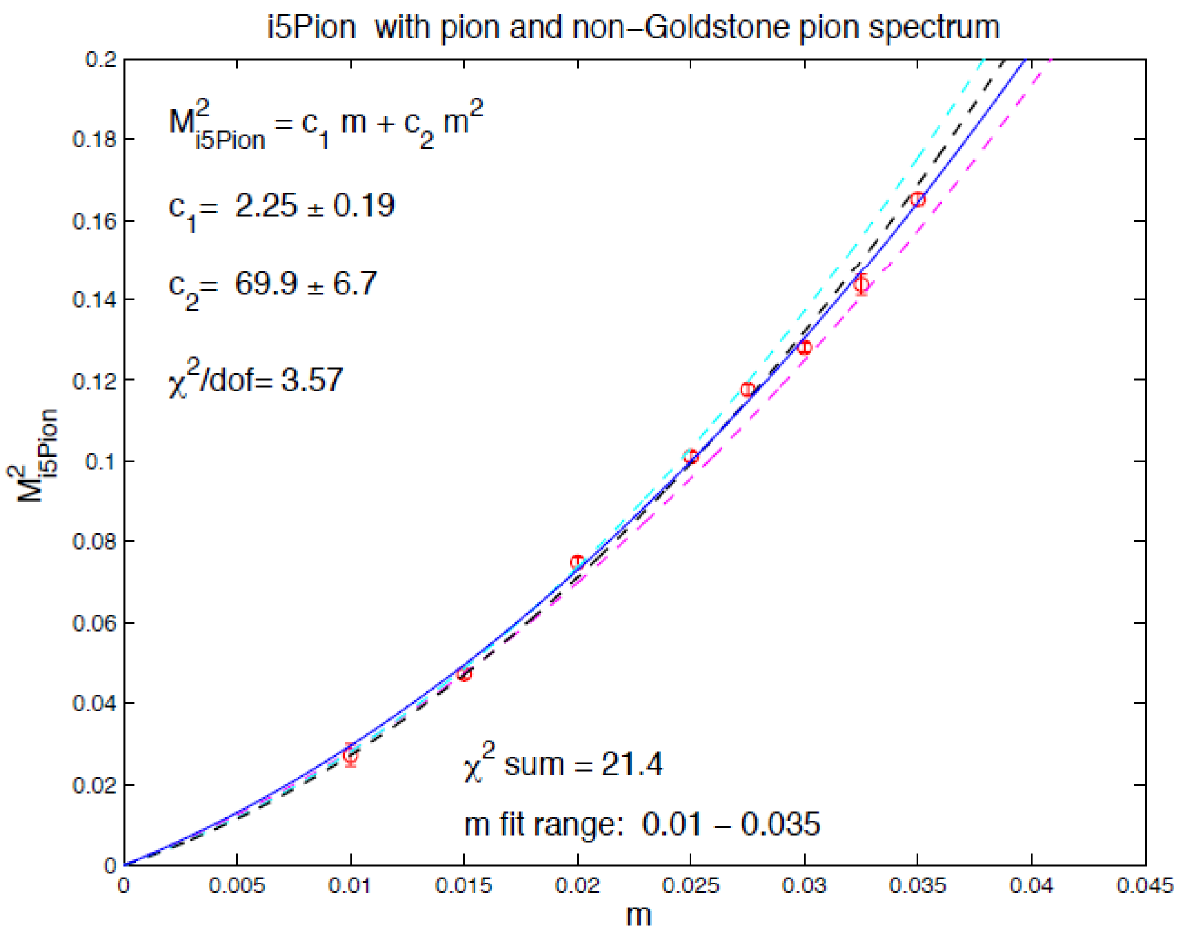
$$\chi^2/\mathrm{dof} = 4.1$$

$$c_0 = 0$$

$$\chi^2/\mathrm{dof} = 28$$

Impossible to reconcile with LHC (Fodor et al.)

LHC Pions



i5Pion with pion and non−Goldstone pion spectrum
$M^2_{i5Pion} = c_1 m + c_2 m^2$
$c_1 = 2.25 \pm 0.19$
$c_2 = 69.9 \pm 6.7$
χ^2/dof= 3.57
χ^2 sum = 21.4
m fit range: 0.01 – 0.035
M^2_{i5Pion}
m
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
0.045
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2

- They also do a FSS fit with $f(x) = c_0 + c_1 x$
- The constant term is a guess.

$$\gamma = 0.44 \pm ?, \quad \chi^2/\mathrm{dof} = 4$$

- All results "preliminary" (Lattice 2011 proc.)
- Follow-up work and further results were presented Mon. (Ohki)

