Gauge-Higgs Unification on the Lattice

Lattice 2012
Cairns, Australia

Nikos Irges
National Technical University of Athens

Based on

N.I., F. Knechtli and K. Yoneyama:

"Mean-Field Gauge Interactions in Five Dimensions II: The Orbifold"

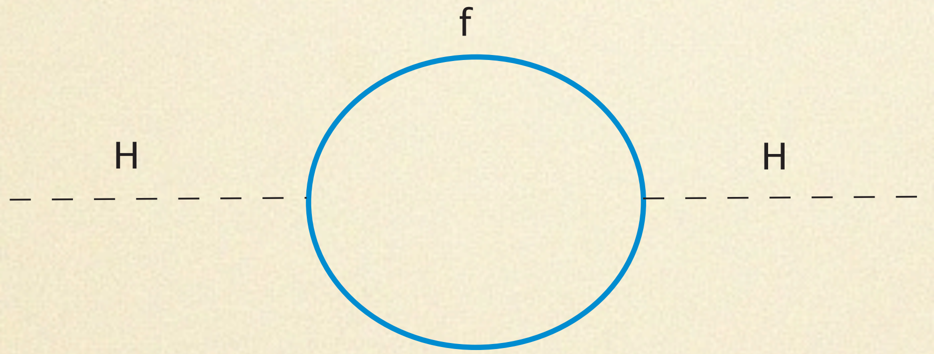
arXiv: 1206.4907 [hep-lat]

Important questions in the Standard Model

- *What is the origin of the Higgs mechanism?*
- *Is the mass of the physical scalar stable under Quantum corrections?*
- *Why is the Higgs heavier than the Z and by how much?*

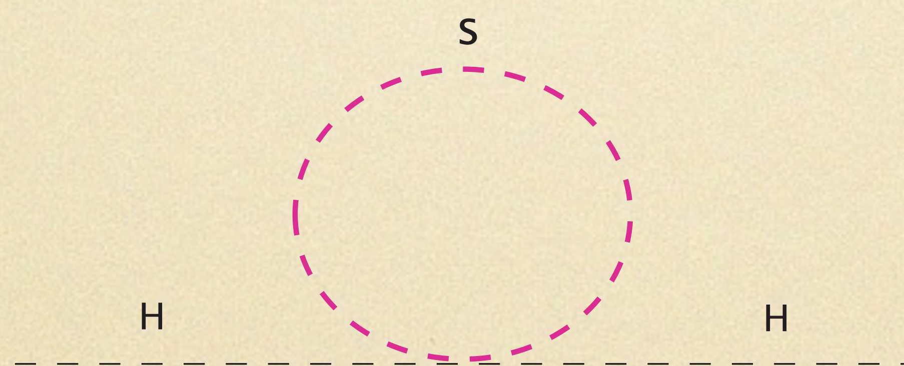
Supersymmetry: two complex scalars vs a Dirac spinor

$$-\lambda_f H \overline{f} f \ :$$



$$\sim \frac{|\lambda_f|^2}{16\pi^2}(-2\Lambda^2 + 6m_f^2 \ln \frac{\Lambda}{m_f})$$

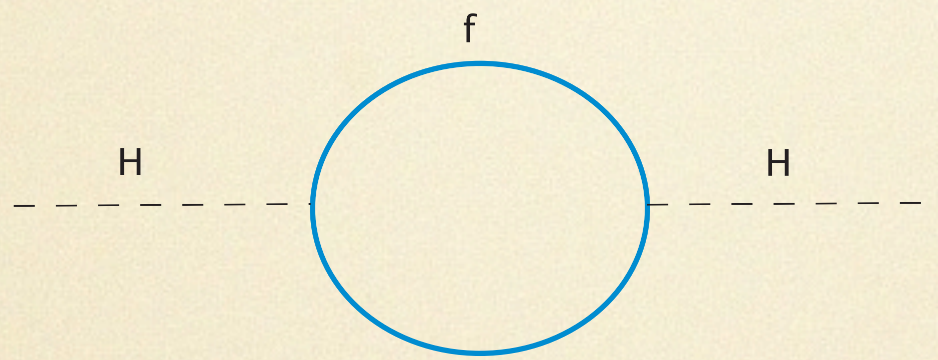
$$-\lambda_s |H|^2 |S|^2 \ :$$



$$\sim \frac{\lambda_s}{16\pi^2}(\Lambda^2 - 2m_s^2 \ln \frac{\Lambda}{m_s})$$

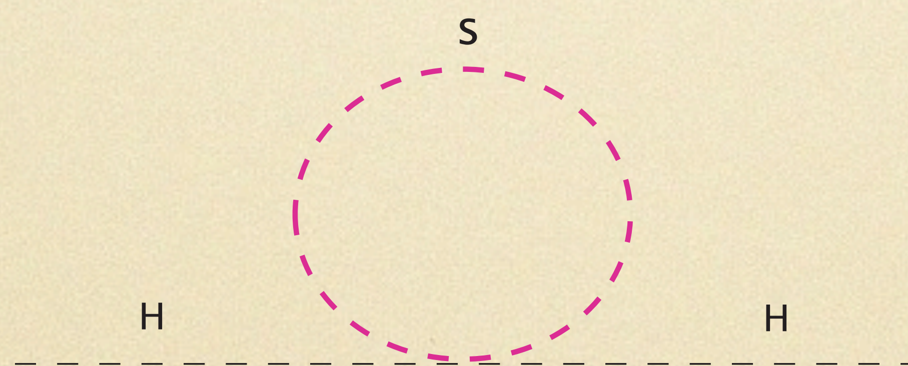
Supersymmetry: two complex scalars vs a Dirac spinor

$-\lambda_f H \overline{f} f$:



$$\sim \frac{|\lambda_f|^2}{16\pi^2}(-2\Lambda^2 + 6m_f^2 \ln\frac{\Lambda}{m_f})$$

$-\lambda_s |H|^2 |S|^2$:



$$\sim \frac{\lambda_s}{16\pi^2}(\Lambda^2 - 2m_s^2 \ln\frac{\Lambda}{m_s})$$

The quadratic cut-off sensitivity cancels...

An alternative: Extra Dimension(s)

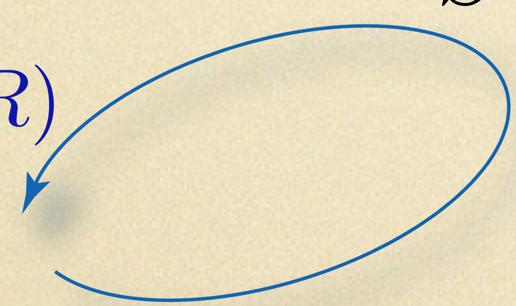
N.S. Manton (1979)
Y. Hosotani (1983)

$$A_M \xrightarrow{\mathcal{M}_5 = E_4 \times S^1} \{A_\mu, A_5\}$$

5D gauge field — *Z: 4D gauge field* — *h: 4D Higgs*

S^1

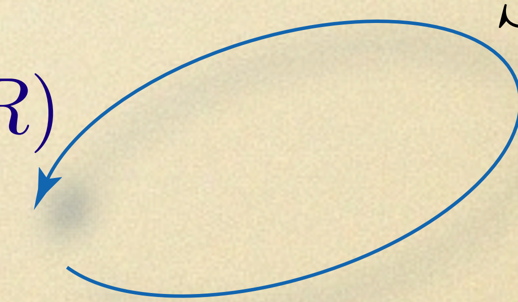
$+\phi(y + 2\pi R)$



$$\phi(y) = \sum_n \phi_n \, e^{iny/R}$$

S^1

$+\psi(y + 2\pi R)$



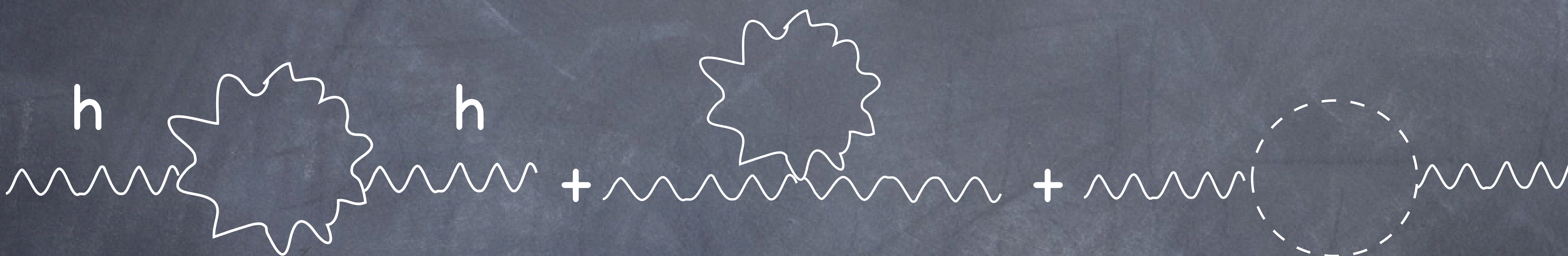
$$\psi(y) = \sum_n \psi_x e^{iny/R}$$

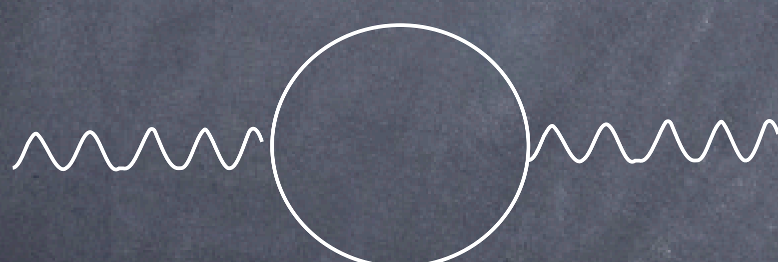
An analytical perturbative computation

G. Gersdorff, N.I., M. Quiros (2002)

H-C Cheng, K. Matchev, M Schmaltz (2002)

$$\mathcal{L} = -\frac{1}{g_5^5} F_{MN} F_{MN} + b.c. \qquad 5D\ SU(2), \;\; h = A_5^1$$