- Appelquist, Fleming, Lin, Neil & Schaich [1106.2148] have considered mass corrected hyperscaling:

$$M_X = C_X m^{1/(1+\gamma)} + D_X m$$

- It is sensible that corrections should be analytic in m.
- Similar eq. for F.
- Condensate: $\langle \bar{\psi}\psi \rangle = A_C m + B_C m^{(3-\gamma)/(1+\gamma)}$
- SD inspired generalization:

$$\langle \bar{\psi}\psi \rangle = A_C m + B_C m^{(3-\gamma)/(1+\gamma)} + C_C m^{3/(1+\gamma)} + D_C m^3$$

- Without the D-terms,

$$\chi^2/\text{dof} = 133/53$$

- With the D-terms and finite volume corrections,

$$\chi^2/\text{dof} = 42/44$$

Sextet

- In the Lattice 2010 proceedings, LHC (FHKNS) report that a collective fit to the pion, F and the chiral condensate produces:
  - $\chi^2/\mathrm{dof} = 1.24$ for χSB
  - $\chi^2/\mathrm{dof} = 6.96$ for IRCFT

- Lattice 2011 proc. LHC results for chiral fit:

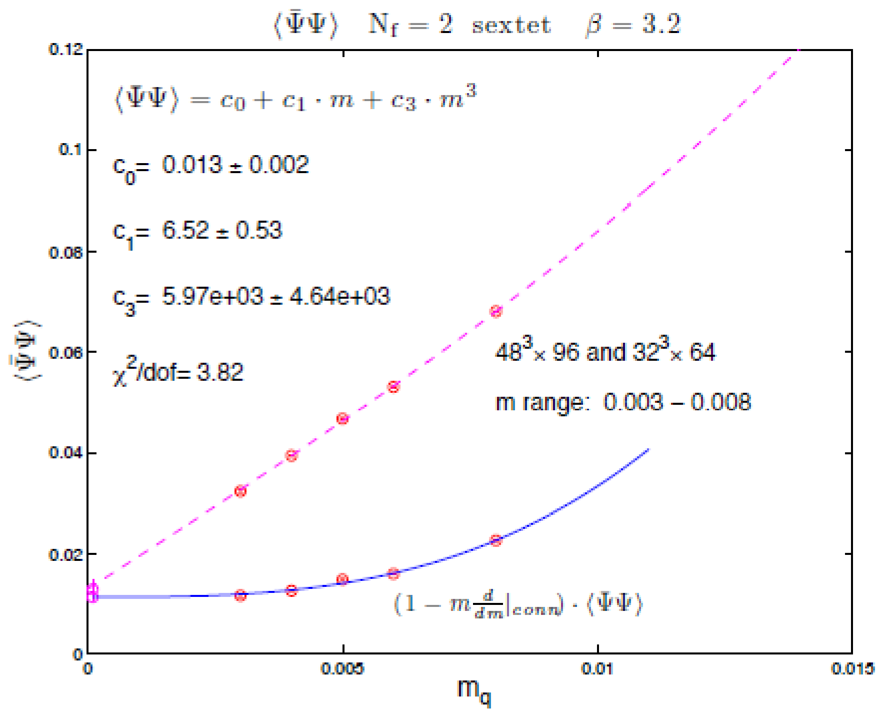
channel	χ^2/dof
pion	1.6
F	0.87
rho	0.56
f0 (connected)	0.48

- Lattice 2011 proc. LHC results for conformal fit

channel	γ	χ^2/dof
pion	1.091(34)	2.0
F	2.13(18)	2.0

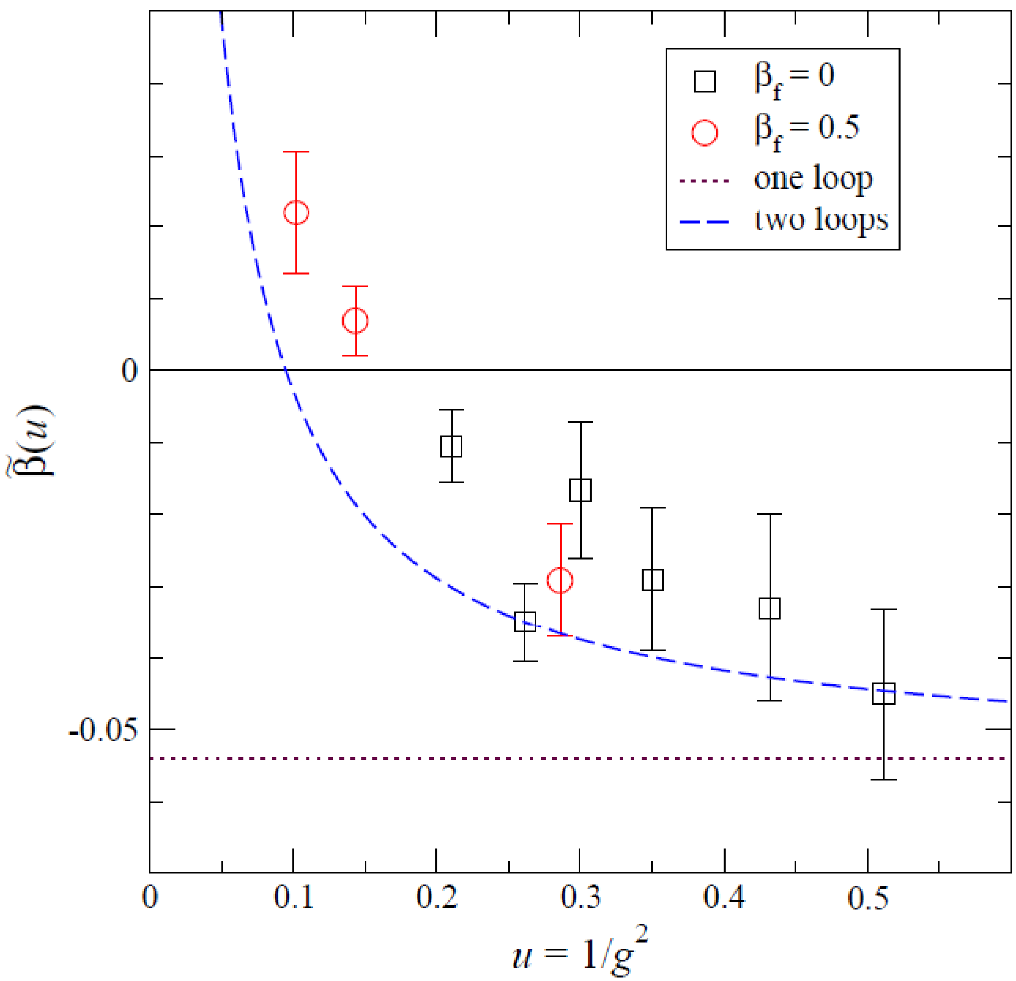
$$M, F \sim m^{1/(1+\gamma)}$$

- And the chiral condensate:



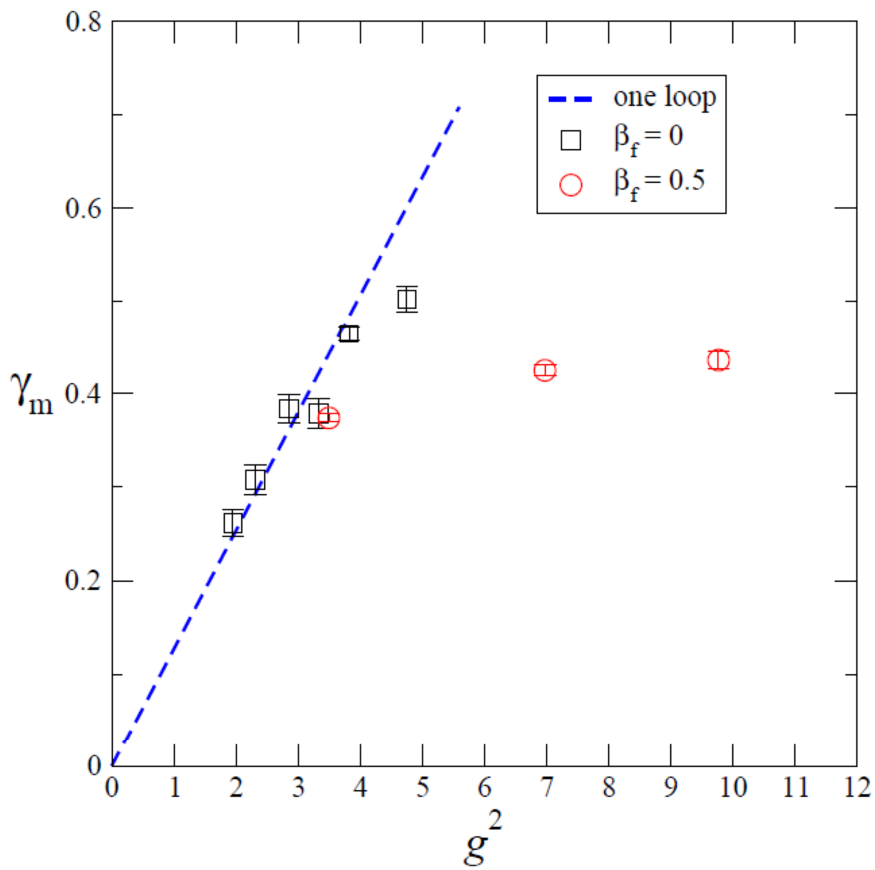
For most recent results, see Kuti talk from Mon.