+  ----> $$m_h R = \frac{c}{\sqrt{N_5 \beta}}$$

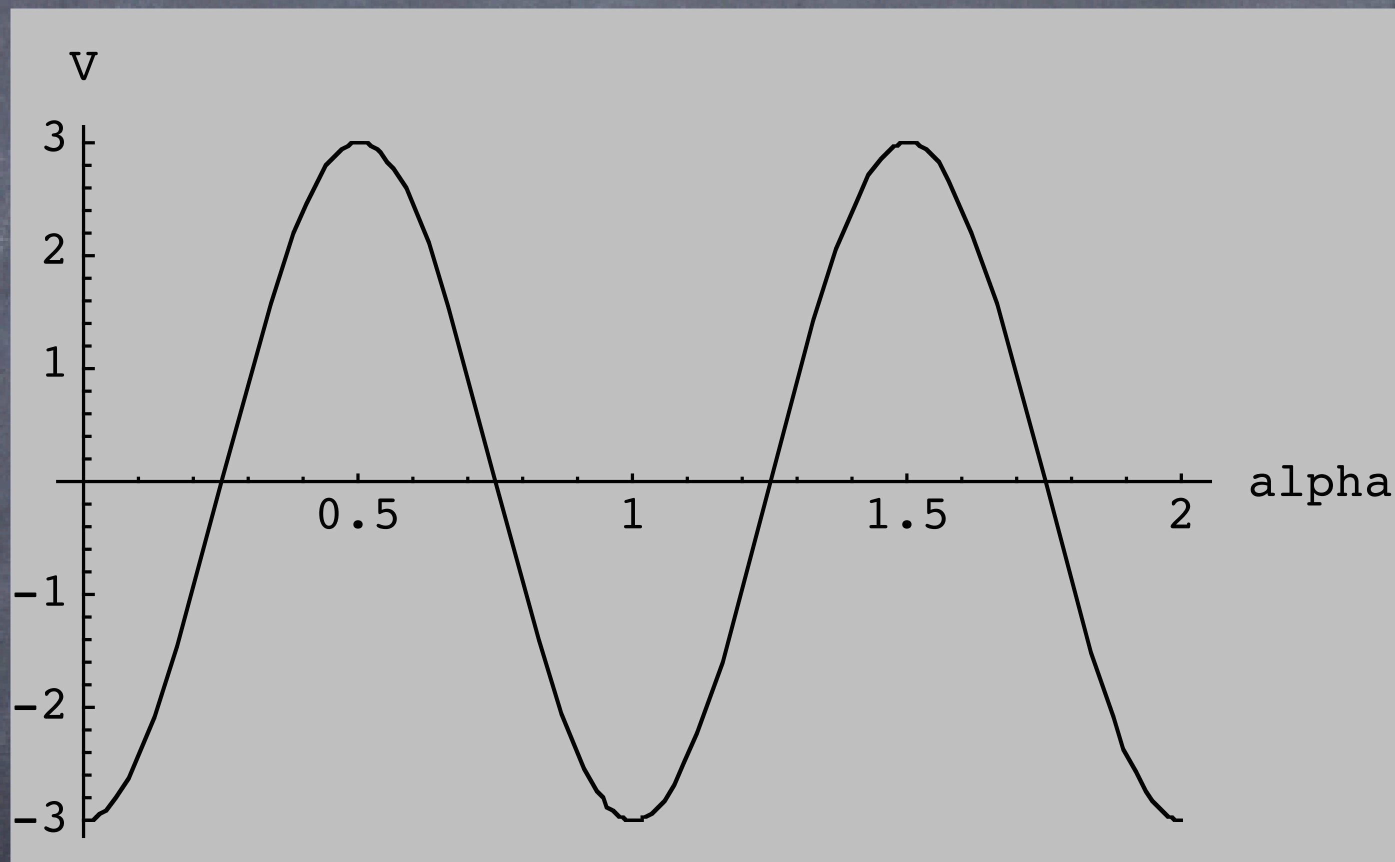
$$N_5 = \pi R \Lambda \qquad \beta = \frac{4}{g_5^2 \Lambda}$$

Finite, as long as $g_5 \equiv g_5(\Lambda = \infty)$!

Coleman-Weinberg computation

Torus: I. Antoniadis, K. Benakli, M Quiros (2001)
Orbifold: M. Kubo, C.S. Lim, H. Yamashita (2002)

$$V = -\frac{3 \cdot P}{64\pi^6 R^4} \sum_{m=1}^{\infty} \frac{\cos(2\pi m \alpha)}{m^5}$$

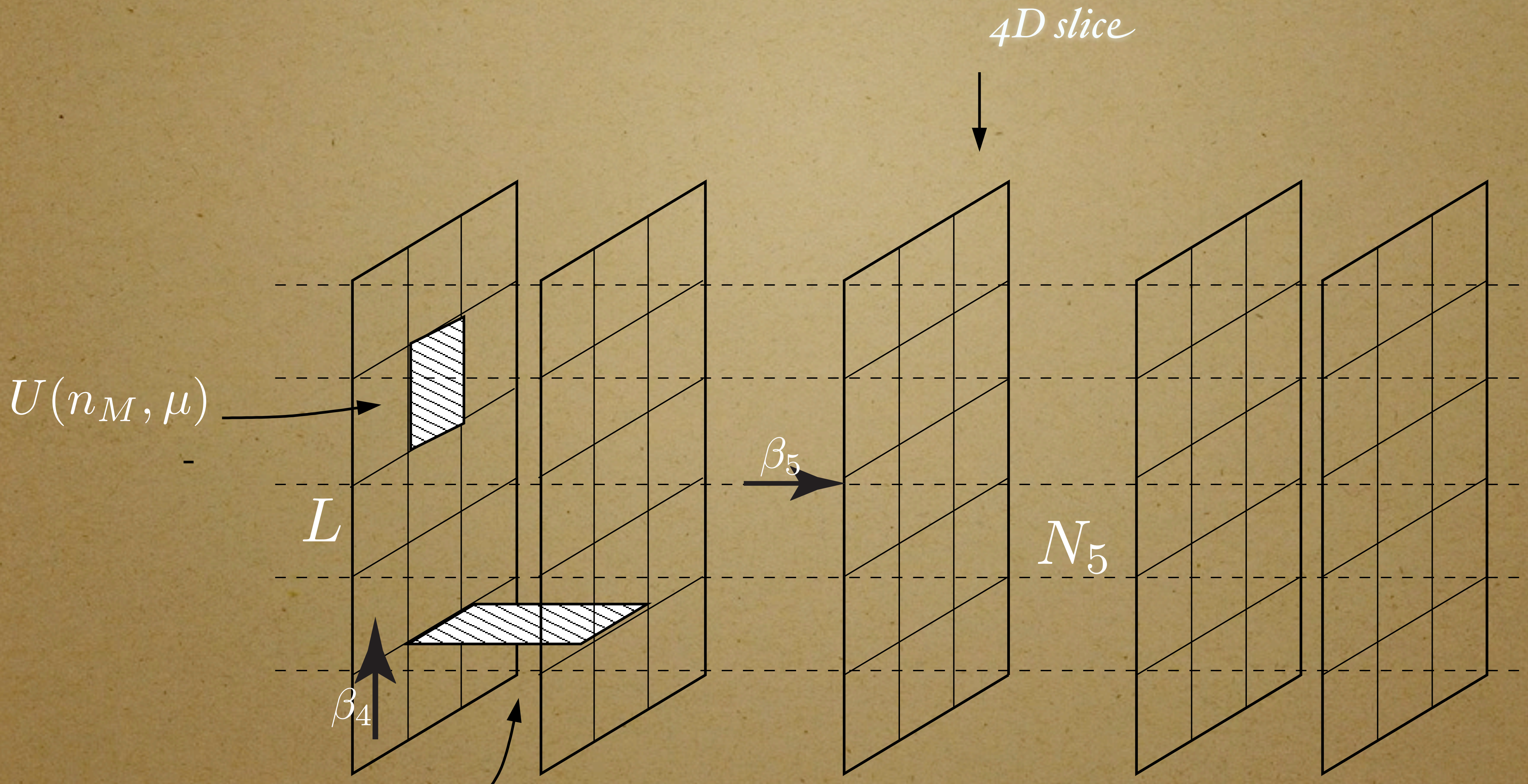


$$\alpha = \frac{g_5 v R}{\sqrt{2\pi R}}$$

$$(m_H R)^2 = \frac{N}{N_5 \beta} R^4 \frac{d^2 V}{d\alpha^2}\Big|_{\alpha_{min}}$$ (same as from pert. theory!)

but no SSB in the pure gauge theory!

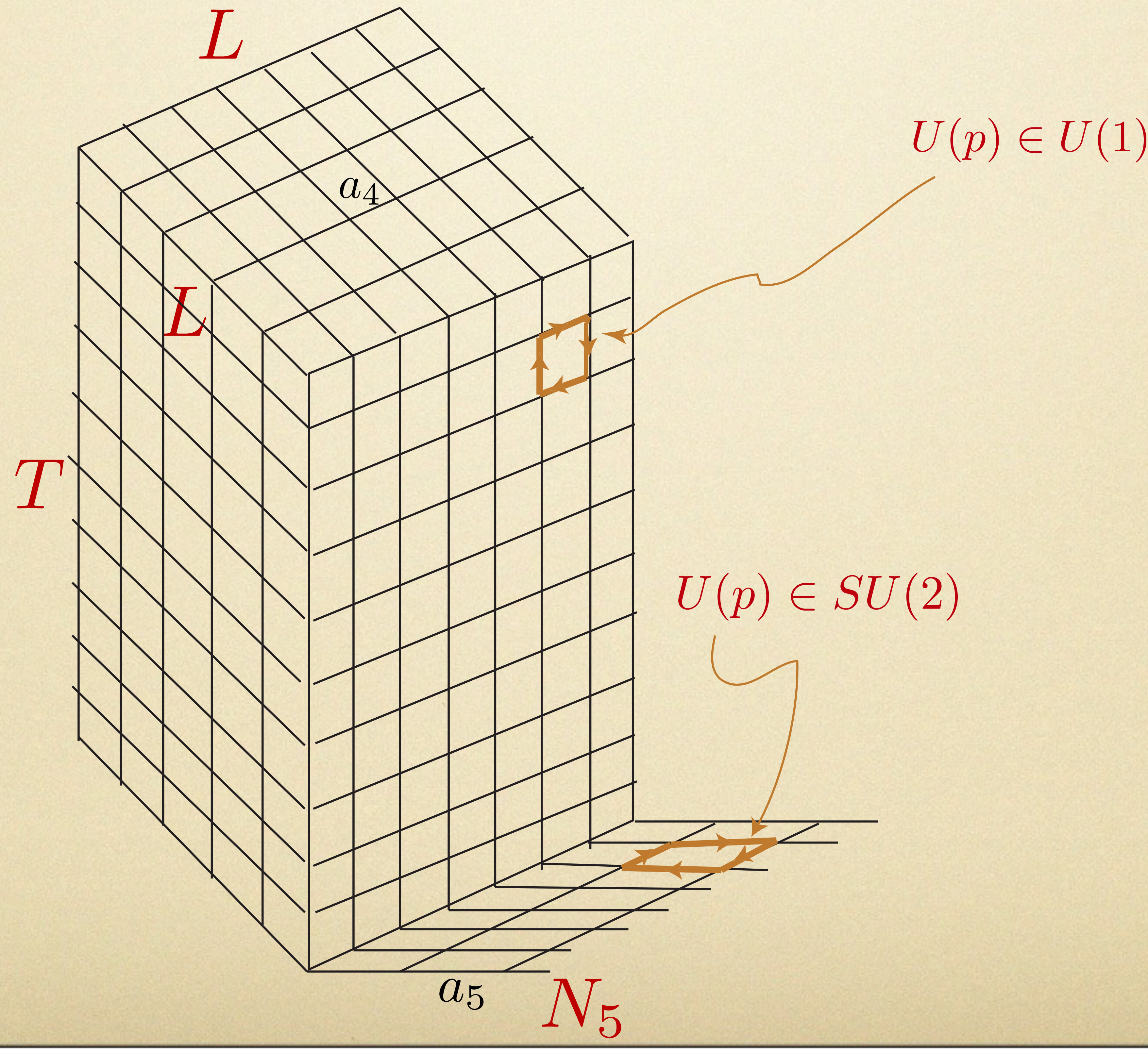
The anisotropic lattice



$U(n_M, 5)$ $\quad \beta_4 = \frac{4a_5}{g_5^2} = \frac{\beta}{\gamma} \quad \beta_5 = \frac{4a_4^2}{g_5^2 a_5} = \beta\gamma \quad \frac{N_5}{L} = \frac{2\pi R}{l}\gamma$