- DeGrand, Shamir and Svetitsky have added a fat link sextet repr. gauge action term which allows them to push into stronger coupling w/o hitting the bulk transition.
- They compute a discrete beta function from differences between two lattices.



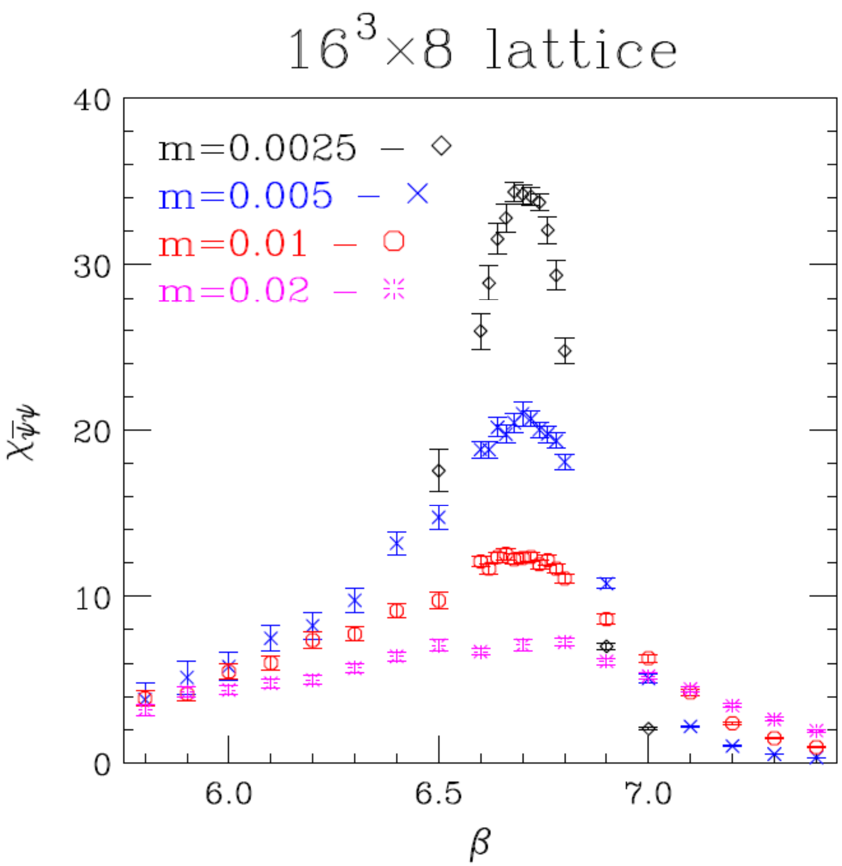
β_f = 0
β_f = 0.5
one loop
two loops
0
-0.05
β̃(u)
0
0.1
0.2
0.3
0.4
0.5
u = 1/g^2