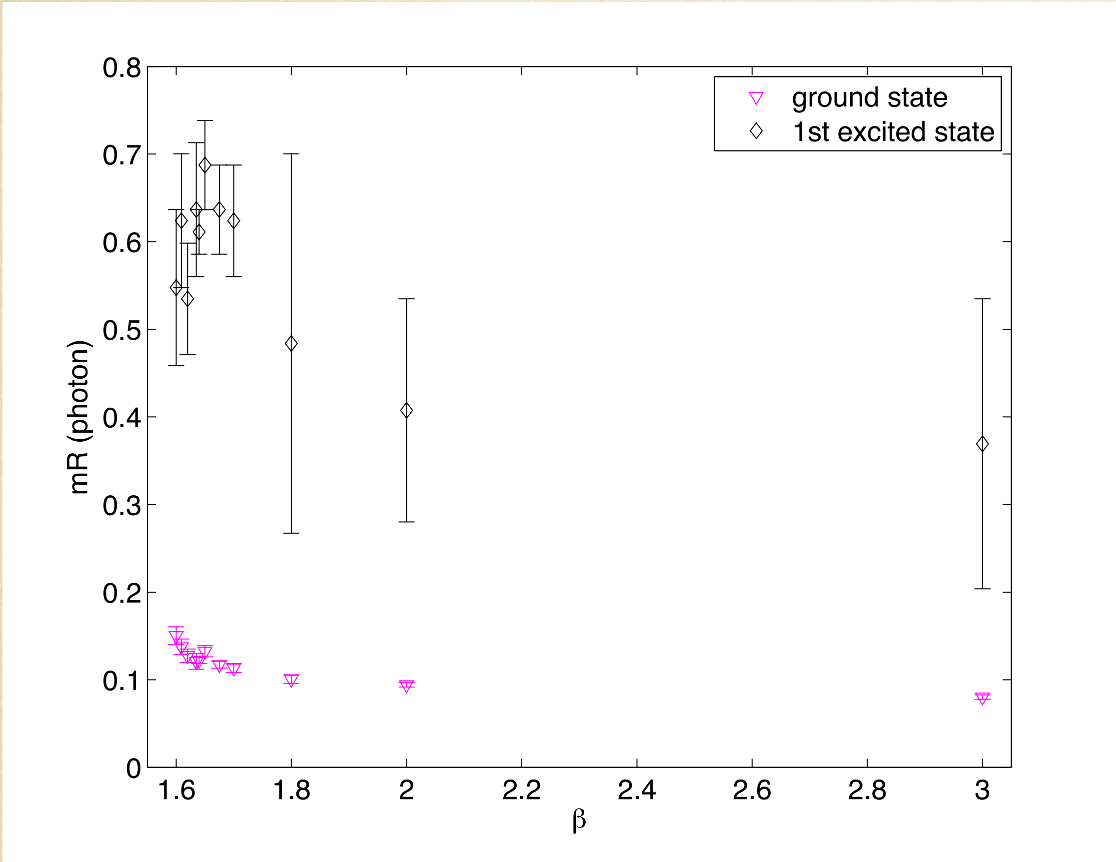
$\gamma =$ anisotropy parameter

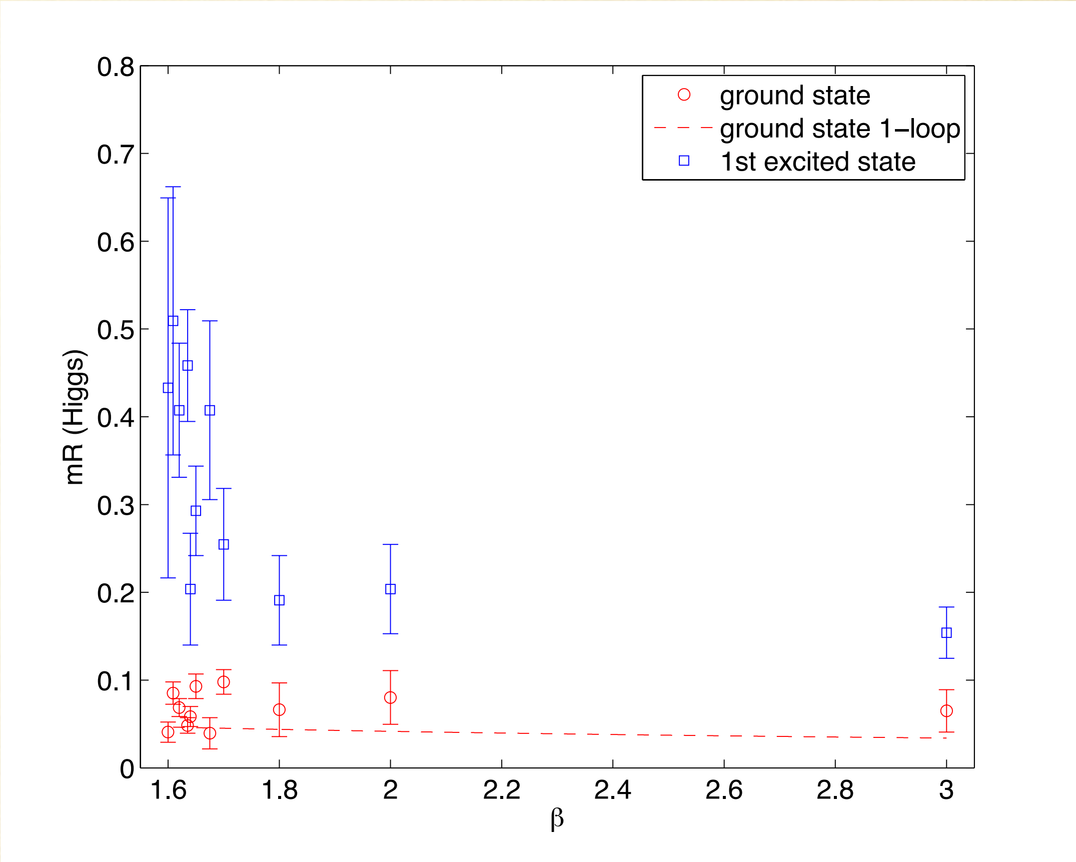
“Orbifold” on the lattice



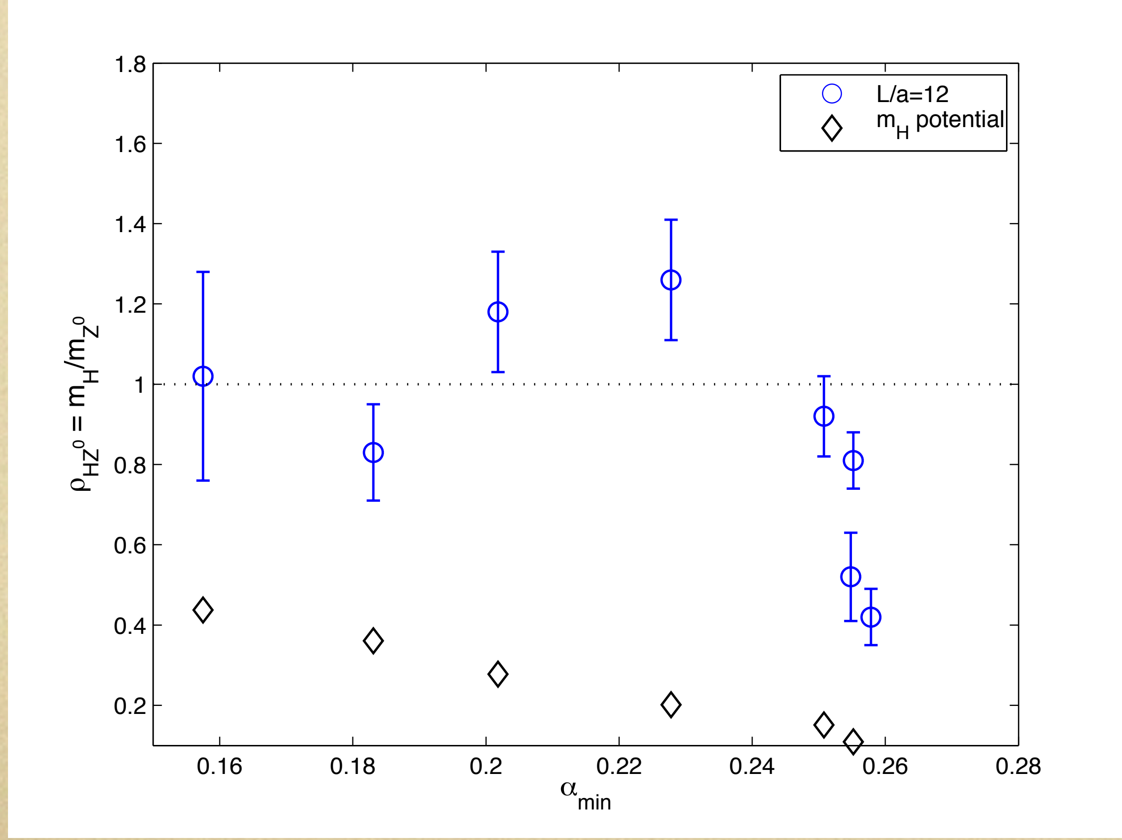
A few early Monte Carlo Orbifold Results

N.I., F. Knechtli (2005, 2006)
N.I., F. Knechtli, M. Luz (2007)





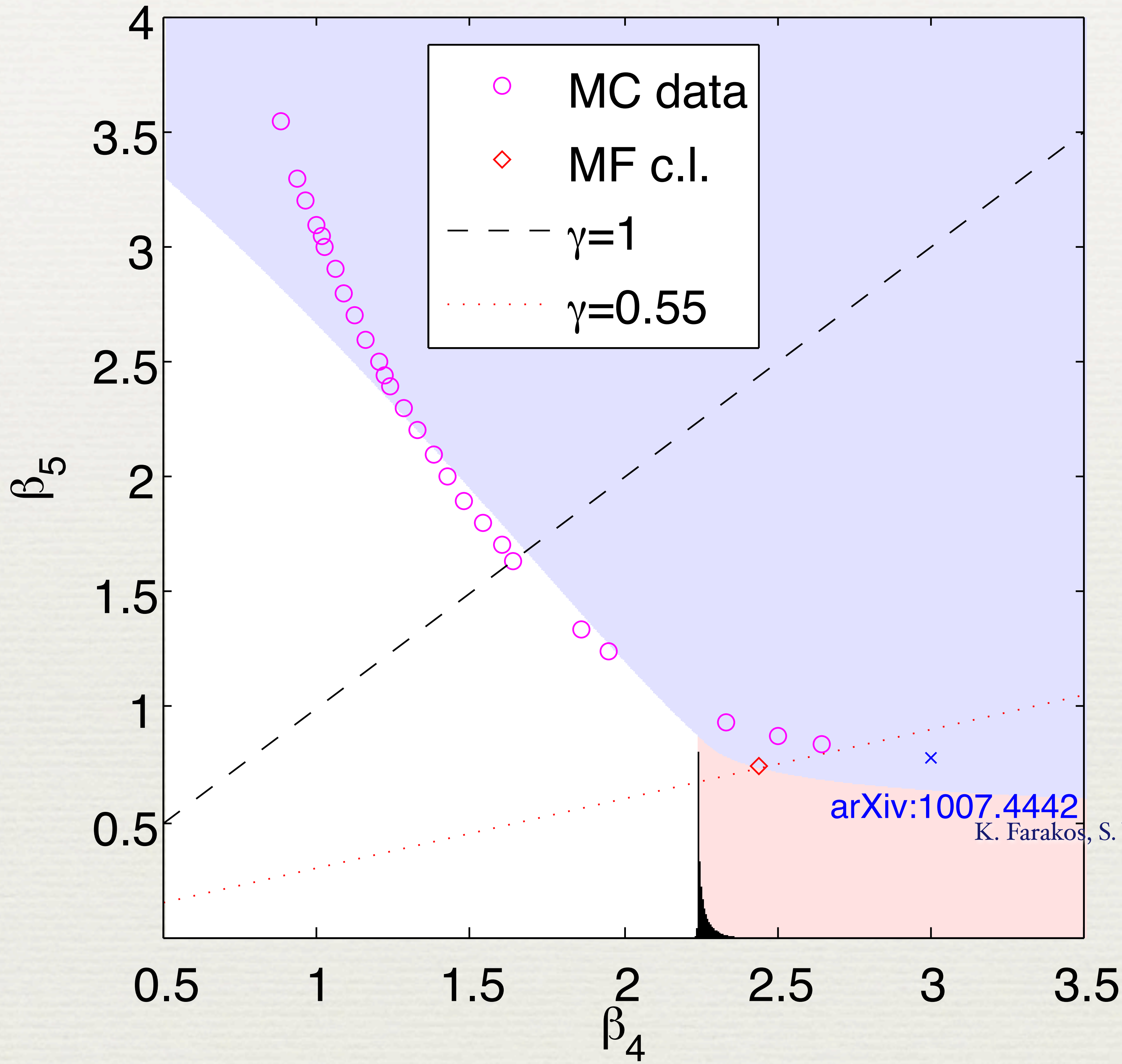
SSB in the pure gauge theory!