- They also measure γ

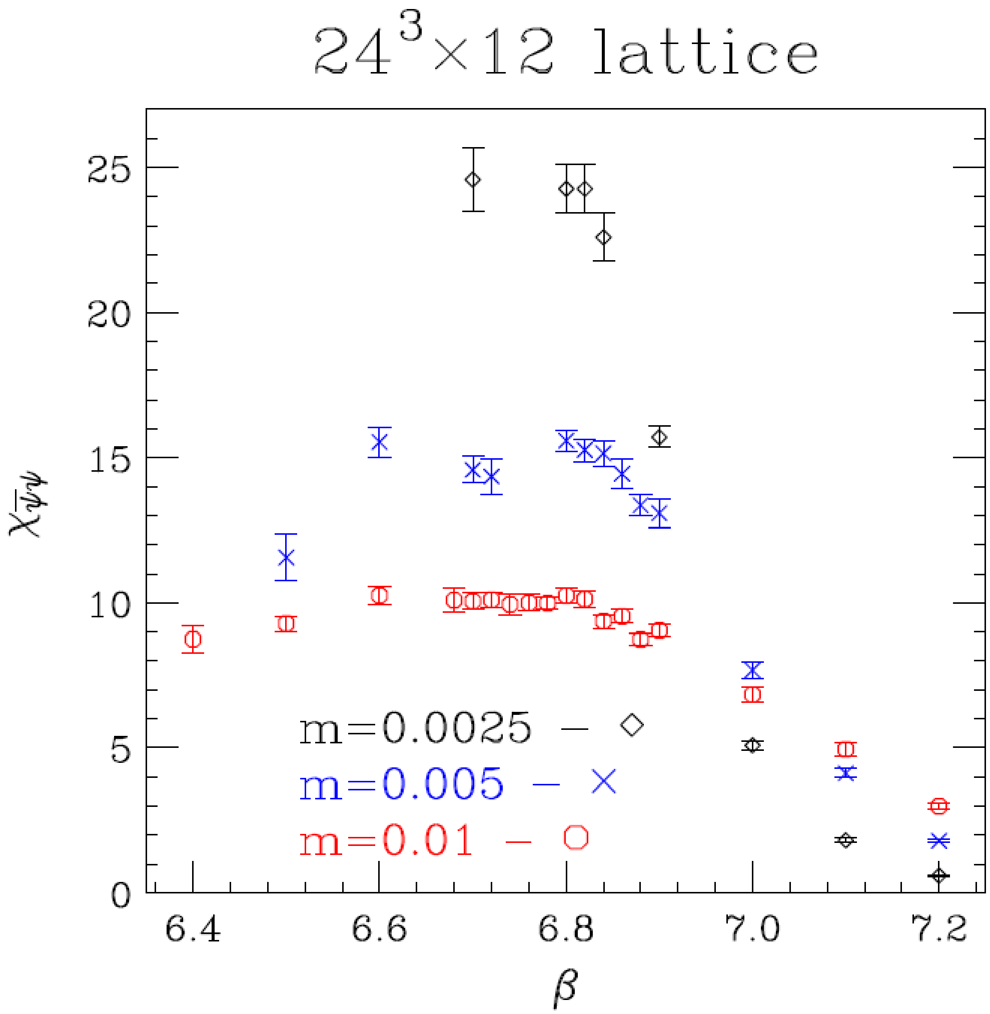


- Similar results by same group for SU(4).

- Kogut and Sinclair have been studying the finite temperature transition in this theory.



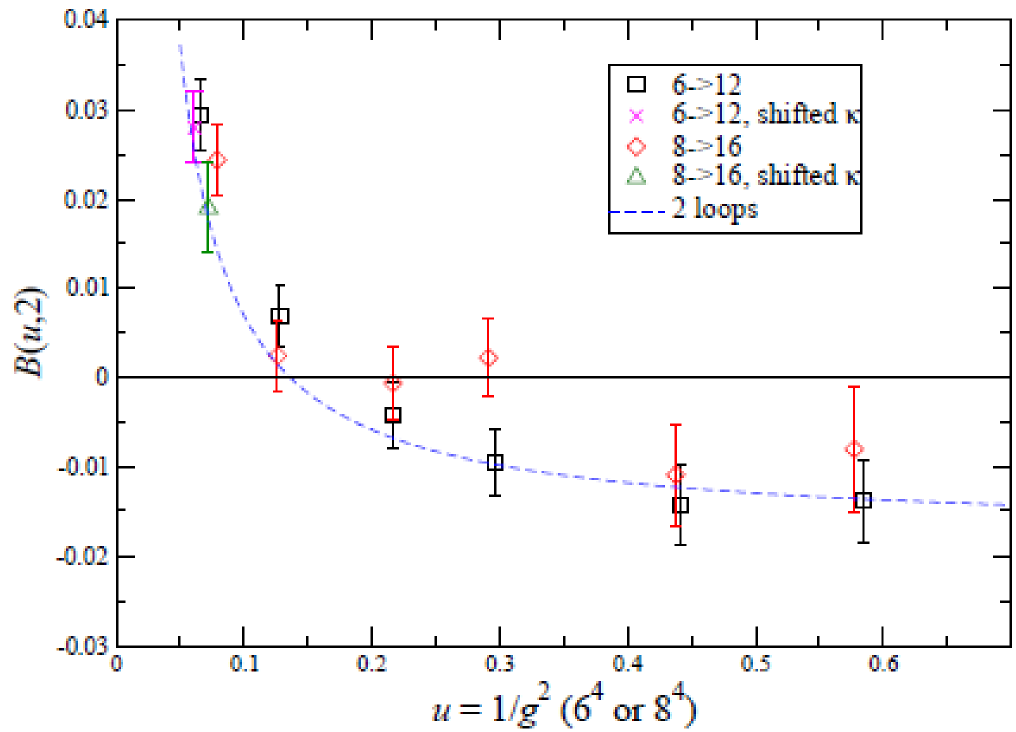
- The scaling of the peak with mass is consistent with a second order finite temperature phase transition and inconsistent with a first order bulk transition.
- Results are too preliminary (smaller mass may be needed) for an accurate estimate of the peak location for $N_t = 12$ (however, see Sinclair's talk from Mon).
- Ultimately $N_t = 18$ will also be needed.