Use a (not perturbative) analytical method:

the Mean-Field Expansion

MC vs MF on the Torus



F. Knechtli, M. Luz, A. Rago (2012)
N. I., F. Knechtli (2009)

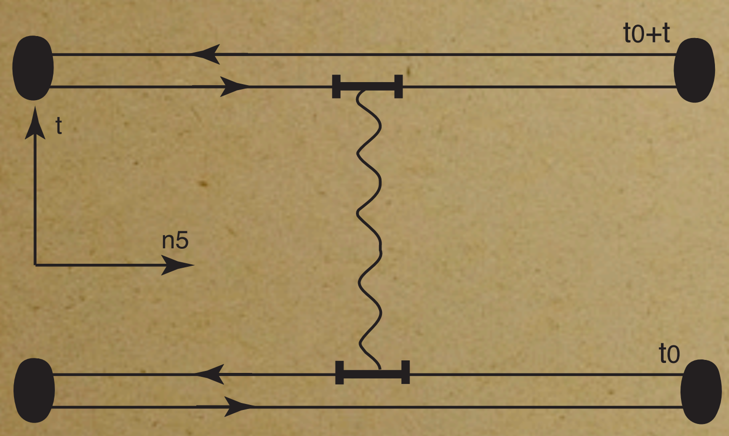
K. Farakos, S. Vrentzos (2007)

MC: M. Creutz (1979), S. Ejiri, J. Kubo, M. Murata (2000), K. Farakos, S. Vrentzos (2007), P. de Forcrand, A. Kurkela, M. Panero (2010), F. Knechtli, M. Luz and A. Rago (2011), L. Del Debbio, A Hart, E. Rinaldi (2012)

MF: N.I . & F. Knechtli, Nucl. Phys. B822 (2009) 1, Phys. Lett. B685 (2010) 86

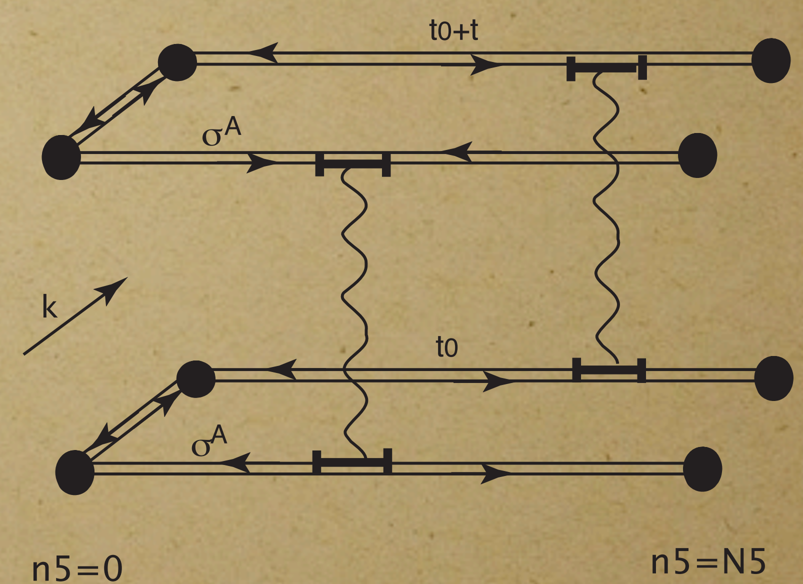
Lattice Observables in the Mean-Field Expansion on the Orbifold

the scalar



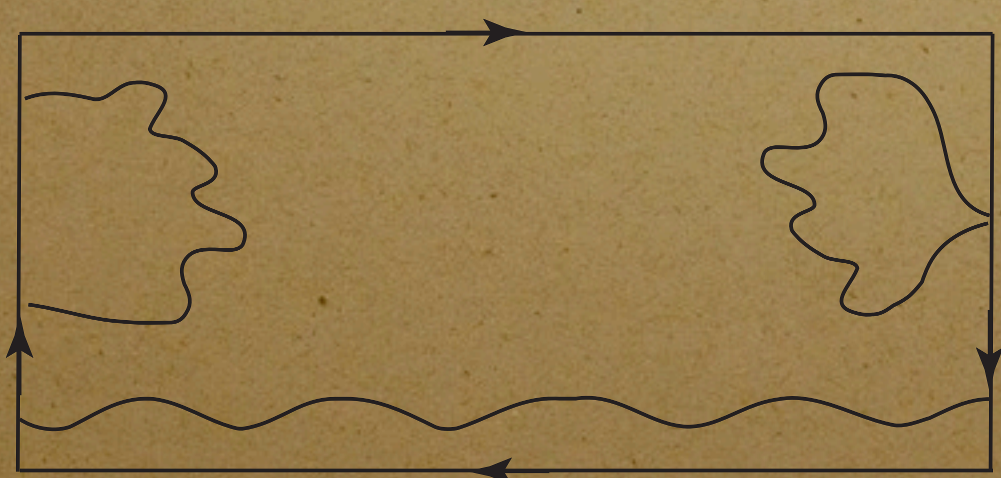
$$m = \lim_{t\to\infty} \ln \frac{C^{(1)}(t)}{C^{(1)}(t-1)}$$

the vector



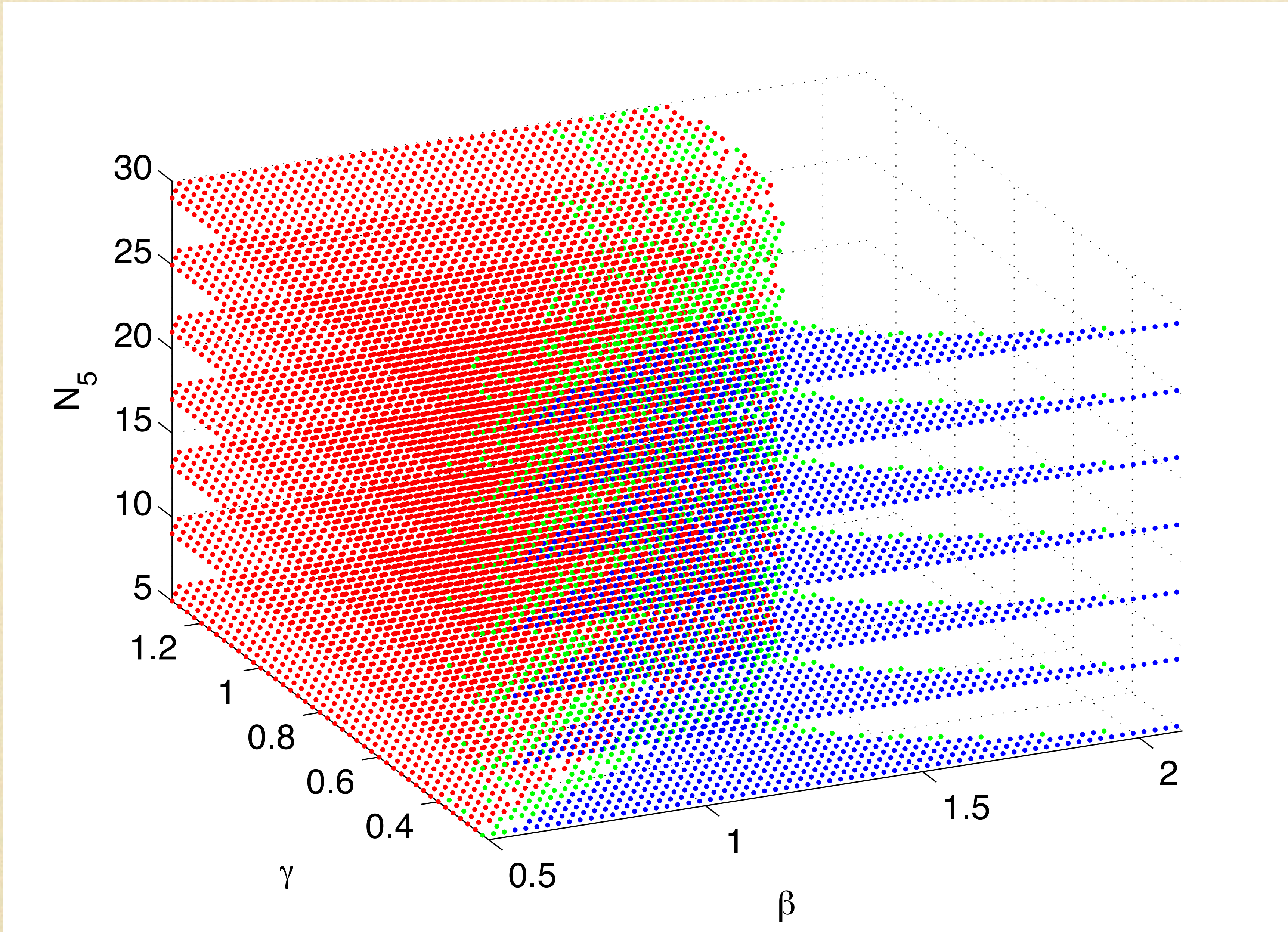
$$m = \lim_{t\to\infty} \ln \frac{C^{(2)}(t)}{C^{(2)}(t-1)}$$