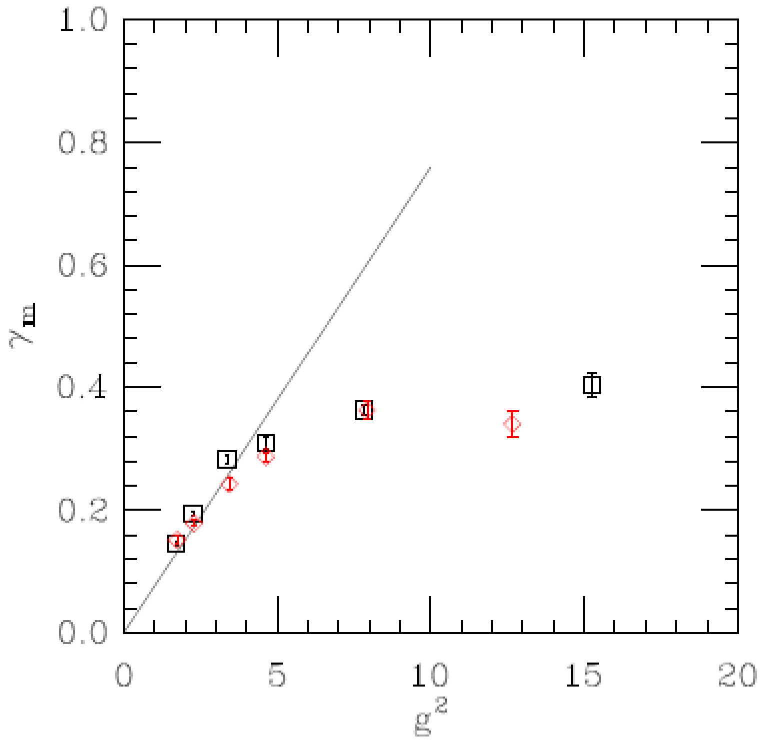


$24^3\times12$ lattice
25
20
15
10
5
0
6.4
6.6
6.8
7.0
7.2
$\chi_{\bar{\psi}\psi}$
β
m=0.0025 – ◇
m=0.005 – ×
m=0.01 – ○

SU(2) N_f=2 Adjoint

- DeGrand, Shamir, Svetitsky results for beta function and γ





1.0
0.8
0.6
0.4
0.2
0.0
γ_m
0
5
10
15
20
g^2

- JG & Weinberg estimate from FSS
- Using approach advocated by DeGrand

Channels and fits

Observable	Quadratic	Log Quad	PWL	Combined
m_π	1.67(93)	1.26(54)	1.51(33)	1.46(27)
m_ρ	1.67(88)	1.37(39)	1.56(31)	1.50(23)
m_{a_1}	1.40(52)	1.42(27)	1.41(22)	1.41(16)
f_π	1.65(22)	1.49(54)	1.60(29)	1.62(17)

$$\gamma = 0.51 \pm 0.16$$

- Patella introduced Dirac mode number approach

$$\nu(\lambda) - \nu(\lambda_0) = cV(\lambda^{4/y_m} - \lambda_0^{4/y_m})$$

- He obtains

$$\gamma = 0.371(20)$$

- (Orthogonal: large-N reduction, Keegan talk Wed., along the lines of Hietanen & Narayanan, Koren, Okawa talks Wed.)

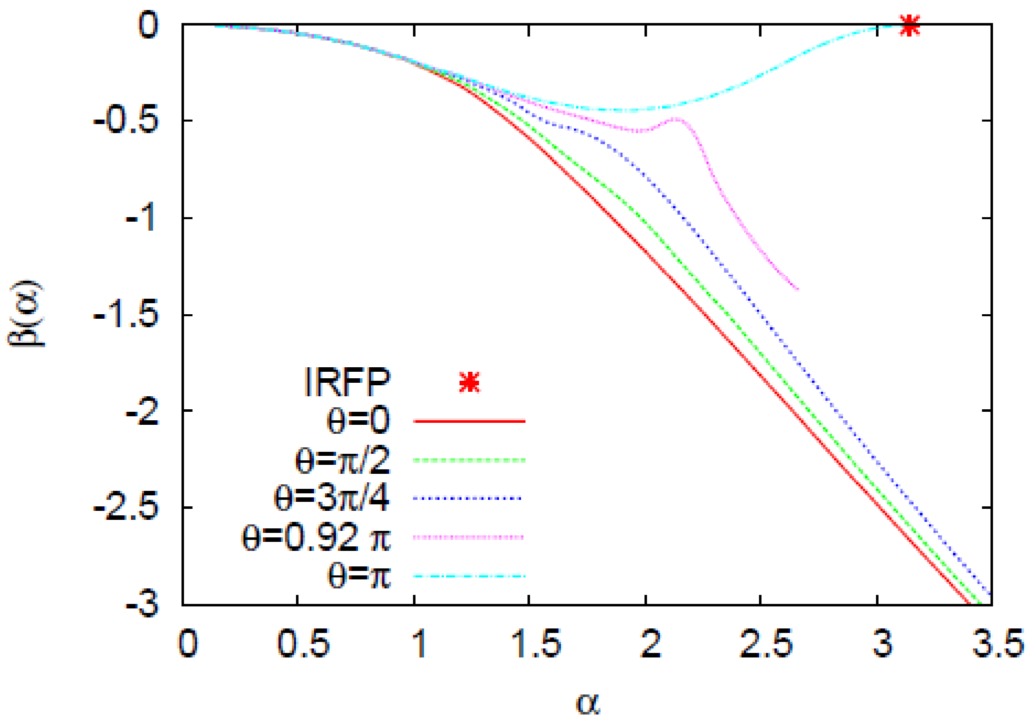
Comparison

Method	γ
SF [Bursa et al. 2009]	$0.05 < \gamma < 0.56$
SF [Degrand et al. 2011]	0.31 ± 0.06
Perturbative 4-loop [Pica & Sannino 2010]	0.500
Schwinger-Dyson [Ryttov & Shrock 2010]	0.653
All-orders hypothesis [Pica & Sannino 2010]	0.46
MCRG [Catterall et al. 2011]	$\|\gamma\| < 0.6$
FSS [Del Debbio et al. 2010]	$0.05 < \gamma < 0.20$
FSS [Del Debbio et al. 2010]	0.22 ± 0.06
FSS [JG & Weinberg 2012]	0.51 ± 0.16
Mode number [Patella 2012]	0.371 ± 0.020

- de Forcrand, Pepe and Wiese [1204.4913] have looked at the 2d O(3) spin model with vacuum angle $\theta \approx \pi$
- This theory was suggested as a proxy by Nogradi [1202.4616]
- This allows them to go arbitrarily close to a CFT, and so have walking behavior (Pepe talk Wed).
- This is an asymptotically free theory, so it is an IRCFT when $\theta = \pi$

$$\alpha(L) = \alpha_* - \frac{1}{C \ln(L/L_0)}$$

- They are able to compute the beta function with high precision:



SU(2) with fundamental flavors

- Finish group [Karavirta (talk Mon.), Rantaharju (talk Tues.), Rummukainen, Tuominen] have recently studied $N_f = 4$, 6 and 10.
- Schrödinger functional with clover fermions.
- $N_f = 4$ appears confining and similar to QCD.
- $N_f = 10$ appears conformal, with Banks-Zaks FP.
- $N_f = 6$ is inconclusive probably because it is right near the bottom of the conformal window.
- Previous work (2010) by Bursa, Del Debbio, Keegan, Pica, Pickup found zero of beta function for constant extrapolation and inconclusive for linear extrapolation.
- Voronov, Hayakawa Wed.?