the Wilson Loop



$$\mathrm{t} \to \infty : \qquad \mathrm{e}^{-Vt} \simeq < \mathcal{O}_W >$$

The Mean-Field Phase Diagram

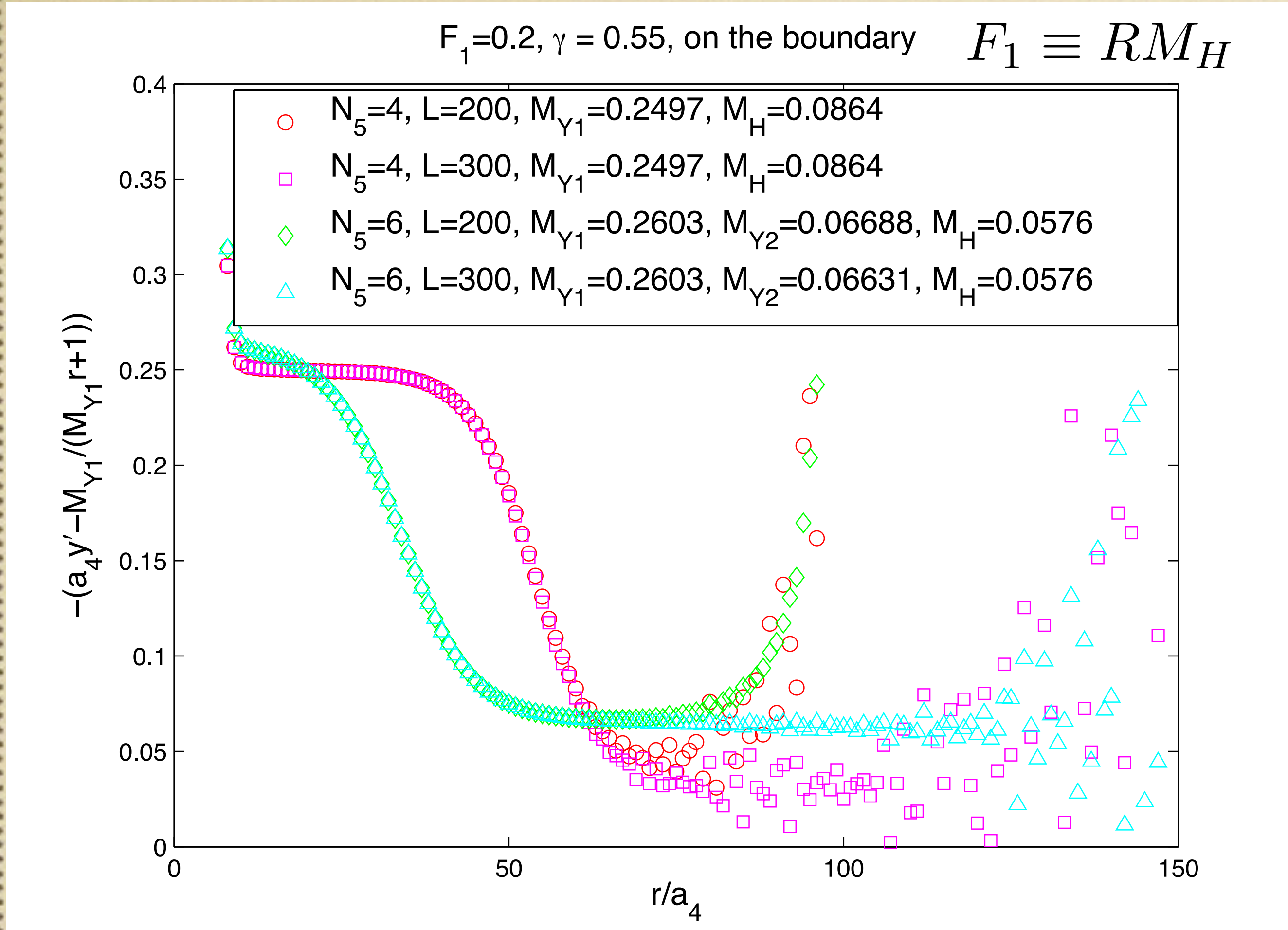


● → confined phase

● → layered phase

● → deconfined phase

Near the boundary between ● and ● and for $\gamma < 1$ the system reduces dimensionally to 4d (via localization); the phase diagram has to be qualified again and the physics must be interpreted since on the boundary we have now a 4d Abelian-Higgs model. Thus, use 4d Yukawa fits to extract the masses.



OBSERVATIONS

- $M_H \, , M_{Y_1} \, , M_{Y_2}$ L-independent
- $M_{Y_1} \, , M_{Y_2}$ mild N5-dependence
- for N5 > 6, vector masses also N5-independent

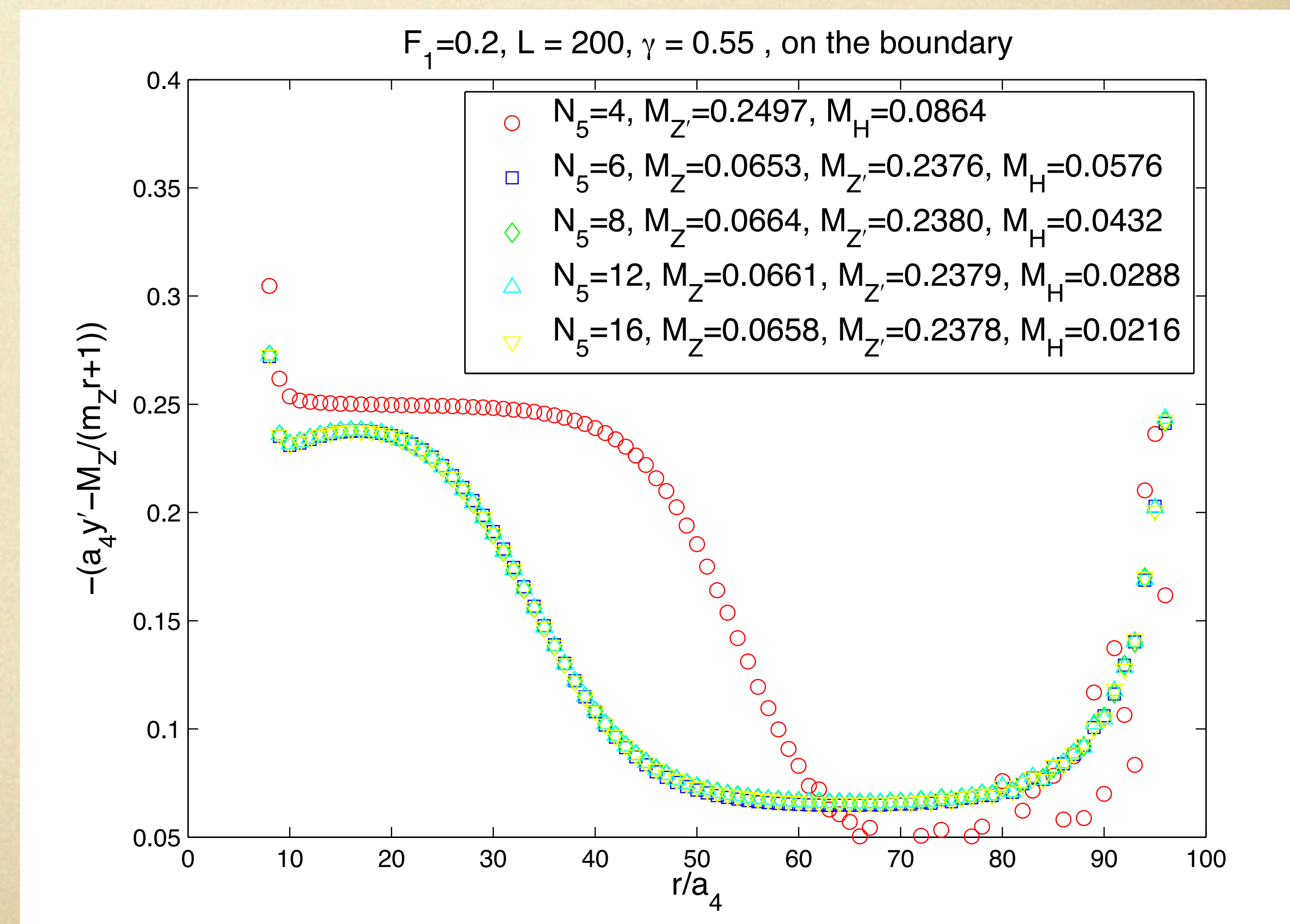
HIGGS PHYSICS

- $M_{Y_2} \equiv M_Z \neq 0 \rightarrow \text{SSB}$

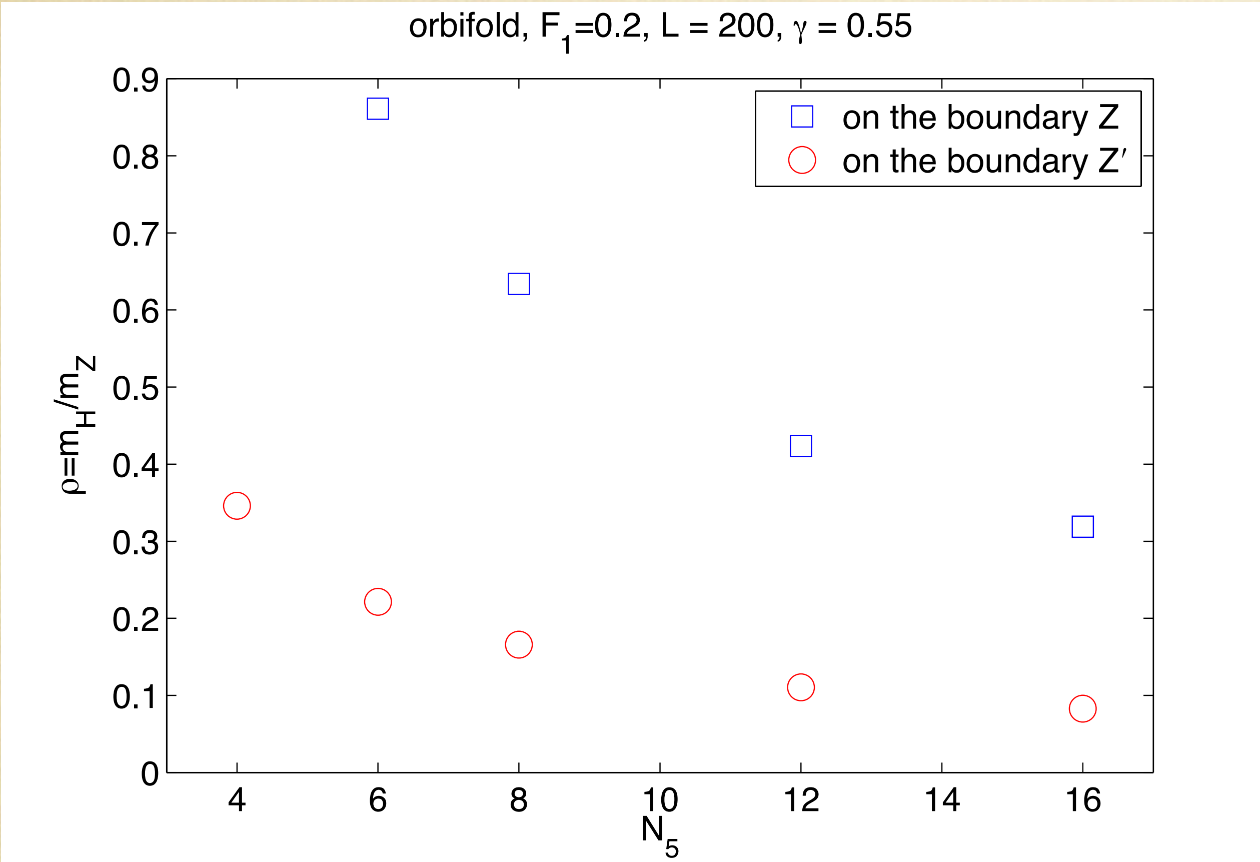
in perturbation theory: NO SSB

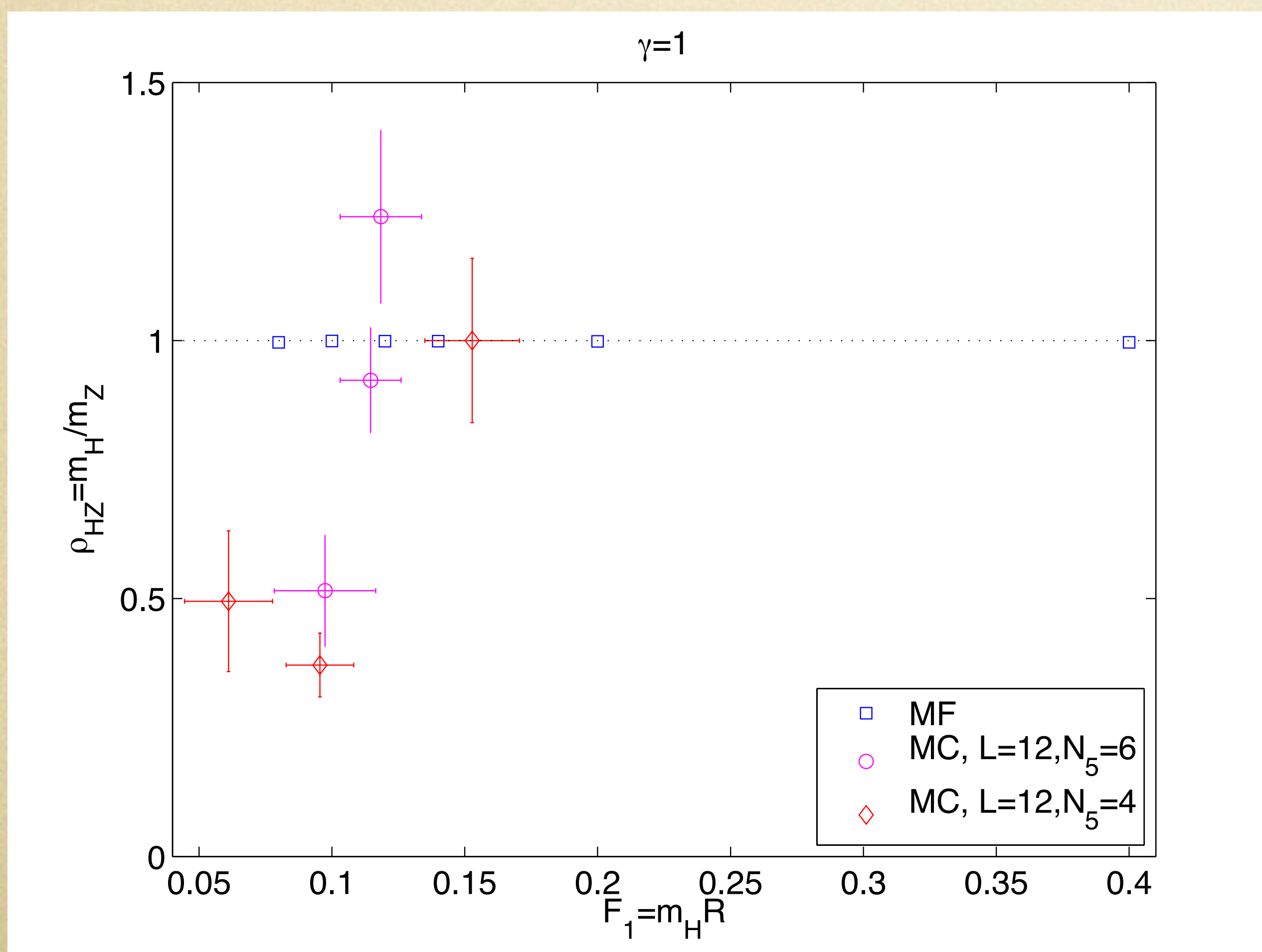
- $M_H = \dfrac{c_H}{N_5}$

recall the perturbative result: $M_H^{PT} = \dfrac{c_H^{PT}}{N_5^{3/2}}$



HIGGS PHYSICS CONTINUED



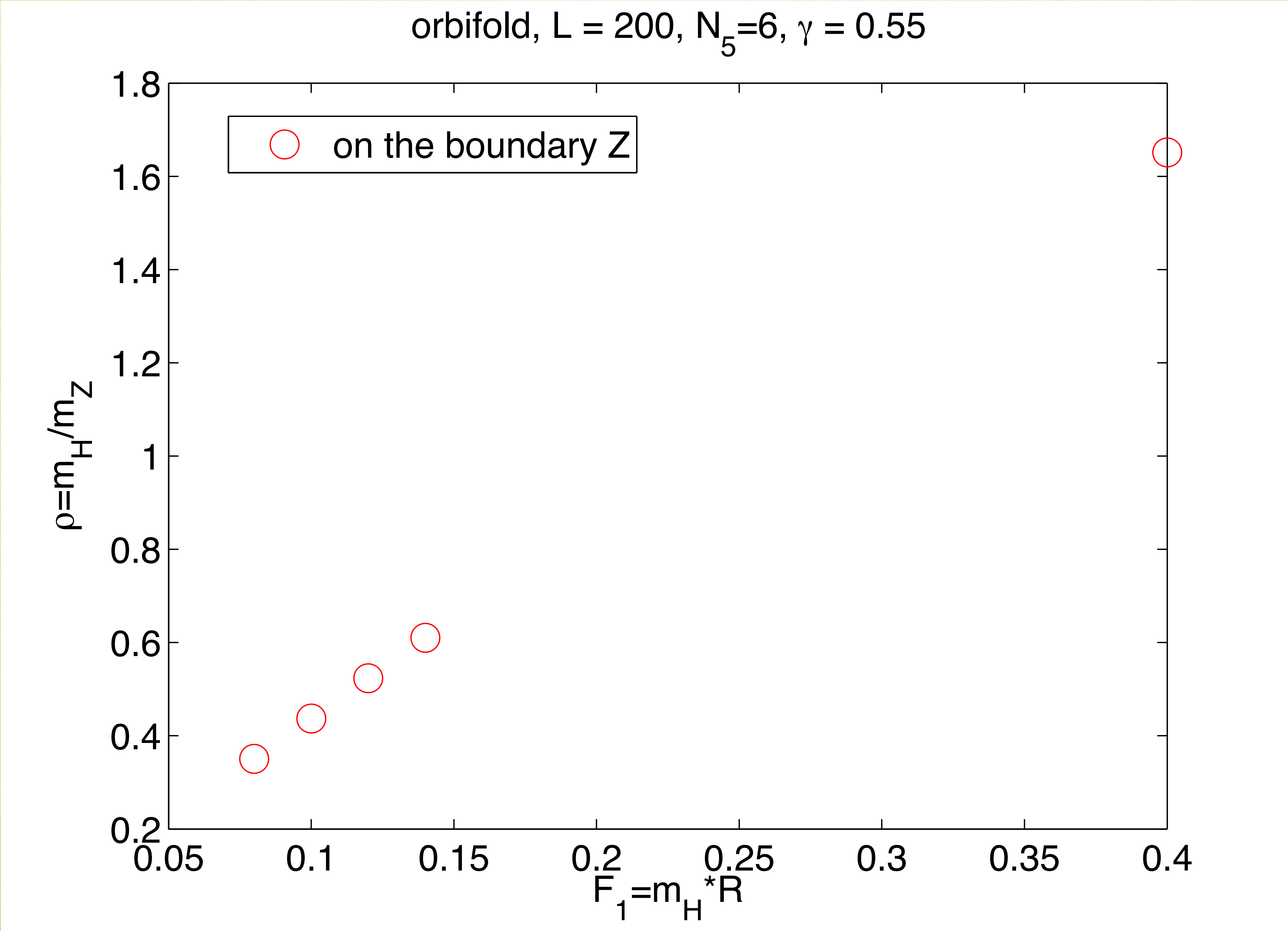


- $\rho_{HZ} \equiv \frac{m_H}{m_Z} \sim O(1)$

typically in perturbation theory: $\rho << 1$

MC vs MF on the Orbifold

Looking for a “SM” HIGGS



CONCLUSIONS AND (NEAR) FUTURE WORK

- Contrary to perturbation theory, non-perturbatively, on the 5d Orbifold, there is SSB already in the pure gauge theory with a Higgs of comparable mass with the Z. This is summarized by $\rho_{HZ} \equiv m_H/m_Z \sim O(1)$. We reach the same conclusion from both Monte Carlo and Mean-Field Expansion methods.

- Is there an LCP of $\rho \simeq 1.385$ and how heavy is the associated Z' ?

N.I., F. Knechtli and K. Yoneyama: work in progress

- How high can we go with ρ ?