SU(3) with N_f=6 fundamentals

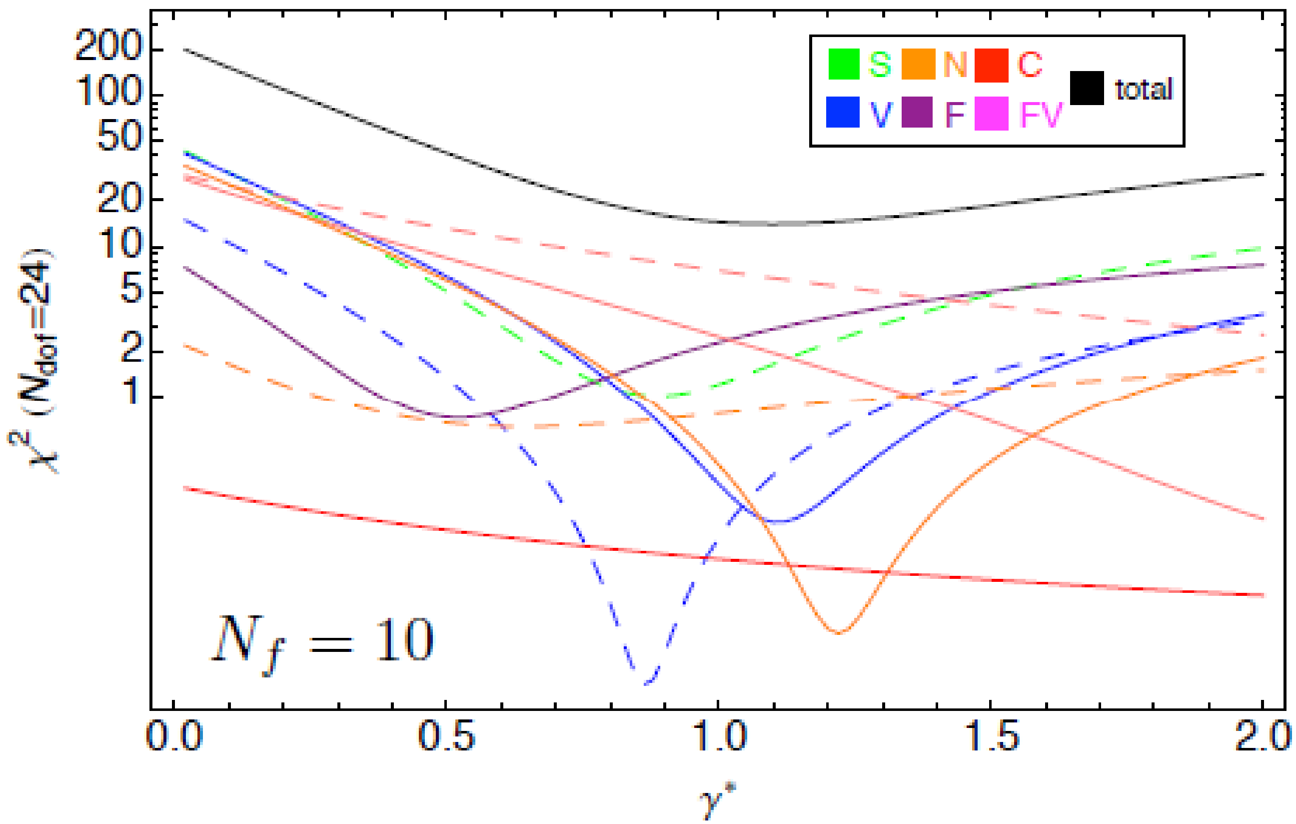
- Recent work of LSD collaboration calculating scattering length.
- Decreased compared to N_f=2.

SU(3) with N_f=10 fundamentals

- Recent work by LSD collaboration using DWF (talk by Fleming Tues afternoon).
- Based on the behavior of m_π/f_π with m_q they don't expect to be able to fit to χPT.
- Hard to reconcile with good chiral fits for N_f=12.
- Hyperscaling + fits similar to 12 flavor fits give

$$\chi^2/\mathrm{dof} \sim 1, \quad \gamma \approx 1$$

- More data on more volumes needs to be added: is there an F problem with FSS?


S
N
C
total
V
F
FV
200
100
50
20
10
5
2
1
χ^2 (N_{dof}=24)
$N_f = 10$
0.0
0.5
1.0
1.5
2.0
γ^*

Four fermion couplings

- It is of interest to see whether these can push a theory out of the conformal window.
- The four fermion coupling could provide a tunable way to be arbitrarily close to that window: as much walking as desired.
- Catterall (Mon talk) has looked at gauged NJL model on lattice.
- No evidence of second order critical line.
- Further work, on models inside the conformal window, needs to be performed.

Conclusions

- Two theories have been studied in depth:
  - SU(3) 12-flavor fundamental
  - SU(2) two-flavor adjoint
- In the first case the controversy has not been resolved and there are claims that cannot be reconciled: more large lattice studies needed, more efforts to understand discrepancies.
- In the second case there is consensus but large lattice spectral studies like LHC should be performed to see how different it is from 12 flavors.
- One theory (10 flavors) may have a γ that is large enough for WTC

- Hyperscaling $M \sim m^{1/y_M}$ is based on crude approximations such as $\gamma = \gamma_*$
- It would be good to have predictions for corrections to these formulas and to be able to fit these corrections from lattice data: different γ's for different channels? (Kurachi talk Thurs.?)
- Similarly, in FSS we should attempt fits to the scaling violations.
- Schrödinger functional studies need to be repeated on larger lattices to reduce lattice artifacts.
- Finite temperature transition needs to be pushed harder.