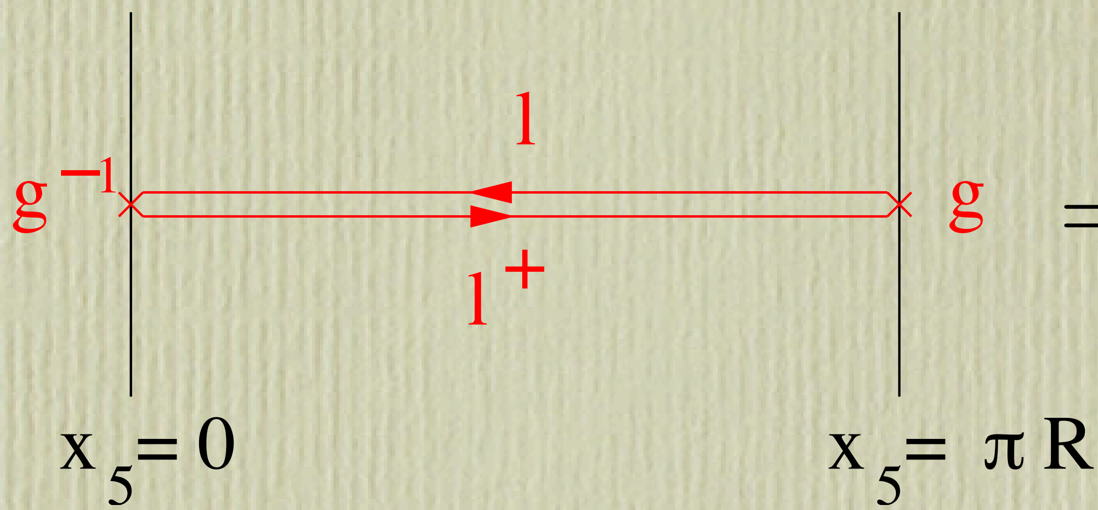
$$\rho_{HZ} = M_H/M_Z \sim O(1)$$

- $\rho_{HZ} \equiv \dfrac{M_H}{M_Z} \sim O(1)$

$$\rho << 1$$

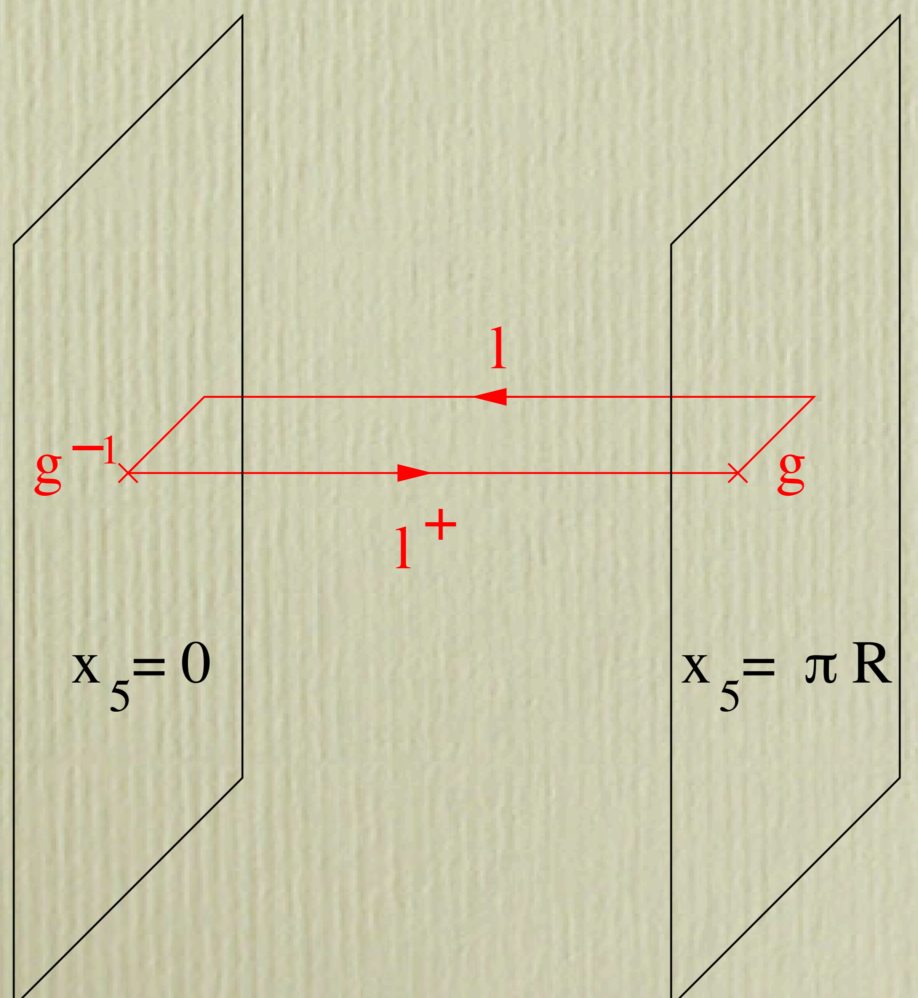
Back-up slides

Non-perturbative Higgs operators



$$= P(n_\mu),$$

the basic Polyakov line



the extended Polyakov line

$$(1 - \Gamma)P = 0$$

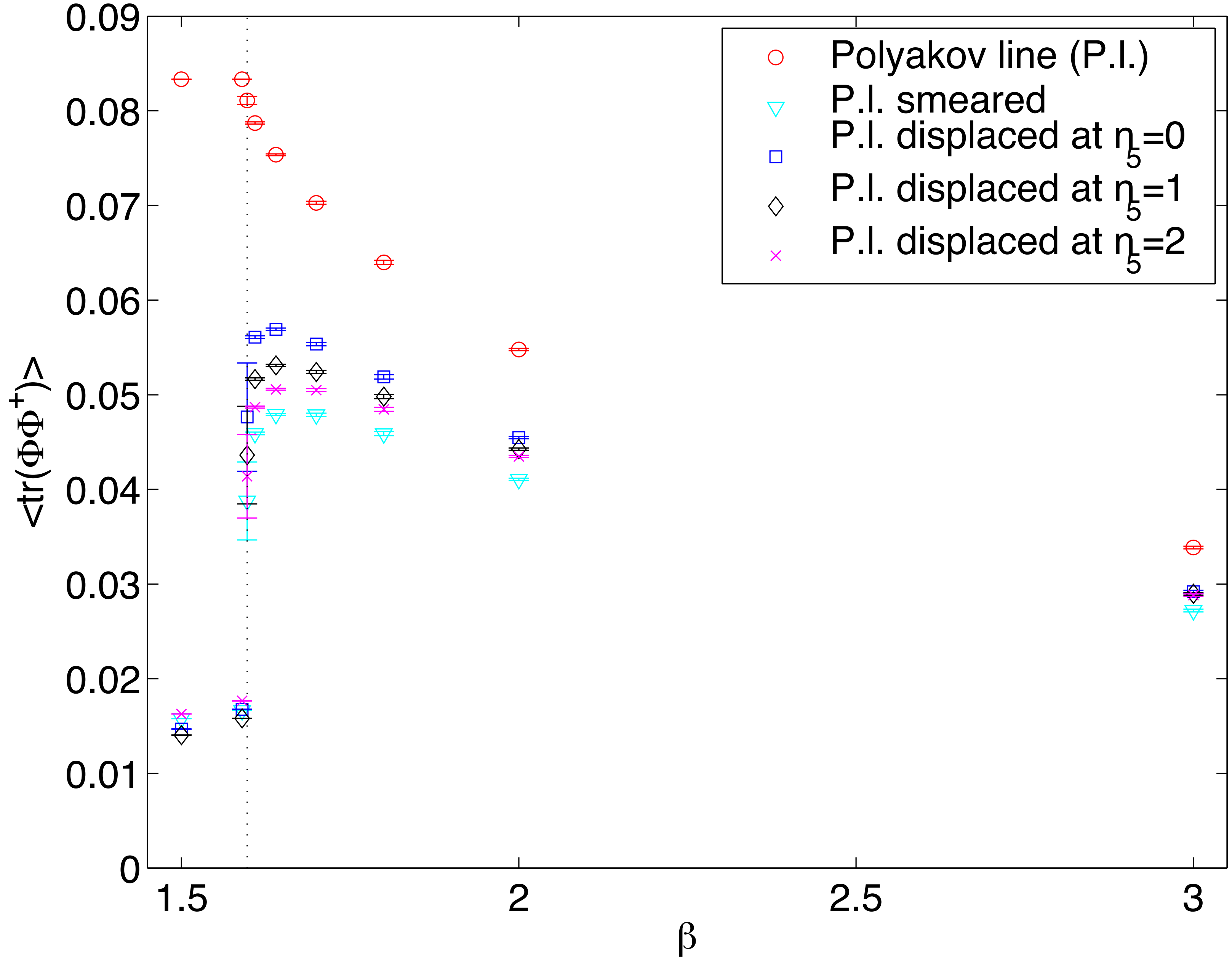
III: Towards an anlytical understanding: A Quantum Higgs mechanism

Coleman-Weinberg:

$$\Gamma = -\log\left[\frac{1}{\sqrt{det[\partial^2 + M^2]}}\right] \longrightarrow$$

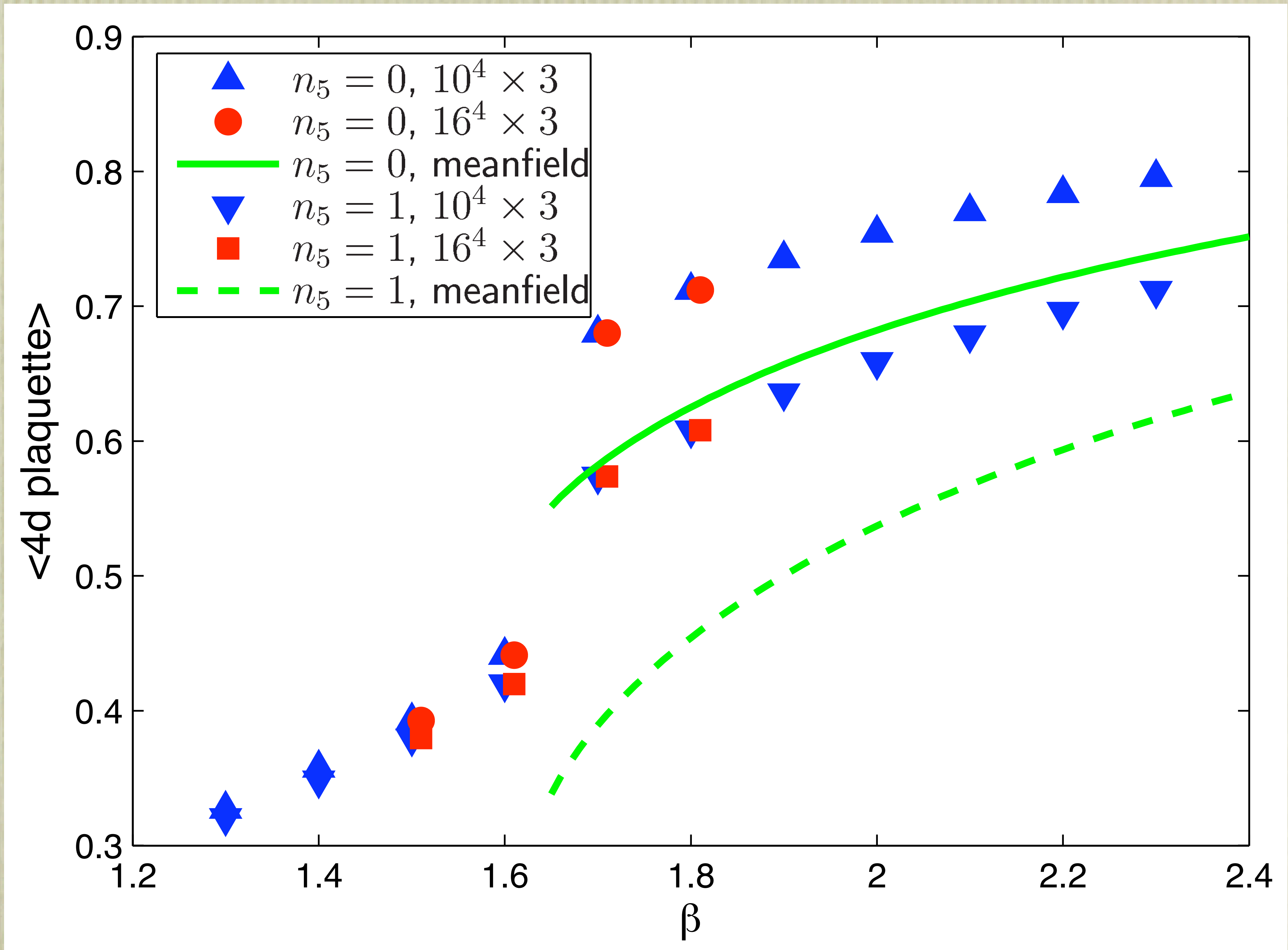
$$V^{S^1}_{1-loop} = \sum_A \sum_n (-)^{F^A} \frac{1}{32\pi^2} \int_0^\infty dl\; l\; e^{-\frac{(n+a^A)^2}{l\,R^2}}$$

A phase transition !

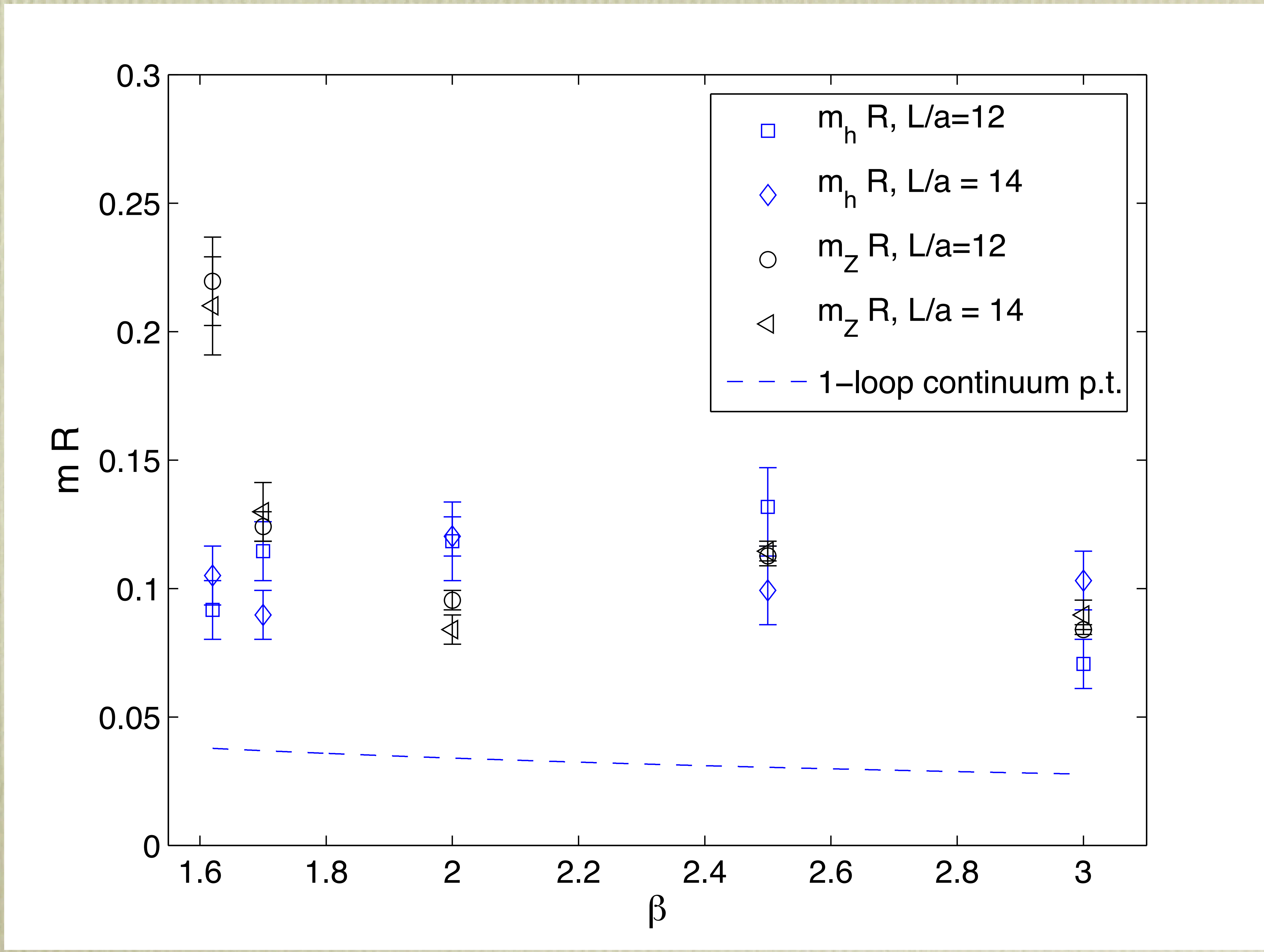


Phase transition at $\beta_c = 1.5975$

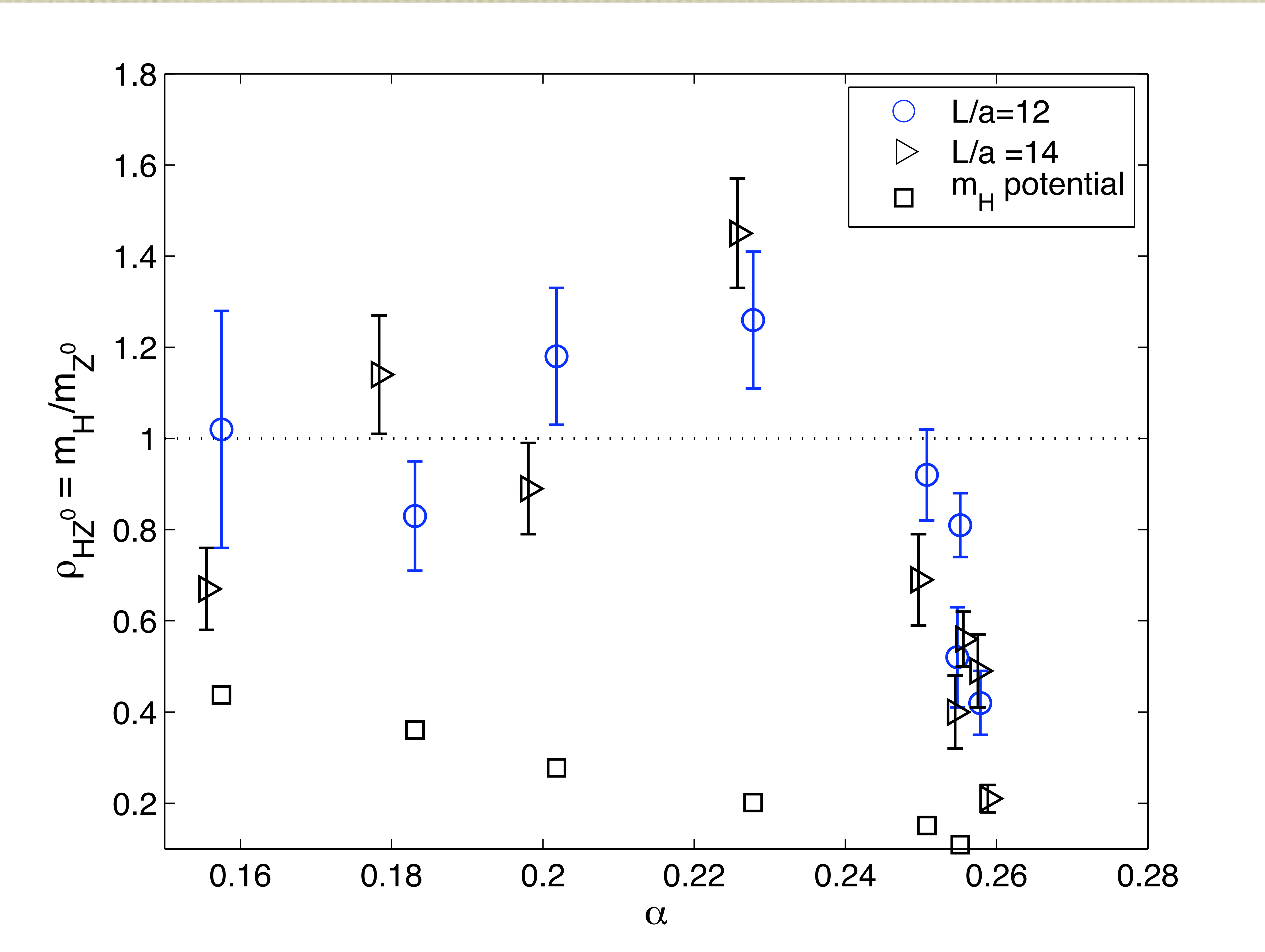
IV: Comparison of results: lattice vs mean-field



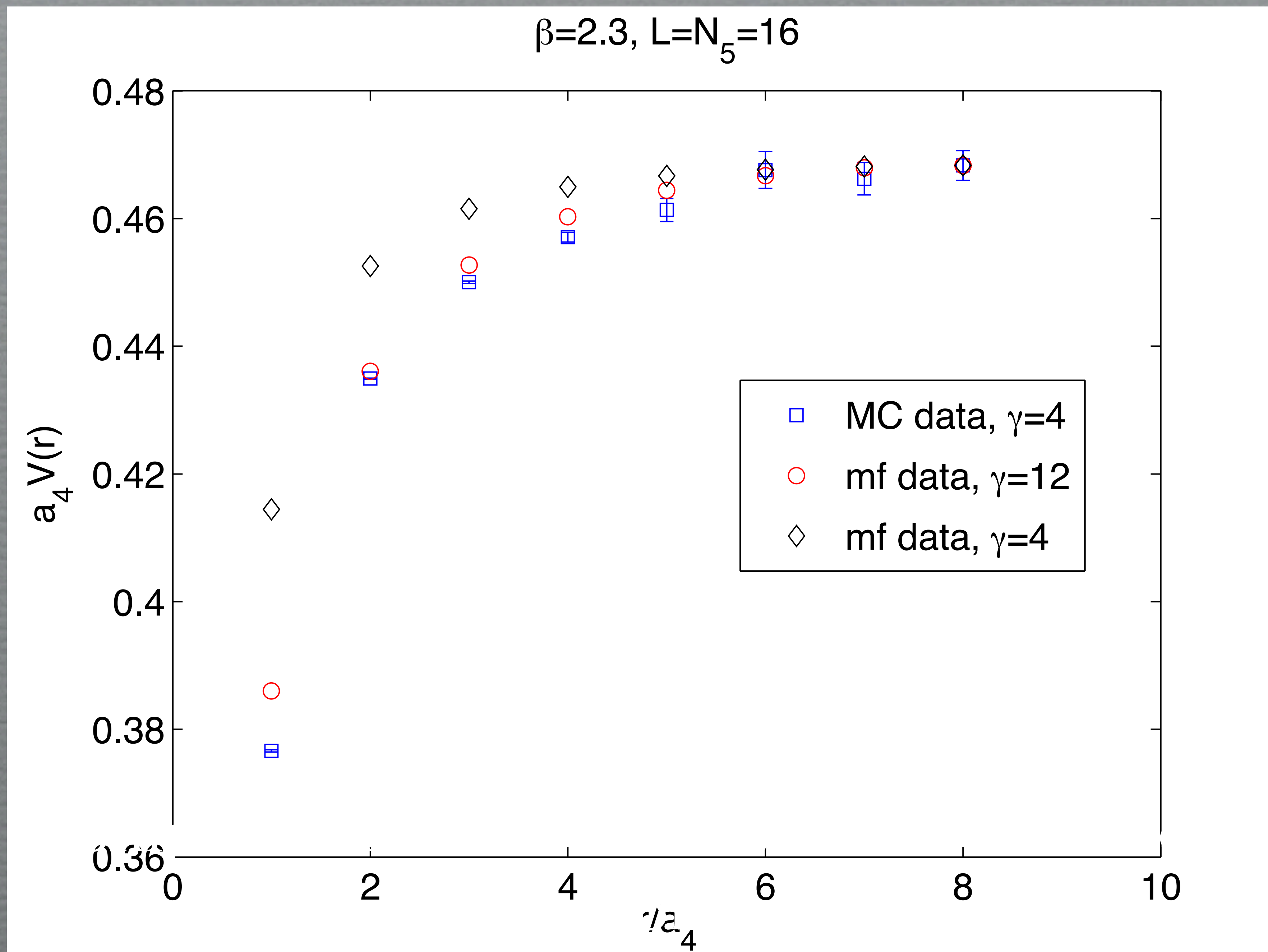
Further comparisons: lattice vs 1-loop pert. theory



The $\rho_{HZ^0} = \frac{m_H}{m_{Z^0}}$ ratio



Comparison Mean-Field vs Monte Carlo: (sample)



MC data generated by M. Luz